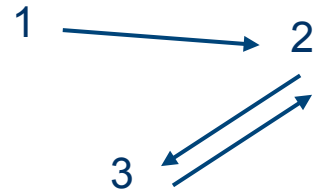
Datalog: Recursive Queries

$\forall$ x,y (T(x,y) $\leftarrow$ G(x,y))

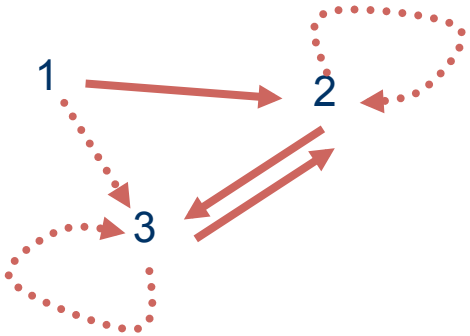
$\forall$ x,y,z (T(x,y) $\leftarrow$ (G(x,z) $\wedge$ T(z,y)))

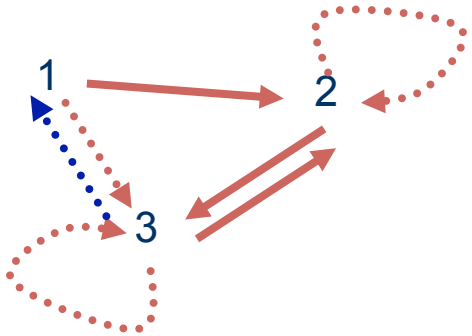
G(1, 2), G(2, 3), G(3, 2)

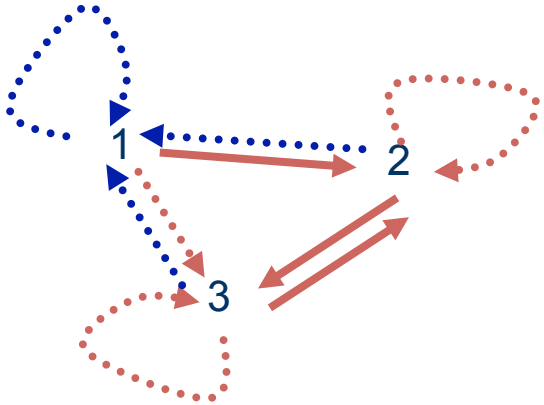
G:



Possible solutions:







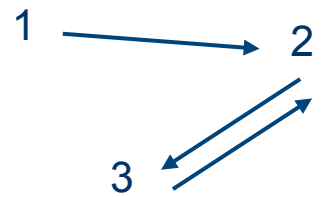
Model Theoretic Approach

$\forall\ x,y($ $\quad T(x,y) \leftarrow G(x,y)\ )$

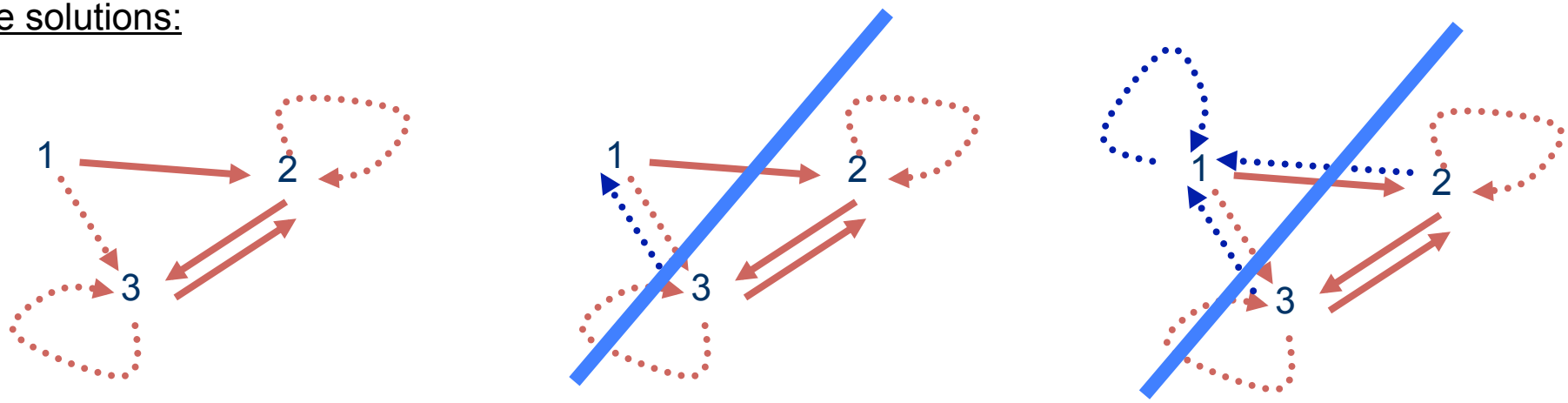
$\forall x,y,z($ $\quad T(x,y) \leftarrow (\ G(x,z) \wedge T(z,y)\ )\ )$

$G(\ 1,2\ ),\ \ G(\ 2,3\ ),\ \ G(\ 3,\ 2\ )$

G:



Possible solutions:



→ ***Chose the minimum model*** ←
T consists of the smallest set of facts that make the sentences true.

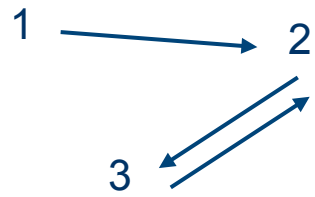
Logic programming revisited

Transitive closure of a graph:

```
T(x,y) :- G(x,y)
T(x,y) :- G(x,z), T(z,y)
```

Intuition

Transitive closure of a graph:




```
T(x,y) :- G(x,y)
T(x,y) :- G(x,z), T(z,y)
```

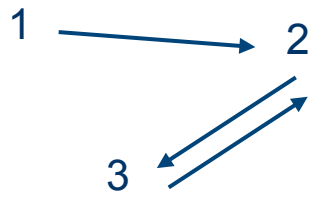
T

G

1	2
2	3
3	2

Intuition

Transitive closure of a graph:




```
T(x,y)  :- G(1,2)
T(x,y)  :- G(x,z), T(z,y)
```

T

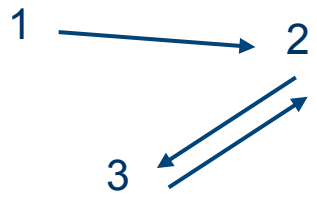
G

1	2
2	3
3	2

(1) Map from instances over the relations in the rule body

Intuition

Transitive closure of a graph:




```
T(1,2)  :- G(1,2)

T(x,y)  :- G(x,z), T(z,y)
```

T

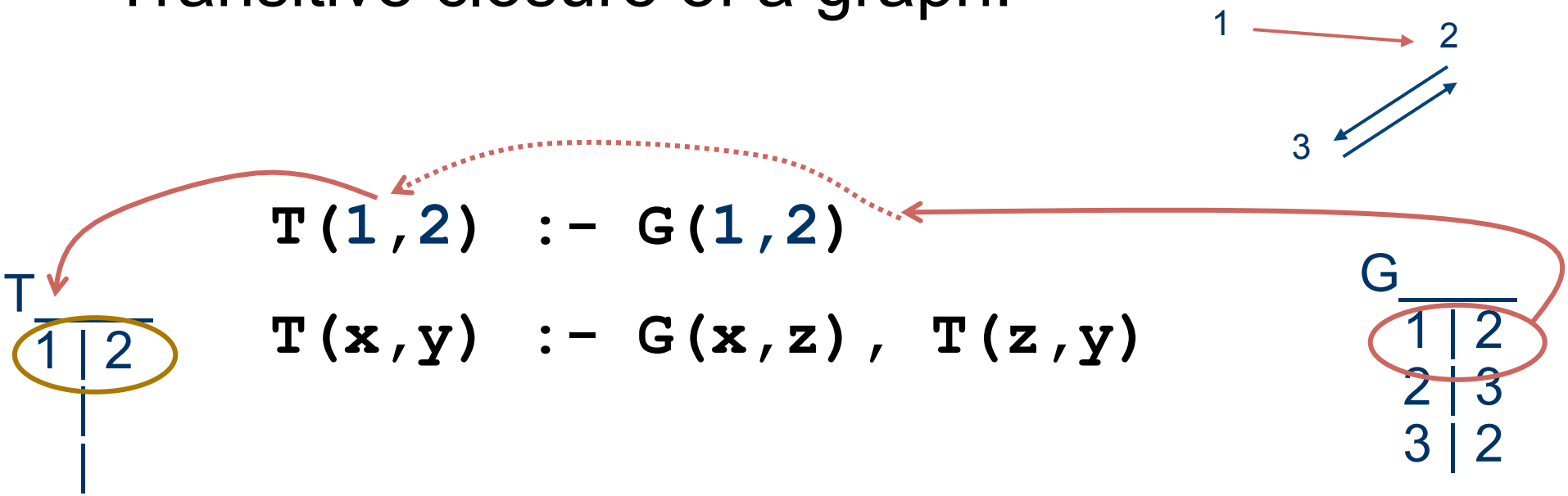
G

1	2
2	3
3	2

(2) ... map to instances over the relations in the rule head

Intuition

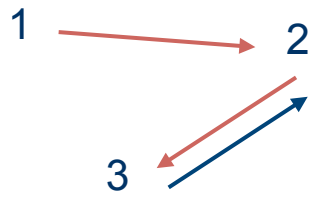
Transitive closure of a graph:



(2) ... map to instances over the relations in the rule head

Intuition

Transitive closure of a graph:




```
T(x,y)  :- G(x,y)
T(x,y)  :- G(x,z), T(z,y)
```

T

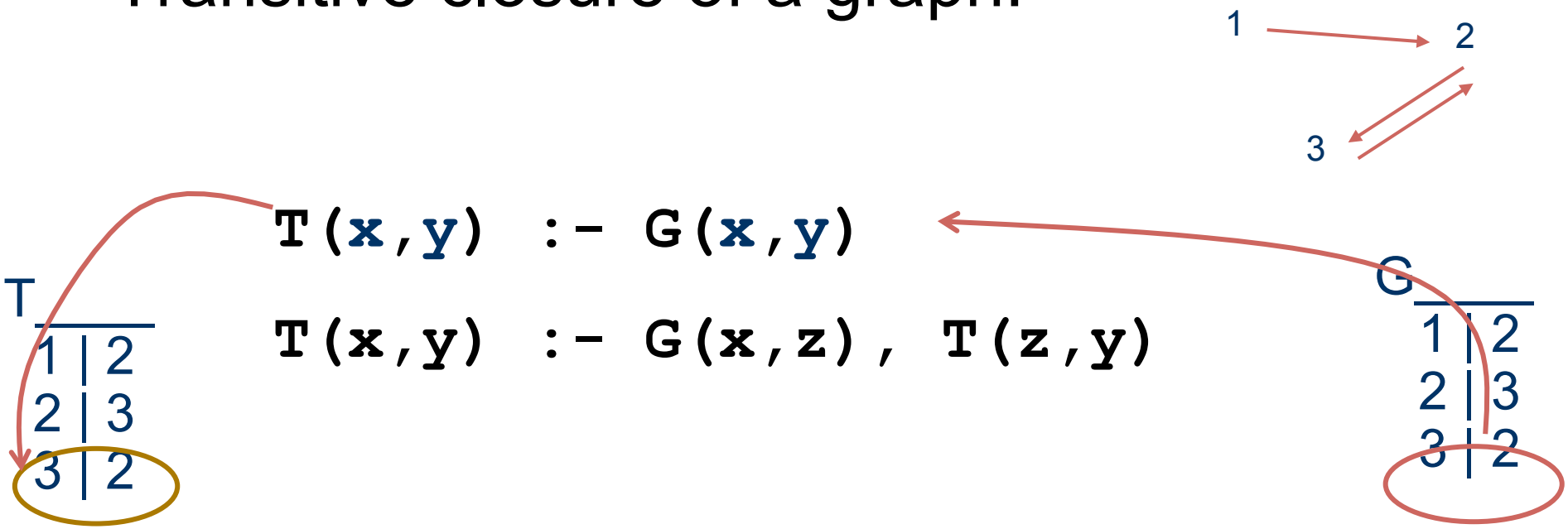
1	2
2	3

G

1	2
2	3
3	2

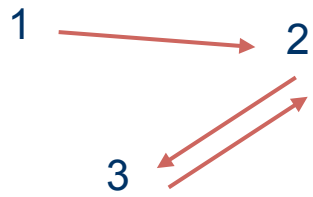
Intuition

Transitive closure of a graph:



Intuition

Transitive closure of a graph:




```
T(x,y) :- G(x,y)
T(x,y) :- G(1,2), T(2,3)
```

T

1	2
2	3
3	2

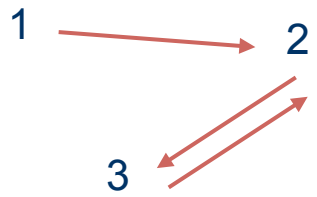
G

1	2
2	3
3	2

(1) Map from instances over the relations in the rule body

Intuition

Transitive closure of a graph:




```
T(x,y) :- G(x,y)
T(1,3) :- G(1,2), T(2,3)
```

T

1	2
2	3
3	2

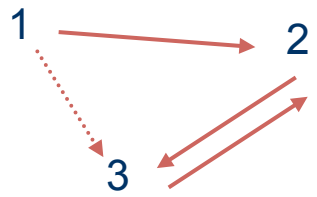
G

1	2
2	3
3	2

(2) ... map to instances over the relations in the rule head

Intuition

Transitive closure of a graph:

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T(x,y)  :- G(x,y)
T(1,3)  :- G(1,2), T(2,3)
```

T

1	2
2	3
3	2
1	3

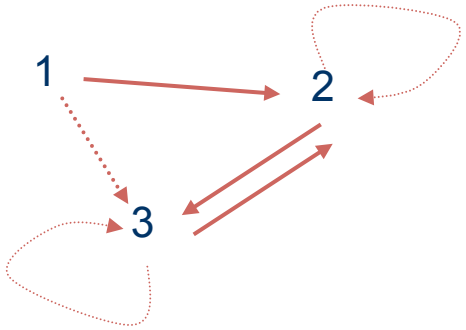
G

1	2
2	3
3	2

(2) ... map to instances over the relations in the rule head

Intuition

Transitive closure of a graph:



T

1	2
2	3
3	2
1	3
2	2
3	3

```
T(x,y)  :- G(x,y)
T(x,y)  :- G(x,z), T(z,y)
```

G

1	2
2	3
3	2

... repeat until fixpoint is reached
(datalog without negation is monotone)

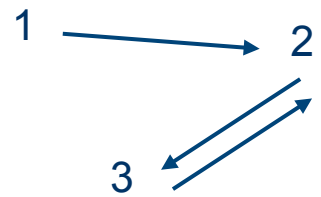
Model Theoretic Approach

ν x,y (T(x,y) $\leftarrow$ G(x,y))

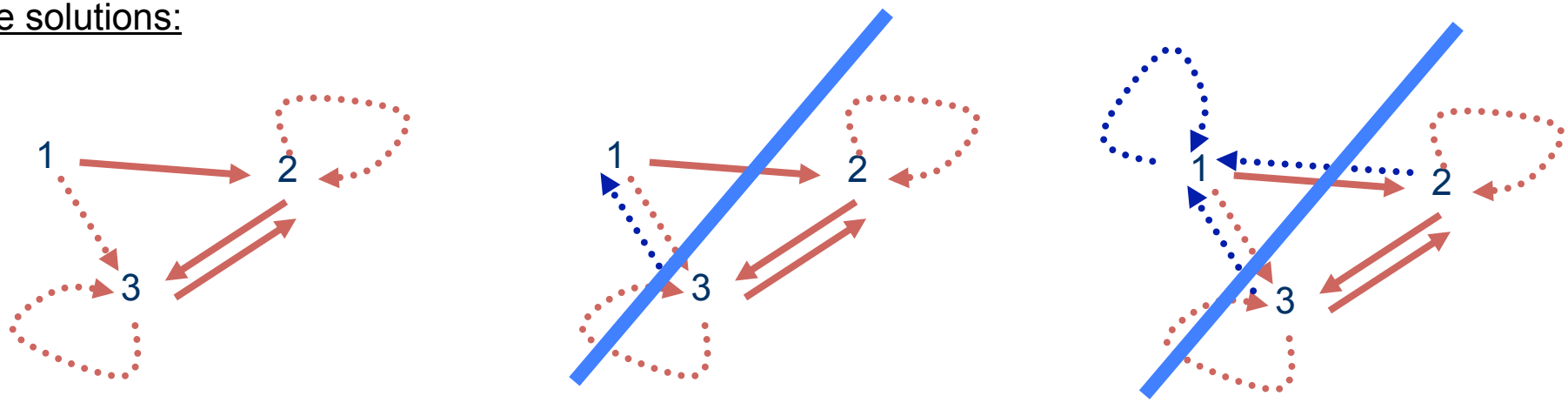
ν x,y $\forall$z (T(x,y) $\leftarrow$ (G(x,z) $\wedge$ T(z,y)))

G(1,2), G(2,3), G(3, 2)

G:



Possible solutions:



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