Multimedia Information Extraction and Retrieval

Similarity

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Acknowledgement

- Slides taken from presentation material for the following book:

Introduction to Information Retrieval

Christopher D. Manning
Stanford University

Prabhakar Raghavan
Yahoo! Research

Hinrich Schütze
University of Stuttgart

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Recap of the last lecture

- Parametric and field searches
  - Zones in documents
- Can apply text queries to images due to interpretation results
- Scoring documents: zone weighting
  - Index support for scoring
- *tf*×*idf* and vector spaces

Indexes: "Postings lists"

- On the query ***bill*** *OR* ***rights*** suppose that we retrieve the following docs from the various zone indexes:

<u>*Author*</u>	***bill***	1 → 2
	rights	
<u>*Title*</u>	***bill***	3 → 5 → 8
	rights	3 → 5 → 9
<u>*Body*</u>	***bill***	1 → 2 → 5 → 9
	rights	3 → 5 → 8 → 9

Recap: tf x idf (or tf.idf)

- Assign a tf.idf weight to each term *i* in each document *d*

$$w_{i,d} = tf_{i,d} \times \log(n / df_i)$$

$$tf_{i,d} = \text{frequency of term } i \text{ in document } j$$
$$n = \text{total number of documents}$$
$$df_i = \text{the number of documents that contain term } i$$

- *Instead of tf, sometimes wf is used:*

$$wf_{t,d} = 0 \text{ if } tf_{t,d} = 0,\ 1 + \log tf_{t,d} \text{ otherwise}$$

This lecture

- Vector space scoring
- Efficiency considerations
  - Nearest neighbors and approximations

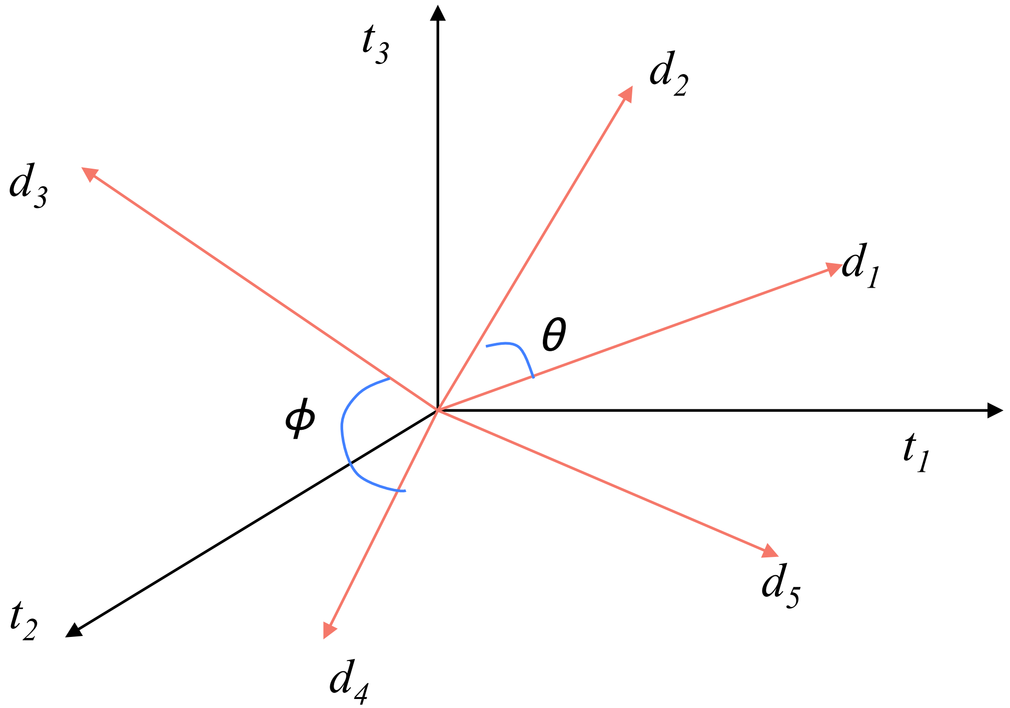
Documents as vectors

- At the end of the last lecture we said:
- Each doc *d* can now be viewed as a vector of *tf*×*idf* values, one component for each term
- So we have a vector space
  - terms are axes
  - docs live in this space
  - even with stemming, may have 50,000+ dimensions

Why turn docs into vectors?

- First application: Query-by-example
  - Given a doc *d*, find others “like” it.
- Now that *d* is a vector, find vectors (docs) “near” it.

Intuition



Postulate: Documents that are "close together" in the vector space talk about the same things.

Desiderata for proximity

- If d_1 is near d_2, then d_2 is near d_1.
- If d_1 near d_2, and d_2 near d_3, then d_1 is not far from d_3.
- No doc is closer to d than d itself.
- Triangle inequality

First cut

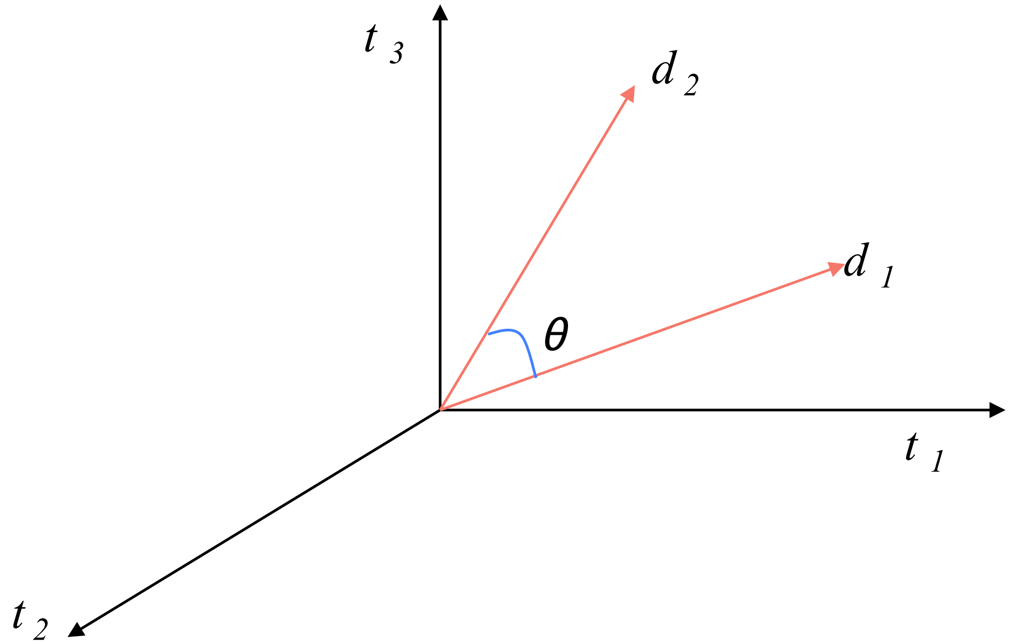
- Idea: Distance between d_1 and d_2 is the length of the vector $d_1 - d_2$.
  - Euclidean distance:

$$\left| d_j - d_k \right| = \sqrt{\sum_{i=1}^{n} \left(d_{i,j} - d_{i,k} \right)^2}$$

- Why is this not a great idea?
- We still haven’t dealt with the issue of length normalization
  - Short documents would be more similar to each other by virtue of length, not topic
- However, we can implicitly normalize by looking at *angles* instead

Cosine similarity

- Distance between vectors d_1 and d_2 *captured* by the cosine of the angle x between them.
- Note – this is *similarity*, not distance
  - No triangle inequality for similarity.



Cosine similarity

- A vector can be *normalized* (given a length of 1) by dividing each of its components by its length – here we use the L_2 norm

$$\|\mathbf{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- This maps vectors onto the unit sphere:
- Then, $$\left|\vec{d}_j\right| = \sqrt{\sum_{i=1}^{n} w_{i,j}^2} = 1$$
- Longer documents don't get more weight

Cosine similarity

$$sim(d_j, d_k) = \frac{\vec{d}_j \cdot \vec{d}_k}{\left|\vec{d}_j\right| \left|\vec{d}_k\right|} = \frac{\sum_{i=1}^{n} w_{i,j} w_{i,k}}{\sqrt{\sum_{i=1}^{n} w_{i,j}^2} \sqrt{\sum_{i=1}^{n} w_{i,k}^2}}$$

- Cosine of angle between two vectors
- The denominator involves the lengths of the vectors.

Normalization

Normalized vectors

- For normalized vectors, the cosine is simply the dot product:

$$\cos(\vec{d}_j, \vec{d}_k) = \vec{d}_j \cdot \vec{d}_k$$

Example

- Docs: Austen's *Sense and Sensibility*, *Pride and Prejudice*; Bronte's *Wuthering Heights*. *Tf* weights

	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6

	SaS	PaP	WH
affection	0.996	0.993	0.847
jealous	0.087	0.120	0.466
gossip	0.017	0.000	0.254

- cos(SAS, PAP) = .996 x .993 + .087 x .120 + .017 x 0.0 = 0.999
- cos(SAS, WH) = .996 x .847 + .087 x .466 + .017 x .254 = 0.889

Cosine similarity exercises

- *Exercise: Rank the following by decreasing cosine similarity. Assume tf-idf weighting:*
  - Two docs that have only frequent words ***(the, a, an, of)*** in common.
  - Two docs that have no words in common.
  - Two docs that have many rare words in common ***(wingspan, tailfin).***

Exercise

- Show that, for normalized vectors, Euclidean distance gives the same proximity ordering as the cosine measure

Queries in the vector space model

Central idea: the query as a vector:

- We regard the query as short document
- We return the documents ranked by the closeness of their vectors to the query, also represented as a vector.

$$sim(d_j, d_q) = \frac{\vec{d}_j \cdot \vec{d}_q}{\left|\vec{d}_j\right|\left|\vec{d}_q\right|} = \frac{\sum_{i=1}^{n} w_{i,j} w_{i,q}}{\sqrt{\sum_{i=1}^{n} w_{i,j}^2}\sqrt{\sum_{i=1}^{n} w_{i,q}^2}}$$

- Note that d_q is very sparse!

Summary: What's the point of using vector spaces?

- A well-formed algebraic space for retrieval
- Key: A user's query can be viewed as a (very) short document.
- Query becomes a vector in the same space as the docs.
- Can measure each doc's proximity to it.
- Natural measure of scores/ranking – no longer Boolean.
    - Queries are expressed as bags of words

Digression: spamming indices

- This was all invented before the days when people were in the business of spamming web search engines. Consider:
  - Indexing a sensible passive document collection vs.
  - An active document collection, where people (and indeed, service companies) are shaping documents in order to maximize scores

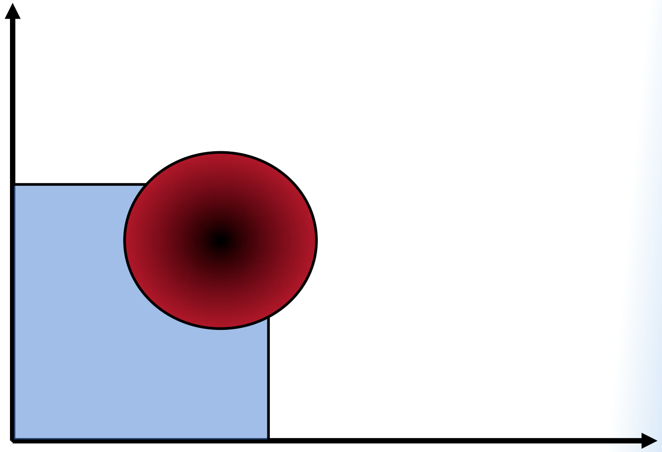
- Vector space similarity may not be as useful in this context.

Interaction: vectors and phrases

- Scoring phrases doesn't fit naturally into the vector space world:
  - ***"tangerine trees" "marmalade skies"***
  - Positional indexes don't calculate or store tf.idf information for ***"tangerine trees"***
- Biword indexes treat certain phrases as terms
  - For these, we can pre-compute tf.idf.
  - Theoretical problem of correlated dimensions
- Problem: we cannot expect end-user formulating queries to know what phrases are indexed
- We can use a positional index to boost or ensure phrase occurrence

Vectors and Boolean queries

- Vectors and Boolean queries really don't work together very well
- In the space of terms, vector proximity selects by <u>spheres</u>: e.g., all docs having cosine similarity ≥0.5 to the query
- Boolean queries on the other hand, select by (hyper-)rectangles and their unions/intersections
- Round peg - square hole



Vectors and wild cards

- How about the query ***tan* marm*?***
  - Can we view this as a bag of words?
  - Thought: expand each wild-card into the matching set of dictionary terms.
- Danger – unlike the Boolean case, we now have *tf*s and *idf*s to deal with.
- Net – not a good idea.

Vector spaces and other operators

- Vector space queries are apt for no-syntax, bag-of-words queries
  - Clean metaphor for similar-document queries
- Not a good combination with Boolean, wild-card, positional query operators
- But …

Query language vs. scoring

- May allow user a certain query language, say
  - Free text basic queries
  - Phrase, wildcard etc. in Advanced Queries.
- For scoring (oblivious to user) may use all of the above, e.g. for a free text query
  - Highest-ranked hits have query as a phrase
  - Next, docs that have all query terms near each other
  - Then, docs that have some query terms, or all of them spread out, with tf x idf weights for scoring

Efficient cosine ranking

- Find the *k* docs in the corpus “nearest” to the query ⇒ *k* largest query-doc cosines.
- Efficient ranking:
  - Computing a single cosine efficiently.
  - Choosing the *k* largest cosine values efficiently.
    - Can we do this without computing all *n* cosines?
      - *n* = number of documents in collection

Efficient cosine ranking

- What we're doing in effect: solving the *k*-nearest neighbor problem for a query vector
- In general, we do not know how to do this efficiently for high-dimensional spaces
- But it is solvable for short queries, and standard indexes are optimized to do this

Computing a single cosine

- For every term *i*, with each doc *j*, store term frequency tf_{ij}.
  - Some tradeoffs on whether to store term count, term weight, or weighted by idf_i.
- At query time, use an array of accumulators $Scores_j$ to accumulate component-wise sum

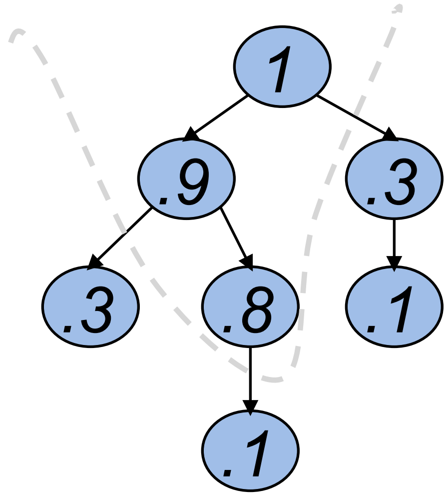
$$sim(\vec{d}_j, \vec{d}_q) = \sum_{i=1}^{m} w_{i,j} \times w_{i,q}$$

- If you're indexing 5 billion documents (web search) an array of accumulators is infeasible

Ideas?

Use heap for selecting top *k*

- Binary tree in which each node's value > the values of children
- Takes *2n* operations to construct, then each of *k* "winners" read off in 2log *n* steps.
- For *n*=1M, *k*=100, this is about 10% of the cost of sorting.



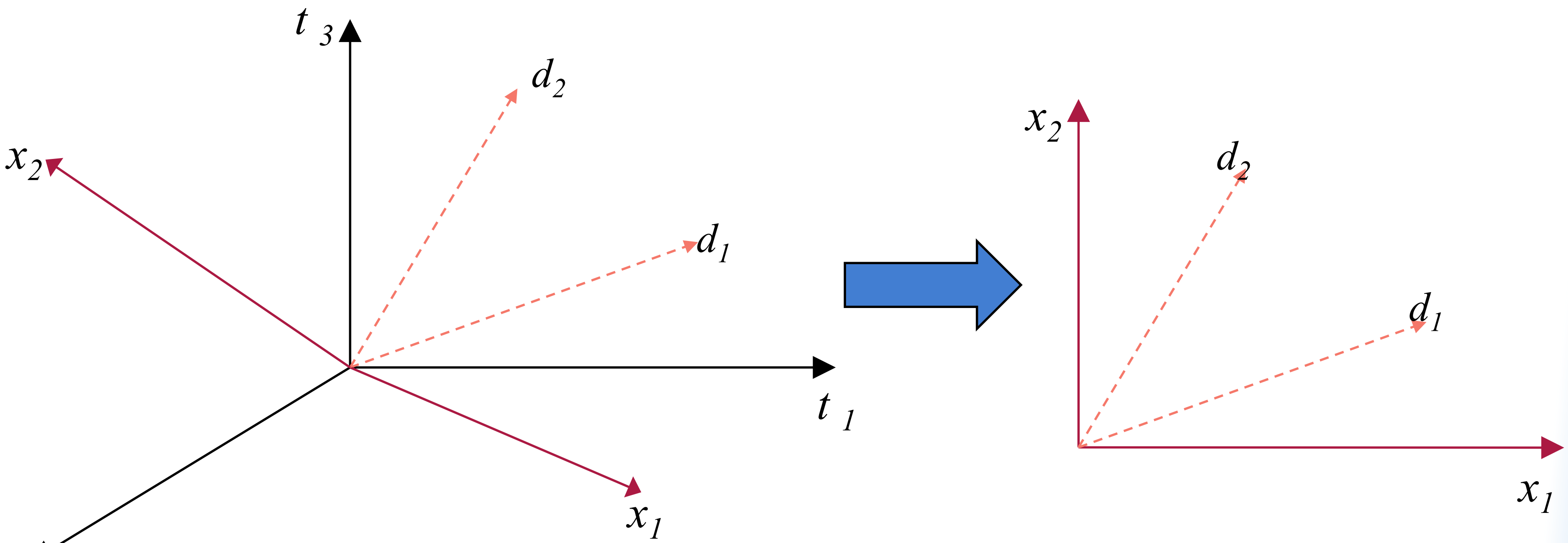
Dimensionality reduction

- What if we could take our vectors and “pack” them into fewer dimensions (say 50,000→100) while preserving distances?
- (Well, almost.)
  - Speeds up cosine computations.
- Two methods:
  - Random projection.
  - “Latent semantic indexing”.

Random projection onto $k \ll m$ axes

- Choose a random direction x_1 in the vector space.
- For $i = 2$ to k,
  - Choose a random direction x_i that is orthogonal to $x_1, x_2, \ldots x_{i-1}$.
- Project each document vector into the subspace spanned by $\{x_1, x_2, \ldots, x_k\}$.

E.g., from 3 to 2 dimensions



x_1 is a random direction in (t_1,t_2,t_3) space.

x_2 is chosen randomly but orthogonal to x_1.

Dot product of x_1 and x_2 is zero.

Guarantee

- With high probability, relative distances are (approximately) preserved by projection
- But: expensive computations

Latent semantic indexing (LSI)

- Another technique for dimension reduction
- Random projection was data-*independent*
- LSI on the other hand is data-*dependent*
  - Eliminate redundant axes
  - Pull together "related" axes – hopefully
    - ***car*** and ***automobile***