Knowledge-Base Revision Using Implications as Hypotheses (Extended Version)

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30.04.2012

Abstract

In semantic integration scenarios, the integration of an assertion from some sender into the knowledge base (KB) of a receiver may be hindered by inconsistencies due to ambiguous use of symbols; hence a revision of the KB is needed to preserve its consistency. This paper analyzes the new family of implication based revision operators, which exploit the idea of revising hypotheses on the semantic relatedness of the receiver's and sender's symbols. In order to capture the specific inconsistency resolution strategy of these operators, the novel concept of uniform sets, which are based on prime implicates, is elaborated. According to two main results of this paper these operators lend themselves to practical use in systems for semantic integration: First, the operators are finitely representable. Second, the non-sceptical versions of these operators can be axiomatically characterized by postulates, which provide a full specification of the operators' effects.

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1 Introduction

Belief revision (Alchourrón et al. (1985)) deals with the problem of integrating an assertion stemming from an agent (sender) into a knowledge base of another agent (receiver). If the receiver trusts the incoming information—and classical belief revision in contrast to non-prioritized belief revision (Hansson (1999a)) assumes he does—the integration may trigger a revision of the knowledge base. The reason is that the trigger may be incompatible with the knowledge base; hence some of its formulas have to be eliminated in order to keep it consistent. Classical belief revision explains the incompatibility with false information in the knowledge base. Therefore, the elimination of formulas in the knowledge base is an adequate means. But if the diagnosis for the incompatibility is not false information but ambiguous use of symbols, a different strategy seems more appropriate. For example, suppose an agent (the receiver) thinks the terminus "frugal" is meant to denote something very rich or praiseful though it really means "poor". He has different sentences in his KB in which he uses "frugal" in this sense. Another agent, the sender, uses "frugal" in a correct sense; so, a trigger sentence stemming from the sender may lead to inconsistencies with the receiver's KB. In order to resolve the inconsistencies, it would not be a good idea to eliminate only one sentence of the KB that contains "frugal" and that is involved in the conflict; because the next time the receiver integrates a (different) trigger from the sender, the false interpretation of "frugal" may again lead to inconsistencies.

An appropriate means to deal with conflicts caused by ambiguous use of symbols between different agents is first to state hypotheses on the semantical relatedness of symbols from different agents and second to eliminate some of the hypotheses that are involved in the conflict. This is the general approach of semantic integration based on semantic mappings (or bridging axioms) for heterogeneous knowledge bases (Noy (2004)). Every KB is assigned a unique name space, and semantic mappings associate symbols of different name spaces. In the case of the example above this means distinguishing between the use of "frugal" in the receiver's name space and in the sender's name space and initially hypothesizing that the uses are equivalent. If the integration of a trigger containing "frugal" into the receiver's KB leads to inconsistency, a proper strategy for resolving the conflict is eliminating the equivalence hypothesis and possibly replacing it by a weaker hypothesis compatible with the trigger (e.g., by hypothesizing that the sender's use is wider (narrower) than the receiver's use).

Based on this strategy for inconsistency resolution, this paper investigates a new class of operators for revising propositional KBs with propositional triggers. The hypotheses used in these operators are implications of the form $p' \rightarrow p$ or $p \rightarrow p'$ where p' stands for the p in the name space of the receiver, and p is the p of the sender. These operators generalize the revision operators of Delgrande and Schaub (2003) who consider biimplications of the form $p \leftrightarrow p'$ only. Using implications rather than biimplications allows for a more fine-grained analysis of what caused the conflict between the sender's trigger and the receiver's KB; hence—as demonstrated with examples in this paper—implication based revision preserves more sentences of the receiver's KB in the revision result than the biimplication based revision. The way in which the hypotheses for elimination are chosen is similar to the way how partial meet revision operators (Alchourrón et al. (1985)) choose formulas for elimination. In fact, as will be shown in this paper, the implication based operators can also be simulated by classical partial meet revision operators, but with a main switch concerning the arguments of the revision: The revised knowledge base is a set of hypotheses and the trigger is a union of an internalized version of the original knowledge base and the original trigger (Proposition 15).

Though the technical definitions of the revision operators of this paper and of Delgrande and Schaub (2003) are similar, the theory developed in this paper deviates considerably from that of Delgrande and Schaub (2003). One of the main innovative features of the theory developed in this paper is a formal specification and analysis of the uniformity property which distinguishes the implication (and biimplication) based operators from classical belief-revision operators: Classical belief revision eliminates only sentences in the KB which are identified as culprits for an inconsistency; on the other hand, the elimination of, e.g., the hypothesis $p' \rightarrow p$ amounts to (uniformly) eliminating a whole bunch of formulas of the KB, namely those containing p positively. The main idea of the analysis is first to equivalently represent the KB by its most atomic components (prime implicates) and then describe the effect of the implication based operators on the prime implicates by uniform closure conditions.

The implication revision operators provide a useful abstract implementation model for semantic integration scenarios in which conflicts caused by ambiguous use of symbol between heterogeneous KBs have to be resolved. Though the definitions of the operators are based on infinite sets, they can be described equivalently by finite operators that are more appropriate for implementation means (see Th. 11). This the first main result of this paper. Moreover, anyone implementing the choice versions of these operators gets a whole declarative specification of their properties (including uniformity): as a second main result (Th. 25) this paper describes a novel set of axiomatic postulates which are fulfilled by the operators and which characterize these operators in the sense that all other operators fulfilling them are representable as implication based choice revision operators.

The paper is structured as follows. The second section provides background on propositional logic and belief revision. The third section discusses the revision operators of Delgrande and Schaub (2003). The following section introduces the implication based revision operators and shows that these are indeed different from the operators of Delgrande and Schaub. Moreover, the finite representability by a partial polarity flipping operator is proved and the representability by classical partial meet revision is shown. The last section before the section on related work and the conclusion gives an axiomatic characterization of non-sceptical implication based revision operators by postulates.

2 Logical Preliminaries

This section introduces notation and concepts from propositional logic and belief revision that are used in the paper.

2.1 Propositional Logic

Let $\mathcal{P}$ be a set of propositional symbols and form($\mathcal{P}$) be the set of propositional logical formulas over $\mathcal{P}$ denoted by lower greek letters α, β.... More formally: All symbols in $\mathcal{P}$ as well as the constants $\top$ and $\bot$ are formulas. And if α, β are formulas, so are $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \rightarrow \beta$ and $\alpha \leftrightarrow \beta$. Finite sets of formulas are called *knowledge bases* or *belief bases* and are denoted by B as well as primed and indexed variants of B (e.g. B_1, B', $\bar{B}$). $\bigwedge B$ denotes the conjunction of all formulas in B. symb(B) is the set of propositional symbols in B. Formulas that are propositional symbols or negations of propositional symbols are called *literals*. A disjunction of literals is called a *clause*. A formula is in *conjunctive normal form (CNF)* iff it is a conjunction of clauses. A clause $p_1 \vee \cdots \vee p_n$ is also represented as set the $\{p_1, \ldots, p_n\}$ of its literals. With respect to this representation a clause is a *subclause* of another clause iff it is a subset of this clause. A disjunction of conjunctions of literals is a formula in *disjunctive normal form (DNF)*.

An *interpretation* or *assignment* $\mathcal{I} \in \mathrm{Int}(\mathcal{P})$ is a function that assigns truth values $0, 1$ to propositional symbols in $\mathcal{P}$. The definition can be recursively extended to formulas α, so that $\mathcal{I}(\alpha)$ denotes the truth value of α under $\mathcal{I}$. If $\mathcal{I}(\alpha) = 1$, then we also write $\mathcal{I} \models \alpha$ and say that $\mathcal{I}$ *models* α or is a model of α. $\mathcal{I}_{[p/x]}$ for $x \in \{0, 1\}$ denotes the variant of $\mathcal{I}$ that maps p to x. $\mathcal{I} \models B$ for a set B is a short notation for $\mathcal{I} \models \bigwedge B$. The set B is *consistent* iff it has a model. A set B *entails* a formula α, denoted $B \models \alpha$, iff all models of B are models of α. Define the set of *consequences* of B over the set of propositional symbols S by $\mathrm{Cn}^S(B) = \{\alpha \in \mathrm{form}(S) \mid B \models \alpha\}$. If the set of propositional symbols is clear, it is not written as an index. Moreover, if the symbol index is left out in some context, then the consequences have to be understood with respect to the maximal set of propositional symbols of discussed in the context. If two sets B_1 and B_2 have the same sets of consequences of formulas in $\mathrm{form}(S)$, write $B_1 \equiv_S B_2$. The truth value of a formula α depends only on the symbols occurring in it, therefore $\mathrm{Int}(S)$ is also considered as a set of interpretations, where $S \subseteq \mathcal{P}$ and $\alpha \in \mathrm{form}(S)$.

Given a formula $\alpha \in \mathrm{form}(\mathcal{P})$ and a subset $S \subseteq \mathcal{P}$ of symbols, the *clausal closure* of α w.r.t. S is the set $\mathrm{clauseCl}^S(\alpha)$ of clauses that have only symbols from S and that follow from α.

2.2 Consequences Relative to a Symbol Set

I define two operators Θ_S and Θ'_S that, given a formula α and a set S of symbols $S \subseteq \mathcal{P}$, compute a formula axiomatizing all consequences of α that do not contain symbols in S. For Θ'_S, the argument α has to be transformed in DNF while Θ_S does not presuppose such a transformation. These operators will be used as technical aids for calculating belief-revision results based on hypothesis.

Let α be a formula, $\mathrm{DNF}(\alpha)$ a formula equivalent to α represented as a set of clauses and $S \subseteq \mathcal{P}$. Furthermore, we assume that $\mathrm{DNF}(\alpha)$ is reduced in the formula that it does not contain a contradictory dual clause. $\Theta'_S(\alpha)$ results from $\mathrm{DNF}(\alpha)$ by substituting all literals over S in $\mathrm{DNF}(\alpha)$ by the logical constant $\top$: or equivalently: delete all literals in $\mathrm{DNF}(\alpha)$ that contain a symbol of S. The empty dual clause is interpreted as $\top$. Note that this is not the same as substituting all occurrences of symbols of S in $\mathrm{DNF}(\alpha)$ by $\top$. This would lead to wrong results and would also not justify the fact that have to transform the result into DNF. For example let $\alpha = (p \wedge q) \vee (r \wedge \neg s)$. Then α is already in DNF and one gets $\Theta'_{\{s\}}(\alpha) = (p \wedge q) \vee (r \wedge \top) \equiv (p \wedge q) \vee r$ and $\Theta'_{\{s,r\}}(\alpha) = (p \wedge q) \vee (\top \wedge \top) \equiv \top$. Θ_S is based on substituting symbols in S by truth value assignments. Let $\mathcal{I} \in \mathrm{Int}(S)$ be given, then the formula $\alpha_{\mathcal{I}}$ is defined as follows: Substitute all occurrences of $p \in S$ in α where $p^{\mathcal{I}} = \mathcal{I}(p) = 1$ by $\top$, else $\perp$ is substituted for p. For example, let $\alpha = (p \wedge q) \vee (r \wedge s)$ and $S = \{p, r\}$ and $\mathcal{I} \in \mathrm{Int}(S)$ with $\mathcal{I} : p \mapsto 1$, $\mathcal{I} : r \mapsto 0$, then $\alpha_{\mathcal{I}} = (\top \wedge q) \vee (\perp \wedge s)$. Now Θ_S is defined as follows: Let $S \subseteq \mathrm{symb}(\alpha)$. Then $\Theta_S : \alpha \mapsto \bigvee_{\mathcal{I} \in \mathrm{Int}(S)} \alpha_{\mathcal{I}}$. For arbitrary $S \subseteq \mathcal{P}$ let $\Theta_S(\alpha) = \Theta_{\mathrm{symb}(\alpha) \cap S}(\alpha)$. The following facts concerning Θ'_S and Θ_S can be easily proved.

Proposition 1. $\alpha \models \Theta'_S(\alpha)$ *and* $\alpha \models \Theta_S(\alpha)$

Proof. Let $\mathcal{I}$ be an assignment for α with $\alpha^{\mathcal{I}} = 1$. Then $(\mathrm{DNF}(\alpha))^{\mathcal{I}} = 1$. Hence there is a dual clause kl in $\mathrm{DNF}(\alpha)$ s.t. $kl^{\mathcal{I}} = 1$. Per definition there is a subclause $kl' \subseteq kl$ in $\Theta'_S(\alpha)$. Therefore $(\Theta'_S(\alpha))^{\mathcal{I}} = 1$. If $\alpha^{\mathcal{I}} = 1$, then $(\alpha_{\mathcal{I}})^{\mathcal{I}} = 1$ and hence $(\Theta_S(\alpha))^{\mathcal{I}} = 1$. □

Proposition 2. *Let* $S \subseteq \mathcal{P}$*. For all formulas* α *over* $\mathcal{P}$ *and* $\Theta_S \in \{\Theta'_S, \Theta_S\}$*:* $\mathrm{Cn}^{\mathcal{P} \setminus S}(\alpha) = \mathrm{Cn}^{\mathcal{P} \setminus S}(\Theta_S(\alpha))$

Proof. Proof of "'$\supseteq$"': Consequence of Prop. 1.

Proof of "'$\subseteq$"': Let $\Theta = \Theta$. Let $\beta \notin \mathrm{Cn}^{P \setminus S}(\Theta'_S(\alpha))$. Then there is an assignment $\mathcal{I}_1$ for $\bigvee_{\mathcal{I} \in \mathrm{Int}(S)} \alpha_v$ such that $\mathcal{I}_1 \models \bigvee_{\mathcal{I} \in \mathrm{Int}(S)} \alpha_{\mathcal{I}}$ and $\mathcal{I}_1 \models \neg\beta$. The first relation implies that there is a $\mathcal{I} \in \mathrm{Int}(S)$ such that $\mathcal{I}_1 \models \alpha_{\mathcal{I}}$. Define a new assignment $\mathcal{I}_2$ with $p^{\mathcal{I}_2} = p^{\mathcal{I}}$ for all $p \in S$ and $p^{\mathcal{I}_2} = p^{\mathcal{I}_1}$ for all other symbols.Then $\mathcal{I}_2 \models \neg\beta$, $\mathcal{I}_2 \models \alpha$ follow and hence $\beta \notin \mathrm{Cn}^{\mathcal{P} \setminus S}(\alpha)$.

Now let $\Theta = \Theta'$. We have to show: For all δ with $\mathrm{symb}(\delta) \subseteq \mathcal{P} \setminus S$ it is the case that if $\alpha \models \delta$, then also $\Theta'_S(\alpha) \models \delta$. This assertion is equivalent to the assertion that for δ with $\mathrm{symb}(\delta) \subseteq \mathcal{P} \setminus S$ it is the case that if $\Theta'_S(\alpha) \cup \{\neg\delta\}$ has a model, $\alpha \cup \{\neg\delta\}$ has a model, too. Let $\mathcal{I}$ be a model of $\Theta'_S(\alpha) \cup \{\neg\delta\}$. That means that there exists a dual clause kl' in $\Theta'_S(\alpha)$ with $(kl')^{\mathcal{I}} = 1$. In $\mathrm{DNF}(\alpha)$ there is a dual clause kl, such that substituting all literals of S in kl by $\top$ results in kl'. Let $\mathcal{I}'$ be a modification of $\mathcal{I}$ defined by: for all $p \notin S$ let $p^{\mathcal{I}'} = p^{\mathcal{I}}$. For all $p \in S$ let $p^{\mathcal{I}'} = 1$ if $p \in kl$ and $p^{\mathcal{I}'} = 0$ if $\neg p \in kl$. (We may assume that not at the same time $p, \neg p \in kl$. Then $kl^{\mathcal{I}'} = 1$ and hence $\alpha^{\mathcal{I}'} = 1$ follows. Because $\mathcal{I}$ changes at most symbols in S , it is also the case that $(\neg\delta)^{\mathcal{I}'} = 1$. $\square$

As a corollary to Proposition 1 and 2 the logical equivalence of $\Theta'_S(\alpha)$ and $\Theta_S(\alpha)$ follows.

Corollary 3. $\Theta'_S(\alpha) \equiv \Theta_S(\alpha)$.

Proof. As $\alpha \models \Theta'_S(\alpha)$ and $\mathrm{Cn}^{P\setminus S}(\alpha) = \mathrm{Cn}^{\mathcal{P}\setminus S}(\Theta_S(\alpha))$, $\Theta_S(\alpha) \models \Theta'_S(\alpha)$. Similarly $\alpha \models \Theta_S(\alpha)$ and $\mathrm{Cn}^{\mathcal{P}\setminus S}(\alpha) = \mathrm{Cn}^{\mathcal{P}\setminus S}(\Theta'_S(\alpha))$ imply that $\Theta'_S(\alpha) \models \Theta_S(\alpha)$. So, $\Theta'_S(\alpha) \equiv \Theta_S(\alpha)$. $\square$

Note, that $\Theta_S(\alpha)$ can be described by the quantified boolean formula (QBF) $\exists S.\alpha$.

2.3 Belief Revision

Belief revision as initiated by the pioneering work of Alchourrón, Gärdenfors and Makinson (AGM) Alchourrón et al. (1985) has evolved into a wide subfield of knowledge representation that deals with the dynamics of knowledge bases in a logical framework. Classical belief-revision functions à la AGM operate on a logically closed set called *belief set* and a formula which triggers the revision of the belief set into a new belief set. In belief-base revision (Hansson (1991)) the objects that are revised are called *belief bases*. They do not have to be logically closed; so, different syntactical representations of equivalent belief bases may lead to different outcomes.

AGM Alchourrón et al. (1985) construct belief-revision functions based on the concept of *remainder sets* Alchourrón and Makinson (1981). Given a set $B \subseteq \mathrm{form}(\mathcal{P})$ and formula $\alpha \in \mathrm{form}(\mathcal{P})$ define B's *remainder sets modulo* α, $B \perp \alpha$, by: $X \in B \perp \alpha$ iff $X \subseteq B$, $X \not\models \alpha$ and for all $\bar{X} \subseteq B$ with $X \subset \bar{X}$ it follows that $\bar{X} \models \alpha$. A dual concept will be used in this paper: Let $B \top \alpha$, *the dual remainder sets modulo* α, denote the set of inclusion maximal subsets X of B that are consistent with α, i.e., $X \in B \top \alpha$ iff $X \subseteq B$, $X \cup \{\alpha\}$ is consistent and for all $\bar{X} \subseteq B$ with $X \subset \bar{X}$ the set $\bar{X} \cup \{\alpha\}$ is not consistent. The notion of dual remainders is extended to arbitrary belief bases B_1 as second argument by defining $B \top B_1$ as $B \top \bigwedge B_1$. An *AGM selection function* γ for B is defined for all α as follows: If $B \top \alpha \neq \emptyset$, then $\emptyset \neq \gamma(B \top \alpha) \subseteq B \top \alpha$. Else set $\gamma(\emptyset) = \{B\}$. If for all $X \in \mathrm{Pow}(\mathrm{Pow}(\mathrm{form}(\mathcal{P})))$ the cardinality of $\gamma(X)$ is 1, then γ is called a *maxi-choice selection function*. Using this notion of selection function and the dual concept of remainder sets, classical *partial meet revision* $*_\gamma$ is defined by: $B * \alpha = \bigcap \gamma(B \top \alpha) \cup \{\alpha\}$.

An analysis of belief-revision functions involves the investigation of postulates that they fulfil and the invention of representation theorems for the functions. That is, for a class of belief-revision functions one seeks a set of postulates such that 1) all belief-revision functions fulfil the postulates; and 2) all other functions fulfilling the postulates can be represented by belief-revision functions from the given class.

Some postulates for belief-base revision operators $*$ that I will refer to in this paper are given below.

(BR1) $B * \alpha \not\models \perp$ if $\alpha \not\models \perp$.

(BR2) $B * \alpha \models \alpha$.

(BR3) $B * \alpha \subseteq B \cup \{\alpha\}$.

(BR4) For all $\beta \in B$ either $B * \alpha \models \beta$ or $B * \alpha \models \neg\beta$.

(BR5) If for all $\bar{B} \subseteq B$: $\bar{B} \cup \{\alpha\} \models \perp$ iff $\bar{B} \cup \{\beta\} \models \perp$, then $(B * \alpha) \cap B = (B * \beta) \cap B$.

Postulate (BR1) is the *consistency postulate* (Alchourrón et al. (1985)); it says that the revision result has to be consistent in case the trigger α is consistent. Postulate (BR2) is the *success postulate* (**?**); the revision must be successful in so far as α has to be in the revision result. (BR3) is called the *inclusion postulate* for belief-base revision (Hansson, 1999b, p. 200). The revision result of operators fulfilling it are bounded from above. Postulate (BR4) is the *tenacity postulate* Gärdenfors (1988); it states that the revision result is complete with respect to all formulas of B (Gärdenfors (1988)). Postulate (BR5) is the *logical uniformity postulate* for belief-base operators (Hansson (1993b)). It says that the revision outcomes are determined by the subsets (in)consistent with the trigger. The logical uniformity postulate generalizes the right extensionality postulate for revision operators, which states that the revision outcomes of equivalent triggers α, β lead to the same revision result: $B * \alpha = B * \beta$ if $\alpha \equiv \beta$. In contrast to Hansson, I specified the postulate as "logical" as I will use the notion of uniformity later on in a different sense.

3 Revision Based on Hypotheses

One example for belief-revision operators that are based on hypotheses are the operators of Delgrande and Schaub (2003). The general idea is to internalize the symbols of the knowledge base thereby dissociating the name spaces of the receiver, who holds the knowledge base, and the sender, who is the holder of a another knowledge base from which the trigger stems. Both name spaces are related by one special form of formula (bridging axiom), namely the biimplication. In order to resolve inconsistencies the internalized knowledge base stays untouched, but some subset of the biimplications are eliminated. After the elimination the name space dissociation is abandoned by retaining only those formulas of the old vocabulary. I recapitulate the definitions of the operators and their properties because the revision operator I will introduces is an extension, which uses implications $p' \rightarrow p$ and $p \rightarrow p'$ as hypotheses.

For a given set of propositional symbols $\mathcal{P}$ let $\mathcal{P}'$ denote the set $\{p' \mid p \in \mathcal{P}\}$ of *internal* or *internalized* propositional symbols. Similarly B' denotes the pendant of B where all symbols p are substituted by the corresponding internalized variant p'. In order to save space, I will use the following abbreviations for $p \in \mathcal{P}$: $\overleftrightarrow{p} = p \leftrightarrow p'$, $\overrightarrow{p} = p \rightarrow p'$ and $\overleftarrow{p} = p' \rightarrow p$. A *belief-change scenario* $\langle B_1, B_2, B_3 \rangle$ consists of three sets B_i ($i \in \{1, 2, 3\}$) of formulas over the set of propositional symbols $\mathcal{P}$. B_1 is the initial knowledge base of the receiver, B_2 is a knowledge base that must be contained in the change result and B_3 is a knowledge base that is not allowed to be in the change result. Classical revision of B with α is modelled by the belief-change scenario $\langle B, \{\alpha\}, \emptyset \rangle$; classical contraction of B with α is modelled by the belief-change scenario $\langle B, \emptyset, \{\alpha\} \rangle$. A *belief-change extension* (Delgrande and Schaub, 2003, p. 9) (*bc extension* for short) of the belief change scenario $\langle B_1, B_2, B_3 \rangle$ is a set of the form

$$\mathrm{Cn}^{\mathcal{P}}(B_1' \cup B_2 \cup EQ_i)$$

where $EQ_i \subseteq EQ = \{\overleftrightarrow{p} \mid p \in \mathcal{P}\}$ is an inclusion maximal set of biimplications (equivalences) fulfilling the following integrity condition

$$\mathrm{Cn}(B_1' \cup B_2 \cup EQ_i) \cap (B_3 \cup \{\perp\}) = \emptyset$$

If no such EQ_i exists, then form($\mathcal{P}$) is set as the only belief-change extension.

In case of classical belief revision represented by $\langle B, \{\alpha\}, \emptyset \rangle$ the set of bc extensions E_i have the form $E_i = \mathrm{Cn}^{\mathcal{P}}(B\sigma \cup EQ_i \cup \{\alpha\})$, where $\perp \notin \mathrm{Cn}(B_1\sigma \cup B_2 \cup EQ_i)$. Let $(E_i)_{i \in I}$ be the family of all bc extensions in the belief-change scenario $\langle B, \{\alpha\}, \emptyset \rangle$. A selection function c over the index set I selects exactly one index $c(I) \in I$. Two revision operators are defined, *choice revision* $\dot{+}_c$ based on a selection function c and *sceptical revision* $\dot{+}$.

Definition 4. (Delgrande and Schaub, 2003, p. 11)

$$\begin{aligned} O \dotplus_c \alpha &= E_k \text{ (for } c(I) = k) \\ O \dotplus \alpha &= \bigcap_{i \in I} E_i \end{aligned}$$

The definition of a selection function according to Delgrande and Schaub (Delgrande and Schaub, 2003, p. 11) differs from the notion of a selection function in AGM style (Alchourrón et al. (1985)). Though it may appear as if Delgrande's and Schaub's selection function c depends not only on B but also on the trigger α, their article seems to suggest the opposite. I reconstruct their notion of selection function in the following way: Consider the power set of all biimplications over the symbols $\mathcal{P}$, i.e., consider $\mathrm{Pow}(EQ)$ and let Ind be an index set for $\mathrm{Pow}(EQ)$. A selection function c is a function that for all non-empty subsets $I \subseteq Ind$ chooses one element of I, i.e., $c(I) \in I$. This definition guarantees that one has to use the same index I set for the belief sets of belief-change scenarios $\langle B, \{\alpha\}, \emptyset\rangle$ and $\langle B, \{\beta\}, \emptyset\rangle$ with equivalent triggers α and β.

Though the revision results under both operators $\dotplus_c, \dotplus$ are not finite, Delgrande and Schaub can show that these operators are finitely representable. That is more formally, operators $\circ^{fin}$ can be defined such that they operate on a finite knowledge base B as there left argument, a formula α as their second argument and output a finite knowledge base $B \circ^{fin} \alpha$. Additionally the following closure condition holds:

$$\mathrm{Cn}(B) \circ \alpha = \mathrm{Cn}(B \circ^{fin} \alpha)$$

The corresponding finite operators are based on substituting propositional symbols by their negation, thereby flipping the polarity of the symbols. Let $\mathcal{B} = \langle B, \{\alpha\}, \emptyset\rangle$ be a bc scenario and EQ_i a set of biimplications. The formula $\lceil \alpha \rceil_i$ results from α by substituting all occurrences of propositional symbols $p \in \mathcal{P} \backslash \mathrm{symb}(EQ_i)$ with their negation $\neg p$. Let $(E_i)_{i \in I}$ be the family of bc extensions over $\mathcal{B}$ and c a selection function with $c(I) = k$. Then Delgrande and Schaub define the flipping operators by $\lceil \mathcal{B} \rceil = \bigvee_{i \in I} \bigwedge_{\beta \in B} \lceil \beta \rceil_i$ and $\lceil \mathcal{B} \rceil_c = \bigwedge_{\beta \in B} \lceil \beta \rceil_k$ and finite revision operators by:

$$\begin{aligned} B \dotplus_c^{fin} \alpha &= \lceil (B, \{\alpha\}, \emptyset) \rceil_c \wedge \alpha \\ B \dotplus^{fin} \alpha &= \lceil (B, \{\alpha\}, \emptyset) \rceil \wedge \alpha \end{aligned}$$

The finite representability is stated in Theorem 5.

Theorem 5. *(Delgrande and Schaub, 2003, p. 17)*

$$\begin{gathered} B \dotplus_c \alpha \equiv B \dotplus_c^{fin} \alpha \\ B \dotplus \alpha \equiv B \dotplus^{fin} \alpha \end{gathered}$$

The reason for the compact representation of $\dotplus_c$ relies on the benign interaction of biimplications with the negation symbol. First one can easily prove that $B \equiv_{\mathcal{P}} B[p/p'] \cup \{\overleftrightarrow{p}\}$. On the other hand, if a biimplication $\overleftrightarrow{p}$ is not in the revision result $B \dotplus_c \alpha$, the maximality of E_k (for $c(I) = k$) underlying the result implies that $B' \cup E_k \cup \{\alpha\} \cup \{\overleftrightarrow{p}\} \models \bot$, i.e., $B' \cup E_k \cup \{\alpha\} \models \neg \overleftrightarrow{p}$. But $\neg \overleftrightarrow{p}$ is equivalent to $\neg p \leftrightarrow p'$, which explains the flipping.

This theorem evokes a new perspective on what has caused the inconsistency between the knowledge base and the trigger: a flip in the polarity of a propositional symbol. By re-flipping the propositional symbols that caused the inconsistency the result becomes consistent. A remarkable point here is that the flip of a propositional symbol concerns all its occurrences in the formula, it is a kind of uniform flipping. This uniformity can be interpreted as a systematic use of the proposition in just the opposite sense. For example, think of an agent (the receiver) who thinks the terminus "frugal" is meant to denote something very rich or praiseful though it really means "poor". If the trigger uses “frugal” in a correct sense, this may lead to inconsistencies. In order to resolve the inconsistencies, all occurrences of “frugal” have to be substituted by its negation.

The operators of Schaub and Delgrande rely on the elimination of biimplications. The biimplications can be considered as hypotheses on the relation of symbols of different name spaces Özçep (2008). Holding to a biimplication $\overleftrightarrow{p} = p \leftrightarrow p'$ means believing that the propositional symbol p of the receiver (holder of B) has the same meaning as the propositional symbol p of the sender. If this biimplication is eliminated during the revision process, this can be interpreted as diagnosing the inter-ambiguity of p between the sender and the agent as the culprit for the inconsistency. This perspective on Delgrande's and Schaub's operators as operators for the revision of hypotheses can be stated even more concretely. It can be shown that for both operators there exist classical partial meet revision operators (Alchourrón et al. (1985)) that act on a set of hypotheses (an enriched set of hypotheses, respectively). This result is given in the following proposition, my first contribution in this paper. It states that the choice revision $\dot{+}_c$ can be modelled as partial meet revision of the set EQ. In case of the sceptical operator $\dot{+}$ the disjunctive closure (Hansson (1993a)) of EQ is used as set of hypotheses that is revised. Hereby, the disjunctive closure of a set B is defined by $\mathrm{DC}(B) = B \cup \{\alpha_1 \vee \cdots \vee \alpha_n \mid \alpha_i \in B\}$. In both cases the result of the partial meet revision has to be relativized to the non-internalized set of symbols $\mathcal{P}$.

Proposition 6. *Let $\langle B, \{\alpha\}, \emptyset\rangle$ be a bc scenario, $(E_i)_{i\in I}$ the set of bc extensions and $(EQ_i)_{i\in I}$ the set of biimplications on which the E_i are based. Let c be a selection function according to Delgrande and Schaub (2003).*

1. *There is a maxichoice selection function γ for EQ s.t.:*
$$B \dot{+}_c \alpha = \mathrm{Cn}^{\mathcal{P}}(EQ *_\gamma (B' \cup \{\alpha\})) \tag{1}$$

2. *There is a selection function γ for $\mathrm{DC}(EQ)$ s.t.:*
$$B \dot{+} \alpha = \mathrm{Cn}^{\mathcal{P}}(\mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\})) \tag{2}$$

Proof. “Representation of $\dot{+}_c$”: Let γ be defined by $\gamma(EQ \top (B' \cup \{\alpha\})) = \{EQ_i\}$ for $i = c(I)$. Then per definition: $EQ *_\gamma (B' \cup \{\alpha\}) = \bigcap \big(\gamma(EQ \top (B' \cup \{\alpha\}))\big) \cup B' \cup \{\alpha\} = EQ_i \cup B' \cup \{\alpha\}$. Hence $\mathrm{Cn}^{\mathcal{P}}(EQ *_\gamma (B' \cup \{\alpha\})) = \mathrm{Cn}(EQ *_\gamma (B' \cup \{\alpha\})) \cap \mathrm{form}(\mathcal{P}) = E_i = B \dot{+}_c \alpha$.
“Representation of $\dot{+}$”: Let $H = \mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\})$ and γ be defined by:

$$\begin{aligned}\gamma(H) \quad = \quad & \{X \in H \mid X \cap EQ \text{ is maximal in} \\ & \{X' \cap EQ \mid X' \in H\}\}\end{aligned}$$

Let $(EQ_j^{\vee})_{j\in J}$ be the family of sets in $\gamma(\mathrm{DC}(EQ) \top (B' \cup \{\alpha\}))$. It follows from the definition of a disjunctive closure and the definition of a dual remainder set that for all $i \in I$ there is a $j \in J$ s.t. $EQ_i \subseteq EQ_j^{\vee}$, $EQ_j^{\vee} \cap EQ = EQ_i$ and

$$EQ_j^{\vee} \subseteq \mathrm{Cn}(EQ_i) \tag{3}$$

On the other hand, from the definition of γ it follows that for all $j \in J$ there is an $i \in I$ such that

$$EQ_j^{\vee} \supseteq EQ_i \tag{4}$$

Now I show “$B \dot{+} \alpha \supseteq \mathrm{Cn}^{P}(\mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\}))$”: Let $\beta \in \mathrm{Cn}^{\mathcal{P}}(\mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\}))$, that is $\beta \in \mathrm{form}(\mathcal{P})$ und $(\bigcap_{j\in J} EQ_j^{\vee}) \cup B' \cup \{\alpha\} \models \beta$. Therefore, for all $j \in J$ we have $EQ_i^{\vee} \cup B' \cup \{\alpha\} \models \beta$ and $EQ_j^{\vee} \models (\bigwedge B' \wedge \alpha) \to \beta$. Together with 3 it follows that for all $i \in I$ we have: $EQ_i \models$

$(\bigwedge B' \wedge \alpha) \rightarrow \beta$, hence $(\bigwedge B' \wedge \alpha) \rightarrow \beta \in \mathrm{Cn}(EQ_i) \subseteq \mathrm{Cn}(EQ_i \cup B' \cup \{\alpha\}) = E_i$ for all $i \in I$. In consequence, for all $i \in I$ one has $\beta \in E_i$ and, finally, $\beta \in \bigcap_{i\in I} E_i = O \dotplus \alpha$.
Now I show the other subset relation "$B \dotplus \alpha \subseteq \mathrm{Cn}^{\mathcal{P}}(\mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\}))$": Let $\beta \in B \dotplus \alpha = \bigcap_{i\in I} E_i$, i.e., $\beta \in \mathrm{form}(\mathcal{P})$, and for all $i \in I$ it is the case that $EQ_i \cup B' \cup \{\alpha\} \models \beta$ and hence $EQ_i \models (\bigwedge B' \wedge \alpha) \rightarrow \beta$. Because of compactness of propositional logic there is a finite subset $EQ_i^f \subseteq EQ_i$ for every $i \in I$ such that $EQ_i^f \models (\bigwedge B' \wedge \alpha) \rightarrow \beta$. As B is finite so is the index set I. Let $I = \{1, \ldots, k\}$. There are only finitely many sets EQ_i and finitely many E_i. Therefor the disjunction $\bigvee_{i\in I} EQ_i^f$ is defined and the following equation holds:

$$\bigvee_{i\in I} EQ_i^f \models (\bigwedge B' \wedge \alpha) \rightarrow \beta \tag{5}$$

For all $i \in I$, let $n_i = |EQ_i|$ denote the cardinality of EQ_i^f and $N_i = \{1, \ldots, n_i\}$. Every set EQ_i, $i \in I$, can be represented by $EQ_i \equiv \bigwedge_{j=1}^{n_i}(p_{ij} \leftrightarrow p'_{ij})$. Using the law of distribution, $\bigvee_{i\in I} EQ_i^f$ can be transformed equivalently to a conjunction of disjunctions of biimplications :

$$\bigvee_{i\in I} EQ_i^f \equiv \bigwedge_{(j_1,\ldots,j_k)\in N_1\times\cdots\times N_k} \bigvee_{i}^{k} (p_{j_i} \leftrightarrow p'_{j_i}) \tag{6}$$

According to (4), for all $j \in J$ there is a $i \in I$ with $EQ_j^\vee \supseteq EQ_i$. But for all $(j_1, \ldots, j_k) \in N_1 \times \cdots \times N_k$ we have $(p_{j_i} \leftrightarrow p'_{j_i}) \in EQ_i$ and therefore for all $(j_1, \ldots, j_k) \in N_1 \times \cdots \times N_k$ we have $\bigvee_i^k (p_{j_i} \leftrightarrow p'_{j_i}) \in EQ_j^\vee$, too. Hence, for all $j \in J$ it is the case that $EQ_j^\vee \models \bigwedge_{(j_1,\ldots,j_k)\in N_1\times\cdots\times N_k} \bigvee_i^k (p_{j_i} \leftrightarrow p'_{j_i})$. Using the equivalence (6) it follows that for all $j \in J$ it is the case that $EQ_j^\vee \models \bigvee_{i\in I} EQ_i^f$. Using the entailment relation (5) one can conclude that $EQ_j^\vee \models (\bigwedge B' \wedge \alpha) \rightarrow \beta$. In the end one gets $\beta \in \mathrm{Cn}^P(\mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\}))$—as desired. □

The representability of $\dotplus$ can be demonstrated with a simple example.

Example 7. Let be given $\mathcal{P} = \{p, q\}$, $EQ = \{\overleftrightarrow{p}, \overleftrightarrow{q}\}$, $B = \{p \wedge q\}$ and $\alpha = \neg p \vee \neg q$. It follows that

$$\begin{aligned} EQ \top (B' \cup \{\alpha\}) &= \{\{\overleftrightarrow{p}\}, \{\overleftrightarrow{q}\}\} \\ B \dotplus \alpha &= \mathrm{Cn}^{\mathcal{P}}(p \leftrightarrow \neg q) \end{aligned}$$

On the other hand we can calculate that

$$\begin{aligned} \gamma(\mathrm{DC}(EQ) \top (B' \cup \{\alpha\})) &= \{\{\overleftrightarrow{p}, \overleftrightarrow{p} \vee \overleftrightarrow{q}\}, \\ &\quad\ \{\overleftrightarrow{q}, \overleftrightarrow{p} \vee \overleftrightarrow{q}\}\} \\ \mathrm{DC}(EQ) *_\gamma (B' \cup \{\alpha\}) &= \{\overleftrightarrow{p} \vee \overleftrightarrow{q}\} \cup B' \cup \{\alpha\} \\ &= \{\overleftrightarrow{p} \vee \overleftrightarrow{q}, p', q', \neg p \vee \neg q\} \end{aligned}$$

Using $\Theta_{\{p',q'\}}$ the consequences w.r.t. $\mathcal{P}$ are computed.

$$\begin{aligned} \mathrm{Cn}^{\mathcal{P}}\left(\{\overleftrightarrow{p} \vee \overleftrightarrow{q}, p', q', \neg p \vee \neg q\}\right) &= \\ \mathrm{Cn}^{\mathcal{P}}\left(\Theta_{\{p',q'\}}[(\overleftrightarrow{p} \vee \overleftrightarrow{q}) \wedge p' \wedge q' \wedge (\neg p \vee \neg q)]\right) &= \\ \mathrm{Cn}^{\mathcal{P}}\left(((p \leftrightarrow \top) \vee (q \leftrightarrow \top)) \wedge \top \wedge \top \wedge (\neg p \vee \neg q)\right) &= \\ \mathrm{Cn}^{\mathcal{P}}(p \leftrightarrow \neg q) = B \dotplus \alpha & \end{aligned}$$

4 Using Implications as Hypotheses

By using the set of implications $\mathit{Impl} = \{\overrightarrow{p}, \overleftarrow{p} \mid p \in \mathcal{P}\}$ as set of hypotheses instead of the set of biimplications $EQ = \{\overleftrightarrow{p} \mid p \in \mathcal{P}\}$ new classes of revision operators result. The notion of belief extension is adapted accordingly, i.e., a set $\mathrm{Cn}^{\mathcal{P}}(B \cup \{\alpha\} \cup X)$ is an *implication based belief extension* iff $X \in \mathit{Impl}\top(B' \cup \{\alpha\})$. (Remember that $\top$ denotes the operator for dual remainder sets defined in Section 2.3). Let $(\mathit{Impl}_i)_{i \in I}$ be the set of all implication based consistent belief set extensions of the bc scenario $\langle B, \{\alpha\}, \emptyset\rangle$ and c be a selection for I with $c(I) = k$. The new operators are defined as follows:

Definition 8. The *implication based choice revision* $\dotplus_c^{\mathit{Impl}}$ and the *implication based sceptical revision* $\dotplus^{\mathit{Impl}}$ are defined by :

$$\begin{aligned} B \dotplus_c^{\mathit{Impl}} \alpha &= \mathit{Impl}_k \text{ (for } c(I) = k) \\ B \dotplus^{\mathit{Impl}} \alpha &= \bigcap_{i \in I} \mathit{Impl}_i \end{aligned}$$

The maximality of the Impl_i has the effect that for every $p \in \mathcal{P}$ at least one of $p \rightarrow p', p' \rightarrow p$ is contained in Impl_i.

Proposition 9. *Let $(\mathit{Impl}_i)_{i \in I}$ be the set of all implication based consistent belief set extensions of the bc scenario $\langle B, \{\alpha\}, \emptyset\rangle$. Then for all $i \in I$ and all $p \in \mathrm{symb}(B)$ one of $\overrightarrow{p} = p \rightarrow p'$, $\overleftarrow{p} = p' \rightarrow p$ is contained in Impl_i.*

Proof. Suppose that neither of of $\overrightarrow{p}, \overleftarrow{p}$ is contained in Impl_i. The maximality of Impl_i implies that $B' \cup \mathit{Impl}_i \cup \{\alpha\} \models \neg\overrightarrow{p} \wedge \neg\overleftarrow{p}$ and so $B' \cup \mathit{Impl}_i \cup \{\alpha\} \models \bot$, which contradicts the fact that $B' \cup \mathit{Impl}_i \cup \{\alpha\}$ is consistent. $\square$

As in the case of the Delgrande/Schaub revision operators I can finitely represent the results by an operation on the knowledge base. For convenience, I assume that only the connectors $\wedge, \vee$ and $\neg$ are allowed in the formulas; this is no real restriction as this set of connectors is functionally complete.

An occurrence of a propositional symbol is *syntactically positive* iff it occurs in the scope of an even number of negation symbols, otherwise the occurrence is *syntactically negative.* I also speak of the *(positive, negative) polarity* of a propositional symbol's occurrence. In contrast to the polarity switching in case of Delgrande's and Schaub's finite operators, the operator of partial flipping does not change the polarity of all occurrences of a symbol p but only of those of a particular polarity—depending on which implication $\overrightarrow{p}$ or $\overleftarrow{p}$ is missing in the given set of implications.

Definition 10. Let $(\mathit{Impl}_i)_{i \in I}$ be the family of belief extensions for a belief-change scenario $\mathcal{B} = (B, \{\alpha\}, \emptyset)$ and let Impl_k be an implication based belief extension chosen by the selection function, $c(I) = k$. Then define the operator of *partial flipping* $\lceil \mathcal{B} \rceil_k^{\mathit{Impl}} = \lceil \mathcal{B} \rceil_c^{\mathit{Impl}}$ in the following way: If $p \rightarrow p' \notin \mathit{Impl}_k$, then switch the polarity of the negative occurrences of p in $\bigwedge B$ (by adding $\neg$ in front of these occurrences). Similarly, if $p' \rightarrow p \notin \mathit{Impl}_k$, then switch the polarity of the positive occurrences of p in $\bigwedge B$ (by adding $\neg$ in front of these occurrences). Let $\lceil \mathcal{B} \rceil^{\mathit{Impl}} = \bigvee_{i \in I} \lceil \mathcal{B} \rceil_i^{\mathit{Impl}}$.

With this definition at hand, the following representation theorem follows:

Theorem 11.

$$\begin{aligned} B \dotplus_c \alpha &\equiv \lceil (B, \{\alpha\}, \emptyset) \rceil_c^{\mathit{Impl}} \wedge \alpha \\ B \dotplus \alpha &\equiv \lceil (B, \{\alpha\}, \emptyset) \rceil^{\mathit{Impl}} \wedge \alpha \end{aligned}$$

case	form in $\lceil \mathcal{B} \rceil_k^{Impl}$	form in B	implications in $Impl_k$
I	p	p	$p' \to p, p \to p'$
II	p	p	$p' \to p$
III	p	$\neg p$	$p' \to p$
IV	$\neg p$	$\neg p$	$p' \to p, p \to p'$
V	$\neg p$	$\neg p$	$p \to p'$
VI	$\neg p$	p	$p \to p'$

Table 1: Cases for literals

Proof. One can assume that B is a formula in DNF. Let $c(I) = k$. I show that $B' \cup Impl_k \cup \{\alpha\} \equiv_{\mathcal{P}} \lceil \mathcal{B} \rceil_k^{Impl} \cup \{\alpha\}$ by proving the two implicit directions.

'Right to left': Let $B' \cup Impl_k \cup \{\alpha\} \models \beta$ for $\beta \in \text{form}(\mathcal{P})$. I have to show $\lceil \mathcal{B} \rceil_k^{Impl} \cup \{\alpha\} \models \beta$. Let be given a model $\mathcal{I} \models \lceil \mathcal{B} \rceil_k^{Impl} \cup \{\alpha\}$. Then there is a dual clause cl in $\lceil \mathcal{B} \rceil_k^{Impl}$ such that $\mathcal{I} \models cl$. For every literal li in cl one of the cases mentioned in Table 1 holds.

So there are 6 different types of literals in cl; this justifies the following representation of cl in $\lceil \mathcal{B} \rceil_k^{Impl}$.

$$\begin{aligned} kl \quad = \quad & p_1^1 \wedge \cdots \wedge p_{n_1}^1 \wedge p_1^2 \wedge \cdots \wedge p_{n_2}^2 \wedge p_1^3 \wedge \cdots \wedge p_{n_3}^3 \\ & \wedge \neg p_1^4 \wedge \cdots \wedge \neg p_{n_4}^4 \wedge \neg p_1^5 \wedge \cdots \wedge \neg p_{n_5}^5 \\ & \wedge \neg p_1^6 \wedge \cdots \wedge \neg p_{n_6}^6 \end{aligned}$$

Define a new interpretation $\mathcal{I}'$ in the following way:

- $\mathcal{I}'(p_i'^1) = \mathcal{I}'(p_i^1) = 1 = \mathcal{I}(p_i^1)$;
- $\mathcal{I}'(p_i'^2) = \mathcal{I}'(p_i^2) = 1 = \mathcal{I}(p_i^2)$;
- $\mathcal{I}'(p_i'^3) = 0 \neq \mathcal{I}(p_i^3) = 1$; $\mathcal{I}'(p_i^3) = \mathcal{I}(p_i^3) = 1$;
- $\mathcal{I}'(p_i'^4) = \mathcal{I}'(p_i^4) = 0 = \mathcal{I}(p_i^4)$;
- $\mathcal{I}'(p_i'^5) = \mathcal{I}'(p_i^5) = 0 = \mathcal{I}(p_i^5)$;
- $\mathcal{I}'(p_i'^6) = 1 \neq \mathcal{I}(p_i^6) = 0$; $\mathcal{I}'(p_i^6) = \mathcal{I}(p_i^6) = 0$;
- if r is a propositional symbol in $\mathcal{P}$ with $r \neq p_i^j$ and $r' \neq p_i'^j$, let $\mathcal{I}'(r') = \mathcal{I}(r)$;

From the construction of $\mathcal{I}$ it follows that $\mathcal{I}'_{\restriction \mathcal{P}} = \mathcal{I}_{\restriction \mathcal{P}}$ and $\mathcal{I}' \models B' \cup Impl_k \cup \{\alpha\}$. So $\mathcal{I}' \models \beta$ and hence $\mathcal{I} \models \beta$.

'Left to right': Now suppose that $\lceil \mathcal{B} \rceil_k^{Impl} \models \beta$ and let $\mathcal{I} \models B' \cup Impl_k \cup \{\alpha\}$. That is, there is a dual clause cl' in B' of the form

$$\begin{aligned} & p_1'^1 \wedge \cdots \wedge p_{n_1}'^1 \wedge p_1'^2 \wedge \cdots \wedge p_{n_2}'^2 \wedge \neg p_1'^3 \wedge \cdots \wedge \neg p_{n_3}'^3 \\ & \wedge \neg p_1'^4 \wedge \cdots \wedge \neg p_{n_4}'^4 \wedge \neg p_1'^5 \wedge \cdots \wedge \neg p_{n_5}'^5 \\ & \wedge p_1'^6 \wedge \cdots \wedge p_{n_6}'^6 \end{aligned}$$

It follows that $\mathcal{I}(p_i^1) = \mathcal{I}(p_i^2) = 1$ and $\mathcal{I}(p_i^2) = \mathcal{I}(p^5) = 0$. (Because of the types of the literals and the fact that the hypotheses are made true.) Moreover, as $p_i^3 \to p_i'^3$ and $p_i'^6 \to p_i^6$ are not in $Impl_k$, the maximality of $Impl_k$ implies $B' \cup Impl_k \cup \{\alpha\} \models p_i^3 \wedge \neg p_i'^3$ and $B' \cup Impl_k \cup \{\alpha\} \models \neg p_i^6 \wedge \neg p_i'^6$. Therefore we also have $\mathcal{I}(p_i^3) = 1$ and $\mathcal{I}(p_i^6) = 0$. Finally, this implies $\mathcal{I} \models \lceil \mathcal{B} \rceil_k \wedge \alpha$, hence $\mathcal{I} \models \beta$. □

A simple example shows that $\dot{+}_c^{\mathit{Impl}}$ is different from the original operators $\dot{+}_c$, $\dot{+}$.

Example 12. Let be given $\mathcal{P} = \{p, q\}$, $B = \{p \leftrightarrow q\}$, and $\alpha = \neg(p \leftrightarrow q)$. The two inclusion maximal sets of biimplications are $EQ_1 = \{\overleftrightarrow{p}\}$ und $EQ_2 = \{\overleftrightarrow{q}\}$. Let $I = \{1, 2\}$ and $c_1(I) = 1$, $c_2(I) = 2$. Using $\Theta_{\{p',q'\}}$ or the representation theorem we can calculate the outcomes: $B \dot{+}_{c_1} \alpha = B \dot{+}_{c_2} \alpha = B \dot{+} \alpha = \mathrm{Cn}^{\mathcal{P}}(p \leftrightarrow \neg q)$

On the other hand, the inclusion maximal sets of implications are as follows. $\mathit{Impl}_1 = \mathrm{Cn}^{\mathcal{P}}(\{\overleftrightarrow{q}, \overrightarrow{p}\})$, $\mathit{Impl}_2 = \mathrm{Cn}^{\mathcal{P}}(\{\overleftrightarrow{q}, \overleftarrow{p}\})$, $\mathit{Impl}_3 = \mathrm{Cn}^{\mathcal{P}}(\{\overrightarrow{q}, \overleftrightarrow{p}\})$, and $\mathit{Impl}_4 = \mathrm{Cn}^{\mathcal{P}}(\{\overleftarrow{q}, \overleftrightarrow{p}\})$. These lead to four different choice revisions. Let $I = \{1, 2, 3, 4\}$ and $c(I) = i$. Then the revision results with respect to the four different implication based revisions are:

$$\begin{aligned} B \dot{+}_{c_1}^{\mathit{Impl}} \{\alpha\} &= B \dot{+}_{c_4}^{\mathit{Impl}} \{\alpha\} = \mathrm{Cn}^{\mathcal{P}}(\neg p \wedge q) \\ B \dot{+}_{c_2}^{\mathit{Impl}} \{\alpha\} &= B \dot{+}_{c_3}^{\mathit{Impl}} \{\alpha\} = \mathrm{Cn}^{\mathcal{P}}(p \wedge \neg q) \end{aligned}$$

For illustration, the calculation of the equation $B_1 := B \dot{+}_{c_1}^{\mathit{Impl}} \{\alpha\} = \mathrm{Cn}^{\mathcal{P}}(\neg p \wedge q)$ is given below.

$$\begin{aligned} B_1 &= \mathrm{Cn}^{\mathcal{P}}(\{p' \leftrightarrow q', \neg(p \leftrightarrow q), \overleftrightarrow{q}, \overrightarrow{p}\}) \\ &= \mathrm{Cn}^{\mathcal{P}}(\Theta_{\{p',q'\}}((p' \leftrightarrow q') \wedge \neg(p \leftrightarrow q) \wedge \overleftrightarrow{q} \wedge \overrightarrow{p})) \\ &= \mathrm{Cn}^{\mathcal{P}}((\neg(p \leftrightarrow q) \wedge q) \vee (\neg(p \leftrightarrow q) \wedge \neg q \wedge \neg p)) \\ &= \mathrm{Cn}^{P}((\neg(p \leftrightarrow q) \wedge q)) = \mathrm{Cn}^{P}(q \wedge \neg p) \end{aligned}$$

In particular, $\dot{+}_{c_1}^{\mathit{Impl}}$ results in outcomes different from the outcomes of $\dot{+}_{c_1}, \dot{+}_{c_2}$ und $\dot{+}$.

As in the case of choice revision, the use of implications as (enhanced) set of hypotheses has different affects on sceptical revision than the use of biimplications. Consider the following example.

Example 13. Let be given B and α as the following formulas in complete DNF

$$\begin{aligned} B &= (\neg p \wedge q \wedge r \wedge \neg t) \vee (p \wedge \neg q \wedge r \wedge t) \\ \alpha &= (p \wedge \neg q \wedge r \wedge \neg t) \vee \underbrace{(\neg p \wedge q \wedge \neg r \wedge t)}_{\mathcal{I}:=} \end{aligned}$$

The maximal sets of biimplications are given by $EQ_1 = \{\overleftrightarrow{r}, \overleftrightarrow{t}\}$ and $EQ_2 = \{\overleftrightarrow{r}, \overleftrightarrow{p}, \overleftrightarrow{q}\}$. For neither of these sets the model corresponding to $\mathcal{I}$ is implied. More concretely, using Theorem 5, one calculates:

$$\begin{aligned} B \dot{+} \alpha &= (\lceil B \rceil_1 \vee \lceil B \rceil_2) \wedge \alpha \\ &= ((p \wedge \neg q \wedge r \wedge \neg t) \vee (\neg p \wedge q \wedge r \wedge t) \vee \\ &\quad (\neg p \wedge q \wedge r \wedge t) \vee (p \wedge \neg q \wedge r \wedge \neg t)) \wedge \alpha \\ &\equiv (p \wedge \neg q \wedge r \wedge \neg t) \end{aligned}$$

Contrary to this, there is a maximal set of implications Impl_1 that together with $B' \cup \{\alpha\}$ implies $\mathcal{I}$, namely $\mathit{Impl}_1 = \{\overleftrightarrow{t}, \overrightarrow{p}, \overleftarrow{q}, \overrightarrow{r}\}$. So one can calculate:

$$\begin{aligned} B \dot{+}_1^{\mathit{Impl}} \alpha &= ((\neg p \wedge q \wedge \neg r \wedge \neg t) \vee (\neg p \wedge q \wedge \neg r \wedge t)) \\ &\quad \wedge \alpha \\ &\equiv \neg p \wedge q \wedge \neg r \wedge t \end{aligned}$$

Now, $B \dot{+}_1^{\mathit{Impl}} \alpha \models B \dot{+}^{\mathit{Impl}} \alpha$; hence $\mathcal{I}$ is a model of $B \dot{+}^{\mathit{Impl}} \alpha$ but not a model of $B \dot{+} \alpha$.

But al least it can be shown that the implication based sceptical revision operator is logically weaker than the biimplication based sceptical revision operator.

Proposition 14. *The implication based sceptical revision operator $\dot{+}^{Impl}$ is logically weaker than the biimplication based sceptical revision operator $\dot{+}$, i.e., $B \dot{+} \alpha \models B \dot{+}^{Impl} \alpha$.*

Proof. The proposition can be proved by using the semantic characterization of Delgrande's and Schaub's operators by the symmetric difference of models (Delgrande and Schaub, 2003, Theorem 4.7, p.14). Let be given a belief-change scenario $\mathcal{B}$, α a formula (the trigger) and let $(Impl_i)_{i \in I}$ be the implication based belief extensions and $(E_j)_{j \in J}$ the (biimplication based) set of belief extensions for $\mathcal{B}$. We can assume that B is in complete DNF so that its disjuncts directly correspond to its models. For example, if $p \wedge q \wedge \neg r$ is a disjunct of B and $\{p, q, r\}$ is the set of all symbols occurring in B, then the corresponding model is $\mathcal{I}(p) = 1, \mathcal{I}(q) = 1, \mathcal{I}(r) = 0$. We can identify $\mathcal{I}$ with the set of positive literals occurring in it, in our example $\mathcal{I}$ corresponds to the set $\{p, q\}$.

We will use a procedure that uses the semantic characterization of Delgrande's and Schaub's operators by the symmetric differences of models for our proof. The symmetric difference of two sets X_1, X_2 is defined by $X_1 \Delta X_2 = X_1 \cup X_2 \setminus (X_1 \cap X_2)$. Let $\Delta_{min}(B, \alpha) = min_{\subseteq}\{\mathcal{I}_1 \Delta \mathcal{I}_2 \mid \mathcal{I}_1 \models B, \mathcal{I}_2 \models \alpha\}$. Then it holds that (Delgrande and Schaub, 2003, Theorem 4.7, p.14):

$$\{\{p \in \mathcal{P} \mid \overleftrightarrow{p} \notin EQ_j\} \mid j \in J\} = \Delta_{min}(B, \alpha) \tag{7}$$

The procedure we consider is a transformation $\rightsquigarrow$. Let a formula in complete DNF be given and let p be a symbol of B. We define $B \overset{\overleftrightarrow{p}}{\rightsquigarrow} B'$ in the following way: For all dual clauses in B that contain p or $\neg p$ add one clause that contains p in the opposite polarity. For example, if $B = (p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$, then $B \overset{\overleftrightarrow{p}}{\rightsquigarrow} B'$, where $B' = B \vee \neg p \wedge q \wedge r \vee p \wedge \neg q \wedge r$. Note, that this has the same effect as applying θ_p to B. Similarly, define $B \overset{\overrightarrow{p}}{\rightsquigarrow} B'$ in the following way: for all dual clauses containing $\neg p$ add one in which $\neg p$ is switched to p. And last define $B \overset{\overleftarrow{p}}{\rightsquigarrow} B'$ in the following way: for all dual clauses containing p add one in which $\neg p$ is switched to p. For every set $EQ_j = EQ \setminus \{\overleftrightarrow{p_1}, \dots \overleftrightarrow{p_k}\}$ has a derivation $B \overset{\overleftrightarrow{p_1}}{\rightsquigarrow} B' \overset{\overleftrightarrow{p_2}}{\rightsquigarrow} B^{(3)} \dots \overset{\overleftrightarrow{p_k}}{\rightsquigarrow} B^{(k)}$ such that B^k contains models of α; additionally every derivation based on a subset of $\{\overleftrightarrow{p_1}, \dots \overleftrightarrow{p_k}\}$ does lead to a formula that has no models of α. A similar remark holds for maximal sets of implications.

Now to the proof of '$B \dot{+} \alpha \models B \dot{+}^{Impl} \alpha$': Let $\mathcal{I} \models B \dot{+} \alpha$. We have to show that $\mathcal{I} \models B \dot{+}^{Impl} \alpha$. There is a set $EQ_j = EQ \setminus \{\overleftrightarrow{p_1}, \dots, \overleftrightarrow{p_k}\}$ such that $B \overset{\overleftrightarrow{p_1}}{\rightsquigarrow} B' \overset{\overleftrightarrow{p_2}}{\rightsquigarrow} B^{(3)} \dots \overset{\overleftrightarrow{p_k}}{\rightsquigarrow} B^{(k)}$ is a derivation of a formula $B^{(k)}$ containing $\mathcal{I}$. So there is at least one $\mathcal{J} \models B$ from which $\mathcal{I}$ was derived by switching $p_1, \dots, p_k$. There is a set $Impl_i$ such that $Impl_i = EQ_j \cup Impl_i'$ where $Impl_i'$ is a set of implications of symbols in $\{p_1, \dots, p_k\}$. $Impl_i'$ does not contain both $\{p_x \to p_x', p_x' \to p_x\}$ for any $x \in \{1, \dots, k\}$ as this would contradict the maximality of EQ_j. Because of this and the fact that all proposition symbols in all dual clauses in B occur either syntactically positive or negative, the derivation induced by $Impl_i$ has the same effect on $\mathcal{J}$ as the derivation induced by EQ_j leading to $\mathcal{I}$. Therefore $\mathcal{I} \models B \dot{+}^{Impl} \alpha$. □

With techniques analogue to the ones in the proof of Proposition 6, the generalized operators $\dot{+}_c^{Impl}$ and $\dot{+}^{Impl}$ can be represented as partial meet revisions of hypotheses—this time of course using the set *Impl* as initial set of hypotheses.

Proposition 15. *Let $\langle B, \{\alpha\}, \emptyset \rangle$ be a bc scenario, $(E_i)_{i \in I}$ the set of bc extensions and $(EQ_i)_{i \in I}$ the set of biimplications on which the E_i are based. Let c be a selection function according to Delgrande and Schaub (2003).*

1. *There is a maxichoice selection function γ for Impl such that:* $B \dotplus_c^{Impl} \alpha = \mathrm{Cn}^{\mathcal{P}}(Impl *_\gamma (B' \cup \{\alpha\}))$

2. *There is a selection function γ for* $\mathrm{DC}(Impl)$*, such that:* $B \dotplus^{Impl} \alpha = \mathrm{Cn}^{\mathcal{P}}(\mathrm{DC}(Impl) *_\gamma (B' \cup \{\alpha\}))$

5 A Representation Theorem for Implication Based Choice Revision

Following the usual approach in classical belief revision (Alchourrón et al. (1985)), I will characterize the non-sceptical implication based revision operators $\dotplus_c^{Impl}$ by a set of postulates. Referring to the usual terminology in the belief revision literature (cf. Hansson (1999b)), the result of this section can be termed as a *representation* result: there is a set of postulates such that the class of revision operators $\dotplus_c^{Impl}$ represents (modulo equivalence) all revision operators fulfilling that set of postulates.

The main distinctive feature of Delgrande's and Schaub's operators $\dotplus_c, \dotplus$ as well as of $\dotplus_c^{Impl}, \dotplus^{Impl}$ is that these operate on a finite set B of formulas as left argument, but do not depend on the specific representation of B. So in contrast to belief-base revision operators they are operators on the knowledge level Newell (1982) and thus should be termed knowledge-base revision operators (Eschenbach and Özçep (2010)). In order to adapt the postulates for belief-base revision one has to replace all references to the set B and its subsets by syntax insensitive concepts.

The key for the adaptation is the use of prime implicates entailed by the knowledge base B. Roughly, prime implicates are the most atomic clauses implied by B. In the following subsection we recapitulate the definition of prime implicates $\mathrm{prime}(B)$ for a knowledge base B and restate the fact that B is equivalent to $\mathrm{prime}(B)$ (Proposition 16). The idea of using the prime implicate representation of a knowledge base has already been worked out in the literature (Pagnucco (2006), Zhuang et al. (2007), Bienvenu et al. (2008)). (A dual approach based on prime implicants is given by Perrussel et al. (2011).) But in contrast to the approach of this paper, the approaches of Pagnucco (2006), Zhuang et al. (2007), Bienvenu et al. (2008) do not use prime implicates in the formulation of the postulates; they define new belief-revision operators based on prime implicates and show that they fulfill some classical postulates in AGM style.

A second adaptation concerns the uniformity of the operators $\dotplus_c, \dotplus$ as well of the operators $\dotplus^{Impl}, \dotplus^{Impl}$. The conflicts between B and the trigger α are handled on the level of symbols and not on the level of formulas. Therefore, in order to mirror this effect on the prime implicates one has to impose a uniformity condition. This will be done implicitly by switching the perspective even further from prime implicates to uniform sets of prime implicates (see Definition 18 below).

5.1 Prime Implicates and Uniform Sets

Let be given a set of propositional symbols $\mathcal{P}$ and a subset $S \subseteq \mathcal{P}$ thereof. Let $\alpha \in \mathrm{form}(\mathcal{P})$. Let α be a non-tautological formula. The set $\mathrm{prime}^S(\alpha)$ of prime implicates of α over S is defined in the following way.

$$\begin{aligned}\mathrm{prime}^S(\alpha) \quad = \quad & \{\beta \in \mathrm{clauseCl}^S(\alpha) \mid \emptyset \not\models \beta \text{ and } \beta \text{ has no} \\ & \text{proper subclause in } \mathrm{clauseCl}^S(\alpha)\}\end{aligned}$$

For tautological formulas α let $\mathrm{prime}^S(\alpha) = \{p \vee \neg p\}$, where p is the first propositional symbol occurring in α with respect to a fixed order of $\mathcal{P}$. For knowledge bases let $\mathrm{prime}^S(B) = \mathrm{prime}^S(\bigwedge B)$. If S is clear form the context, then just write $\mathrm{prime}(\alpha)$. The conjunction of all formulas in $\mathrm{prime}(\alpha)$

is called the *Dual Blake Canonical Form (DBCF)* of α Armstrong et al. (1998). (The definition of prime implicates according to Armstrong et al. (1998) does not explicitly exclude tautologies; but their examples do not contain tautologies. Therefore we excluded tautological clauses, too). An alternative definition which does not face the problems with the vocabulary and tautologies is given by Marquis (2000). He defines the set of prime implicates as the set of the logically strongest clauses (these are not restricted to a vocabulary). By choosing a representative for every prime clause the set can be kept finite.)

Clearly, the set of prime implicates of a knowledge base B is equivalent to B itself. In the following, let $\mathrm{prime}(\cdot) = \mathrm{prime}^{\mathcal{P}}(\cdot)$ and $B, B_1, B_2 \subseteq \mathrm{form}(\mathcal{P})$.

Proposition 16. *For knowledge bases B:* $\mathrm{prime}(B) \equiv B$.

Proof. Every formula can be transformed into CNF. Therefore $\mathrm{clauseCl}(B) \equiv B$ and so it is sufficient to show $\mathrm{prime}(B) \equiv \mathrm{clauseCl}(B)$. Clearly, $\mathrm{prime}(B) \subseteq \mathrm{clauseCl}(B)$ and so trivially $\mathrm{clauseCl}(B) \models \mathrm{prime}(B)$. In order to show $\mathrm{prime}(B) \models \mathrm{clauseCl}(B)$ we show that for every clause $\beta \in \mathrm{clauseCl}(B)$ there is a $pr \in \mathrm{prime}(B)$ fulfilling $pr \models \beta$. If β is a tautology, then $\emptyset \models \beta$. Therefore suppose that β is not tautological. If $\beta \in \mathrm{prime}(B)$, then set $pr = \beta$. If $\beta \notin \mathrm{prime}(B)$, there is a $\beta' \in \mathrm{clauseCl}(B)$, s.t. β' is a proper subclause of β. Because β is finite, it has only finitely many proper subclause s.t. a minimal subclause β' can be chosen. There is a $\beta' \in \mathrm{clauseCl}(B)$ with: β' is a proper subclause of β and there is no proper subclause $\beta'' \in \mathrm{clauseCl}(B)$ of β'. In the end, $\beta' \in \mathrm{prime}(B)$. □

An additional simple fact is given in the following proposition. It justifies the perspective on the set of prime implicates as a canonical representation for the knowledge contained in the knowledge base.

Proposition 17. *If $B_1 \equiv B_2$, then* $\mathrm{prime}(B_1) = \mathrm{prime}(B_2)$.

Proof. Assume for contradiction, e.g., $\mathrm{prime}(B_1) \nsubseteq \mathrm{prime}(B_2)$. (The other case is proved similarly.) Then there is a prime implicate pl of B_1 that is not a prime implicate with respect to B_2. But $B_2 \models pl$, so there must be a prime implicate $pl' \subsetneq pl$ of B_2. In particular $B_2 \models pl'$, but then also $B_1 \models pl'$, which results in the contradicting assertion that pl cannot be a prime implicate of B_1. □

The notion of uniform sets is introduced in order to capture the conflict resolution strategy by the implication based revision operators. If, e.g., the hypothesis $p' \rightarrow p$ is eliminated in the conflict resolution process, then formulas of the knowledge base B, in which p occurs positively, are not preserved in the revision result. In general, if a set of implication based hypotheses Im is given, then $B' \cup Im$ preserves a subset of prime implicates of B which fulfills some closure condition concerning the polarities of symbols. These sets of prime implicates can be characterized as uniform sets according to the following definition.

Definition 18. Let $B \subseteq \mathrm{form}(\mathcal{P})$ be a knowledge base. A set $X \subseteq \mathrm{prime}(B)$ is called *uniform w.r.t. to B and implications*, $X \in U^{Impl}(B)$ for short, iff the following closure condition holds: If $pr \in \mathrm{prime}(B)$ is such that (a) $\mathrm{symb}(pr) \subseteq \mathrm{symb}(X)$ and (b) for all symbols p in pr there is a $pr_p \in X$ that contains p in the same polarity, then pr is contained in X, i.e., $pr \in X$.

Example 19. Let $B = \{p \vee q, p \vee r \vee s, r \vee t, s \vee u\}$. Then $\mathrm{prime}(B) = B$. Now, among all subsets $X \subseteq \mathrm{prime}(B)$ only the set $Y := \{p \vee q, r \vee t, s \vee u\}$ is not uniform as it would have to contain $p \vee r \vee s$, too. Formally, $U^{Impl}(B) = \mathrm{Pow}(\mathrm{prime}(B)) \setminus \{\{p \vee q, r \vee t, s \vee u\}\}$. Note that only in case of the non-uniform set Y one cannot find a set of implication based hypotheses Im such that Y is exactly the set of prime implicates of B which are preserved by $B' \cup Im$.

The following observation on the closure of the set of uniform sets follows directly from their definition.

Proposition 20. *For all $X, Y \in U^{Impl}(B)$ it is the case that $X \cap Y \in U^{Impl}(B)$.*

The union of uniform sets may not be uniform again. So let $X \cup' Y$ denote the smallest uniform set containing the uniform sets X and Y.

The interaction of uniform sets with implications, which will be used for the representation theorem below, is stated in the following propositions. The first proposition states how prime implicates interact with substitutions. (Uniform substitutions σ are (partial) functions from propositional symbols to propositional formulas; $\sigma(\alpha)$ or $\alpha\sigma$ results from α by substituting for all propositional symbols p all (!) its occurrences in α by the formula $\sigma(p)$.)

Proposition 21. *Let $\mathcal{P}$ and $\mathcal{P}'$ be disjoint sets of propositional symbols. Let B be a knowledge base and σ be a uniform injective substitution for some subset $S = \{p_1, \ldots, p_n\} \subseteq \mathcal{P}$ such that $\sigma(S) = \{p'_1, \ldots, p'_n\} \subseteq \mathcal{P}'$. Then:* $\text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B\sigma) \equiv_{\mathcal{P}} \text{prime}^{\mathcal{P}}(B\sigma)$

Proof. Because $\text{prime}^{\mathcal{P}}(B\sigma) \subseteq \text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B\sigma)$ it follows that for every $\beta \in \text{form}(\mathcal{P})$: If $\text{prime}^{\mathcal{P}}(B\sigma) \models \beta$, then $\text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B\sigma) \models \beta$. In order to show the other direction assume that $\beta \in \text{form}(\mathcal{P})$ and $\text{prime}^{\mathcal{P}}(B\sigma) \not\models \beta$. For reading convenience let $\Gamma_A = \text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B\sigma)$ and $\Gamma_B = \text{prime}^{\mathcal{P}}(B\sigma)$. It needs to be shown that $\text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B\sigma) \not\models \beta$. There is a model $\mathcal{I} \models \Gamma_B \cup \{\neg\beta\}$. So one has to show that there is a model $\mathcal{I}'$ of $\Gamma_A \cup \{\neg\beta\}$, too. The intended model can be constructed inductively by constructing interpretations $\mathcal{I}_i$ such that:

$$\begin{array}{rcl} \mathcal{I} = \mathcal{I}_0 & \models & \Gamma_B \cup \{\neg\beta\} \\ \mathcal{I}_1 & \models & \text{prime}^{\mathcal{P}\cup\{p'_1\}}(B\sigma) \cup \{\neg\beta\} \\ & \ldots & \\ \mathcal{I}' = \mathcal{I}_n & \models & \text{prime}^{\mathcal{P}\cup\{p'_1,\ldots,p'_n\}}(B\sigma) \cup \{\neg\beta\} \\ & = & \Gamma_A \cup \{\neg\beta\} \end{array}$$

The interpretation $\mathcal{I}_i$ is constructed from $\mathcal{I}_{i-1}$ just by modifying only the interpretation of p'_i in a minimal way. Let X denote all prime consequences in $\text{prime}^{\mathcal{P}\cup\{p'_1,\ldots,p'_i\}}(B\sigma)$ that do contain p'_i at most positively. If $\mathcal{I}_{i-1}(p'_i) = 1$, then $\mathcal{I}_i(p'_i) = 1$. Otherwise $\mathcal{I}_{i-1}(p'_i) = 0$. If there is an $\alpha \in X$ such that $\mathcal{I}_{i-1} \models \neg\alpha$, then define $\mathcal{I}_i(p'_i) = 1$. Else let $\mathcal{I}_i(p'_i) = 0$. Clearly $\mathcal{I}_{i-1} \models \neg\beta$ (as only the interpretation of p'_i may have changed). Per definitionem $\mathcal{I}_i \models X$. So the only thing to show is that $\mathcal{I}_i \models pr$ for all prime implicates in $\text{prime}^{\mathcal{P}\cup\{p'_1,\ldots,p'_i\}}(B\sigma)$ with a negative occurrence of p'_i. Let $pr = \neg p'_i \vee M$ where M is a disjunction of literals not containing p'_i. Assume that $\mathcal{I}_i \models p'_i$, i.e., $\mathcal{I}_i(p'_i) = 1$. We have to show $\mathcal{I}_i \models M$. There are two cases: $\mathcal{I}_{i-1}(p'_i) = 1$, then $\mathcal{I}_{i-1} \models M$ and hence $\mathcal{I}_i \models M$. Otherwise $\mathcal{I}_{i-1}(p'_i) = 0$ and there is a $\alpha = p'_i \vee N \in X$ (where N denotes a disjunction of literals) such that $\mathcal{I}_{i-1} \models \neg\alpha$, i.e., $\mathcal{I}_{i-1} \models \neg p'_i \wedge \neg \bigwedge N$ and so $\mathcal{I}_i \models \neg N$. Resolving α with pr gives the clause $cl = N \vee M$ that does not contain p'_i. So there is a prime clause pr'' in $\text{prime}^{\mathcal{P}\cup\{p'_1,\ldots,p'_i\}}(B\sigma)$ that is a subclause of cl. But $\mathcal{I}_{i-1} \models pr''$ and so $\mathcal{I}_i \models pr''$. As $\mathcal{I}_i \models \neg N$, one concludes $\mathcal{I}_i \models M$. $\square$

The interaction of a set of implications Im with prime implicates is captured in the following proposition. It states that all prime implicates of $B' \cup Im$ that do not contain primed symbols are prime implicates of B.

Proposition 22. *Let $\mathcal{P}$ and $\mathcal{P}'$ be disjoint sets of propositional symbols. Let B be a knowledge base and σ be a uniform injective substitution for a set $S = \{p_1, \ldots, p_n\} \subseteq \mathcal{P}$ such that $\sigma(S) = \{p'_1, \ldots, p'_n\} \subseteq \mathcal{P}$ and let Im be a set of implication based hypotheses containing at most primed symbols of $\sigma(S)$. Then:* $\text{prime}^{\mathcal{P}}(B\sigma \cup Im) \subseteq \text{prime}^{\mathcal{P}}(B)$.

Proof. The proof rests on a lemma (see below), which I mention only in the context of this proof due to its technicality. The lemma refers to the function $g(\cdot,\cdot)$ which is defined in the following way: Let im be an implication of the form $\overrightarrow{p}$ or $\overleftarrow{p}$. Let α be a formula. If $im = \overrightarrow{p}$, then $g(im,\alpha)$ stands for the assertion "p occurs semantically negative or not at all in α". If $im = \overleftarrow{p}$, then $g(im,\alpha)$ stands for the assertion "p occurs semantically positive in α or not at all". An occurrence is semantically positive (negative, resp.) in α iff for all interpretations $\mathcal{I}$: If $\mathcal{I}_{[p/0]} \models \alpha$ ($\mathcal{I}_{[p/1]} \models \alpha$, resp.), then $\mathcal{I}_{[p/1]} \models \alpha$ ($\mathcal{I}_{[p/0]} \models \alpha$, resp.).

Lemma 23. *Let $S = \{p_1, \dots, p_n\}$ and let $S_n = \mathcal{P} \cup \mathcal{P}' \setminus \{p'_1, \dots, p'_n\}$. Let $U \subseteq S$ be the symbols $p_i \in S$, such that $\{\overrightarrow{p_i}, \overleftarrow{p_i}\} \subseteq Im$. For all $p_i \in (S \cap \mathrm{symb}(Im)) \setminus U$ let $im(p_i)$ denote the implication (either $\overrightarrow{p_i}$ or $\overleftarrow{p_i}$) contained in Im. Let $Z = clauseCl^{S_n}(B\sigma \cup Im)$ for short. Then:*

$$
\begin{aligned}
Z \;=\; & \{\beta \in clauseCl^{S_n}(B) \mid \textit{There is a clause } \epsilon \textit{ with:} \\
& \epsilon \in clauseCl^{S_n}(B\sigma \cup Im); \epsilon \models \beta; \epsilon \textit{ does not} \\
& \textit{contain any symbol of } S \setminus \mathrm{symb}(Im) \textit{ and} \\
& \textit{for all } p_i \in (S \cap \mathrm{symb}(Im)) \setminus U\textit{: } g(im(p_i), \epsilon)\}
\end{aligned}
$$

We may assume that for all implications in Im there is no implication of the other direction, so $U = \emptyset$. Let $Im = \{im_1, \dots, im_k\}$. Proof of $\supseteq$: Let $\beta \in clauseCl^{S_n}(B)$ and let ϵ be a clause, s.t.: $\epsilon \in clauseCl^{\mathcal{V}_n}(B\sigma \cup Im)$, $\epsilon \models \beta$, ϵ has no symbol in $\{p_{k+1}, \dots, p_n\}$ and for $1 \le i \le k$ it holds that $g(ba(p_i), \epsilon)$. Hence $\beta \in clauseCl^{\mathcal{V}_n}(B\sigma \cup Im)$.

Proof. Proof of $\subseteq$: Let $\beta \in clauseCl^{S_n}(B\sigma \cup Im)$. Because $\beta \in \mathrm{form}(S_n)$, $(B\sigma\sigma^{-1} \cup (Im)\sigma^{-1} \models \beta$ follows, so $B \models \beta$; hence $\beta \in clauseCl^{S_n}(B)$. We have to show that an $\epsilon \in clauseCl^{S_n}(B\sigma \cup Im)$ exists that fulfils the mentioned conditions. Let $\tilde{Im}$ be an equivalent CNF of Im and let $\tilde{B}$ be an equivalent CNF of B and let $(B\sigma \tilde{\cup} Im)$ be the formula $\tilde{B}\sigma \wedge \tilde{Im}$. Assume β has the form $\beta = (li_1 \vee \dots \vee li_q)$. Because $(B\sigma \tilde{\cup} Im) \models \beta$, $(B\sigma \tilde{\cup} Im) \cup \{\neg\beta\}$ is inconsistent. So $\tilde{B} \wedge \tilde{Im} \wedge \neg li_1 \wedge \dots \wedge \neg li_q$ can be resolved to the empty clause.

If β is already the clause ϵ which fulfils the desired conditions, then set $\epsilon = \beta$. Else β contains a symbol p for which (i) $p \in \{p_{k+1}, \dots, p_n\}$ or there is i, $1 \le i \le k$, s.t. $p = p_i$ and not $g(im_i, \beta)$. Let as call such a symbol p a bad symbol. Let r denote the number of bad symbols in β. By induction on the number j of bad symbols one can construct a sequence $\langle\beta_j\rangle_{0 \le j \le r}$ of clauses $\beta_j \in clauseCl(B\sigma \cup Im)$ such that:

$$B\sigma \cup Im \models \beta_r \models \ldots \models \beta_1 \models \beta_0 = \beta$$

and every β_j has exactly $r - j$ bad symbols; in particular, β_r has no bad symbols so that β_r is the desired ϵ. Assume that we have already constructed β_j and in particular assume $B\sigma \cup Im \models \beta_j$. Let p be a bad symbol of β_j. W.l.o.g we may assume that β_j is not a tautology. We first consider the case that $p \in \{p_{k+1}, \dots, p_n\}$. No literal $\neg li_j$ containing p, can be resolved with $\tilde{B}\sigma \wedge \tilde{Im}$; resolving $\neg li_j$ with a complementary clause in $(\neg li_1 \wedge \dots \wedge \neg li_q)$ would be possible only if β_j were a tautology. Similarly clauses with p are not used for the derivation of the empty clause. So there is a clause β_{j+1}, which is obtained from β_j by eliminating literals containing p and for which $B\sigma \cup Im \models \beta_{j+1}$ and $\beta_{j+1} \models \beta_j$. Moreover β_{j+1} has exactly $r - j - 1$ bad symbols.

In the second case β_j contains a symbol p_i, $1 \le i \le k$ for which $g(im_i, \beta_j)$ does not hold. W.l.o.g. assume $im_i = p_i \rightarrow p'_i$. So β_j does not contain p semantically negative. In particular β_j contains a literal li_j that contains p_i syntactically positive. Again $B\sigma \cup Im \cup \{\neg\beta_j\}$ is inconsistent and so a derivation of the empty clause exists. The clause $\neg li_j$ contains p_i negatively. It cannot resolve with a clause in $(\tilde{B\sigma} \wedge \tilde{Im})$. A resolution with a clause in in $\neg li_1 \wedge \dots \wedge \neg li_q$ is not possible either—otherwise β_j would be a tautology. The clause β_{j+1} is obtained from β_j by removing the literal p_i. Again $\beta_{j+1} \models \beta$ and $B\sigma \cup Im \models \beta_{j+1}$ and β_{j+1} $r - j - 1$ bad symbols. □

Now to the proof of the proposition. Let $pr \in \text{prime}^{\mathcal{P}}(B\sigma \cup Im)$. Then $pr \in clause^{\mathcal{P}}(B)$. We have to show $pr \in \text{prime}^{\mathcal{P}}(B)$. Assume that not $pr \in \text{prime}^{\mathcal{P}}(B)$. That would mean that there is a clause $cl \in clauseCl^{\mathcal{P}}(B)$ that is a proper subclause of pr. There are two cases: a) $cl \in clauseCl^{\mathcal{P}\cup\mathcal{P}'}(B\sigma \cup Im)$; b) $cl \notin clauseCl^{\mathcal{P}\cup\mathcal{P}'}(B\sigma \cup Im)$. Both cases result in a contradiction. Case a) contradicts the fact that pr is prime with respect to $(B\sigma \cup Im)$. In case b) it holds that $B\sigma \cup Im \not\models cl$ and $cl \models pr$. The first assertion and the lemma imply that cl contains a symbol p (i) for which no hypothesis is contained in Im or (ii) for which a hypothesis is contained in the false direction.

Case (i): The lemma implies that there is a clause cl' such that $cl' \in clauseCl^{\mathcal{P}}(B\sigma \cup Im)$; p does not occur in cl' and $cl' \models pr$. Let pr' be a clause resulting from pr by removing all literals containing p. Then $B\sigma \cup Im \models cl' \models pr'$. But this contradicts the primeness of pr w.r.t. $B\sigma \cup Im$.

Case (ii): W.l.o.g. assume that $p' \to p \in Im$. Then cl contains a syntactically negative occurrence of p. Because of the lemma there is a clause $cl' \in clauseCl^{\mathcal{P}}(B\sigma \cup Im)$ such that cl' contains p only positively and $B\sigma \cup Im \models cl' \models pr$. The symbol p can occur in pr at most positively. Otherwise, it would be the case that the clause pr', which results from pr by eliminating all literals $\neg p$, is implied by $B\sigma \cup Im$—contradicting the primeness of pr w.r.t. $B\sigma \cup Im$. But as $cl \models pr$, also $cl[p/\bot] \models pr[p/\bot]$. As p occurs syntactically negative in cl, $cl[s/\bot]$ is a tautology; but then $pr[s/\bot]$ is a tautology, too—contradicting the primeness of pr w.r.t. $B\sigma \cup Im$. □

As a corollary to the propositions, one can deduce the main result of this subsection, Theorem 24. It is a proper justification for Definition 18—in the sense that it really captures the intended concept. The theorem shows that for all B, Im one can find a uniform set X that is equivalent to $B' \cup Im$. The set X exactly describes the collection of logical atoms (prime implicates) of the receiver's KB B that are preserved after dissociating the name spaces of the sender and receiver (step from B to B') and adding hypotheses on the semantical relatedness in Im.

Theorem 24. *Let $\mathcal{P}$ and $\mathcal{P}'$ be disjoint sets of propositional symbols. Let B be a knowledge base and σ be a uniform injective substitution for some subset $S = \{p_1, \ldots, p_n\} \subseteq \mathcal{P}$ such that $\sigma(S) = \{p'_1, \ldots, p'_n\} \subseteq \mathcal{P}$ and let Im be a set of implication based hypotheses containing at most primed symbols of $\sigma(S)$. Then there is a uniform set $X \in U^{Impl}(B)$ such that: $B' \cup Im \equiv_{\mathcal{P}} X$.*

Proof. Due to Proposition 17 we have $B' \cup Im \equiv_{\mathcal{P}\cup\mathcal{P}'} \text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B' \cup Im)$. Now, in order to use Proposition 21 we have to present $(B' \cup Im)$ as a set $B_1\sigma$. The problem is that σ will substitute all occurrences of the same symbol in B_1, so we cannot set $B_1 = B \cup Im$, as then also the non-primed symbols of Im would be substituted. So we proceed in the following way: For all symbols s in B we take a new symbol s_n. Let σ be the substitution where $\tau_1(s) = s_n$, $\tau_2(s') = s_n$ for symbols s in B. Now the set $B_1 = B\tau_1 \cup Im\tau_2$. Let σ be the substitution such that for any s_n is substituted by s'. Then $B_1\sigma = B' \cup Im$. Now because of Proposition 21 we get $\text{prime}^{\mathcal{P}\cup\mathcal{P}'}(B_1\sigma) \equiv_{\mathcal{P}} \text{prime}^{\mathcal{P}}(B_1\sigma)$. But $\text{prime}^{\mathcal{P}}(B_1\sigma)$ is $\text{prime}^{\mathcal{P}}(B' \cup Im)$ and according to Proposition 22 this is a subset of $\text{prime}^{\mathcal{P}}(B)$. Hence we set $X = \text{prime}^{\mathcal{P}}(B' \cup Im)$ which is easily seen to be a uniform set.

□

5.2 Postulates for Implication Based Revision

The following postulates for revision operators $*$ are adapted variants of the postulates mentioned in the section on logical preliminaries. They are exactly the ones that characterize the implication based choice revision operators.

(R1) $B * \alpha \not\models \bot$ if $B \not\models \bot$ and $\alpha \not\models \bot$.

(R2) $B * \alpha \models \alpha$.

(R3) There is a set $H \subseteq U^{\mathit{Impl}}(B)$ such that $B * \alpha \equiv \bigwedge \bigcup' H \wedge \alpha$ or $B * \alpha \equiv \bigwedge \bigcup' H$.

(R4) For all $X \in U^{\mathit{Impl}}(B)$ either $B * \alpha \models X$ or $B * \alpha \models \neg \bigwedge X$.

(R5) For all $Y \subseteq U^{\mathit{Impl}}(B)$: If $\bigcup Y \cup \{\alpha\} \models \bot$ iff $\bigcup Y \cup \{\beta\} \models \bot$, then $\{X \in U^{\mathit{Impl}}(B) \mid B * \alpha \models X\}$ $= \{X \in U^{\mathit{Impl}}(B) \mid B * \beta \models X\}$.

Postulate (R1) can be termed the postulate of weak consistency; it says that the revision result has to be consistent (satisfiable) in case both the trigger α and the knowledge base B are consistent. The consistency postulate for AGM belief revision and belief base revision (BR1) is stronger as it demands the consistency also in the case where only α is consistent. Postulate (R2) is a weak success postulate; the revision must be successful in so far as the result has to imply α. It is weaker than the postulate (BR2) for belief bases. (R3) is an adapted version of the inclusion postulate for belief base revision (BR3). The classical inclusion postulate can be rewritten as: There is a $\bar{B} \subseteq B$ such that $B * \alpha = \bar{B} \cup \{\alpha\}$ or $B * \alpha = \bar{B}$. In (R3) B is replaced by the set of uniform sets w.r.t. B, and set identity is shifted to equivalence. Note that due to the definition of the $\cup'$-closure operator the set $\bigcup H'$ is a uniform set and hence the postulate (R3) can be reformulated as:

(R3') There is a set $H \in U^{\mathit{Impl}}(B)$ such that $B * \alpha \equiv \bigwedge H \wedge \alpha$ or $B * \alpha \equiv \bigwedge H$.

Postulate (R4) can be called uniform tenacity. It is a very strong postulate, which states that all uniform sets w.r.t. to B either follow from the result or are falsified. This postulate will capture the maximality of the operator $\dot{+}_c^{\mathit{Impl}}$. Postulate (R5) is an adaptation of the logical uniformity postulate for belief-base operators (BR5). It says that the revision outcomes w.r.t. to the revision operator $*$ are determined by the uniform sets implied by the revision result.

As the representation theorem below shows, postulates (R1)–(R5) are sufficient to represent the class of implication based choice revision operators modulo equivalence.

Theorem 25. *A revision operator $*$ fulfils the postulates (R1)–(R5) iff it can be equivalently described as $\dot{+}_c^{\mathit{Impl}}$ for some selection function c.*

Proof. 'Left to right': Let B, α be given. Clearly $\dot{+}_c^{\mathit{Impl}}$ fulfils (R1) and (R2). Let Im_k denote the set of implications underlying the belief extension chosen by c and let H_k be the set of prime implicates corresponding to Im_k according to Theorem 24. The fulfilment of (R3) follows by letting $H = \{H_k\}$. (R4) is fulfilled because $B \dot{+}_k \alpha \models H_k$ and for all other uniform sets H the maximality of H_k implies $B \dot{+}_k \alpha \models \neg \bigwedge H$. (R5) holds because if α and β are consistent with the same set of uniform sets, they are consistent with same set of implications. The definition of selection function guarantees that for α and β the same set of consistent implications and thus the same uniform set is implied.

'Right to left': Let B, α be given. Let $(\mathit{Impl}_i)_{i \in I}$ be the set of belief extensions to the given bc scenario. I show, there is a selection function c s.t. $B * \alpha \equiv B \dot{+}_c^{\mathit{Impl}} \alpha$. It can be assumed that B, α is consistent. According to (R3') there is $H \in U^{\mathit{Impl}}(B)$ such that $B * \alpha \equiv H \wedge \alpha$ or $B * \alpha \equiv H$. As (R2) is fulfilled, $B * \alpha \models \alpha$ and so $B * \alpha \equiv H \wedge \alpha$. The set of implications $(\mathit{Impl}_i)_{i \in I}$ induces a set $(H_i)_{i \in I}$ of uniform sets w.r.t. B. This follows from Theorem 24. Because $B * \alpha$ is consistent (according to (R1)) it follows that $\bigwedge H \wedge \alpha$ is consistent. Hence there is a H_k such that $H \subseteq H_k$, because all H_i are maximal uniform sets consistent with α. Because of tenacity $B * \alpha \models H_k$ or $B * \alpha \models \neg \bigwedge H_k$. But in the last case one would have $H \wedge \alpha \models \neg \wedge H_k$ or equivalently $\bigwedge H \wedge \bigwedge H_k \models \neg \alpha$ or equivalently $H_k \models \neg \alpha$, contradicting the consistency of H_k with α. Therefore $B * \alpha \models \bigwedge H_k \wedge \alpha$ and $\bigwedge H_k \wedge \alpha \models B * \alpha$. So one can set $c(I) = k$. Then $B * \alpha \equiv B \dot{+}_c^{\mathit{Impl}} \alpha$. Now if β is such that the belief change scenario $\langle B, \beta, \emptyset \rangle$ has exactly the same set $(\mathit{Impl}_i)_{i \in I}$ of belief extensions, then one has to guarantee that one chooses again Impl_k. Here comes uniformity to the rescue: The set of uniform sets w.r.t. B that are consistent with $B * \beta$ and the set of uniform sets consistent with $B * \alpha$ are the same. Therefore the logical uniformity postulate (R5) implies that the same H_k is chosen. $\square$

6 Related Work

The work described in this paper follows in general the belief-revision tradition as initiated by the pioneering work of Alchourrón, Gärdenfors and Makinson (AGM) (Alchourrón et al. (1985)), but has main differences due to a different explanation of the inconsistencies. Moreover, classical belief-revision functions à la AGM operate on a logically closed set called *belief set* and a formula which triggers the revision of the belief set into a new belief set. In belief-base revision (Hansson (1991)) the revised KB is allowed to be an arbitrary not necessarily closed (finite) set of sentences called *belief bases*. The negative property of belief-base revision of being syntax sensitive is remedied in the case of *knowledge-base revision operators* which are exemplified by the revision operators of this paper as well as those of Delgrande and Schaub (2003) and Dalal (1988).

The revision operators of this paper are based on the elimination of hypotheses that have the role of semantic mappings (Noy (2004)). The idea of using belief revision techniques to revise semantic mappings has already been worked in the literature Meilicke and Stuckenschmidt (2009), Qi et al. (2009). But these approaches consider the set of semantic mappings as the object of revision, while the approach of this paper considers the semantic mappings as revision aids that are deleted after the revision.

The notion of a prime implicate is used in the approaches of Pagnucco (2006), Zhuang et al. (2007), Bienvenu et al. (2008). In contrast to the approach of this paper, these do not use prime implicates in the formulation of the postulates; they (only) define new belief-revision operators based on prime implicates and show that they fulfill some classical postulates.

The implication based revision operators exhibit a symbol-oriented rather than a sentence-oriented strategy for conflict resolution. A different symbol-oriented approach is described in the work of Lang and Marquis (2010). Their revision operators are not based on hypotheses but on the well-known concept of forgetting (Lin and Reiter (1994)).

7 Conclusion and Outlook

I have presented a new type of revision operator, which resulted as a generalization of Delgrande's and Schaub's operators (Delgrande and Schaub (2003)) by considering implications rather then biimplications as hypotheses. The resulting operators are finitely representable (Proposition 11) and can be simulated by classical partial meet revision operators that operate on the set of hypotheses as left argument (Proposition 15). Moreover, implication based choice revision can be characterized by a set of postulates (Theorem 25).

I motivated the perspective to consider the sets of biimplications and implications as hypotheses on the semantical relatedness of symbols belonging to different name spaces. This perspective leads naturally to the question what other initial sets of hypotheses on the semantical relatedness could be used as a basis for new revision operators. In fact one could consider bridging axioms like $p' \leftrightarrow q$, which relate symbols hypothesized to be synonyms. Using a set H of such creative hypothesis may induce operators that are quite different from classical revision operators as the former may not be conservative: $B' \cup H$ may imply formulas $\beta \in \text{form}(\mathcal{P})$ that do not already follow from B. Such creative behavior does not occur for $H = Im_i$ or $H = EQ_i$.

The idea of hypothesis based revision can also be applied to more complex logics like description logics or predicate logic. (Confer the operators of Qi et al. (2009) for revising semantic mappings.) The general idea is to make hypotheses about the relations of the predicate symbols and constants in the different name spaces by stating, e.g., the equivalence of the unary predicate symbol P' and P by $\forall x P'(x) \leftrightarrow P(x)$. The revision operators become more complex ; moreover, it cannot be guaranteed that the conflicts can be solved by disambiguation—the knowledge bases of the sender and the receiver may be *reinterpretation incompatible* because they imply different cardinalities for their domains (Özçep (2008)). But the notion of uniform sets can also be defined for predicate logics and its fragments—though the prime implicate concept may not be purely semantical (Özçep (2009)).

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