Web-Mining Agents

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Structural Causal Models

slides prepared by Özgür Özçep

Part II: Intervention

Literature

- J.Pearl, M. Glymour, N. P. Jewell: Causal inference in statistics – A primer, Wiley, 2016.

  (Main Reference)
- J. Pearl: Causality, CUP, 2000.

Intervention

- Important aim of SCMs for given data: Where to intervene in order to achieve desired effects.

Examples

- Data on wildfires: How to intervene in order to decrease wildfires?
- Data on TV and aggression: How to intervene in order to lower aggression of children?

- How to model intervention and their effects within SCMs and their graphs?

Randomized Controlled Experiment

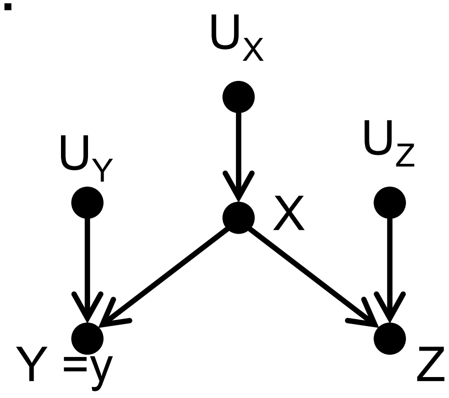
- Randomized contolled experiment gold standard
  - Aim: Answer question whether change in RV X has an effect on some target RV Y with an experiment
  - If outcome of experiment is yes, X is a RV to intervene upon
  - Test condition: all variables different from X are static (fixed) or vary fully randomly.
- Problem: Cannot always set up such an experiment
  - Example: cannot control wether in order to test variables influencing wildfire
- Instead: use observational data & causal model

Example (SCM 5; Intervention)

(X = Temperature, Y = Ice cream Sale, Z = Crime)

- Would intervention on ice cream sales (Y) lead to decrease of crime (Z)?

- What does it mean to intervene on Y?
  - Fix value of Y in the sense of inhibiting the natural influences on Y according to SCM (here of U_Y and X)
  - Leads to change of the SCM



Intervention vs. Conditioning

- Intervention denoted by $do(Y = y)$

  $P(Z = z \mid do(Y = y)) =$

  probability of event $Z = z$ on intervening upon Y by setting $Y = y$

Intervention changes the data generation mechanism

- In contrast

  $P(Z = z \mid Y = y) =$

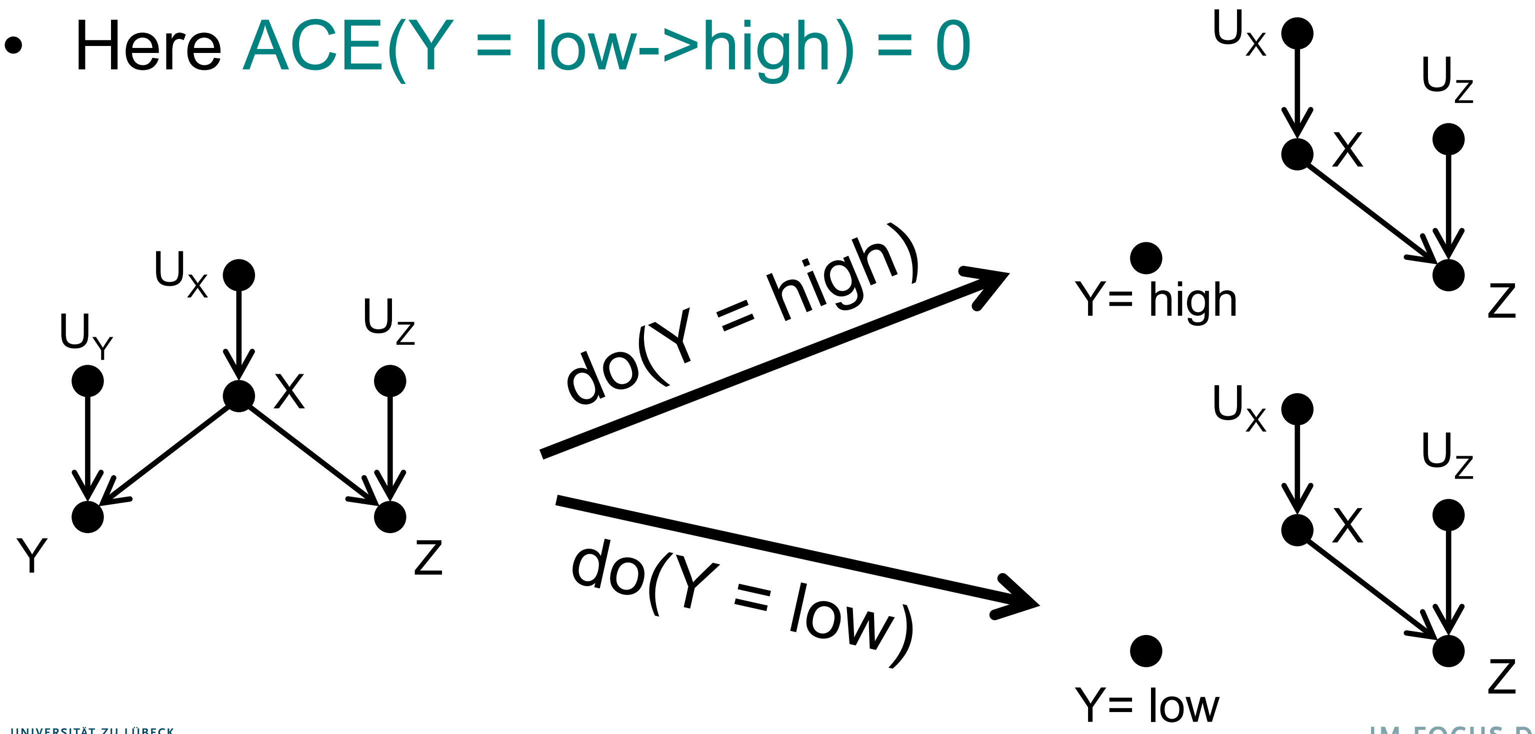
  probability of event $Z = z$ when knowing that $Y = y$

Conditioning only does filtering on the data

Average Causal Effect (ACE)

- Would intervention on ice cream sales (Y) by increasing Y lead to decrease of crime (Z)?
- Causal Effect Difference/average causal effect (ACE)

  $P(Z = low \mid do(Y = high)) - P(Z = low \mid do(Y = low))$
- Here ACE(Y = low->high) = 0



General Causal Effect

- How effective is drug usage for recovery?

  ACE = P(Y = 1 | do(X = 1)) – P(Y = 1 | do(X = 0))

- Need to compute general causal effect

Definition

The general causal effect of X on Y is given by

$P(Y = y \mid do(X = x)) = P_m(Y = y \mid X = x)$

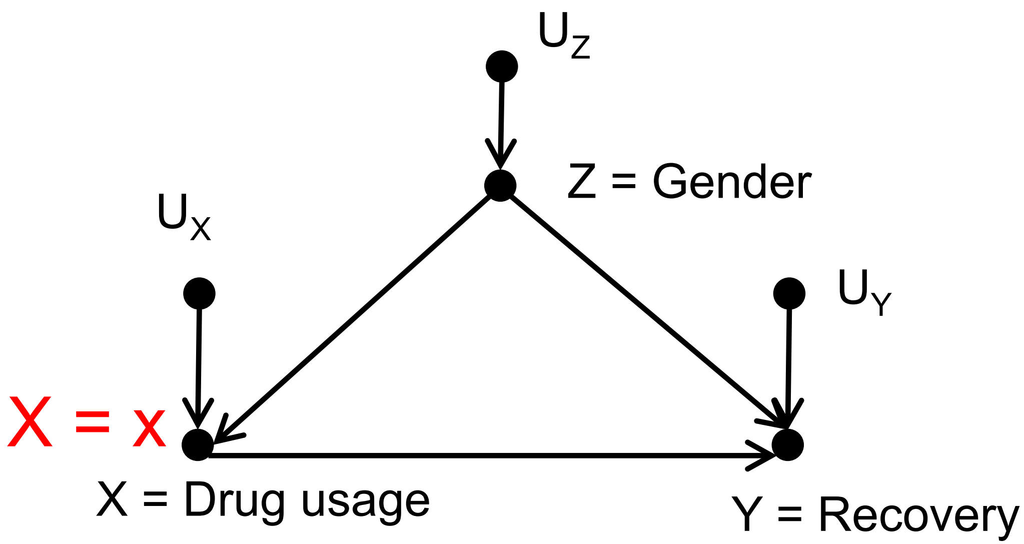
= probability in **m**anipulated graph

Example (drug-recovery effect)

- How effective is drug usage for recovery?

  $ACE = P(Y = 1 \mid do(X = 1)) - P(Y = 1 \mid do(X = 0))$

- $P(Y = y \mid do(X = x)) \quad = P_m(Y = y \mid X = x)$



Intervention (alternatively)

- The definition of intervention with the manipulated graph is not the only possibility

- Model intervention do(X=x) with force variable F
  - F is parent of X,
  - Dom(F) = {do(X=x‘) | x in dom(X)} ∪ {idle}
  - pa‘(X) = pa(X) ∪ {F}
  - New ``CPT“ for X

$$P(X = x \mid pa'(X)) = \begin{cases} P(X=x \mid pa(X)) & \text{if } F = \text{idle} \\ 0 & \text{if } F = do(X=x') \text{ and } x \neq x' \\ 1 & \text{if } F = do(X=x') \text{ and } x = x' \end{cases}$$

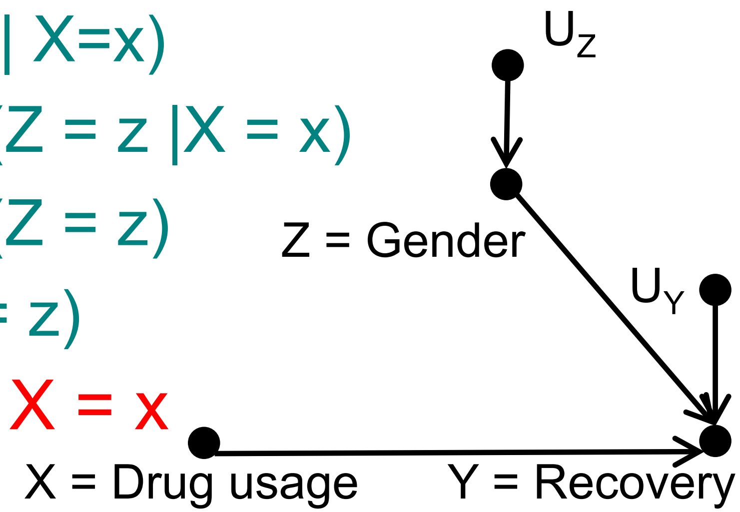
Z value not effected by intervention on x: f_Z: $Z = f(U_Z)$

Example (drug-recovery effect)

- $P_m(Y = y \mid X = x) = ?$
- Need to reduce to probabilities w.r.t. original graph

1. $P_m(Z = z) = P(Z = z)$
2. $P_m(Y = y \mid Z = z, X = x) = P(Y = y \mid Z = z, X=x)$
3. Summing out

$$P(Y = y \mid do(X = x)) = P_m(Y = y \mid X=x)$$
$$= \sum_z P_m(Y = y \mid X = x, Z=z)\, P_m(Z = z \mid X = x)$$
$$= \sum_z P_m(Y = y \mid X = x, Z=z)\, P_m(Z = z)$$
$$= \sum_z P(Y = y \mid X = x, Z=z)\, P(Z = z)$$



Y value not effected by intervention on x, f_Y: $Y = f(x,y,u_y)$

Adjustment

Definition

The adjustment formula (for single parent Z of X) for the calculation of the GCE is given by

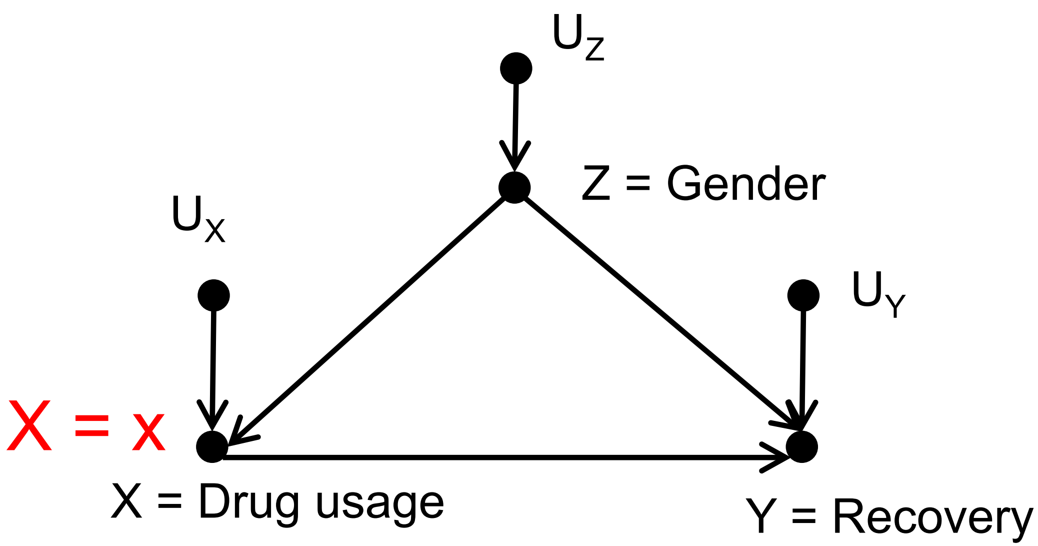
$$P(Y = y \mid do(X = x)) = \sum_z P(Y = y \mid X = x, Z=z)\, P(Z = z)$$

Wording: „Adjusting for Z" or „controlling Z"

Simpson's Paradox

- How effective is drug usage for recovery?

  ACE = $P(Y = 1 \mid do(X = 1)) - P(Y = 1 \mid do(X = 0))$
- $P(Y = y \mid do(X = x)) = P_m(Y = y \mid X = x)$



Reminder: Simpson's Paradox

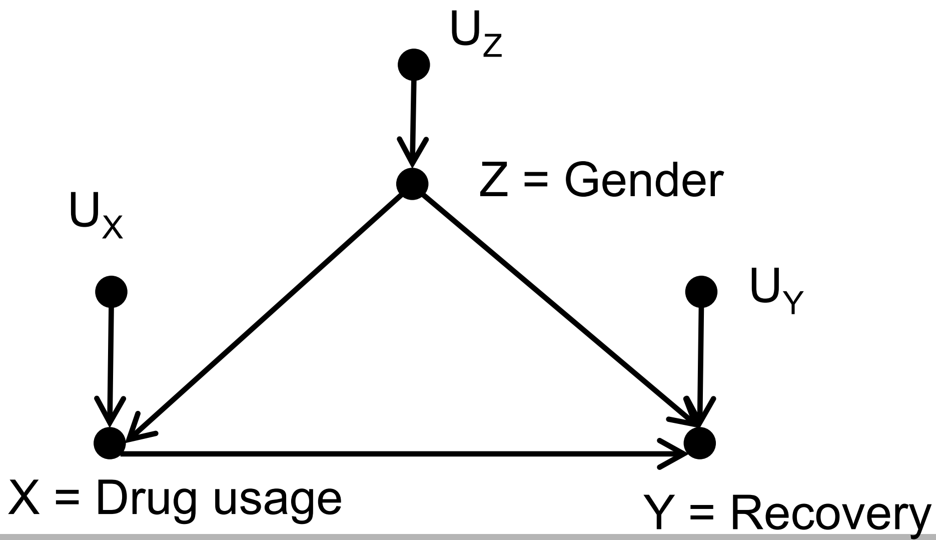
- Record recovery rates of 700 patients given access to a drug

	Recovery rate with drug	**Recovery rate without drug**
Men	81/87 (93%)	234/270 (87%)
Women	192/263 (73%)	55/80 (69%)
Combined	273/350 (78%)	289/350 (83%)

- Paradox:
  - For men, taking drugs has benefit
  - For women, taking drugs has benefit, too.
  - But: for all persons taking drugs has no benefit

Resolving the Paradox (Formally)

- We have to understand the causal mechanisms that lead to the data in order to resolve the paradox
- Formally: What is the general causal effect of drug usage X on recovery Y?
  - P(Y = y | do(X = x)) = ?
  - ACE= P(Y =1 | do(X =1)) – P(Y=1 |do(X=0)) = ?



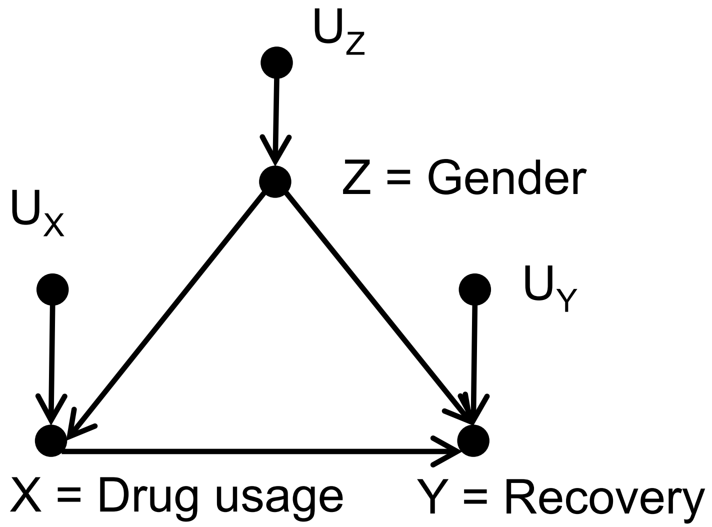
Resolving the Paradox (Formally)

- $P(Y=1 \mid do(X=1)) =$ (using adjustment formula)
- $= P(Y=1 \mid X=1, Z=1)P(Z=1) + P(Y=1 \mid X=1, Z=0)P(Z=0)$

$= 0.93(87 + 270)/700 + 0.73(263 + 80)/700 = 0.832$

- $P(Y=1 \mid do(X=0)) = 0.7818$
- $ACE = 0.832 – 0.7818 = 0.0502 > 0$
- One has to seggregate the data w.r.t. Z (adjust for Z)

	Recovery rate with drug	**Recovery rate without drug**
Men	81/87 (93%)	234/270 (87%)
Women	192/263 (73%)	55/80 (69%)
Combined	273/350 (78%)	289/350 (83%)



Simpson Paradox (Again)

- Record recovery rates of 700 patients given access to a drug w.r.t. blood pressure (BP) segregation

	Recovery rate Without drug	**Recovery rate with drug**
Low BP	81/87 (93%)	234/270 (87%)
High BP	192/263 (73%)	55/80 (69%)
Combined	273/350 (78%)	289/350 (83%)

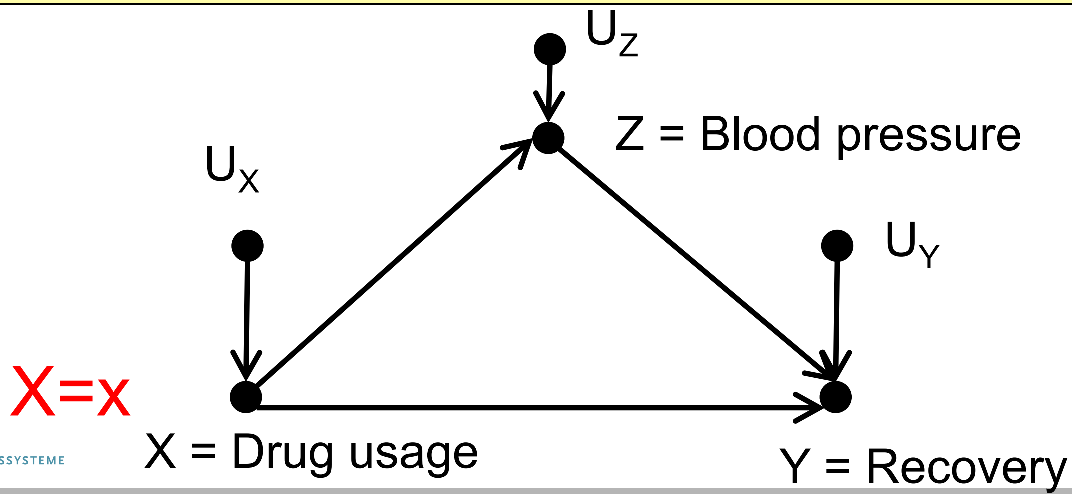
- BP recorded at end of experiment
- This time segregated data recommend not using drug whereas aggregated does

Resolving the Paradox (Formally)

- We have to understand the causal mechanisms that lead to the data in order to resolve the paradox
- Formally: What is the general causal effect of drug usage X on recovery Y?
  - $P(Y = y \mid do(X = x)) = ?$

    $= P_m(Y = y \mid X = x) = P(Y = y \mid X = x)$

So: Do not adjust for/seggregate w.r.t. any variable



Causal Effect for Multiple Adjusted Variables

Rule (Calculation of causal effect)

$P(Y = y \mid do(X = x)) =$

$$\sum_z P(Y = y \mid X = x, Pa(X) = z)\, P(Pa(X) = z)$$

- $Pa(X)$ = parents of X
- z = instantiation of all parent variables of X

Rule (Calculation of Causal Effect Rule (alternative))

$P(Y = y \mid do(X = x)) =$

$$\sum_z P(Y = y, X = x, Pa(X) = z) / P(X = x \mid Pa(X) = z)$$

Truncated Product Formula

- Handling of multiple interventions straightforward
- Joint prob. distribution on all other variables $X_1, \dots, X_n$ after intervention on $Y_1,\dots,Y_m$

That is all variablesare partitioned in Xis andYjs

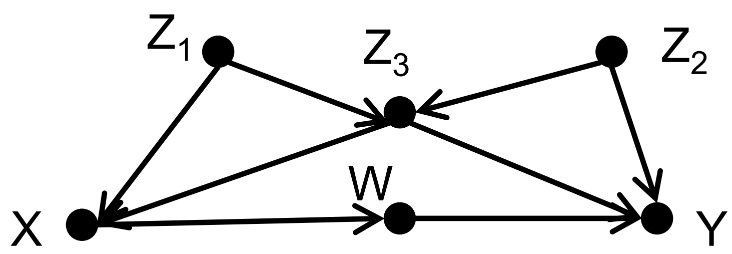
Definition (Truncated product formula (g-formula))

$P(x_1, \dots, x_n \mid do(Y_1=y_1, \dots, Y_m=y_m)) = \prod_{1 \leq j \leq n} P(x_i \mid pa(X_i))$

$pa(X_i)$ = sub-vector of $(x_1, ..x_n, y_1, ...y_m)$ constrained to parents of X_i

Example 1

$P(z_1,z_2,w,y \mid do(X=x, Z_3=z_3))$
$= P(z_1)P(z_2)P(w|x)P(y|w,z_3,z_2)$



Truncated Product Formula

Definition (Truncated product formula (g-formula))

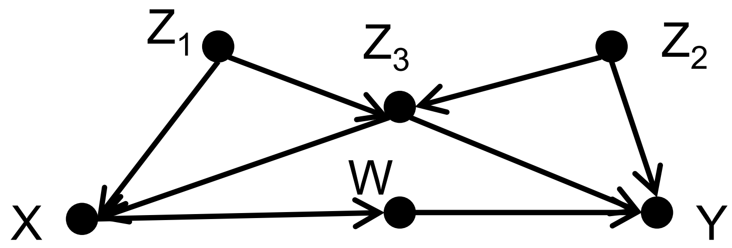
$P(x_1, \ldots, x_n \mid do(Y_1=y_1, \ldots, Y_m=y_m)) = \prod_{1 \leq j \leq n} P(x_i \mid pa(X_i))$

Example 2 (summing out)

$P(w,y \mid do(X=x, Z_3=z_3))$

$= \sum_{z1,z2} P(z_1)P(z_2)P(w|x)P(y|w,z_3,z_2)$

Can check that this is compatible with the adjustment formula



Backdoor Criterion (Motivation)

- Intervention on X requires adjusting parents of X
- But sometimes those variables not measurable (though perhaps represented in graph)

- Need general criterion to identify adjustment variables
  1. Block all spurious paths between X and Y
  2. Leave all directed paths from X to Y unperturbed
  3. Do not create new spurious paths

Backdoor Criterion (Formulation)

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Can adjust for Z satisfying backdoor criterion

$$P(Y = y \mid do(X = x)) = \sum_z P(Y = y \mid X = x, Z = z)P(Z=z)$$

Backdoor Criterion (Intuition)

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Ad 1.: Descendants are effects of X, should not be conditioned on

(compare drug usage X and blood pressure Z)

- Ad 2.: One is interested in effects of X on Y, not vice versa. Effects of Y on X should be blocked.

Backdoor Criterion Generalizes Adjustment

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Z = Pa(X)
- For any W in Z both conditions fulfilled
  - W is not a descendant (as D**A**G)
  - Z blocks every path as every path into X must go trough a parent of X

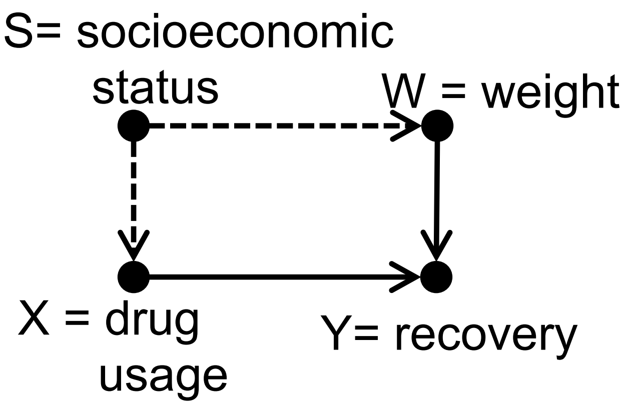
Backdoor Criterion (Example 1)

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Causal effect of X on Y?
- S is not recorded in the data
- Use {W} as Z fulfills backdoor
  - W not descendant of X
  - Blocks backdoor path



Backdoor Criterion (Example 1 (cont'd))

Definition

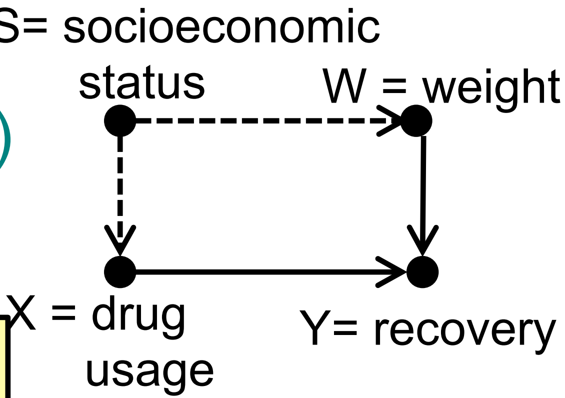
Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Causal effect of X on Y?

$P(y \mid do(x)) = \sum_w P(Y=y|X=x, W=w)P(W=w)$

$= \sum_s P(Y=y|X=x, S=s)P(S=s)$



Conditioning on different variables S vs. W with same effect calculation

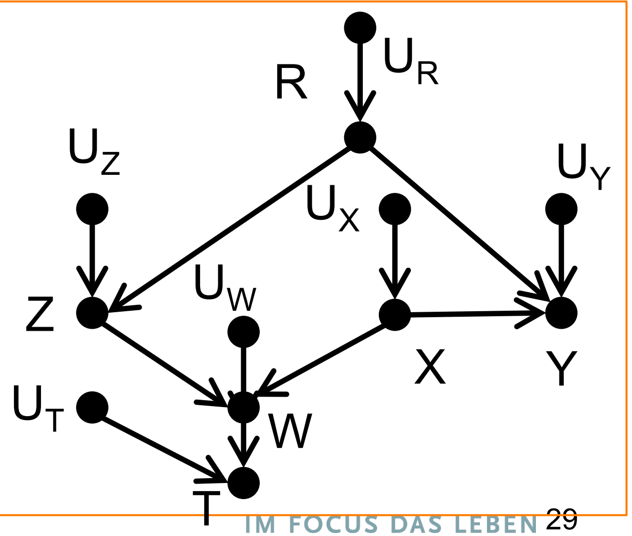
Backdoor Criterion (Example 2a)

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Causal effect of X on Y?
- No backdoor paths
  - Can use Z = {}
  - P(y | do(x)) = P(y | x)



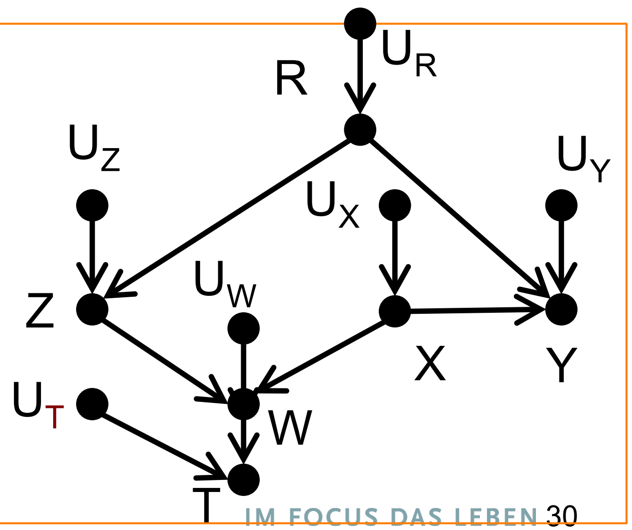
Backdoor Criterion (Example 2b)

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- Causal effect of X on Y?
- No backdoor paths
- Can one adjust for W?
  - No, collider W not blocking spurious path



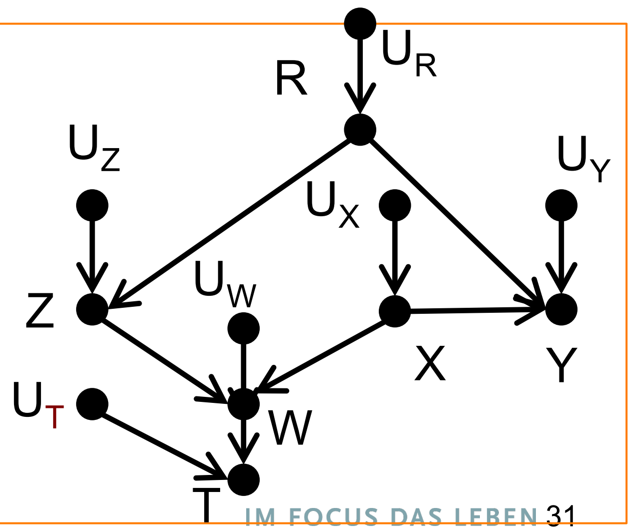
Backdoor Criterion (Example 2c)

Definition

Set of variables Z satisfies backdoor criterion relative to pair (X,Y) of variables iff

1. No node in Z is a descendant of X and
2. Z blocks every path between X and Y that contains an arrow into X

- From 2b we know: effect of X on Y not via conditioning on W.
- But how to calculate w-specific causal effect: P(Y = y | do(X =x), W = w) = ?



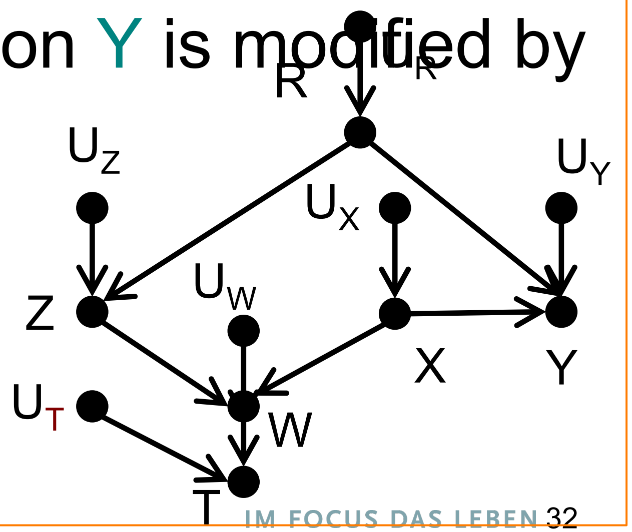
Backdoor Criterion (Example 2c (cont'd))

- W-specific causal effect $P(Y = y \mid do(X = x), W = w) = ?$
- Use fork R to condition on

$$P(Y = y \mid do(X = x), W = w) = \sum_r P(Y=y|X=x,W=w,R=r)P(R=r|X=x,W=w)$$

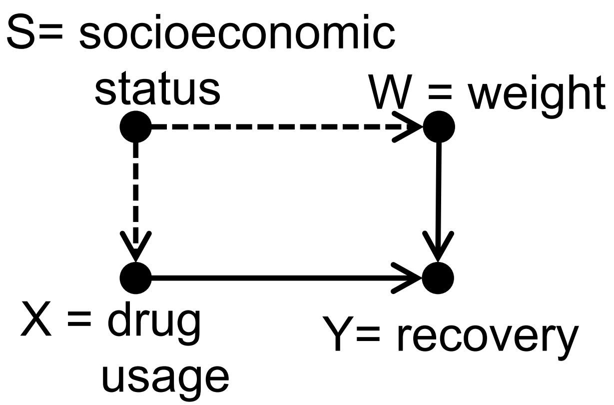
- Degree to which causal effect of X on Y is modified by values of W is called

  effect modification or moderation



Backdoor Criterion (Example 3)

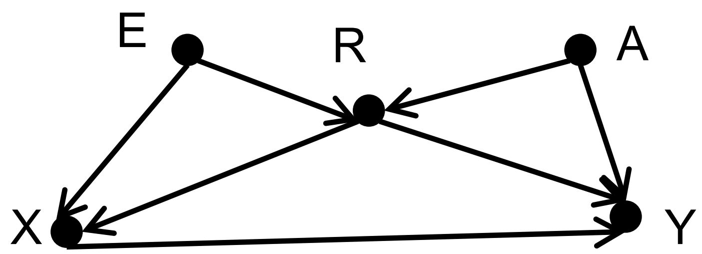
- What is effect modification for X on Y by W in drug example?
- Compare $P(Y = y \mid do(X = x), W = w)$ and
  $P(Y = y \mid do(X = x), W = w')$
- Here: As W blocks backdoor
  - $P(Y = y \mid do(X = x), W = w) = P(Y = y \mid X = x, W = w)$
  - $P(Y = y \mid do(X = x), W = w') = P(Y = y \mid X = x, W = w')$



Backdoor Criterion (Example 4)

- Sometimes also need to condition on colliders
- There are four backdoor paths from X to Y
  1. $X \leftarrow E \rightarrow R \rightarrow Y$
  2. $X \leftarrow E \rightarrow R \leftarrow A \rightarrow Y$
  3. $X \leftarrow R \rightarrow Y$
  4. $X \leftarrow R \leftarrow A \rightarrow Y$
- R needed to block 3. path
- But R collider on 2. path, hence need further blocking variable
- Can use as blocking set Z

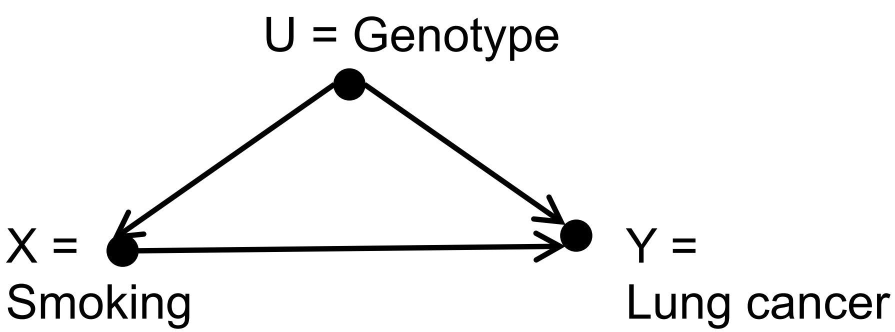
  {E,R}, {R,A} or {E,R,A}



Front-door Criterion (Motivating Example)

Example

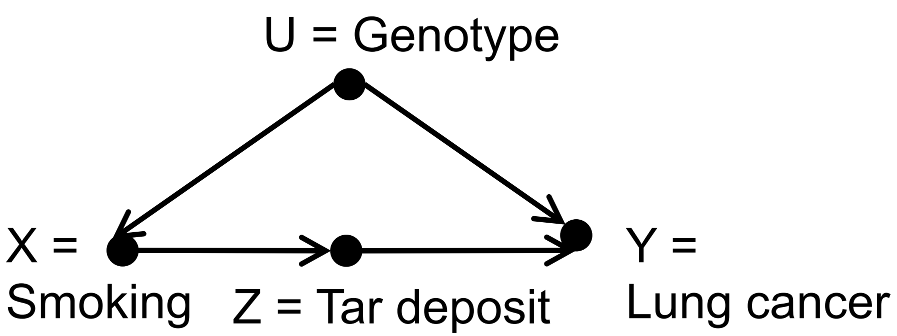
- Sometimes backdoor criterion not applicable
  - P(y | do(x)) = ?
  - Genotype U not observed in data
  - Hence conditioning on U does not help



Front-door Criterion (Motivating Example)

Example

- Sometimes backdoor criterion not applicable
  - P(y | do(x)) = ?
  - Genotype U not observed in data
  - Hence conditioning on U does not help
  - But sometimes a mediating variable helps



Front-door Criterion (Motivating Example)

	Tar (400)		No tar (400)		All subjects (800)	
	Smokers (380)	Nonsmokers (20)	Smokers (20)	Nonsmokers (380)	Smokers (400)	Nonsmokers (400)
No cancer	323 (85%)	1 (5%)	18 (90%)	38 (10%)	341 (85%)	39 (9.75%)
Cancer	57 (15%)	19 (95%)	2 (10%)	342 (90%)	59 (15%)	361 (92.25%)

Tobacco industry:

- 15% of smokers w. cancer < 92.25% nonsmokers w. cancer
- Tar: 15% smokers cancer < 95% nonsmoker cancer
- Non tar: 10% smokers cancer < 90% nonsmoker cancer

Front-door Criterion (Motivating Example)

	Smokers (400)		Nonsmokers (400)		All subjects (800)	
	Tar (380)	No tar (20)	Tar (20)	No tar (380)	Tar (400)	No tar (400)
No cancer	323 (85%)	18 (90%)	1 (5%)	38 (10%)	324 (81%)	56 (19%)
Cancer	57 (15%)	2 (10%)	19 (95%)	342 (90%)	76 (9%)	344 (81%)

Who is right?

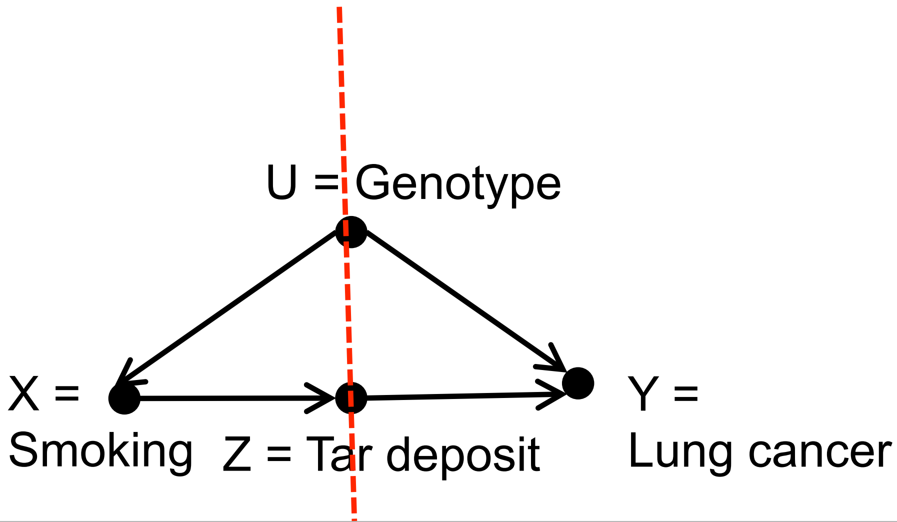
Antismoking lobby

- Choosing to smoke increases chances of tar deposit (95%)
- Effect of tar deposit: look separately at smokers vs. Non-smokers
  - Smokers: 10 % cancer $\xrightarrow{+tar}$ 15 % cancer
  - Nonsmokers: 90 % cancer $\xrightarrow{+tar}$ 95% cancer

Front-door Criterion (Intuition)

- Separate effect of X on Y:

Effect of X on Y = effect of X on Z + effect of Z on Y



Front-door Criterion (Intuition)

- Effect of X on Z:

  $P(Z = z \mid do(X = x)) = P(Z= z \mid X = x)$

  (No unblocked X-Z backdoor path)

- Effect of Z on Y:

  $P(Y = y \mid do(Z = z)) = \sum_x P(Y = y \mid Z = z, X = x)P(X=x)$

  (X blocks Z-Y-backdoorpath)

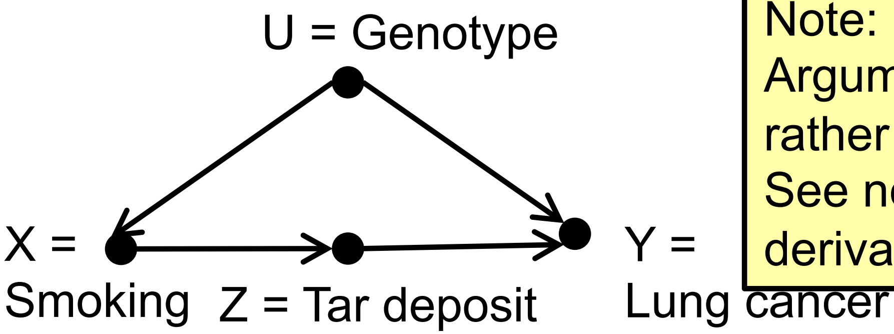
- Effect of X on Y:

  (Chaining and summing out)

  $P(Y = y \mid do(X=x))$

  $= \sum_z P(Y=y|do(Z=z))P(Z=z|do(X=x))$

  $= \sum_z \sum_{x'} P(Y=y|Z=z,X=x')P(X=x')P(Z=z|X=x)$



Note:
Argument in last step rather intuitive
See next slide for formal derivation

More detailed derivation

$P(y|do(X=x))$

$= \sum_u P(Y=y|x,u)P(u)$ (conditioning on U)

$= \sum_u \sum_z P(Y=y|z,x,u)P(z|x,u)P(u)$ (conditioning on Z)

$= \sum_u \sum_z P(Y=y|z,x,u)P(z|x)P(u)$ (Z independent of U given X by (d-separation))

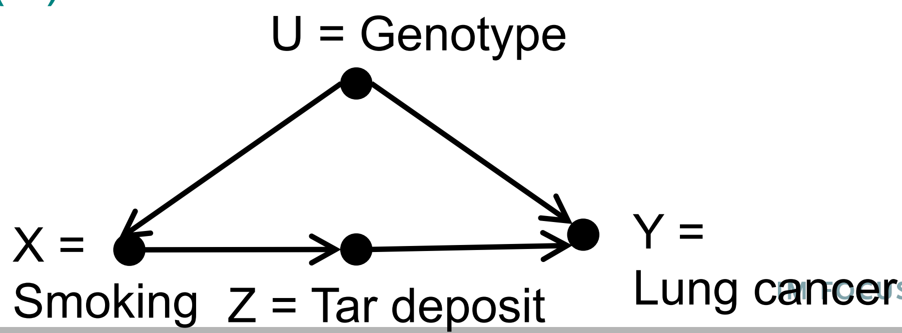
$= \sum_z P(z|x) \sum_u P(Y=y|z,x,u) P(u)$ (by commuting)

$= \sum_z P(z|x) \sum_u P(Y=y|z,u) P(u)$ (Y independent of X given Z,U)

$= \sum_z P(z|x) P(Y|do(z))$ (definition of do())

$= \sum_z P(z|x) \sum_{x'} P(Y|x',z) P(x')$ (adjustment via X)

$= \sum_z \sum_{x'} P(z|x) P(Y|x',z) P(x')$



Front-door Criterion (Formulation & Theorem)

Definition

Set of variables Z satisfies front-door criterion w.r.t. pair of variables (X,Y) iff

1. Z intercepts all directed paths from X to Y
2. Every backdoorpath from X to Z is blocked (by collider))
3. All Z-Y backdoor paths are blocked by X

Theorem (Front-door adjustment**)**

If Z fulfills front-door criterion w.r.t. (X,Y) and $P(x,z) > 0$

then $P(y|do(x)) = \sum_z P(z|x) \sum_{x'} P(y|z, x')P(x')$

Conditional Interventions (Example)

Example (conditioned drug administering)

- Administer drug ($X = 1$) if fever $Z > z$
- Formally:

$$P(Y = y \mid do(X = g(Z)))$$

where $g(Z) = 1$ if $Z > z$ and $g(Z) = 0$ otherwise

- Can be reduced to calculating z-specific effect

$$P(Y = y \mid do(X = x), Z = z)$$

Conditional Interventions (Rule)

Rule (z-specific effect)

If there is set S of variables s.t. $S \cup Z$ satisfies backdoor criterion

then the z-specific effect is given by

$$P(y \mid do(x), z) = \sum_s P(y \mid x,s,z)\, P(s \mid z)$$

Reduction of conditional intervention to z-specific effect:

$P(Y = y \mid do(X = g(Z))) =$

$$= \sum_z P(Y= y \mid do(X = g(Z), Z=z)\, P(Z=z \mid do(X = g(Z)))$$

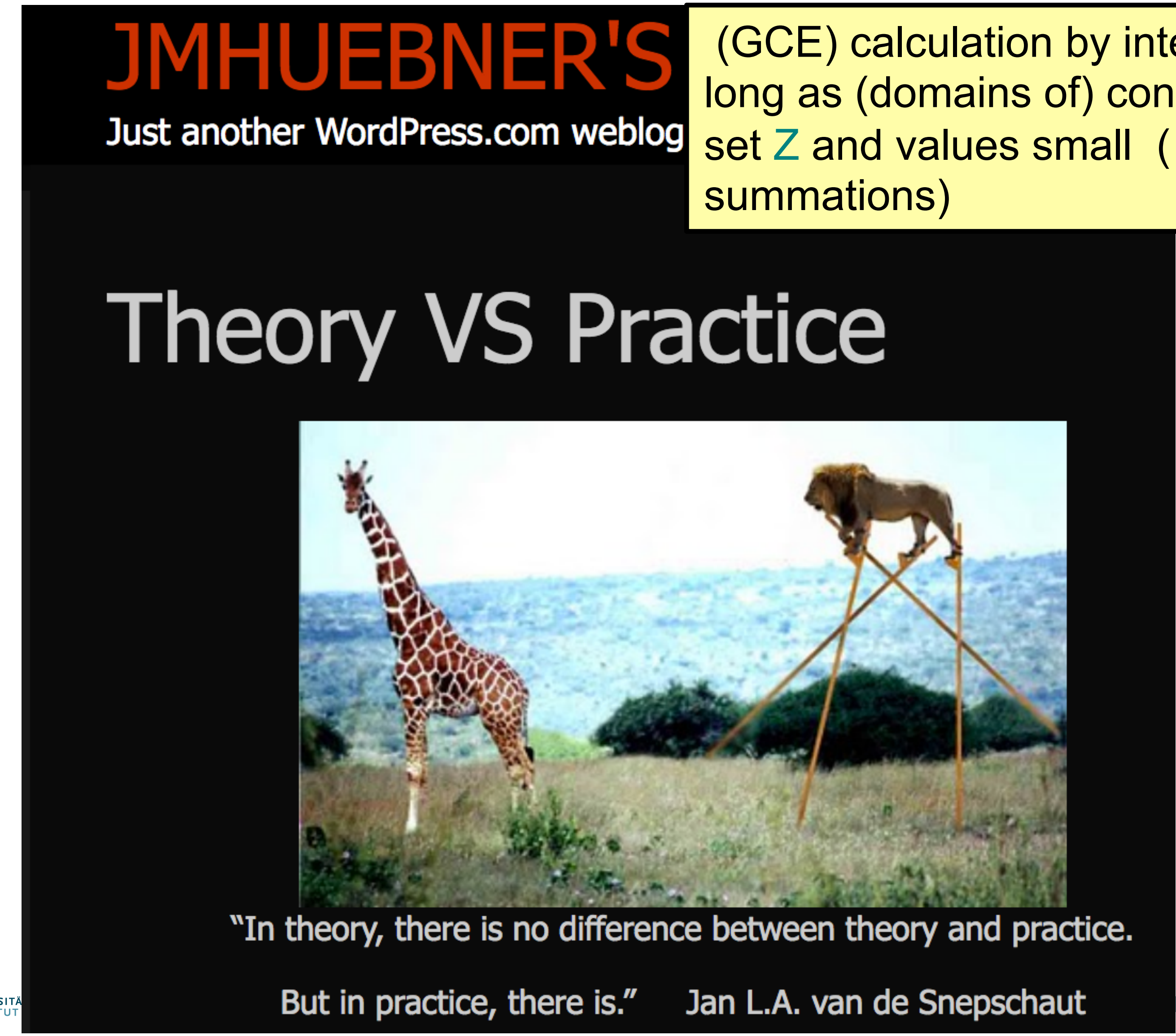
(conditioning on Z)

$$= \sum_z P(Y= y \mid do(X = g(Z), Z=z)\, P(Z=z) \qquad (Z \text{ before } X)$$

$$= \sum_z P(Y= y \mid do(X = x), z)_{|x=g(z)}\, P(Z=z)$$

Intervention Calculation in Practice?

(GCE) calculation by intervention useful as long as (domains of) conditioned variable set Z and values small (i.e. few summations)



Inverse Probability Weighing

- Inverse probability weighing gives estimation of GCE on small sample size << Z.
- Estimation with propensity score $P(X=x|Z=z)$
  - Propensity score can be estimated similarly as in linear regression
  - Weigh small sample set with propensity
  - Estimation of $P(y|do(x))$ by counting all events for y for each stratum $X=x$. (No summation over all instances of Z required)

Inverse Probability Weighing

- Filtering-Case $P(Y=y,Z=z|X=x)$: Evidence leads to re-normalization of full joint probability
  - $P(Y=y,Z=z|X=x) = P(Y=y, Z=z, X=x)/P(X=x)$
  - Have to weight (Y,Z,X) samples by $1/P(X=x)$
- Intervention-Case $P(y|do(x))$: Weighing by propensity
  - $P(y \,|do(x))$

$$= \sum_z P(Y= y \mid X=x, Z=z)\, P(Z=z)$$

$$= \sum_z P(Y= y \mid X=x, Z=z)\, P(Z=z)\, P(X=x|Z=z) \,/\, P(X=x|Z=z)$$

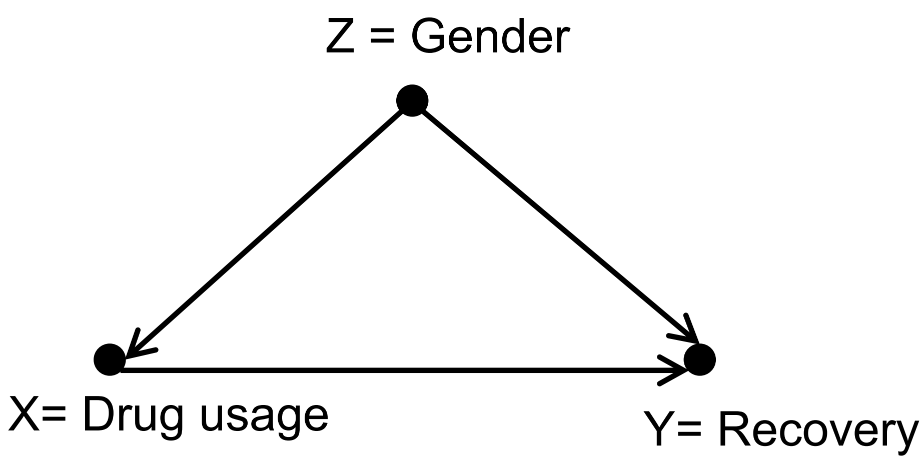
$$= \sum_z P(X=x,Y=y, Z=z) \,/\, P(X=x|Z=z)$$

Weighing joint distribution by inverse propensity

Inverse Probability Weighing (Example)

	Recovery rate with drug	Recovery rate without drug
Men	81/87 (93%)	234/270 (87%)
Women	192/263 (73%)	55/80 (69%)
Combined	273/350 (78%)	289/350 (83%)

- Rewrite table to get % of population for each (X,Y,Z) instance
- Example:
  %(yes,yes,male) = 81/700 = 0.116



Sample percentages

	Recovery rate with drug	Recovery rate without drug
Men	81/87 (93%)	234/270 (87%)
Women	192/263 (73%)	55/80 (69%)
Combined	273/350 (78%)	289/350 (83%)

X	Y	Z	% of population
yes	yes	male	0.116
yes	yes	female	0.274
yes	no	male	0.01
yes	no	female	0.101
no	yes	male	0.334
no	yes	female	0.079
no	no	male	0.051
no	no	female	0.036

Weighing when Filtering for X=yes

X	Y	Z	% of population
yes	yes	male	0.116
yes	yes	female	0.274
yes	no	male	0.01
yes	no	female	0.101
no	yes	male	0.334
no	yes	female	0.079
no	no	male	0.051
no	no	female	0.036

Consider X = yes & weigh (X,Y,Z) with 1/P(X=yes) = 0.116+0.274+0.01+0101

X	Y	Z	% of population
yes	yes	male	0.232
yes	yes	female	0.547
yes	no	male	0.02
yes	no	female	0.202

Weighing when Intervening do(X=yes)

X	**Y**	**Z**	**% of population**
yes	yes	male	0.116
yes	yes	female	0.274
yes	no	male	0.01
yes	no	female	0.101
no	yes	male	0.334
no	yes	female	0.079
no	no	male	0.051
no	no	female	0.036

Consider X = yes & weigh (X,Y,Z) with 1/P(X=yes|Z=z)
P(X=yes|Z=male) = (0.116 + 0.01)/(0.116+0.01 + 0.334 + 0.051)
P(X=yes|Z=female) = (0.274 + 0.101)/(0.274+0.101 + 0.079 + 0.036)

In this example no real savings!
These come into play when
dom(Z) >> sample size

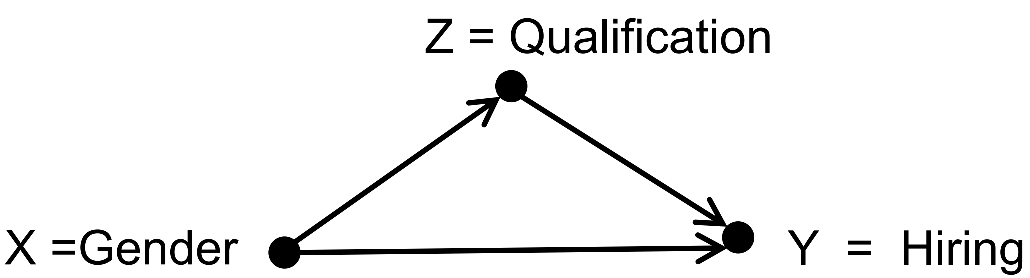
X	**Y**	**Z**	**% of population**
yes	yes	male	0.476
yes	yes	female	0.357
yes	no	male	0.042
yes	no	female	0.132

Mediation (Motivation)

- There may be indirect effects of X on Y via a mediating RV Z
- Interested in direct effect of X on Y

Example

- Gender may effect hiring directly or via qualification
- How to determine direct effect?
- Have to ``fix'' influence of mediators by intervention



The Human Mediator

Car on lhs is broken and is pushed to car workshop by car on rhs mediated by human in the middle

https://www.cnnturk.com/turkiye/yer-zonguldak-gorenler-gozlerine-inanamadi?page=1

Controlled Direct Effect

Definition The controlled direct effect (CDE) on Y of changing X from x to x' is defined by

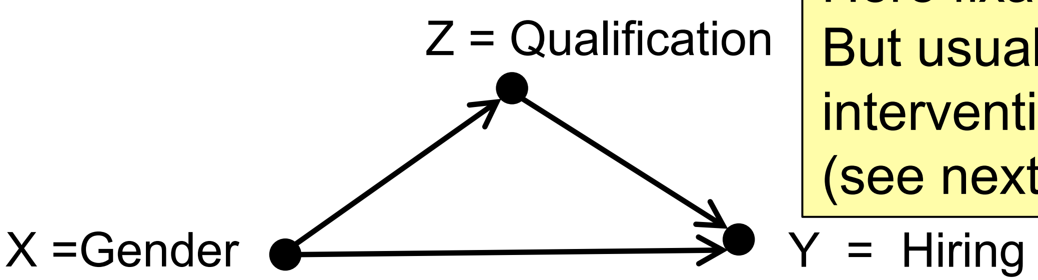
$$P(Y = y \mid do(X=x), do(Z=z)) - P(Y = y \mid do(X=x'), do(Z=z))$$

Example (CDE in Hiring SCM)

- $P(Y = y \mid do(X=x), do(Z=z))$

  $= P(Y = y \mid X=x, do(Z=z))$ (there is no X-Y-backdoor)

  $= P(Y = y \mid X=x, Z=z)$ (Z-Y backdoor blocked by X)
- $CDE = P(Y = y \mid X=x, Z=z) - P(Y=y \mid X=x', Z=z)$



Here fixation by conditioning. But usually fixation by intervention required (see next example)

Controlled Direct Effect (Extended Example)

P(Y= y| do(X=x), do(Z=z))

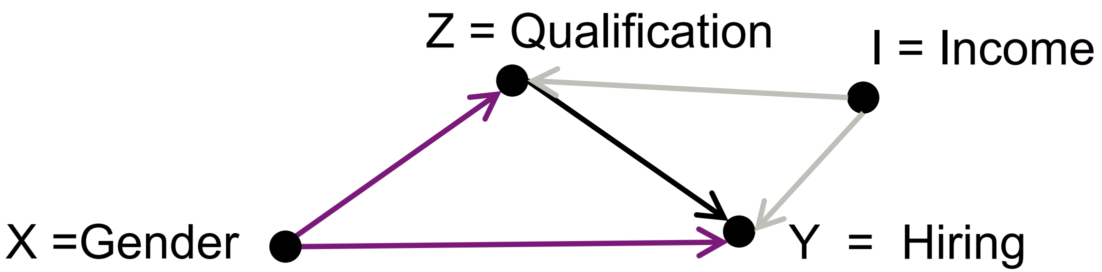
= P(Y= y| X=x, do(Z=z)) (there is no X-Y-backdoor)

= $\sum_i$ P(Y = y|X =x,Z=z,I=i)(P(I=i)

(first Z-Y backdoor blocked by X)

(second Z-Y backdoor blocked by I)

CDE = $\sum_i$ [P(Y = y|X =x,Z=z,I=i) - P(Y = y|X =x',Z=z,I=i)]P(I=i)



Controlled Direct Effect (Rule)

Rule (CDE identification)

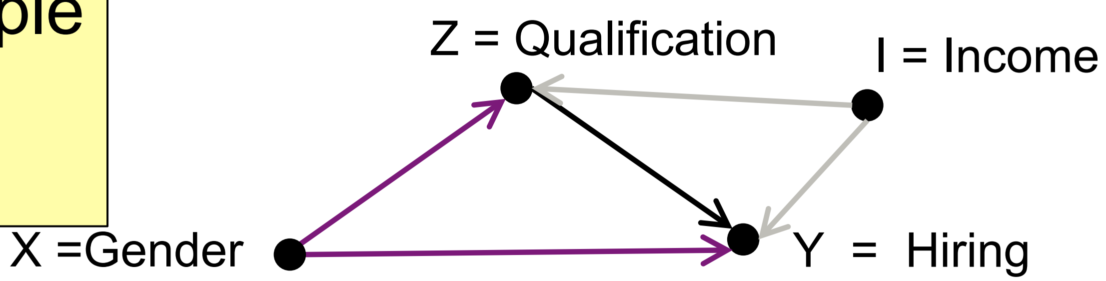
The CDE on Y for X changing from x to x' is given by

$$\sum_{s1,s2} [\, P(Y = y|X = x, Z=z, S_1=s_1, S_2=s_2) - P(Y = y|X = x', Z=z, S_1=s_1, S_2=s_2)\,] P(s1,s2)$$

Here S_1 and S_2 are sets of variables fulfilling

- S_1 blocks all Z-Y backdoor paths and
- S_2 blocks all X-Y backdoor paths after deleting all arrows entering Z

In Example
$S_1 = \{I\}$
$S_2 = \{\}$



Indirect Effects?

- Indirect effects not easily determinable
  - Cannot condition away direct effects of X and Y
  - In general (e.g. for non-linear correlations):

    Indirect effect ≠ total effect + direct effect
- But there is good news:
  - For linear SCMs simpler (next lecture)
  - With framework of counterfactuals one can determine indirect effects (lecture thereafter)