Adaptive Inference on Probabilistic Relational Models

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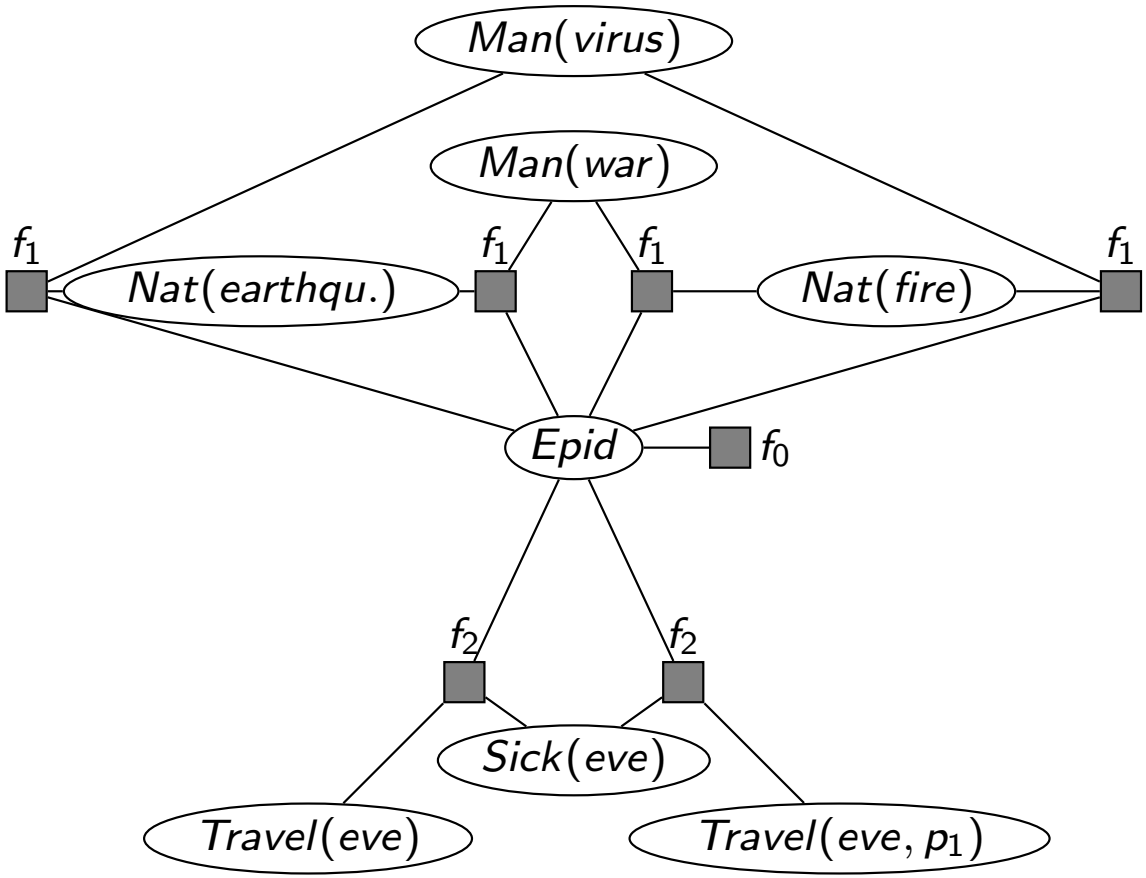
Probabilistic Graphical Models

Factor Graph F: Compact Encoding of Full Joint Distribution $P_F = \frac{1}{Z} \prod_i f_i$



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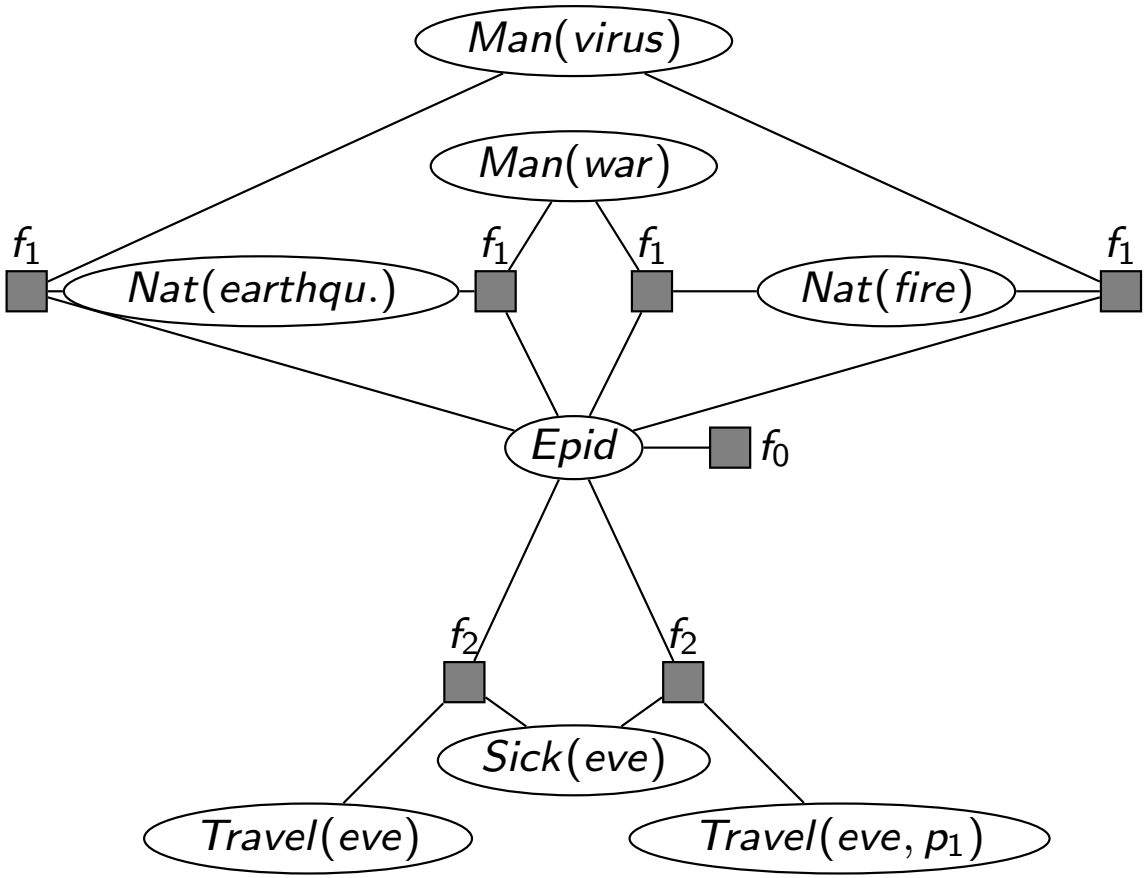

Query answering (QA): Eliminate all non-query variables
avoiding building P_F

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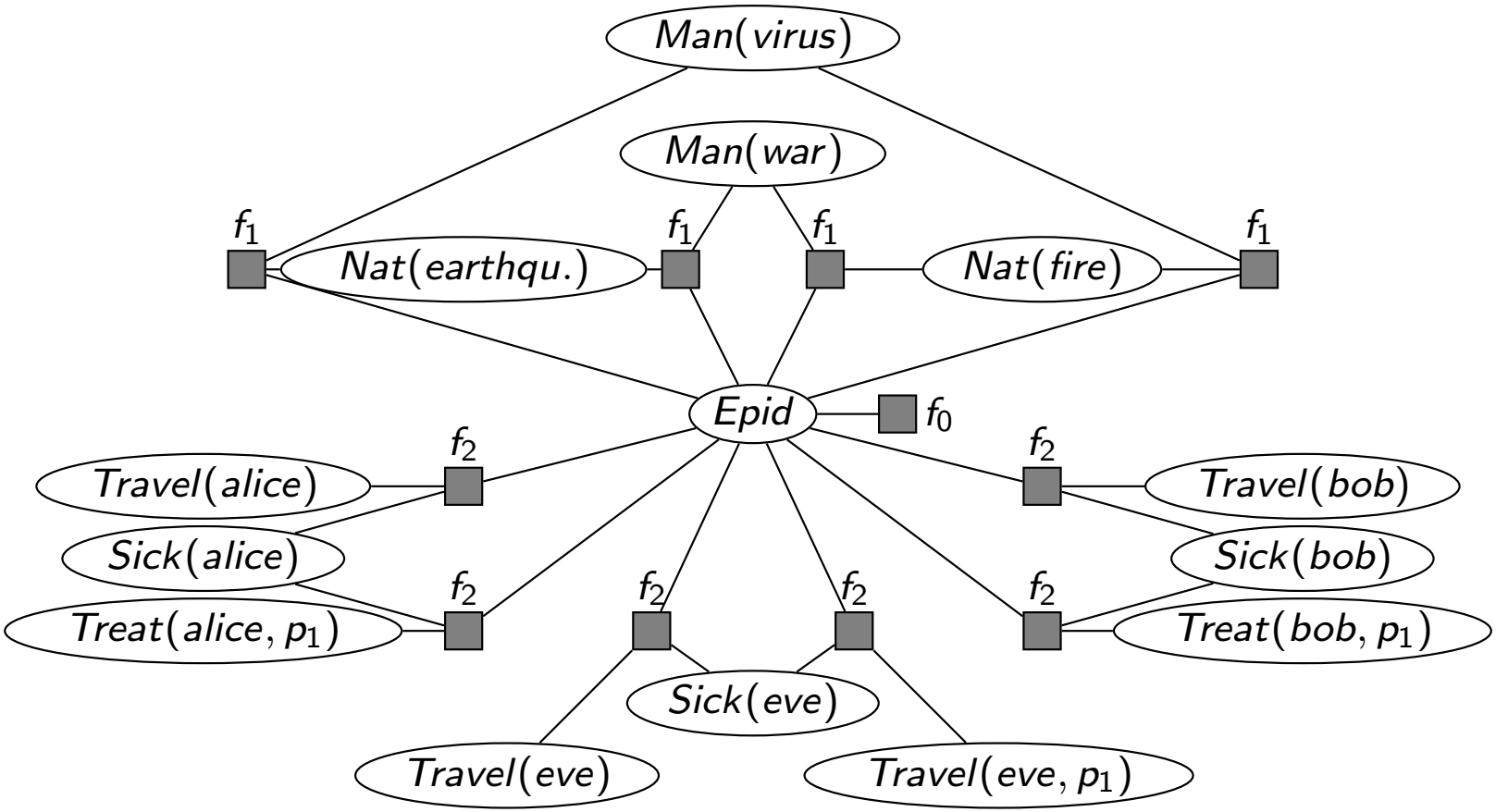

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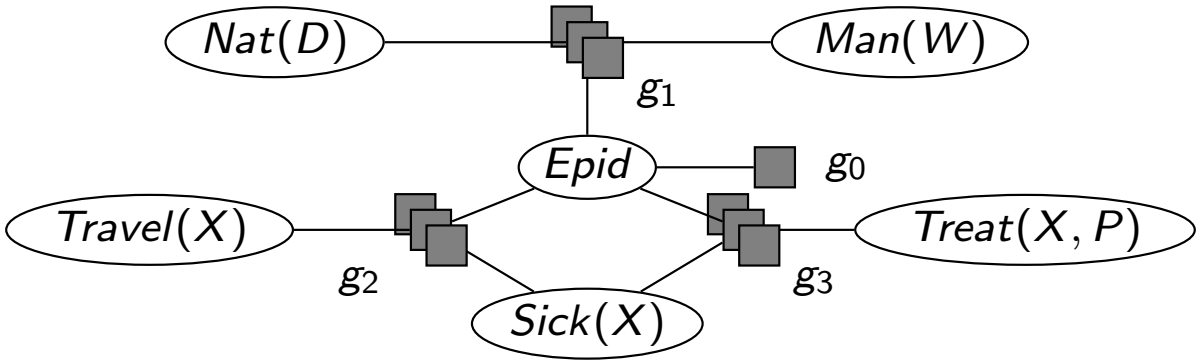
Probabilistic Relational and Lifted Models

Parfactor Graph G: Compact Encoding of Full Joint Distribution $P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$



Probabilistic Relational and Lifted Models

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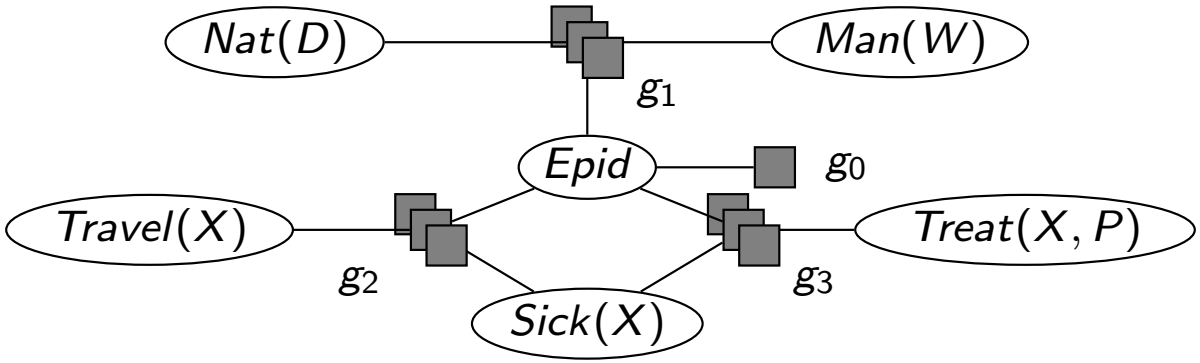
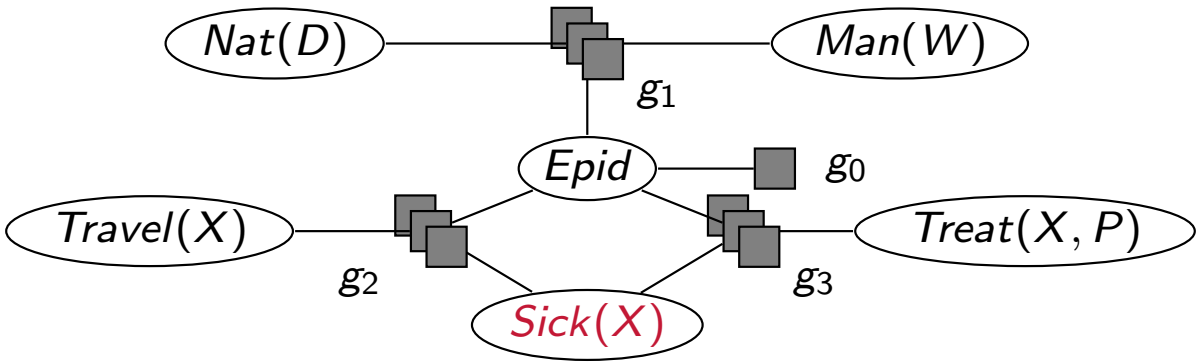


QA: Eliminate all non-query variables
while **avoiding building P_F and avoiding groundings**

QA: Lifted Variable Elimination (LVE)

Poole (2003), de Salvo Braz et al. (2005,2006), Milch et al. (2008), Taghipour et al. (2013)



$P(Sick(eve)) \propto$

$\sum_V$ indicates a sum over the values of V; # indicates a count conversion

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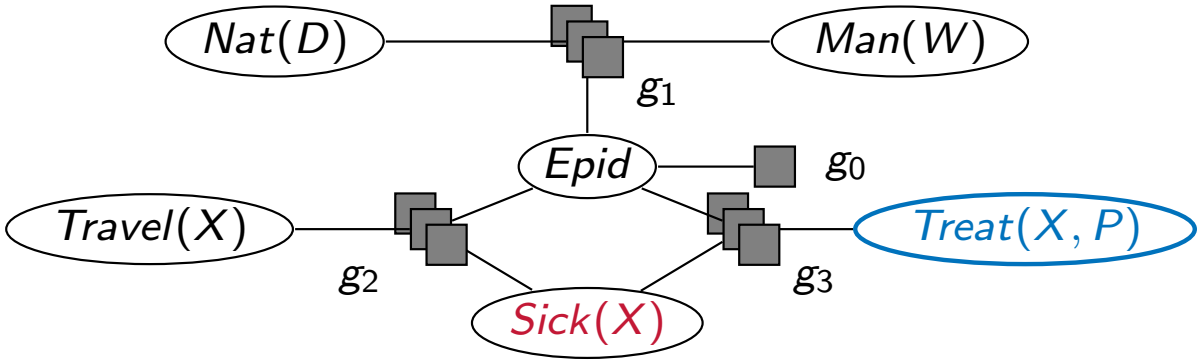

$$P(Sick(eve)) \propto \sum_{Treat(X,P)} g_3$$

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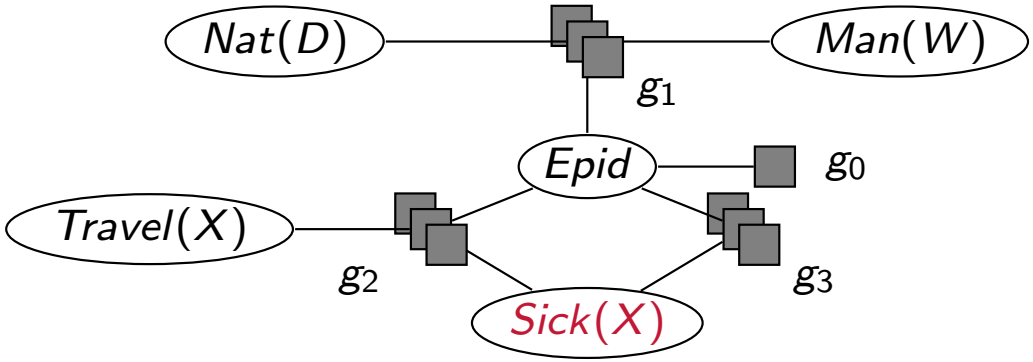

$$P(Sick(eve)) \propto \left(\sum_{Treat(X,P)} g_3 \right)^{|dom(P)|}$$

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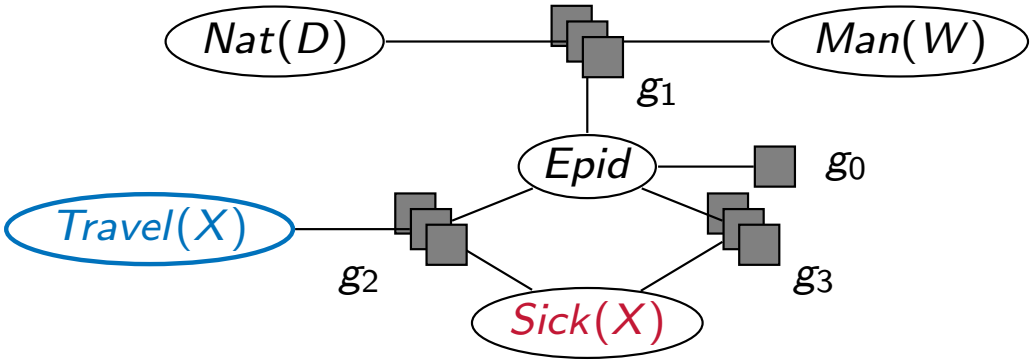

$$P(Sick(eve)) \propto \sum_{Travel(X)} g_2 \left(\sum_{Treat(X,P)} g_3 \right)^{|dom(P)|}$$

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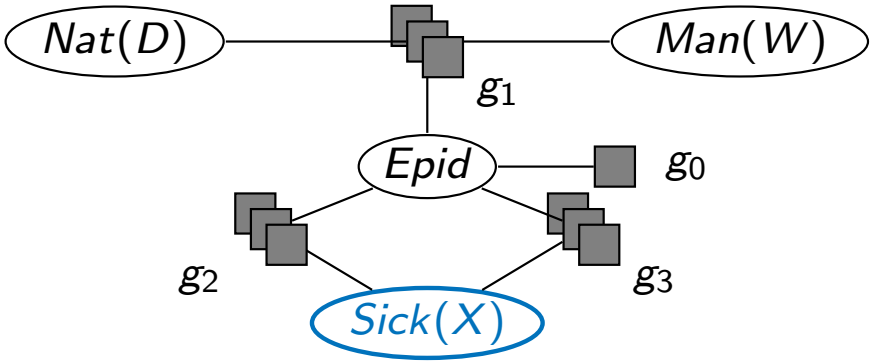

$$P(Sick(eve)) \propto \sum_{\substack{Sick(X) \\ X \neq eve}} \sum_{Travel(X)} g_2 \left(\sum_{Treat(X,P)} g_3 \right)^{|dom(P)|}$$

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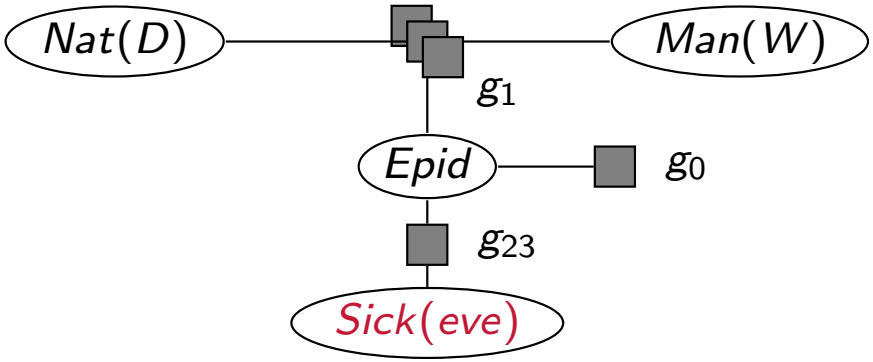

$$P(Sick(eve)) \propto \left(\sum_{\substack{Sick(X) \\ X \neq eve}} \sum_{Travel(X)} g_2 \left(\sum_{Treat(X,P)} g_3 \right)^{|dom(P)|} \right)^{|dom(X)|_{X \neq eve}}$$

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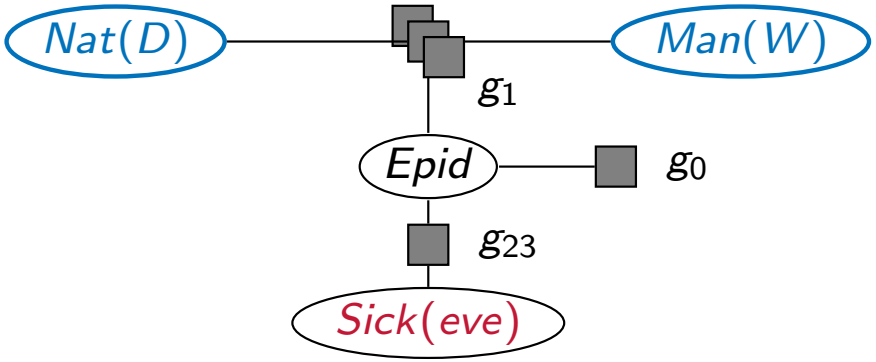

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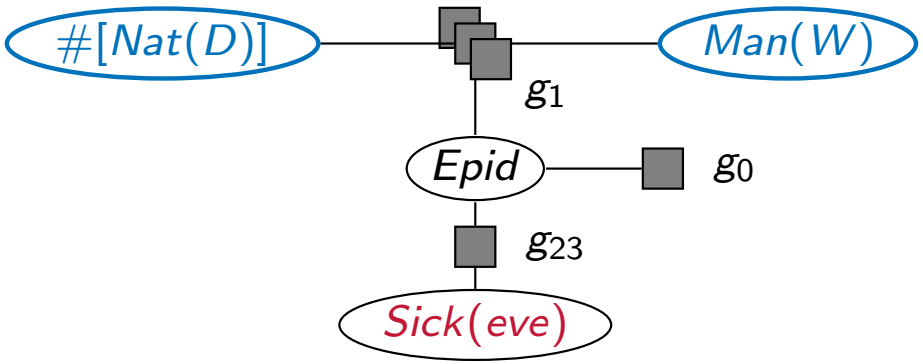

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$$g_1^{\#}$$

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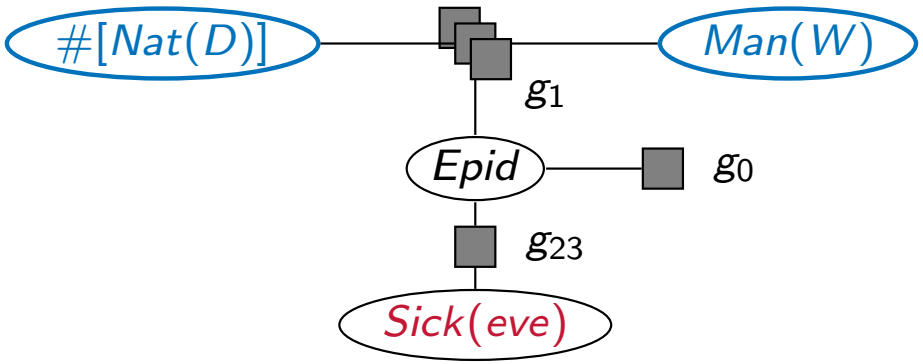

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$$\sum_{Man(W)} g_1^{\#}$$

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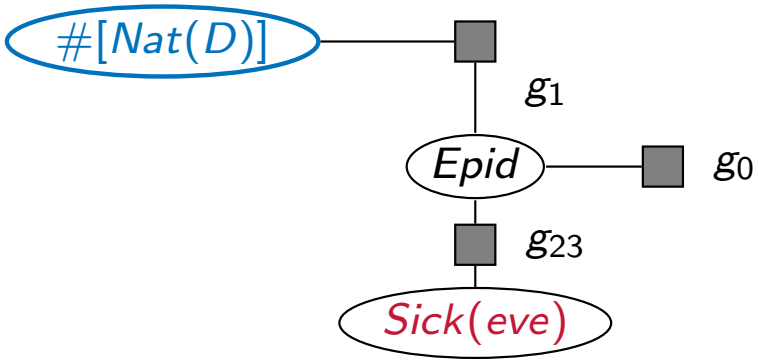

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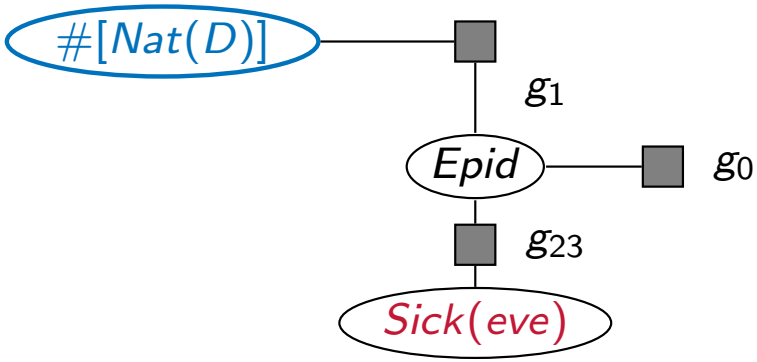

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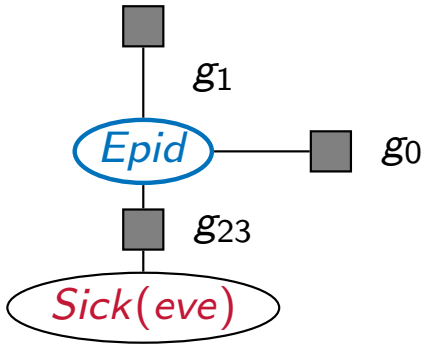

$$P(Sick(eve)) \propto \sum_{Epid} g_0 \left(\sum_{\substack{Sick(X) \\ X \neq eve}} \sum_{Travel(X)} g_2 \left(\sum_{Treat(X,P)} g_3 \right)^{|dom(P)|} \right)^{|dom(X)|_{X \neq eve}} \sum_{\#_D[Nat(D)]} \left(\sum_{Man(W)} g_1^{\#} \right)^{|dom(W)|}$$

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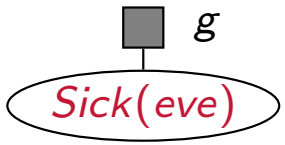

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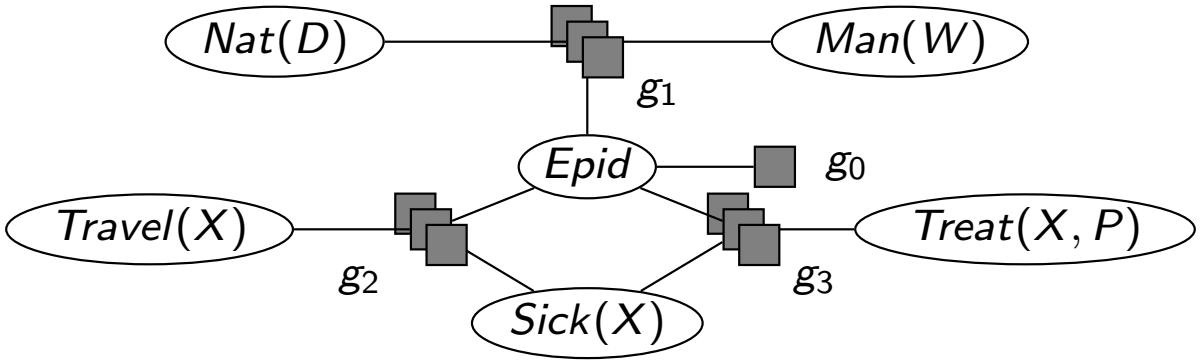
Repeated Inference

Answer Many Queries for Marginal Distributions



Solve each query more efficiently than starting from scratch for each query

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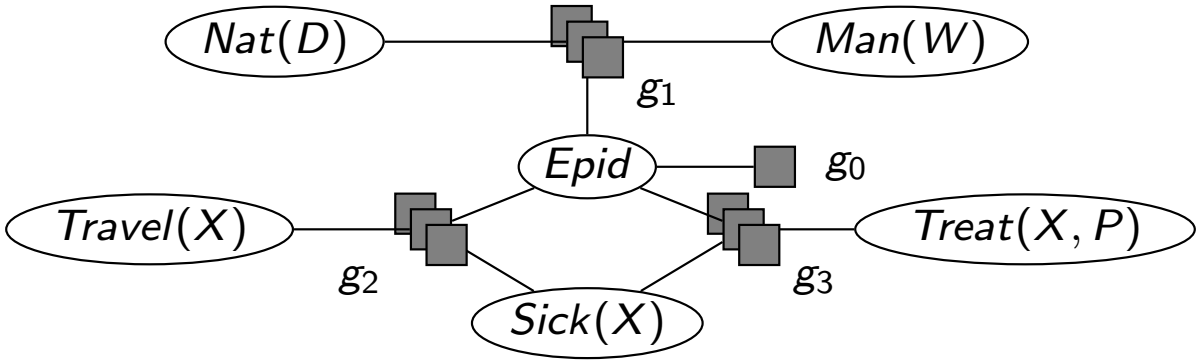


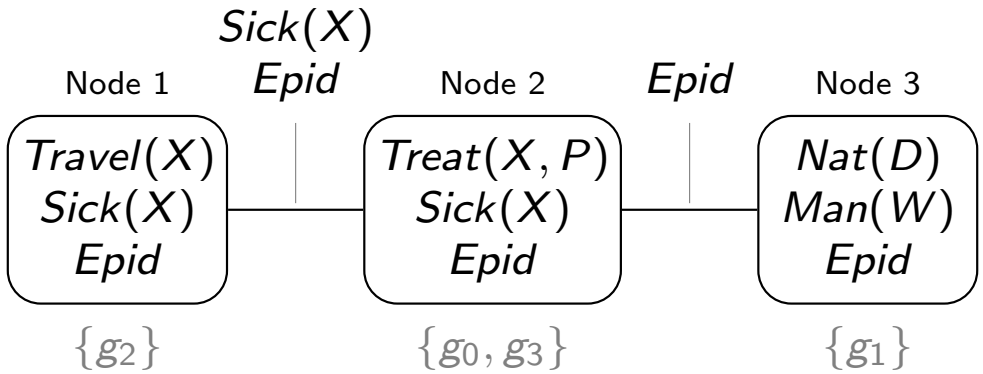
Solve each query more efficiently than starting from scratch for each query

General Idea

1. Build a helper structure.
2. Precompute as much computations as possible, including evidence **E**.
3. Using the precomputed information, answer individual queries.

Lifted Junction Tree Algorithm (LJT)

Braun and Möller (2016)



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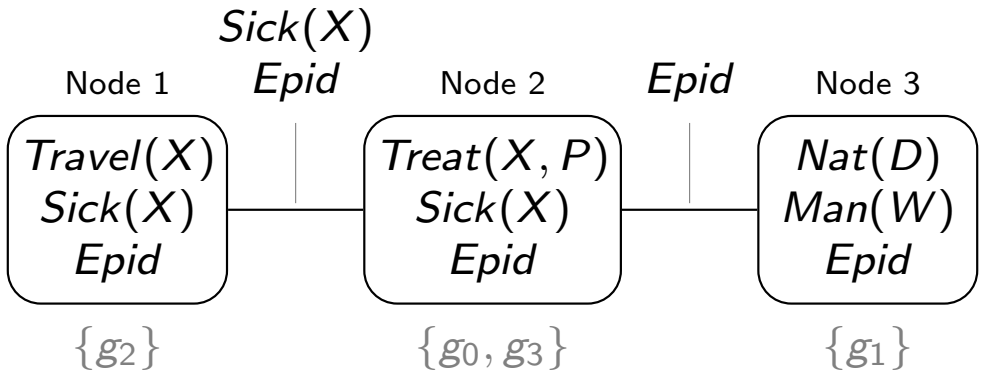

1. LJT constructs a first-order junction tree for model
2. LJT passes messages (independent of query)
   - Message = query on edge variables

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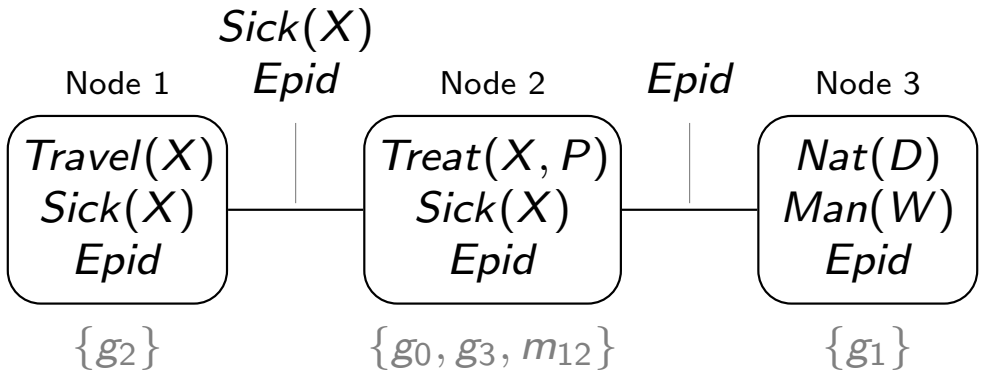

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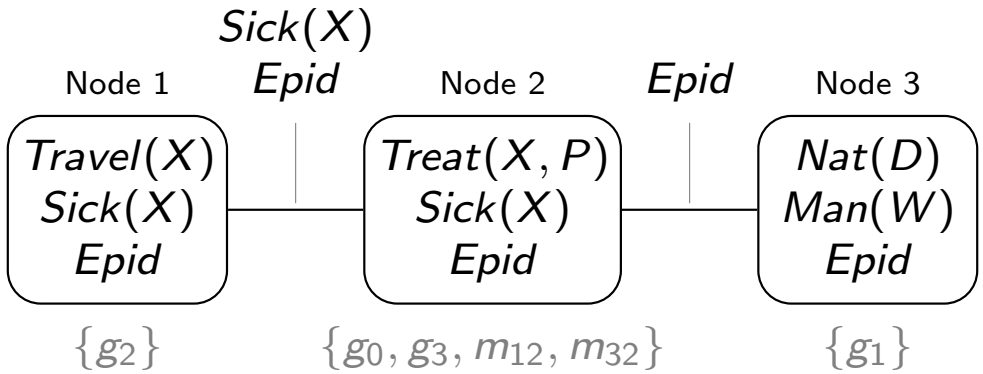

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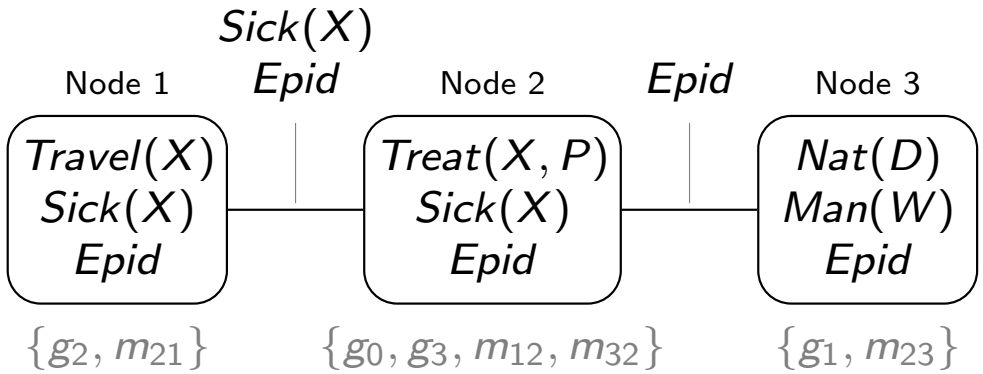

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   - Messages from node 2

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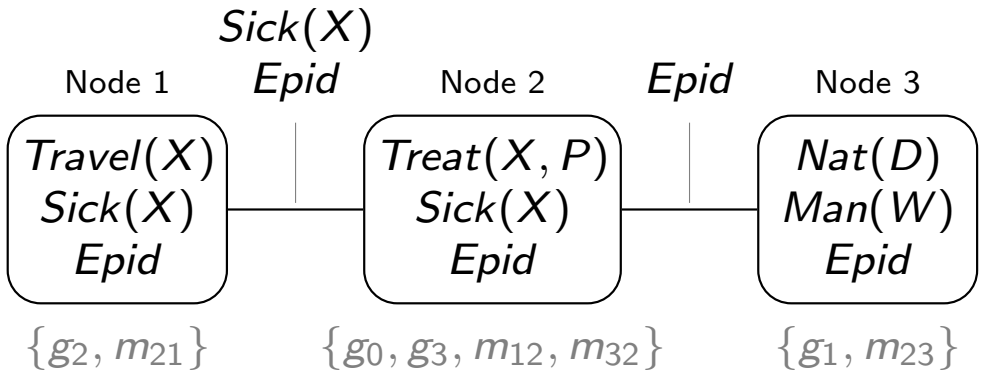

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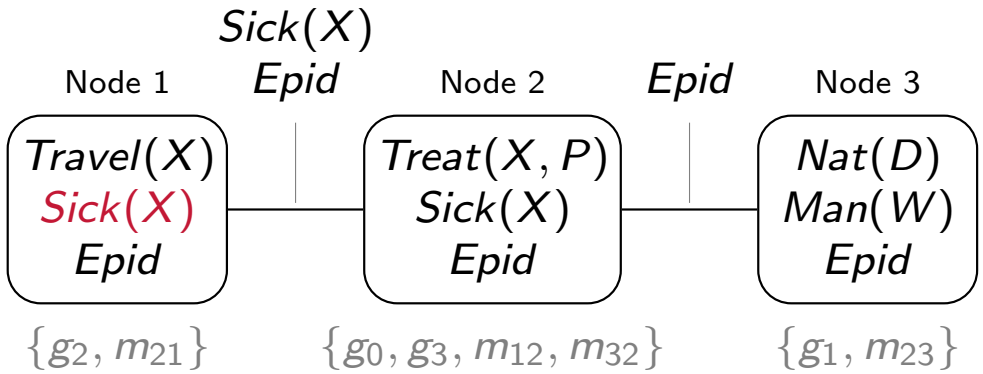

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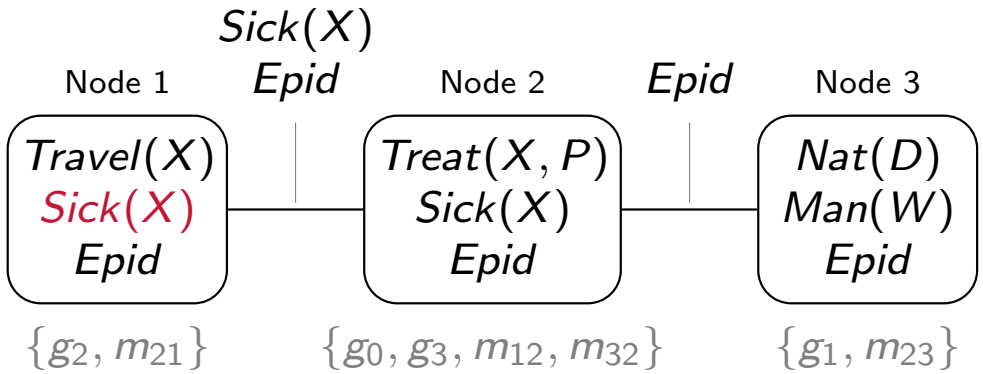

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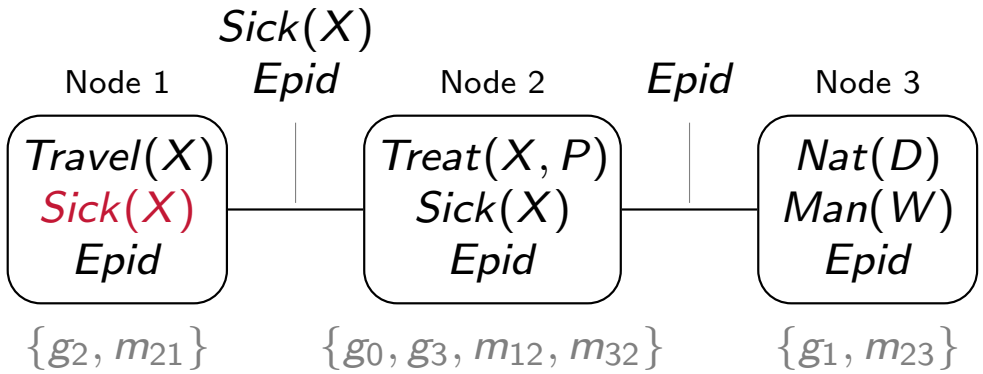

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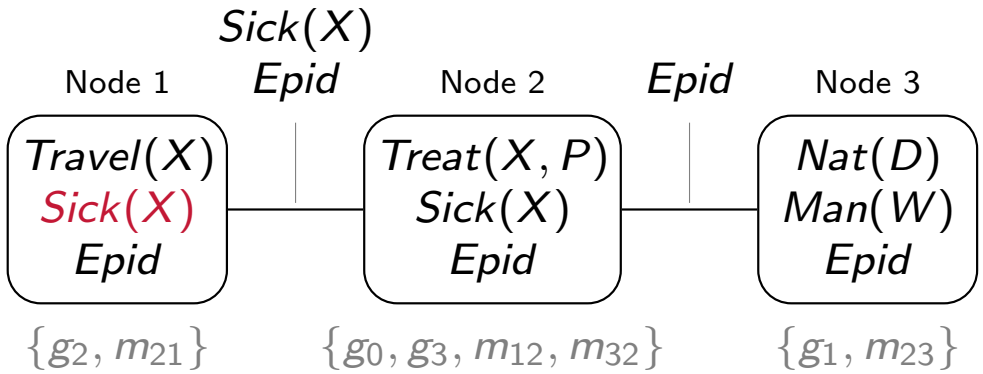

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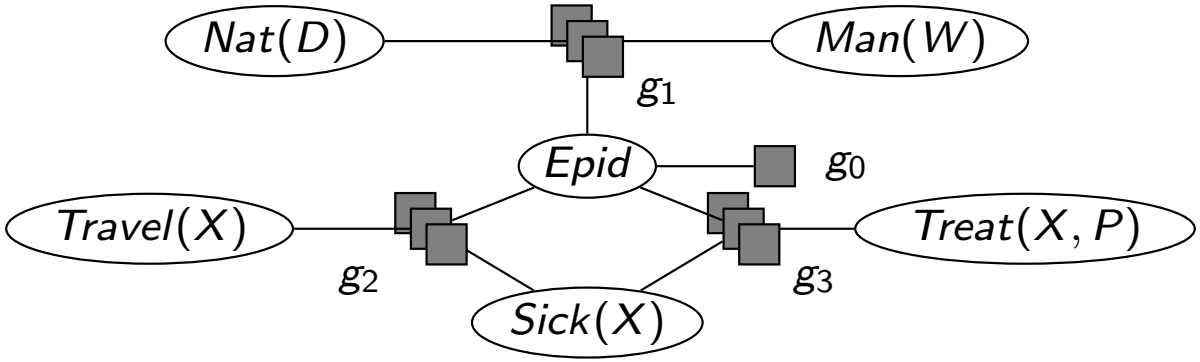
     - Eliminate $Epid$ from g_{21}'

Adaptive Inference

Answer Queries for Marginal Distributions in an Incrementally Changing Model



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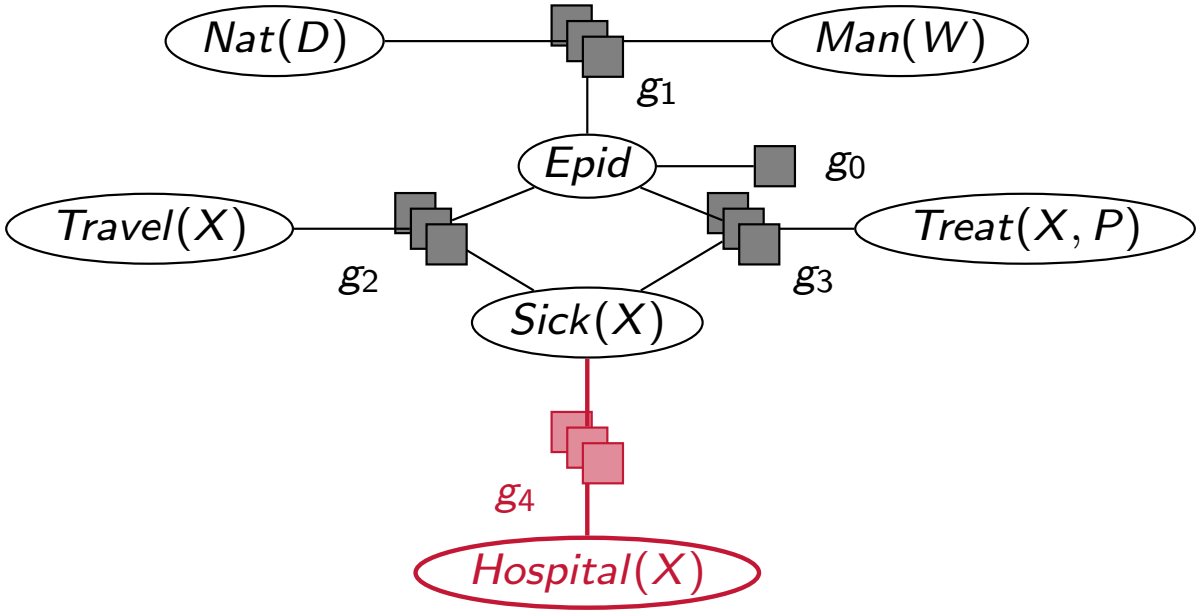

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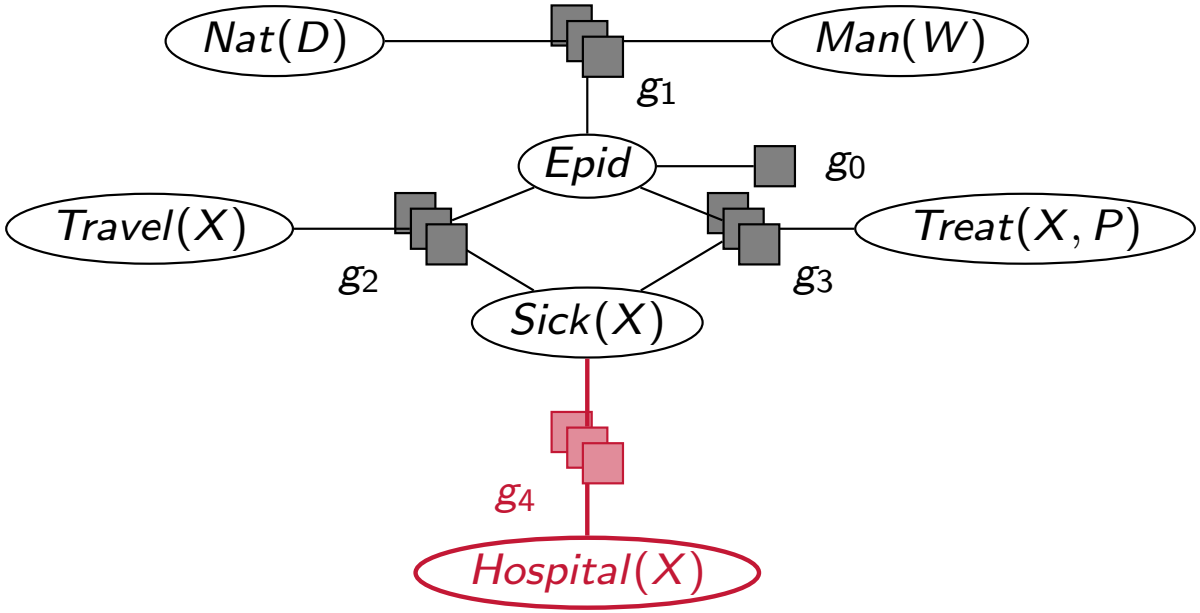

Evidence changes (new/retracted)

$$Sick(alice) = true$$
$$Sick(bob) = true$$

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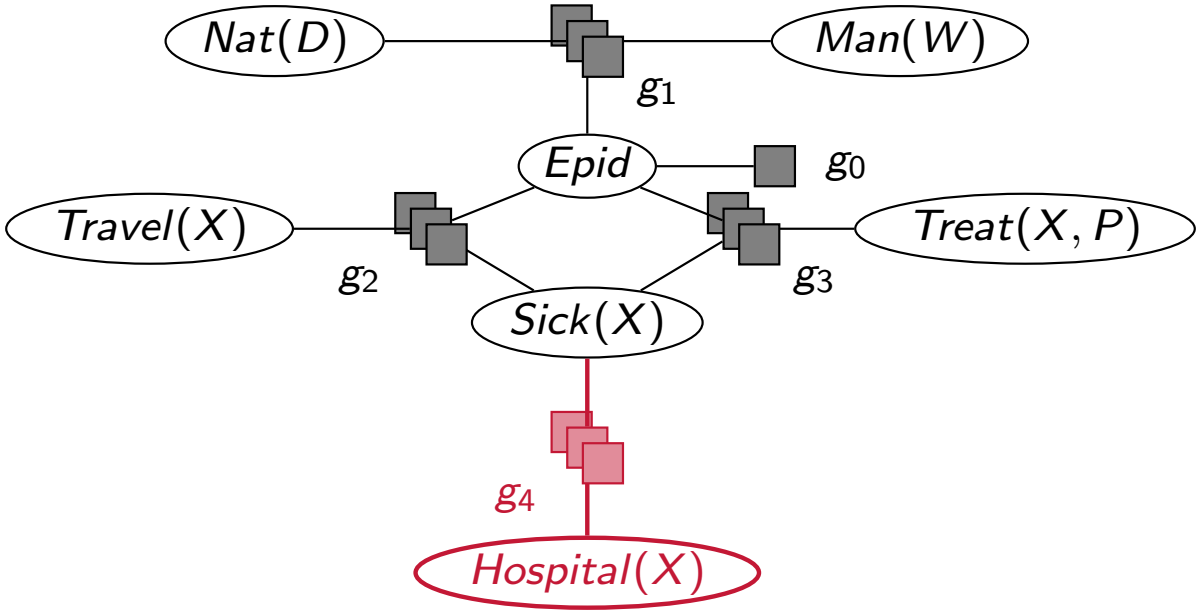


Evidence changes (new/retracted)

$Sick(alice) = true$

$Sick(bob) = true$

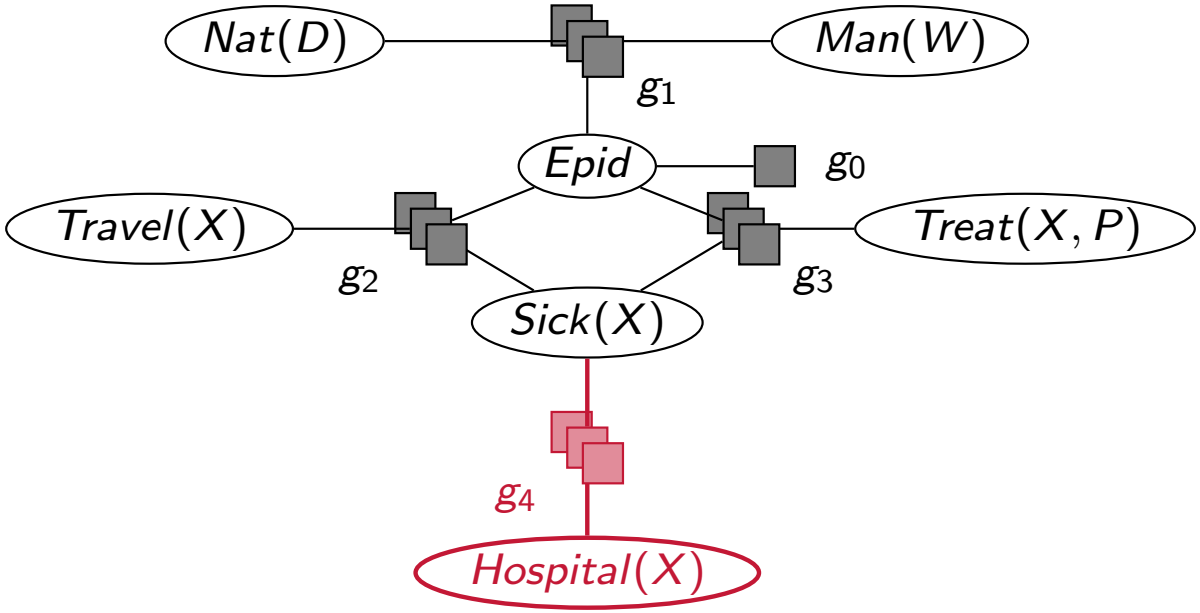
Domains change (new constants)

$X \in \{alice, eve, bob\}$

$\cup\{charlie\}$

Adaptive Inference

Answer Queries for Marginal Distributions in an Incrementally Changing Model



Evidence changes (new/retracted)

$Sick(alice) = true$

$Sick(bob) = true$

Domains change (new constants)

$X \in \{alice, eve, bob\}$

$\cup \{charlie\}$

Solve each query more efficiently than starting from scratch for each query

Conference Contribution

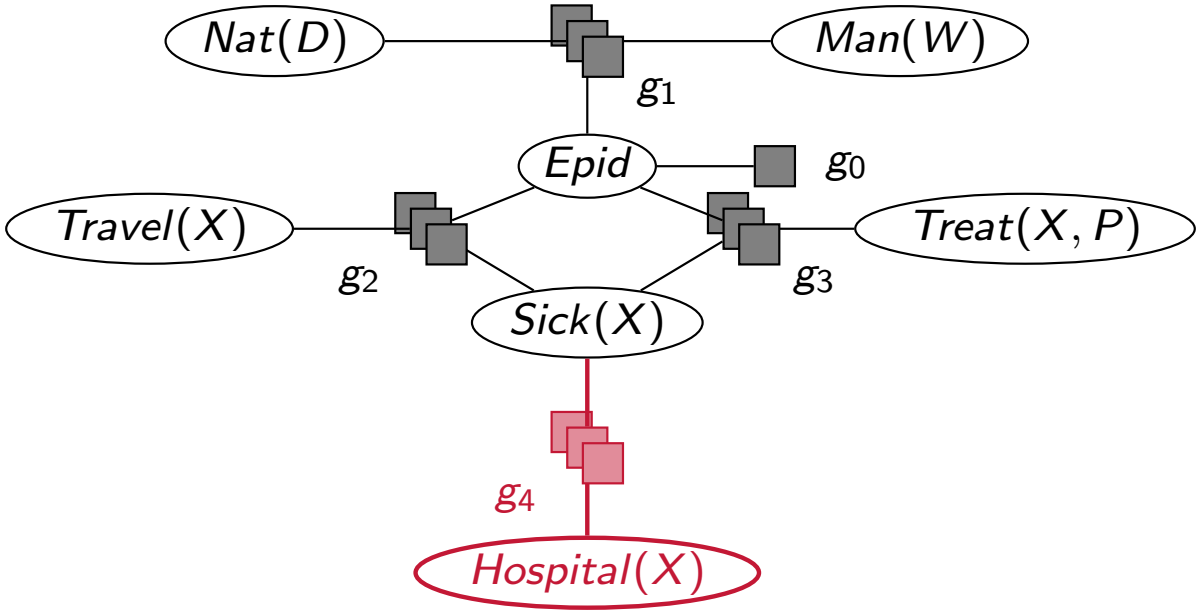
Adaptive Inference with LJT

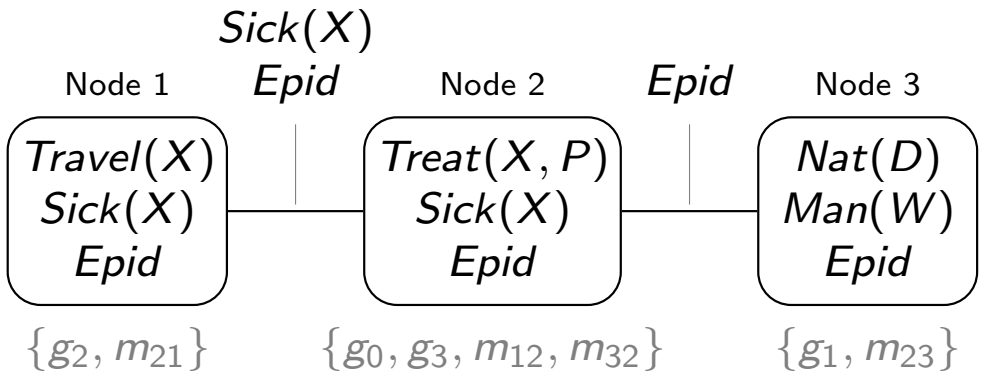
- Adjust a first-order junction tree to structural changes
  $\rightarrow$ While maintaining a valid tree
- Adapt local information to changes
  $\rightarrow$ Valid local information
- Adapt messages to changes
  $\rightarrow$ Valid outside information

Efficient, exact adaptive inference in probabilistic relational models

Adaptive Inference with LJT

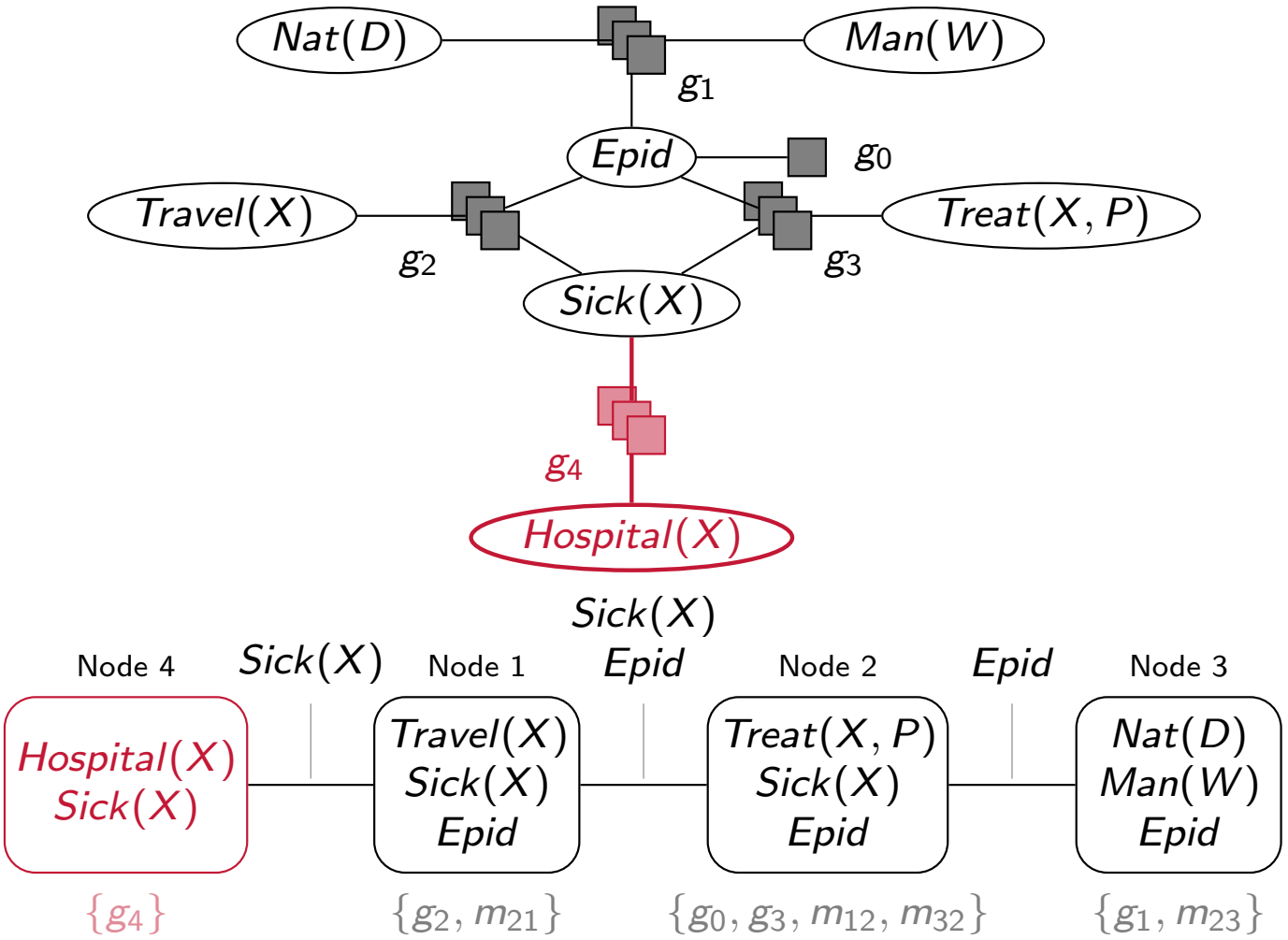
Structural Change: Adding a Parfactor with a New Variable





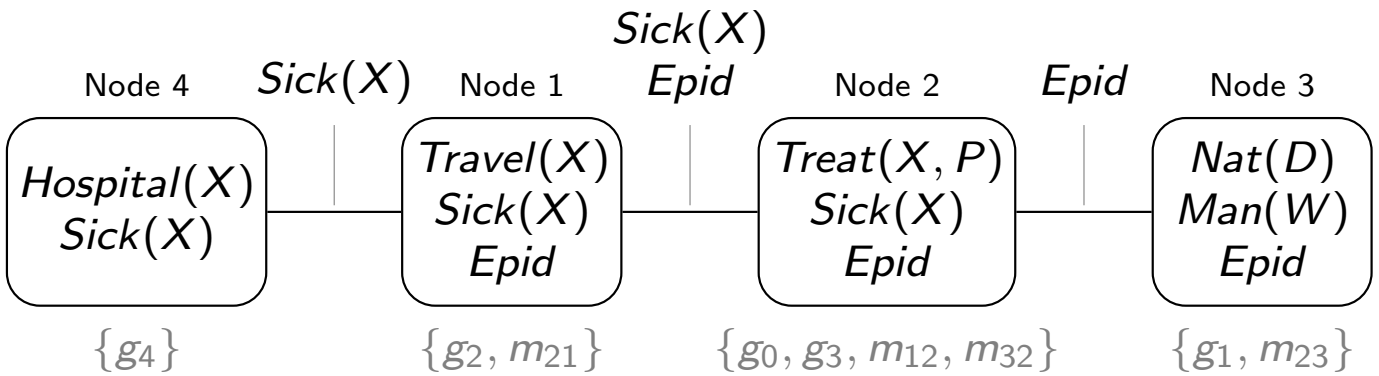
Adaptive Inference with LJT

Structural Change: Adding a Parfactor with a New Variable



Adaptive Inference with LJT

Adaptive Message Passing

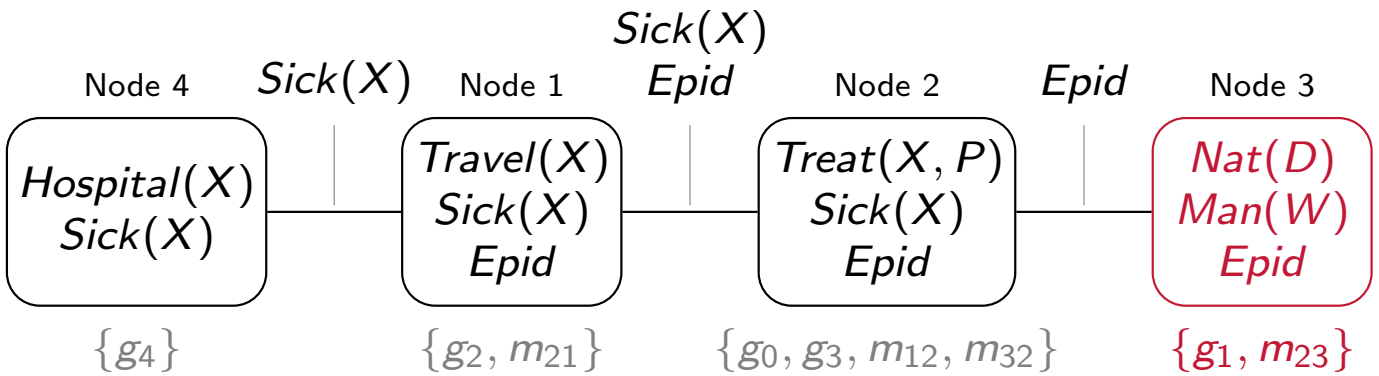


A node calculates a message if changes occur

- in its structure, local model, or in a message

Adaptive Inference with LJT

Adaptive Message Passing



A node calculates a message if changes occur

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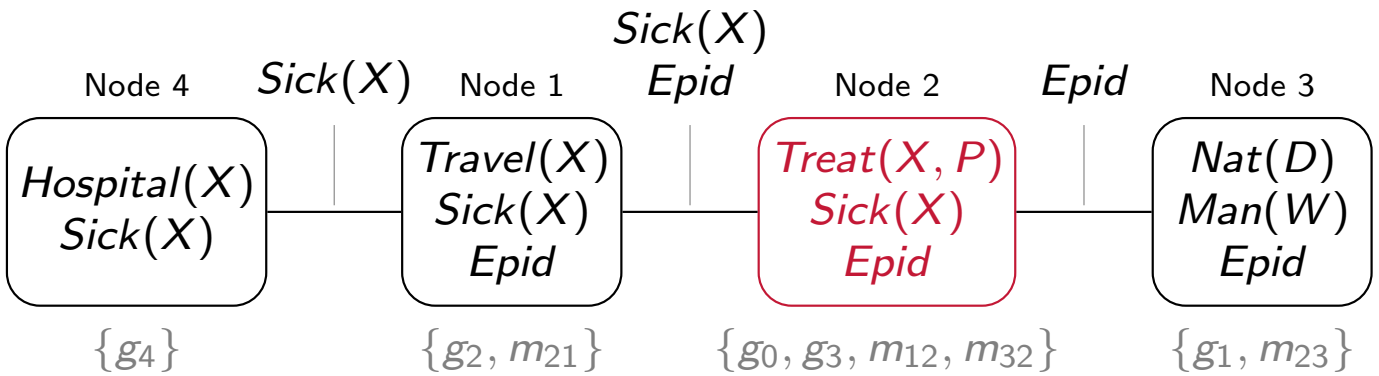
Messages

- $3 \longrightarrow 2$: ✓

Adaptive Inference with LJT

Adaptive Message Passing



A node calculates a message if changes occur

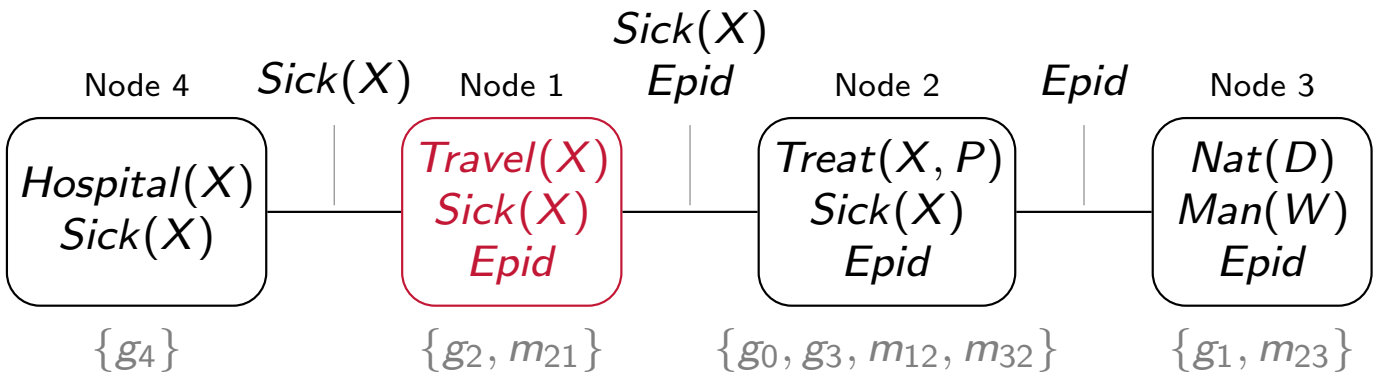
- in its structure, local model, or in a message

Messages

- $3 \longrightarrow 2$: ✓
- $2 \longrightarrow 1$: ✓

Adaptive Inference with LJT

Adaptive Message Passing



A node calculates a message if changes occur

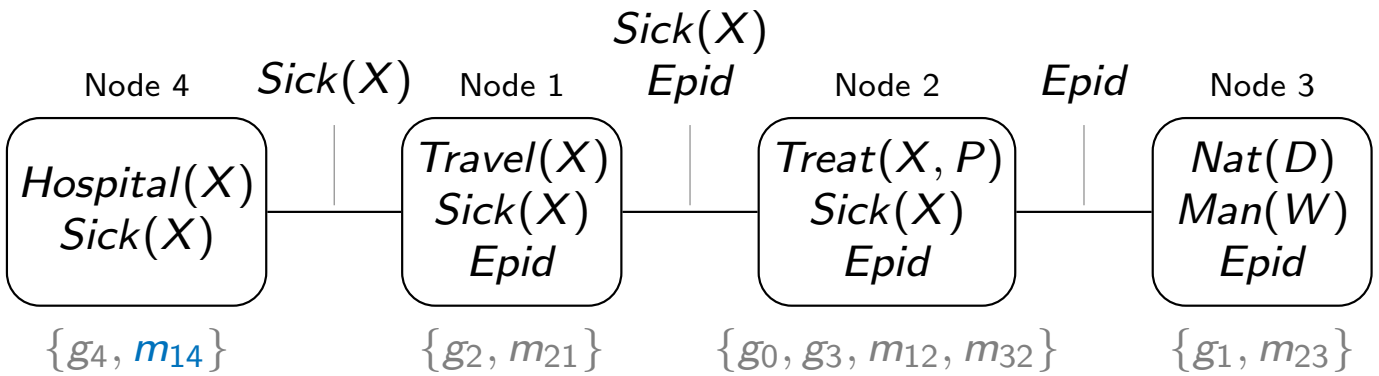
- in its structure, local model, or in a message

Messages

- $3 \longrightarrow 2$: ✓
- $2 \longrightarrow 1$: ✓
- $1 \longrightarrow 4$: new neighbour

Adaptive Inference with LJT

Adaptive Message Passing



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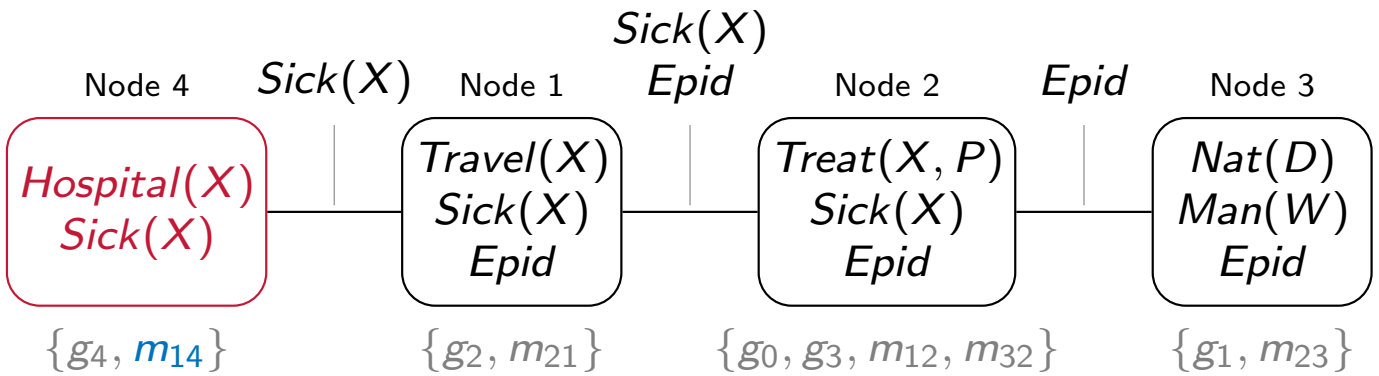
Adaptive Inference with LJT

Adaptive Message Passing



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Messages

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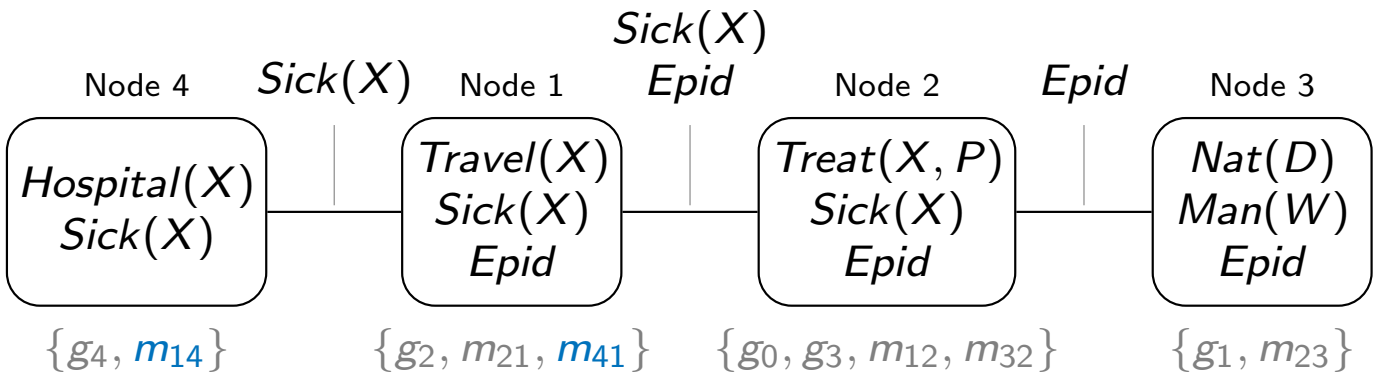
- $4 \longrightarrow 1$: new node

Adaptive Inference with LJT

Adaptive Message Passing



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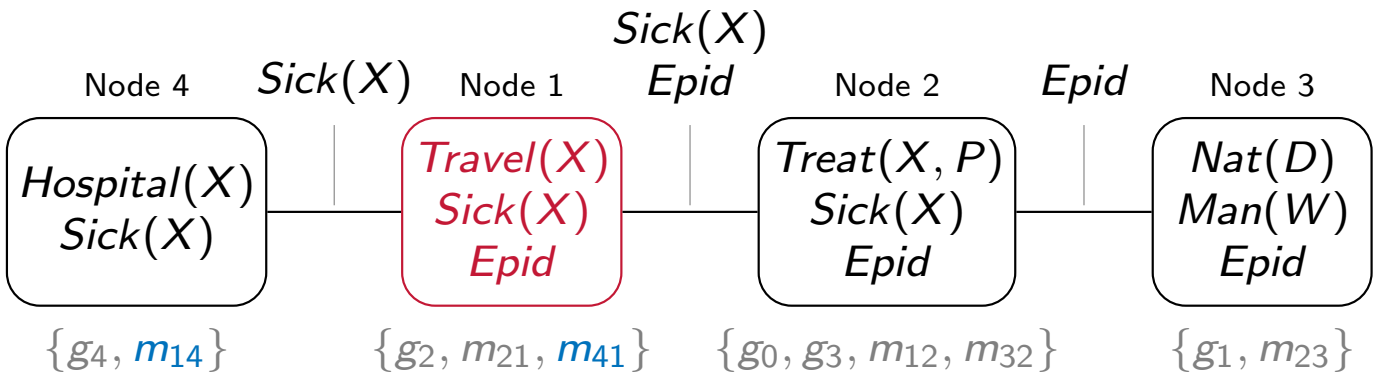
Adaptive Inference with LJT

Adaptive Message Passing



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Messages

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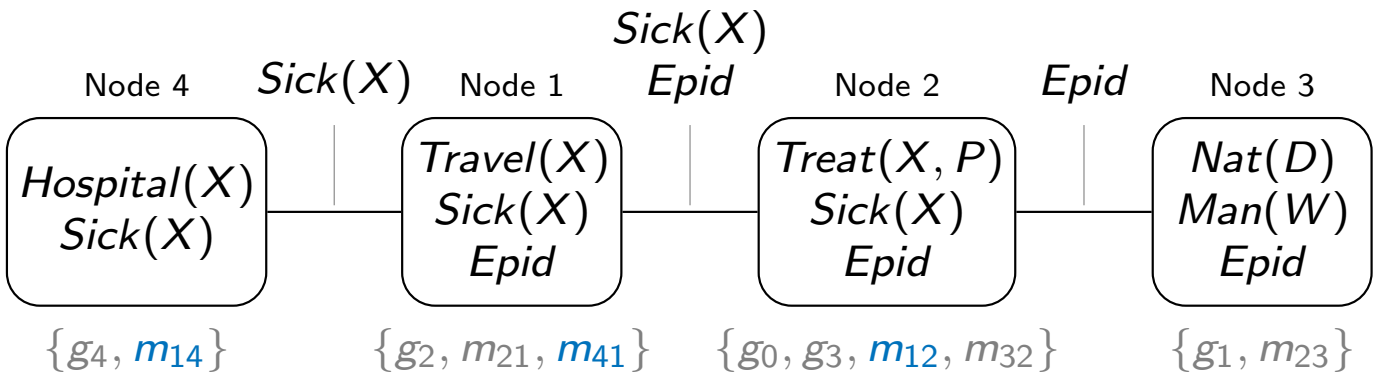
Adaptive Inference with LJT

Adaptive Message Passing



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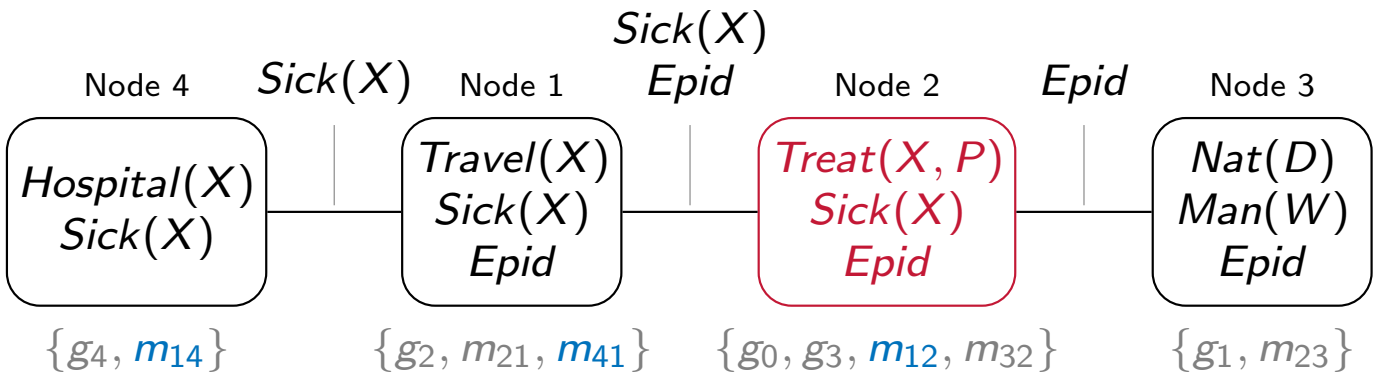
Adaptive Inference with LJT

Adaptive Message Passing



A node calculates a message if changes occur

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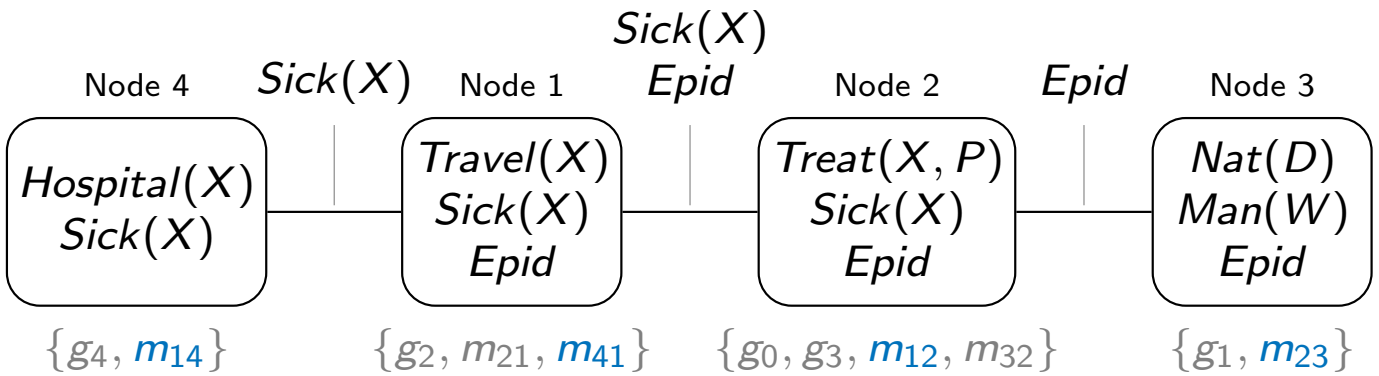
Messages

- 3 $\longrightarrow$ 2: ✓
- 2 $\longrightarrow$ 1: ✓
- 1 $\longrightarrow$ 4: new neighbour
- 4 $\longrightarrow$ 1: new node
- 1 $\longrightarrow$ 2: new message
- 2 $\longrightarrow$ 3: msg. changed

Adaptive Inference with LJT

Adaptive Message Passing



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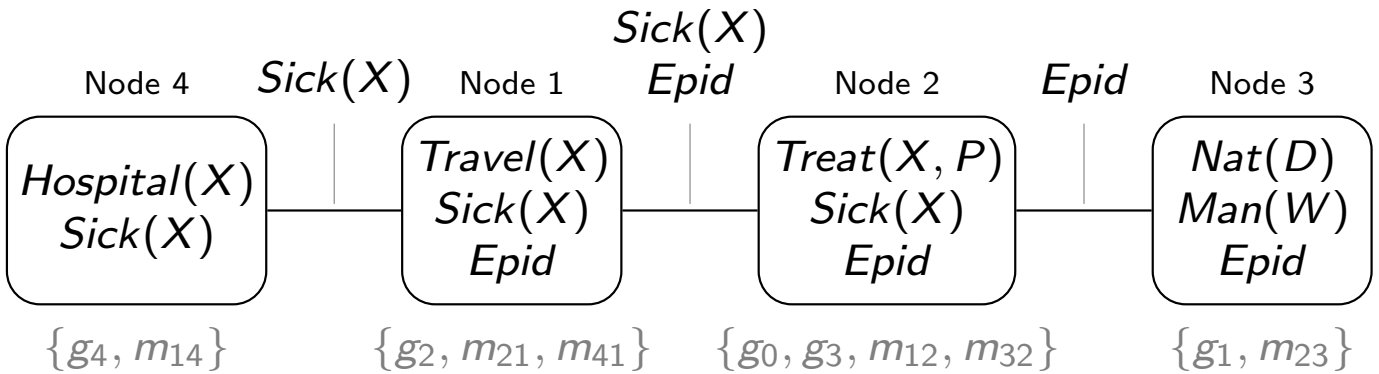
Adaptive Inference with LJT

Adaptive Evidence Entering

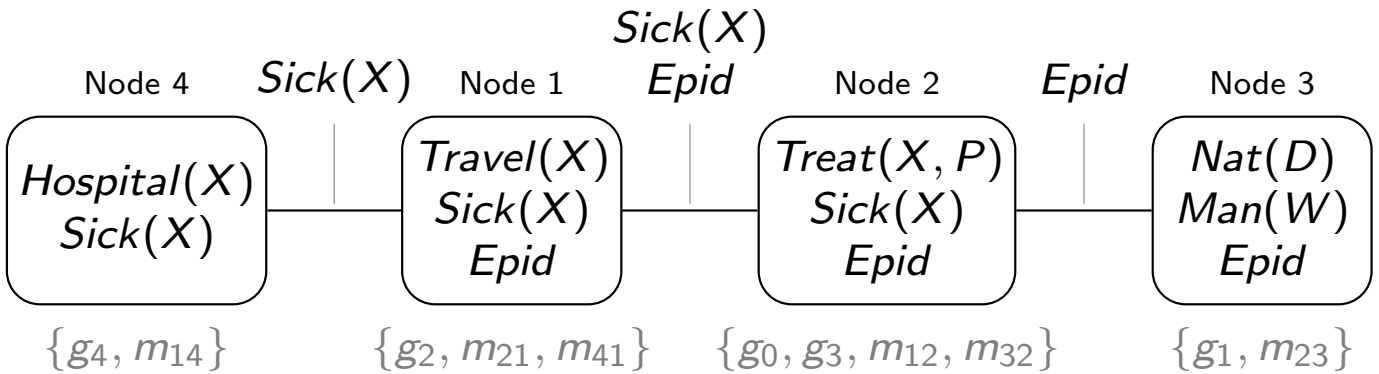


A node enters evidence if changes occur

- for its own variables

Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

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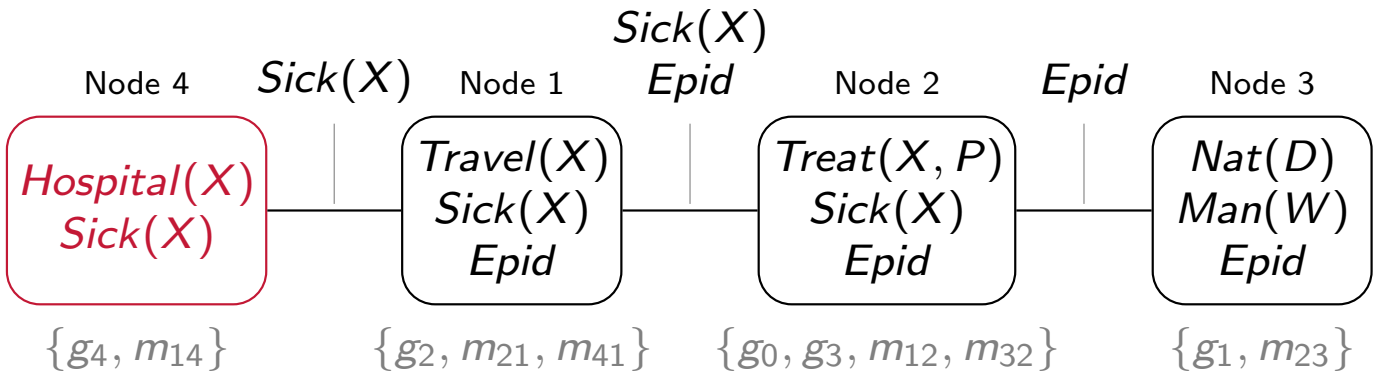
Evidence: $Hospital(eve) = true$

Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

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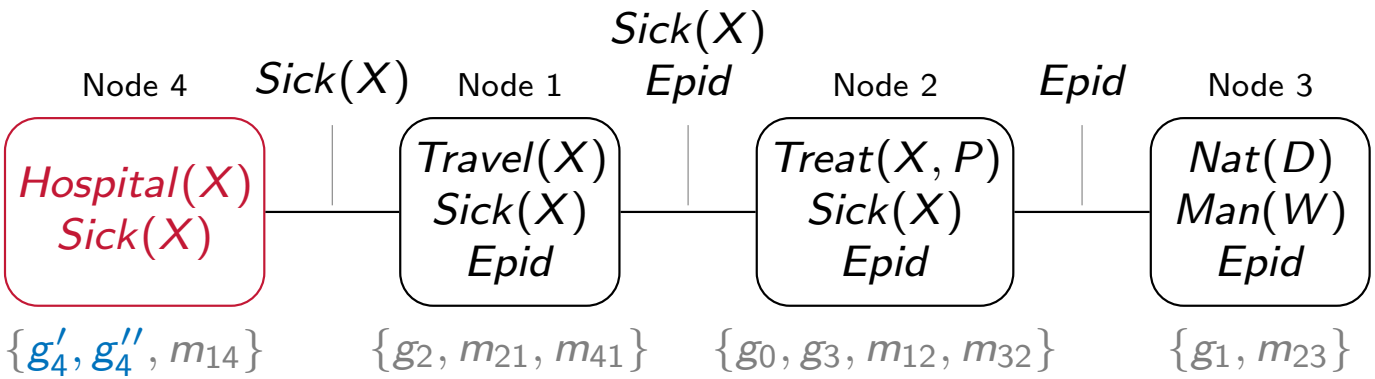
Evidence: $Hospital(eve) = true$

- Node 4: enter evidence

Adaptive Inference with LJT

Adaptive Evidence Entering



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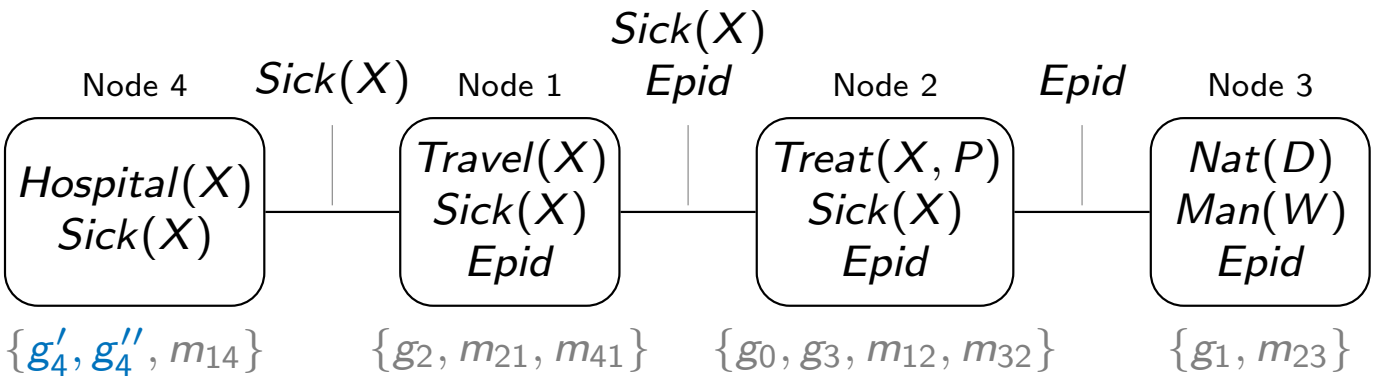


A node enters evidence if changes occur

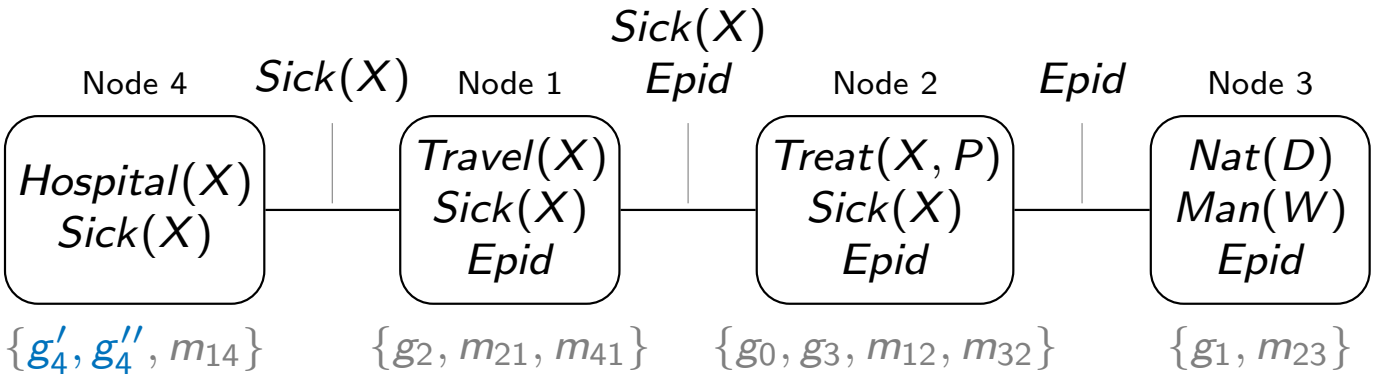
- for its own variables

Evidence: $Hospital(eve) = true$

- Node 4: enter evidence
- All other nodes: ✓

Adaptive Inference with LJT

Adaptive Evidence Entering



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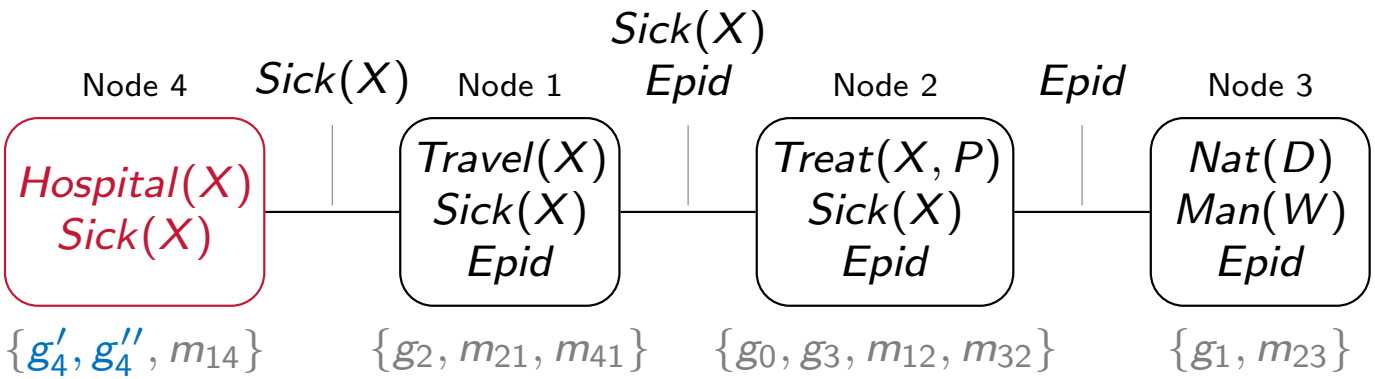
Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

- for its own variables

Evidence: $Hospital(eve) = true$

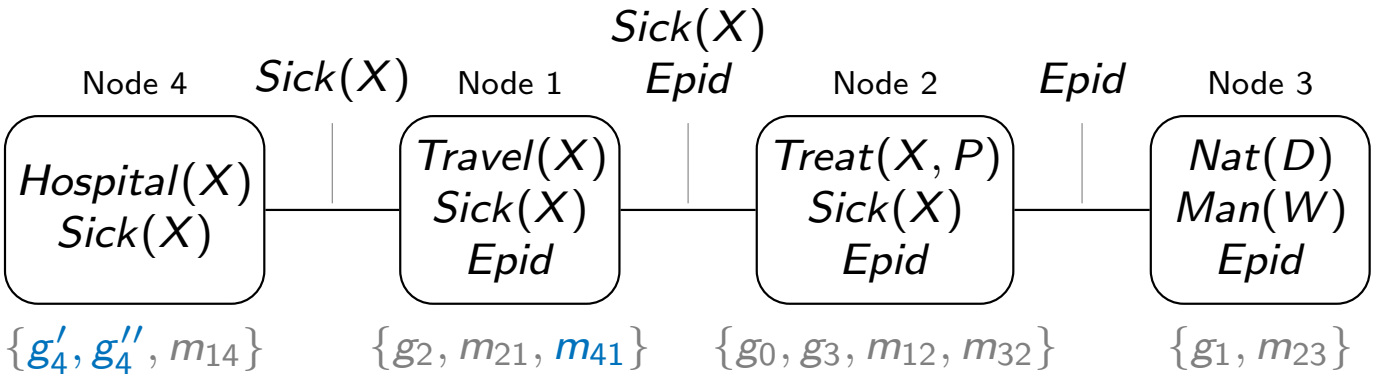
- Node 4: enter evidence
- All other nodes: ✓

Messages

- $4 \longrightarrow 1$: new evidence

Adaptive Inference with LJT

Adaptive Evidence Entering



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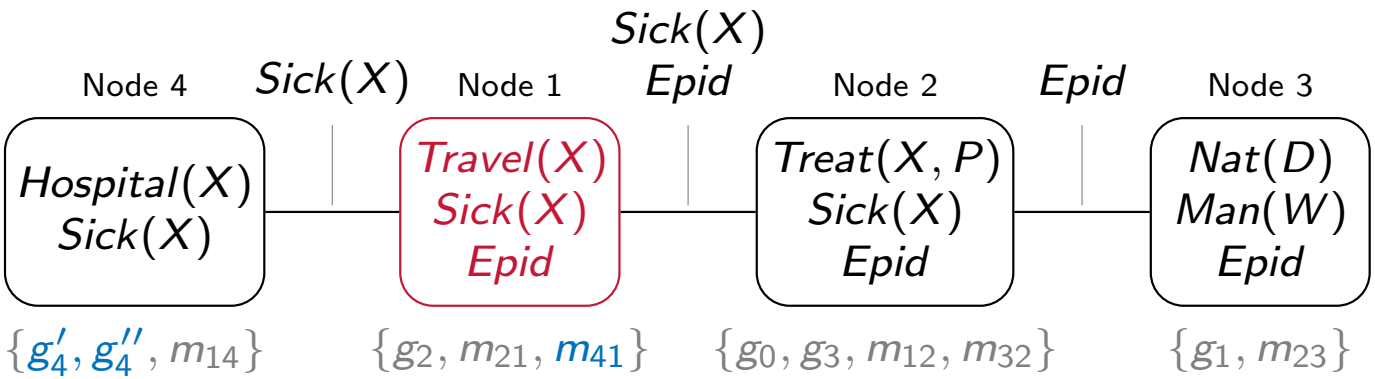
Messages

- $4 \longrightarrow 1$: new evidence

Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

- for its own variables

Evidence: $Hospital(eve) = true$

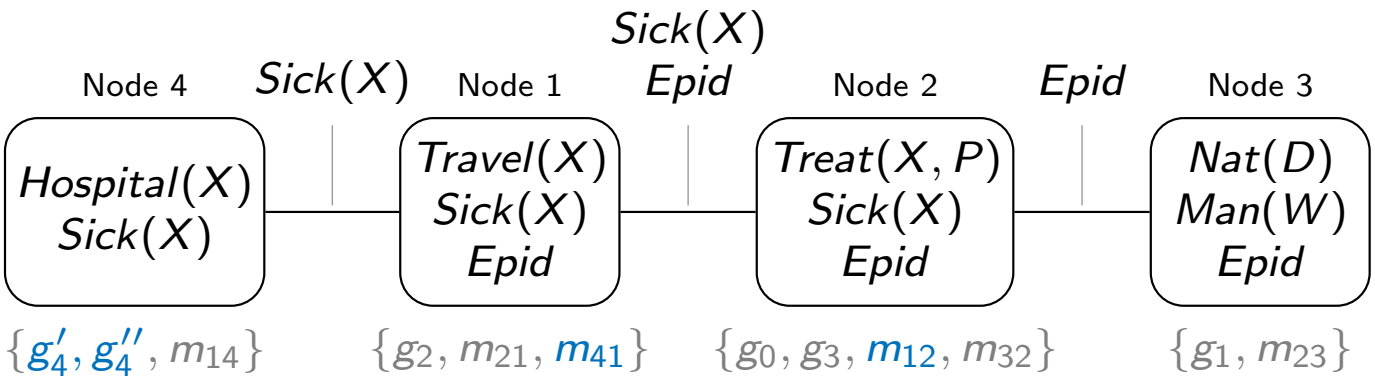
- Node 4: enter evidence
- All other nodes: ✓

Messages

- $4 \longrightarrow 1$: new evidence
- $1 \longrightarrow 2$: msg. changed

Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

- for its own variables

Evidence: $Hospital(eve) = true$

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Messages

- $4 \longrightarrow 1$: new evidence
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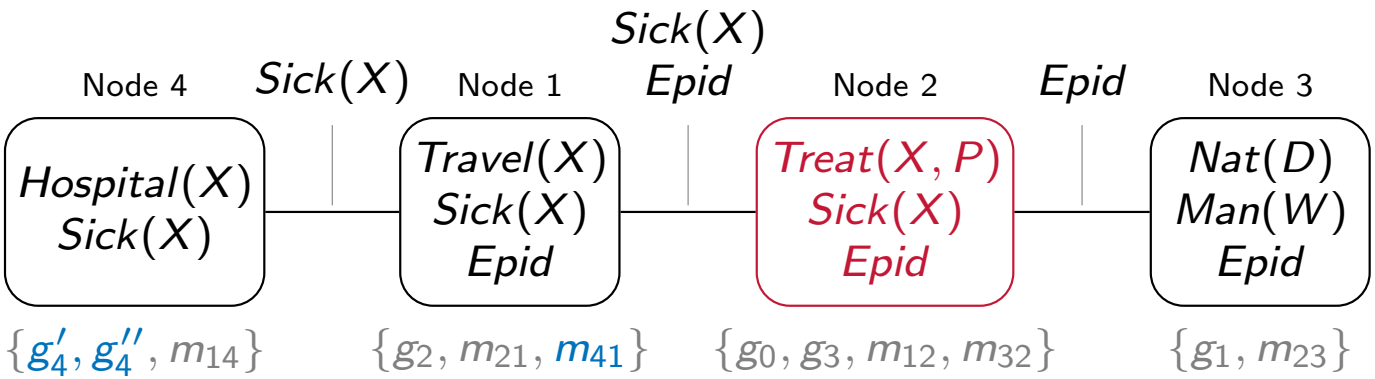
Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

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Evidence: $Hospital(eve) = true$

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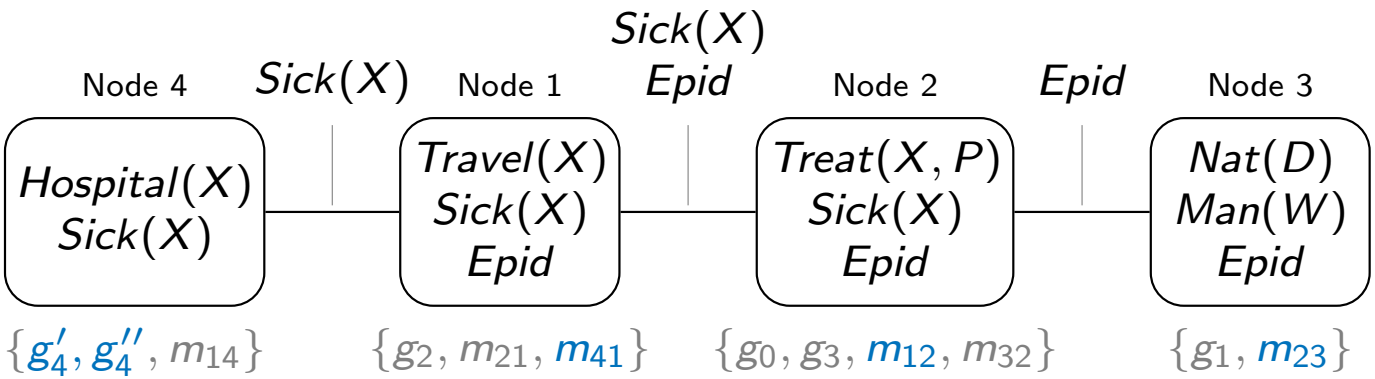
Messages

- $4 \longrightarrow 1$: new evidence
- $1 \longrightarrow 2$: msg. changed
- $2 \longrightarrow 3$: msg. changed

Adaptive Inference with LJT

Adaptive Evidence Entering



A node enters evidence if changes occur

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Evidence: $Hospital(eve) = true$

- Node 4: enter evidence
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Messages

- $4 \longrightarrow 1$: new evidence
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Test Run

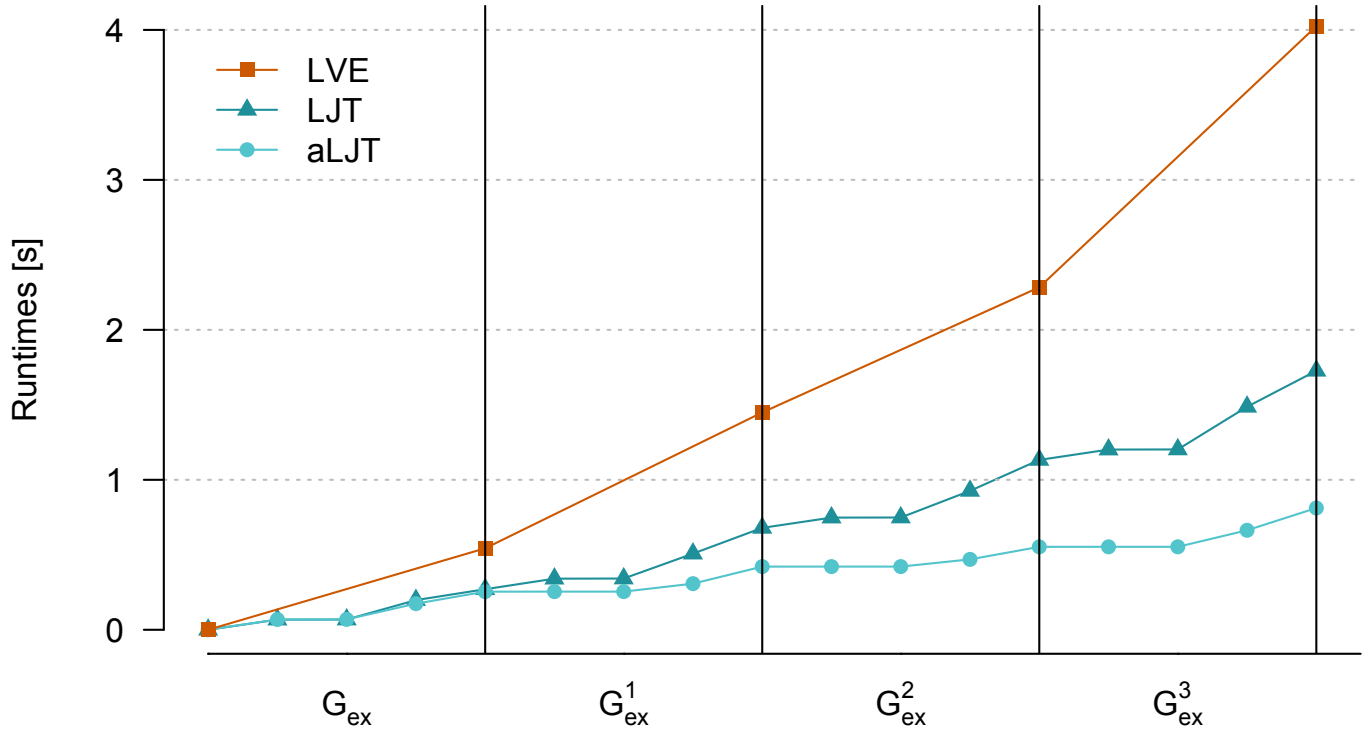

Figure: Runtimes [ms] for two queries and three changes (points connected for readability)

`lve`: Implementation by Taghipour (2013)

`(a)ljt`: Our implementation of (adaptive) LJT

Analysis: Adaptive Inference with LJT

Algorithm steps

1. Adaptation of tree to changes
2. Adaptive evidence entering
3. Adaptive message passing
4. Query answering

Ensure correct information

$\rightarrow$ Redo what is necessary

Adaptive steps

$\rightarrow$ Save up to 50% of message passing runtime (most expensive step)

Changes too drastic/too many

$\rightarrow$ Reconstruct tree from scratch

Conference Contribution

Adaptive Inference with LJT

- Adapt a first-order junction tree to structural changes
  $\rightarrow$ Valid representation of model
- Adapt local information to changes
  $\rightarrow$ Valid local information
- Adapt messages to changes
  $\rightarrow$ Valid overall information

Efficient, exact adaptive inference in probabilistic relational models

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Efficient, exact adaptive inference in probabilistic relational models

Future work

- Learning lifted representations
- Modelling decision making