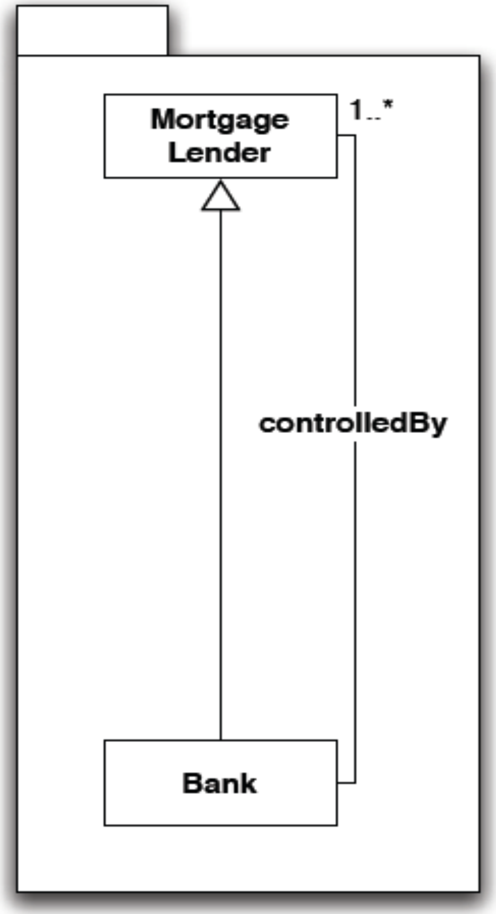
Conceptual Modeling and Query Answering

MortgageLender

Bank
HLB

controlledBy	
BLB	HVB



- Query:
  { (X) | controlledBy(X, Y) }

Description Logics

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1. Motivation and introduction to Description Logics

2. Tableau-based reasoning procedures

Description Logics

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Literature:

The Description Logic Handbook

edited by F. Baader, D. Calvanese, D. McGuinness, D. Nardi, P. Patel-Schneider

Cambridge University Press

The description language

prototypical DL $\mathcal{ALC}$

Set N_C of concept names and disjoint set N_R of role names.

$\mathcal{ALC}$-concept descriptions are defined by induction:

- If $A \in N_C$, then A is an $\mathcal{ALC}$-concept description.
- If C, D are $\mathcal{ALC}$-concept descriptions, and $r \in N_R$, then the following are $\mathcal{ALC}$-concept descriptions:
  - $C \sqcap D$ (conjunction)
  - $C \sqcup D$ (disjunction)
  - $\neg C$ (negation)
  - $\forall r.C$ (value restriction)
  - $\exists r.C$ (existential restriction)

Abbreviations:
- $\top := A \sqcup \neg A$ (top)
- $\bot := A \sqcap \neg A$ (bottom)
- $C \Rightarrow D := \neg C \sqcup D$ (implication)

The description language

examples of $\mathcal{ALC}$-concept descriptions

$\text{Person} \sqcap \text{Female}$

$\text{Person} \sqcap \exists \text{attends.Course}$

$\text{Person} \sqcap \forall \text{attends.}(\text{Course} \sqcap \neg \text{Easy})$

$\text{Person} \sqcap \exists \text{teaches.}(\text{Course} \sqcap \forall \text{topic.DL})$

$\text{Person} \sqcap \forall \text{teaches.}(\text{Course} \sqcap \exists \text{topic.}(\text{DL} \sqcup \text{NMR}))$

The description language

semantics of $\mathcal{ALC}$-concept descriptions

An interpretation $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ consists of a non-empty domain $\Delta^\mathcal{I}$ and an interpretation function $\cdot^\mathcal{I}$:

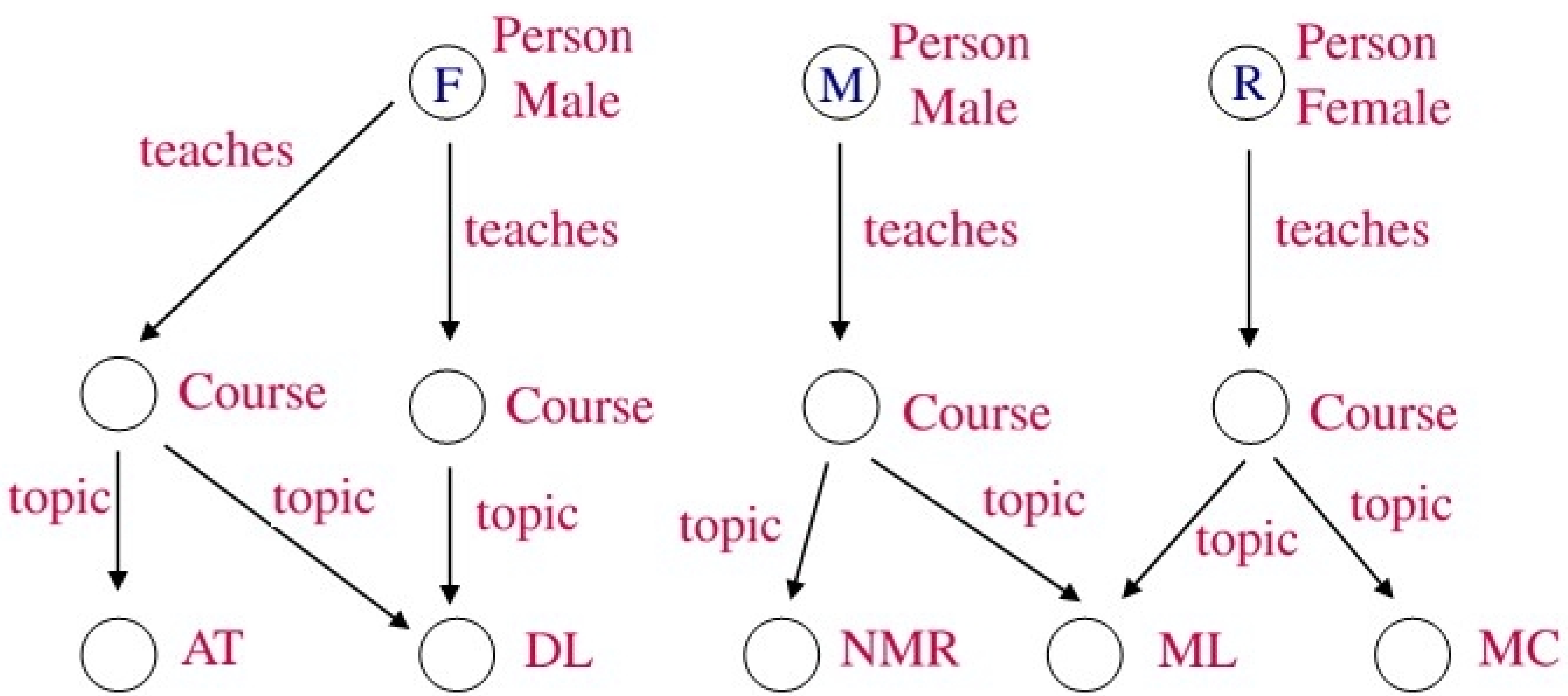
- $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$ for all $A \in N_C$, — concepts interpreted as sets
- $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$ for all $r \in N_R$. — roles interpreted as binary relations

The interpretation function is extended to $\mathcal{ALC}$-concept descriptions as follows:

- $(C \sqcap D)^\mathcal{I} := C^\mathcal{I} \cap D^\mathcal{I}$
- $(C \sqcup D)^\mathcal{I} := C^\mathcal{I} \cup D^\mathcal{I}$
- $(\neg C)^\mathcal{I} := \Delta^\mathcal{I} \setminus C^\mathcal{I}$
- $(\forall r.C)^\mathcal{I} := \{d \in \Delta^\mathcal{I} \mid \text{for all } e \in \Delta^\mathcal{I} : (d,e) \in r^\mathcal{I} \text{ implies } e \in C^\mathcal{I}\}$
- $(\exists r.C)^\mathcal{I} := \{d \in \Delta^\mathcal{I} \mid \text{there is } e \in \Delta^\mathcal{I} : (d,e) \in r^\mathcal{I} \text{ and } e \in C^\mathcal{I}\}$

Example

of an interpretation



$$(\ \text{Person} \sqcap \exists \text{teaches}.(\text{Course} \sqcap \forall \text{topic}.\text{DL})\ )^{\mathcal{I}} = \{F\}$$

$$(\ \text{Person} \sqcap \forall \text{teaches}.(\text{Course} \sqcap \exists \text{topic}.(\text{DL} \sqcup \text{NMR}))\ )^{\mathcal{I}} = \{F, M\}$$

Relationship with First-order Logic

$\mathcal{ALC}$ can be seen as a fragment of first-order logic:

- Concept names are unary predicates, and role names are binary predicates.
- Concept descriptions C yield formulae with one free variable $\tau_x(C)$:
  - $\tau_x(A) := A(x)$ for $A \in N_C$
  - $\tau_x(C \sqcap D) := \tau_x(C) \wedge \tau_x(D)$
  - $\tau_x(C \sqcup D) := \tau_x(C) \vee \tau_x(D)$
  - $\tau_x(\neg C) := \neg \tau_x(C)$
  - $\tau_x(\forall r.C) := \forall y.(r(x,y) \rightarrow \tau_y(C))$
  - $\tau_x(\exists r.C) := \exists y.(r(x,y) \wedge \tau_y(C))$

  y variable different from x

C and $\tau_x(C)$ have the same semantics:

$$C^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \mathcal{I} \models \tau_x(C)[x \leftarrow d]\}$$

Relationship with First-order Logic

$\mathcal{ALC}$ can be seen as a fragment of first-order logic:

- Concept names are unary predicates, and role names are binary predicates.
- Concept descriptions C yield formulae with one free variable $\tau_x(C)$:

These formulae belong to known decidable subclasses of first-order logic:

– two-variable fragment

– guarded fragment

$$\begin{aligned}\tau_x(\forall r.(A \sqcap \exists r.B)) &= \forall y.(r(x,y) \rightarrow \tau_y(A \sqcap \exists r.B)) \\ &= \forall y.(r(x,y) \rightarrow (A(y) \wedge \exists z.(r(y,z) \wedge B(z))))\end{aligned}$$

Additional constructors

$\mathcal{ALC}$ is only an example of a description logic.

DL researchers have introduced and investigated many additional constructors.

Example

Number restrictions: $(\geq n\, r.C)$, $(\leq n\, r.C)$ with semantics

$$(\geq n\, r.C)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid \text{card}(\{e \mid (d,e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\}) \geq n\}$$

$$(\leq n\, r.C)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid \text{card}(\{e \mid (d,e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\}) \leq n\}$$

Persons that attend at most 3 courses, of which at least 2 have the topic DL:

$$\text{Person} \sqcap (\leq 3\, \text{attends.Course}) \sqcap (\geq 2\, \text{attends.}(\text{Course} \sqcap \exists \text{topic.DL}))$$

Additional constructors

In addition to concept constructors, one can also introduce role constructors.

Example

Inverse roles: if r is a role, then r^{-1} denotes its inverse

$$(r^{-1})^{\mathcal{I}} := \{(e,d) \mid (d,e) \in r^{\mathcal{I}}\}$$

Inverse roles can be used like role names in value and existential restrictions.

Teacher of a boring course:

$$\text{Person} \sqcap \exists \text{teaches}.(\text{Course} \sqcap \forall \text{attends}^{-1}.(\text{Bored} \sqcup \text{Sleeping}))$$

Terminologies

introduce names for complex descriptions

A concept defintion is of the form $A \equiv C$ where

- A is a concept name;
- C is a concept description.

A TBox is a finite set of concept definitions that

- does not contain multiple definitions;
- does not contain cyclic definitions.

$$\begin{array}{lcl} A & \equiv & C \\ A & \equiv & D \end{array} \quad \text{for } C \neq D$$

$$\begin{array}{lcl} A & \equiv & B \sqcap \forall r.P \\ B & \equiv & P \sqcap \forall r.C \\ C & \equiv & \exists r.A \end{array}$$

Defined concept occurs on left-hand side of a definition

Primitive concept does not occur on left-hand side of a definition

Terminologies

semantics and example

An interpretation $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ if it satisfies all its concept definitions:

$$A^{\mathcal{I}} = C^{\mathcal{I}} \text{ for all } A \equiv C \in \mathcal{T}$$

$$\begin{array}{rcl}
\text{Woman} & \equiv & \text{Person} \sqcap \text{Female} \\
\text{Man} & \equiv & \text{Person} \sqcap \neg\text{Female} \\
\text{Course} & \equiv & \exists\text{topic}.\top \\
\text{Lecturer} & \equiv & \text{Person} \sqcap \exists\text{teaches}.\text{Course} \\
\text{Student} & \equiv & \text{Person} \sqcap \exists\text{attends}.\text{Course} \\
\text{BusyLecturer} & \equiv & \text{Lecturer} \sqcap (\geq 3\,\text{teaches}.\text{Course}) \\
\text{BadLecturer} & \equiv & \text{Lecturer} \sqcap \forall\text{teaches}.(\forall\text{attends}^{-1}.(\text{Bored} \sqcup \text{Sleeping}))
\end{array}$$

Terminologies

beyond concept definitions

Modern DL systems allow their users to state more general constraints for the interpretation of concepts.

A general concept inclusion axiom (GCI) is of the form

$C \sqsubseteq D$ where C, D may be complex concept descriptions.

general TBox

An interpretation $\mathcal{I}$ is a model of a set of GCIs $\mathcal{T}$ if it satisfies all its concept inclusions:

$$C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \text{ for all } C \sqsubseteq D \in \mathcal{T}$$

$\text{Course} \sqcap \forall \text{attends}^{-1}.\text{Sleeping} \sqsubseteq \text{Boring}$

$\text{Lecturer} \sqcap \text{Student} \sqsubseteq \bot$

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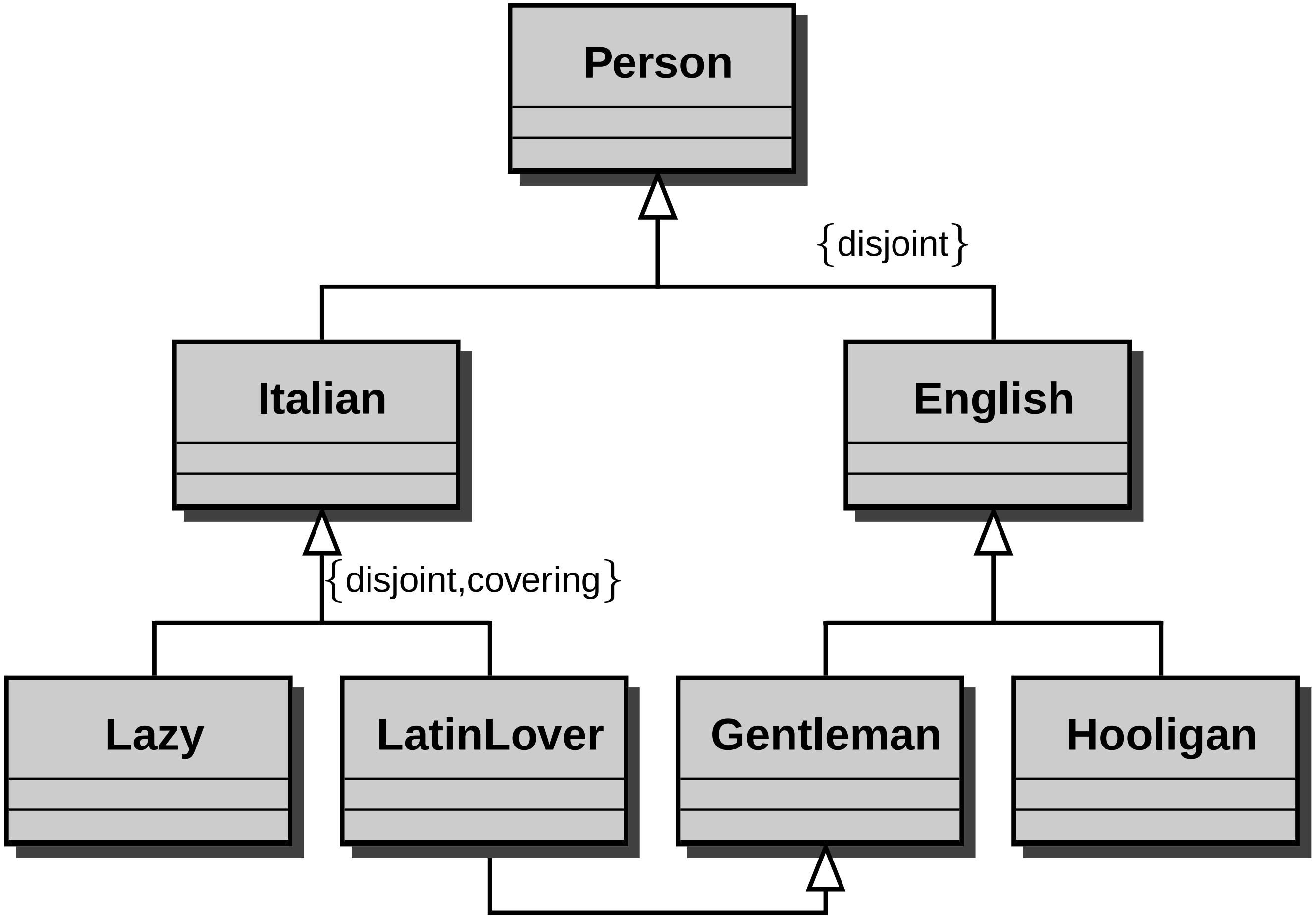
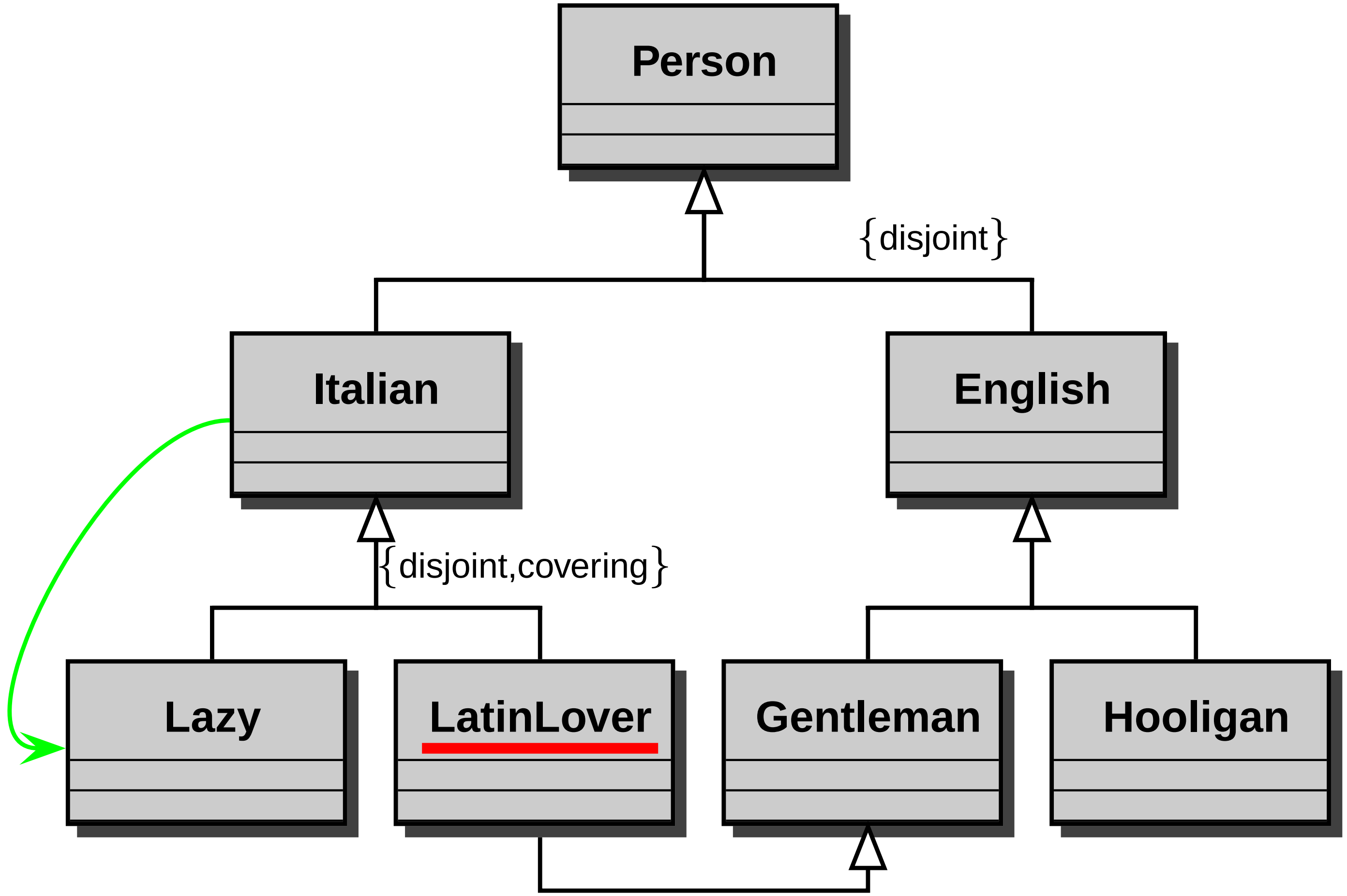
Simple reasoning example



Simple reasoning example

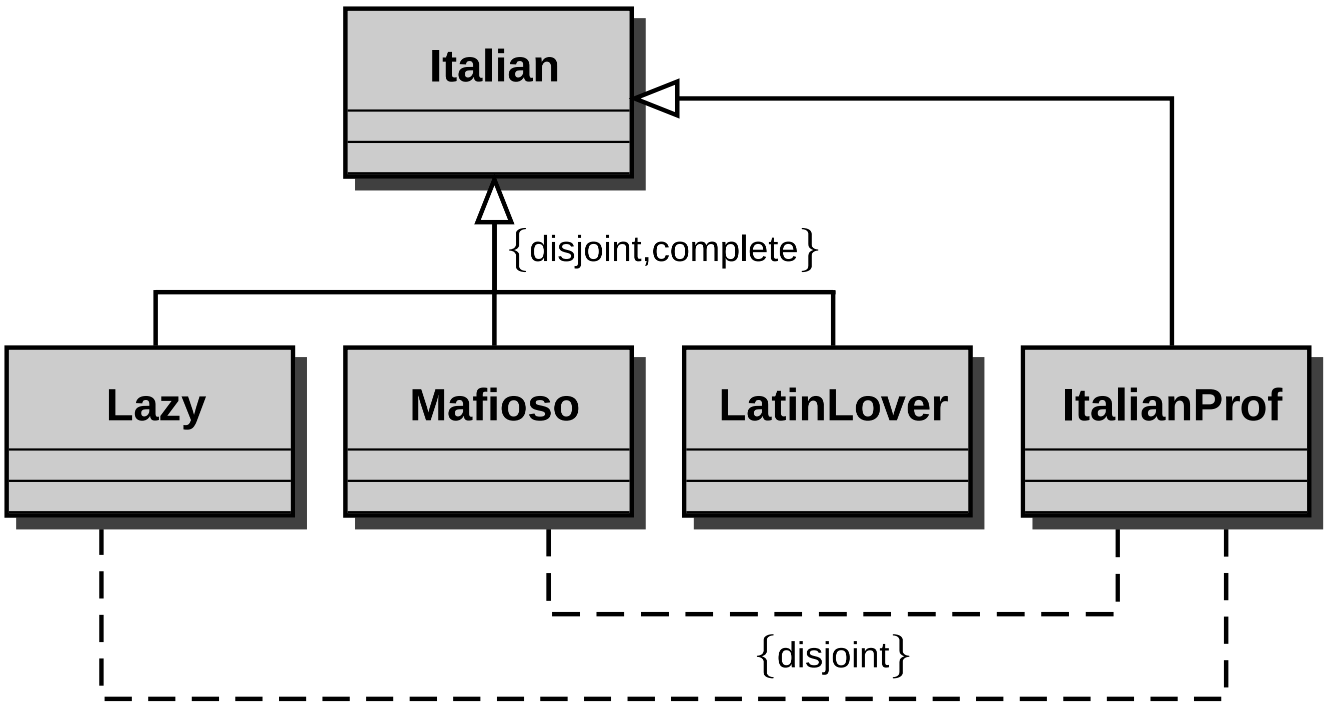


implies

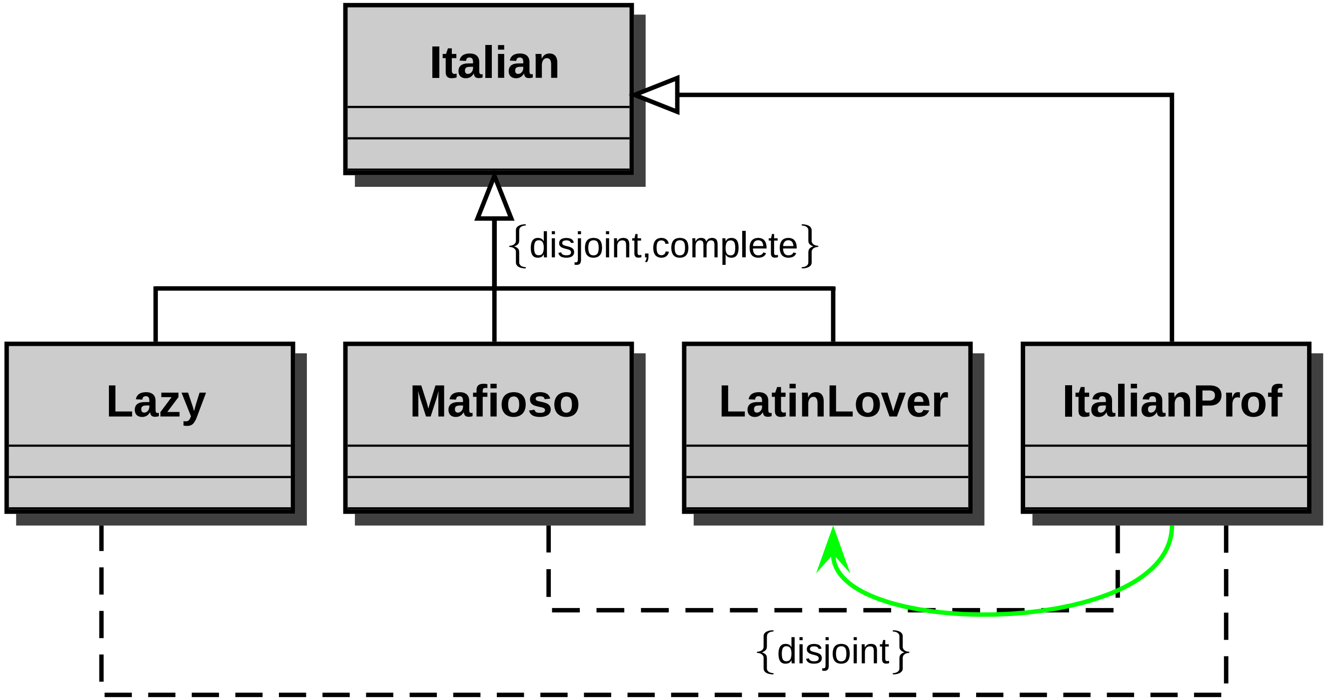
LatinLover $= \emptyset$

Italian $\subseteq$ Lazy – Italian $\equiv$ Lazy

Reasoning by cases



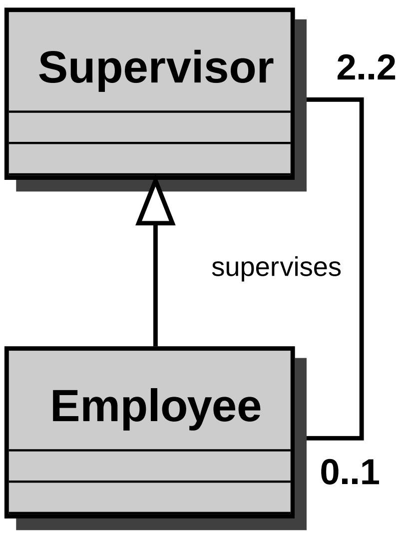
Reasoning by cases



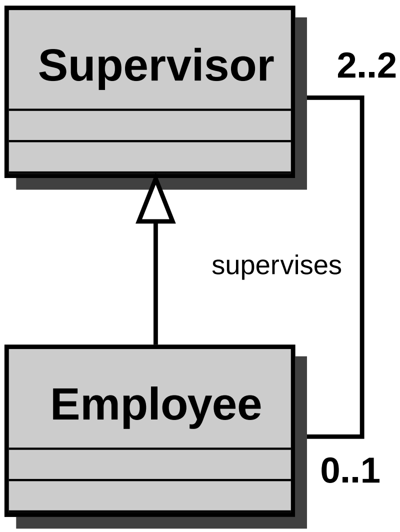
implies

ItalianProf $\subseteq$ LatinLover

Infinite worlds: the democratic company



Infinite worlds: the democratic company



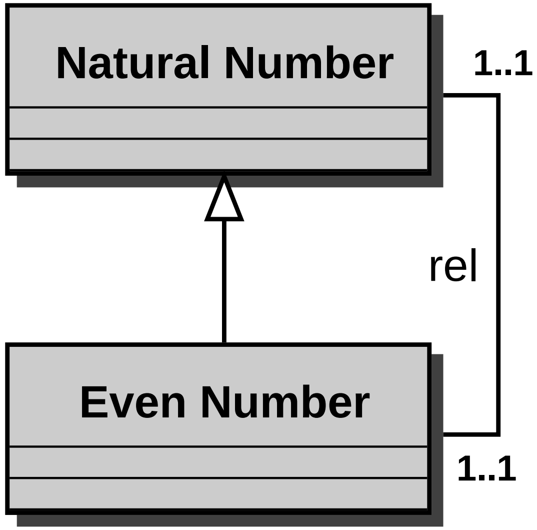
implies

“the classes Employee and Supervisor contain an infinite number of instances”.

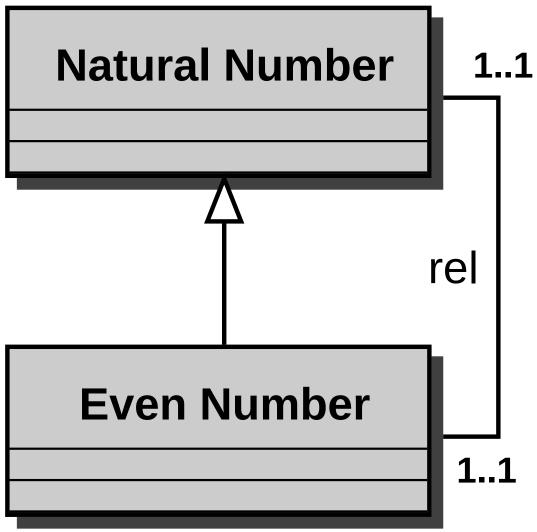
If the domain is finite:

Therefore, the schema is inconsistent.

Bijection: how many numbers



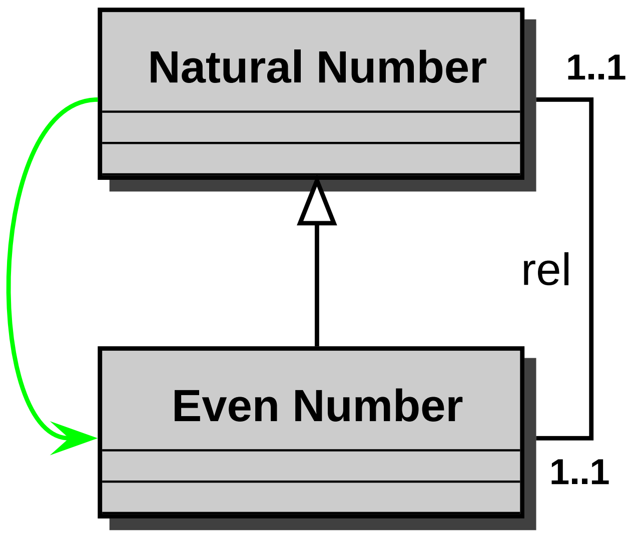
Bijection: how many numbers



implies

“the classes *’Natural Number’* and *’Even Number’* contain the same number of instances”.

Bijection: how many numbers



implies

"the classes *'Natural Number'* and *'Even Number'* contain the same number of instances".

If the domain is finite: Natural Number $\equiv$ Even Number

ABox assertions

state properties of individuals

An assertion is of the form

$C(a)$ (concept assertion) or $r(a, b)$ (role assertion)

where C is a concept description, r is a role, and a, b are individual names from a set N_I of such names.

An ABox is a finite set of assertions.

An interpretation $\mathcal{I}$ is a model of an ABox $\mathcal{A}$ if it satisfies all its assertions:

$$a^{\mathcal{I}} \in C^{\mathcal{I}} \quad \text{for all } C(a) \in \mathcal{A}$$
$$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}} \quad \text{for all } r(a, b) \in \mathcal{A}$$

$\mathcal{I}$ assigns elements of $\Delta^{\mathcal{I}}$ to individual names

Lecturer(FRANZ),	teaches(FRANZ, C1),
Course(C1),	topic(C1, T1),
DL(T1)	

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Reasoning

makes implicitly represented knowledge explicit, provided as service by the DL system, e.g.:

Subsumption: Is C a subconcept of D?

$C \sqsubseteq_{\mathcal{T}} D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$ of the TBox $\mathcal{T}$.

Satisfiability: Is the concept C non-contradictory?

C is satisfiable w.r.t. $\mathcal{T}$ iff $C^{\mathcal{I}} \neq \emptyset$ for some model $\mathcal{I}$ of $\mathcal{T}$.

Consistency: Is the ABox $\mathcal{A}$ non-contradictory?

$\mathcal{A}$ is consistent w.r.t. $\mathcal{T}$ iff it has a model that is also a model of $\mathcal{T}$.

Instantiation: Is e an instance of C?

$\mathcal{A} \models_{\mathcal{T}} C(e)$ iff $e^{\mathcal{I}} \in C^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{T}$ and $\mathcal{A}$.

polynomial reductions

in presence of negation

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Reductions

between inference problems

Subsumption to satisfiability:

$C \sqsubseteq_{\mathcal{T}} D$ iff $C \sqcap \neg D$ is unsatisfiable w.r.t. $\mathcal{T}$

Satisfiability to subsumption:

C is satisfiable w.r.t. $\mathcal{T}$ iff not $C \sqsubseteq_{\mathcal{T}} \bot$

Satisfiability to consistency:

C is satisfiable w.r.t. $\mathcal{T}$ iff $\{C(a)\}$ is consistent w.r.t. $\mathcal{T}$

Instance to consistency:

a is an instance of C w.r.t. $\mathcal{T}$ and $\mathcal{A}$ iff $\mathcal{A} \cup \{\neg C(a)\}$ is inconsistent w.r.t. $\mathcal{T}$

Consistency to instance :

$\mathcal{A}$ is consistent w.r.t. $\mathcal{T}$ iff a is not an instance of $\bot$ w.r.t. $\mathcal{T}$ and $\mathcal{A}$

Reduction

getting rid of the TBox

Expansion of concepts:

For a given TBox $\mathcal{T}$ and concept description C, the expansion $C^{\mathcal{T}}$ of C w.r.t. $\mathcal{T}$ is obtained from C by

- replacing defined concepts by their definitions
- until no more defined concepts occur.

$$\mathcal{T} \quad \begin{array}{|lcl|} \hline \text{Woman} & \equiv & \text{Person} \sqcap \text{Female} \\ \text{Course} & \equiv & \exists \text{topic}.\top \\ \text{Lecturer} & \equiv & \text{Person} \sqcap \exists \text{teaches}.\text{Course} \\ \hline \end{array}$$

Woman $\sqcap$ Lecturer expands to

$$\text{Person} \sqcap \text{Female} \sqcap \text{Person} \sqcap \exists \text{teaches}.(\exists \text{topic}.\top)$$

Reduction

getting rid of the TBox

Since TBoxes are acyclic, expansion always terminates,

but the expanded concept may be exponential in the size of $\mathcal{T}$.

$$
\begin{array}{rcl}
A_0 & \equiv & \forall r.A_1 \sqcap \forall s.A_1 \\
A_1 & \equiv & \forall r.A_2 \sqcap \forall s.A_2 \\
 & \vdots & \\
A_{n-1} & \equiv & \forall r.A_n \sqcap \forall s.A_n
\end{array}
$$

The size of $\mathcal{T}$ is linear in n,

but the expansion $A_0^{\mathcal{T}}$ contains A_n 2^n times.

Reductions:

- C is satisfiable w.r.t. $\mathcal{T}$ iff $C^{\mathcal{T}}$ is satisfiable w.r.t. the empty TBox $\emptyset$.
- $C \sqsubseteq_{\mathcal{T}} D$ iff $C^{\mathcal{T}} \sqsubseteq_{\emptyset} D^{\mathcal{T}}$.
- Consistency and the instance problem can be treated similarly.

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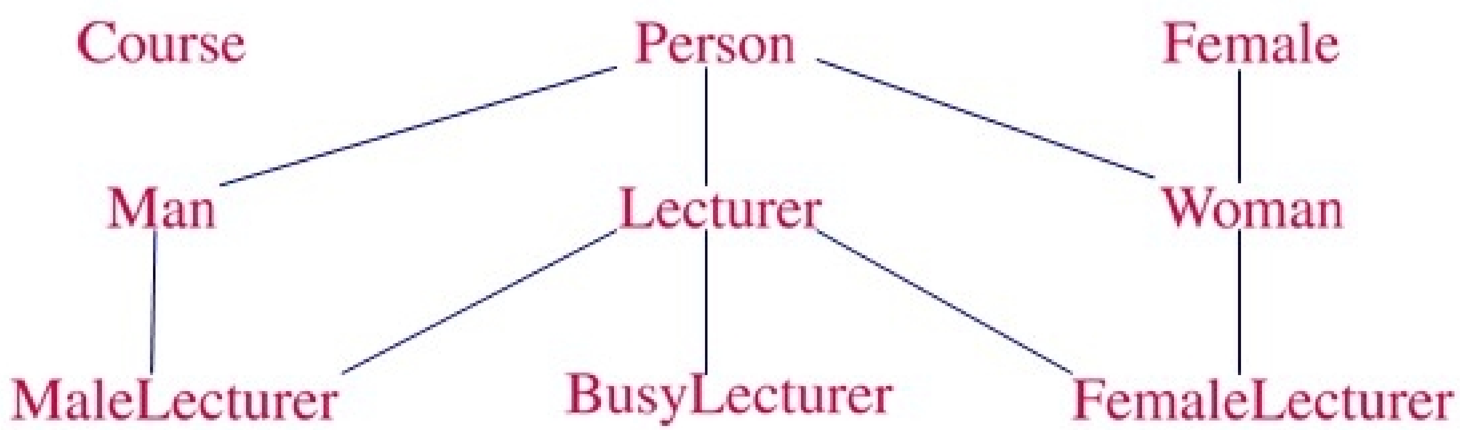
Classification

Computing the subsumption hierarchy of all concept names occurring in the TBox.

$$
\begin{aligned}
\mathsf{Man} &\equiv \mathsf{Person} \sqcap \neg\mathsf{Female} \\
\mathsf{Woman} &\equiv \mathsf{Person} \sqcap \mathsf{Female} \\
\mathsf{MaleLecturer} &\equiv \mathsf{Man} \sqcap \exists \mathsf{teaches}.\mathsf{Course} \\
\mathsf{FemaleLecturer} &\equiv \mathsf{Woman} \sqcap \exists \mathsf{teaches}.\mathsf{Course} \\
\mathsf{Lecturer} &\equiv \mathsf{FemaleLecturer} \sqcup \mathsf{MaleLecturer} \\
\mathsf{BusyLecturer} &\equiv \mathsf{Lecturer} \sqcap (\geq 3\, \mathsf{teaches}.\mathsf{Course})
\end{aligned}
$$



Realization

Computing the most specific concept names in the TBox to which an ABox individual belongs.

$$
\begin{array}{rcl}
\text{Man} & \equiv & \text{Person} \sqcap \neg\text{Female} \\
\text{Woman} & \equiv & \text{Person} \sqcap \text{Female} \\
\text{MaleLecturer} & \equiv & \text{Man} \sqcap \exists\text{teaches.Course} \\
\text{FemaleLecturer} & \equiv & \text{Woman} \sqcap \exists\text{teaches.Course} \\
\text{Lecturer} & \equiv & \text{FemaleLecturer} \sqcup \text{MaleLecturer} \\
\text{BusyLecturer} & \equiv & \text{Lecturer} \sqcap (\geq 3\, \text{teaches.Course})
\end{array}
$$

Man(FRANZ), teaches(FRANZ, C1), Course(C1)

FRANZ is an instance of Man, Lecturer, $\underbrace{\text{MaleLecturer}}_{\text{most specific}}$.

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Task

- Solve the following puzzle:
  - There are three chairs on a stage
  - On the left chair sits a woman
  - On the right chair sits a man
  - Assuming that any person sits on the middle chair, does a man sit next to a woman on the stage?

Use description logics to model the scenario and then Racer to solve the puzzle!

Task

Task

(equivalent Person (or Male Female))

(disjoint Male Female)

(instance a Female)

(instance c Male)

(instance b Person)

(related a b nextto)

(related b a nextto)

(related b c nextto)

(related c b nextto)

(instance s Stage)

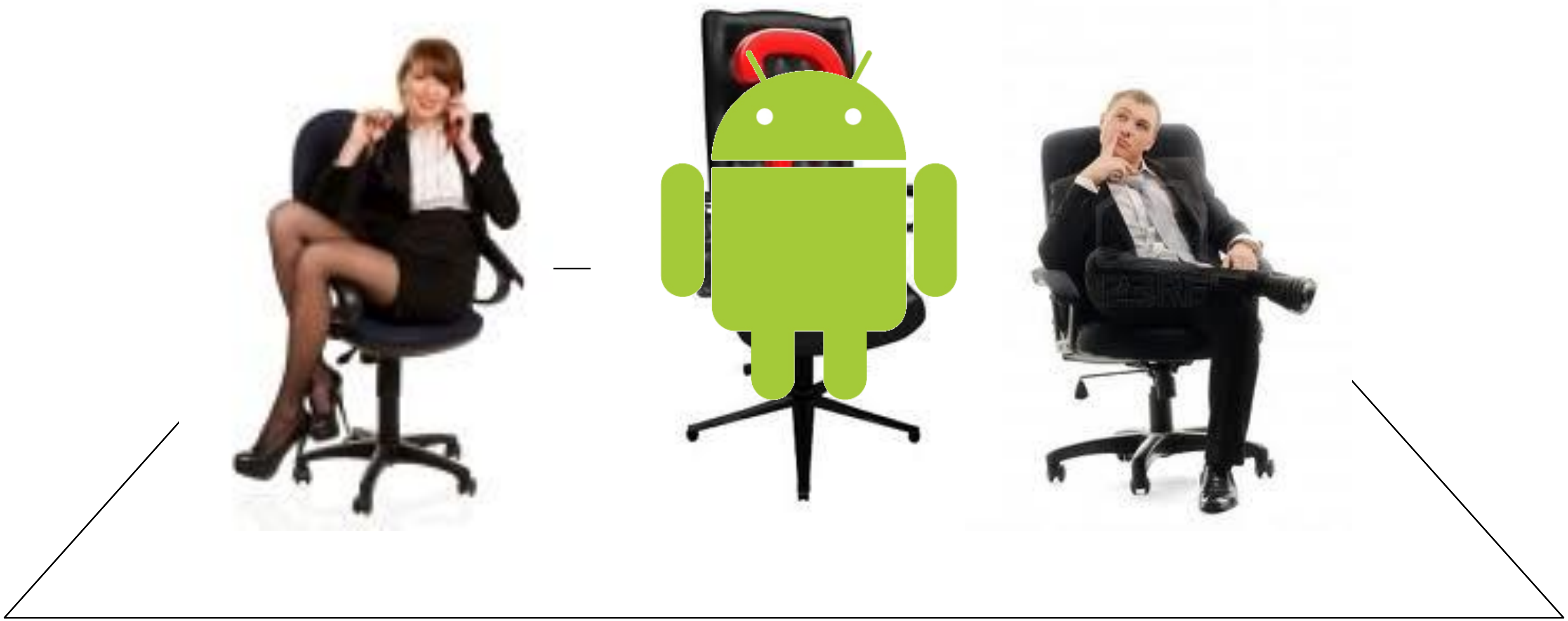
(related a s onstage)

(related b s onstage)

(related c s onstage)

(concept-instances (and Stage (some (inv onstage) (and Male (some nextto Female)))))

Task



- It works, because all models agree
- Please note that we cannot retrieve “the” man sitting next to a woman.