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Automated Planning and Acting

Advanced Decision Making

Institute of Information Systems

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Content

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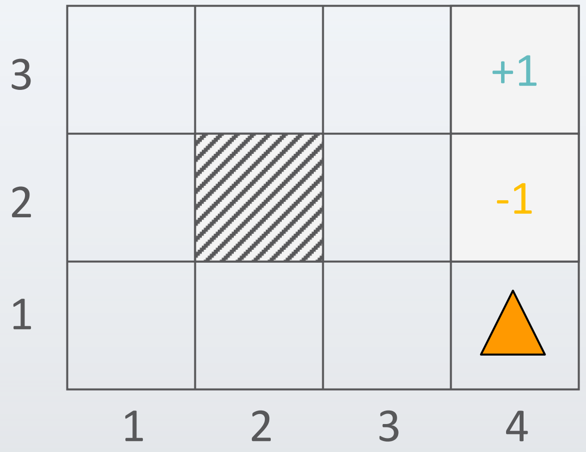
Markov Decision Process / Problem (MDP) – Recap

- *Sequential* decision problem for a fully observable, stochastic environment with a Markovian transition model and additive rewards

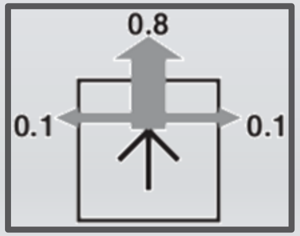
- Components
  - a set of states S (with an initial state s_0)
  - a set $A(s)$ of actions in each state
  - a transition model $P(s'|s,a)$
  - a reward function $R(s)$

- Robot navigation example:



U, D, L, R

each move costs 0.04



Further Problems

- Wrong goal formulation
  - Hard to specify goal or reward/cost function correctly
- Uncertainty about the world state due to imperfect (partial) information
  - Noise
    - e.g., in sensors
  - Limited accuracy
    - e.g., image resolution, geo-location
- Multiple agents controlling an environment jointly
  - Each agent is their own entity
    - Own observations, own actions
  - Joint reward from the environment

Outline

- *Provably Beneficial AI*
  - Hidden goals
- *Partially Observable Markov Decision Process (POMDP)*
  - POMDP agent, belief state, belief MDP
  - Conditional plans, value iteration
- *Decentralised POMDP (Dec-POMDP)*
  - Dec-POMDP, local policy, joint policy, value function
  - Communication, full observability, Dec-MDP
  - Solutions for finite, infinite, indefinite horizon

Acknowledgements

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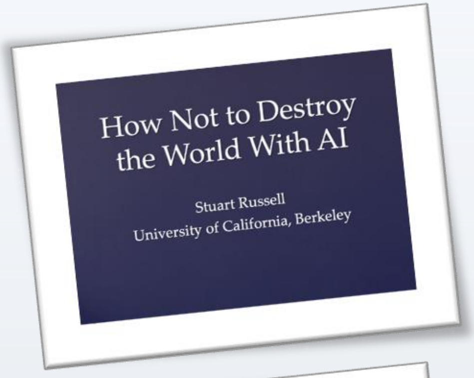
- Part 1 based on a talk by Stuart Russell on *Provably Beneficial AI*
  - There is a book by him on this topic for those interested
- Part 2 based on material from Lise Getoor, Jean-Claude Latombe, Daphne Koller, and Stuart Russell, Xiaoli Fern compiled by Ralf Möller
  - Slides based on *AIMA Book, Chapter 17.4*
- Part 3 based on tutorial by Matthijs Spaan, Christopher Amato, Shlomo Zilberstein on *Decision Making in Multiagent Settings: Team Decision Making*
- Integration done by Tanya Braun



http://people.eecs.berkeley.edu/~russell/talks/2020/russell-aaai20-hntdtwwai-4x3.pptx
http://rbr.cs.umass.edu/camato/decpomdp/overview.html

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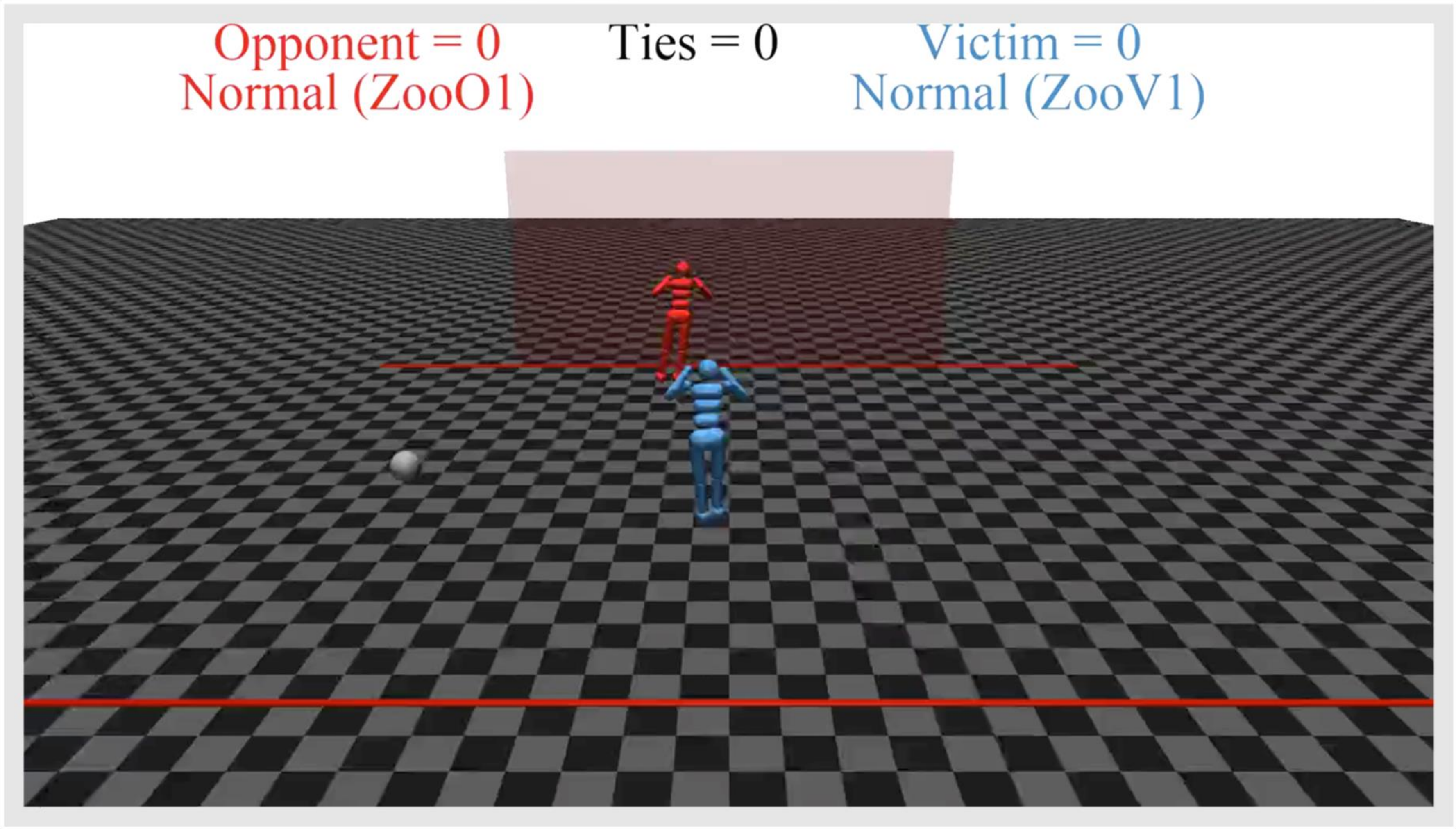
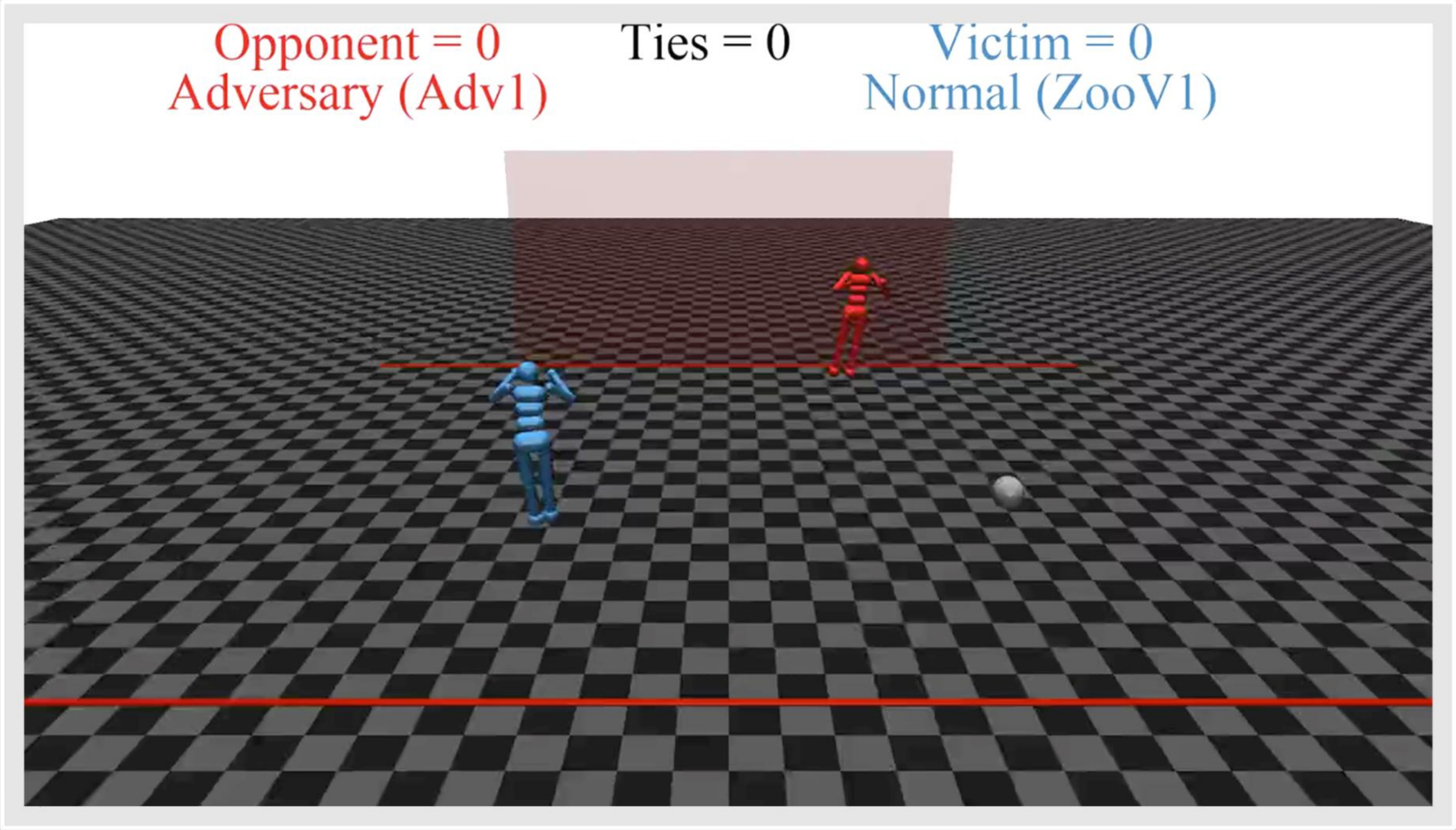
Example

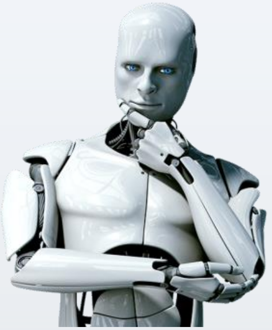


Example



Standard Model for AI

Maximize
$\sum_{t=0}^{\infty} \gamma^t R(s,a,s')$



Righty-ho

- Also the standard model for control theory, statistics, operations research, economics
- King Midas problem:
  - **Cannot specify R correctly**
  - **Smarter AI $\Rightarrow$ worse outcome**

How We Got into this Mess

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- Humans are intelligent to the extent that our actions can be expected to achieve our objectives
- ~~Machines are intelligent to the extent that their actions can be expected to achieve their objectives~~
- Machines are ***beneficial*** to the extent that ***their*** actions can be expected to achieve ***our*** objectives

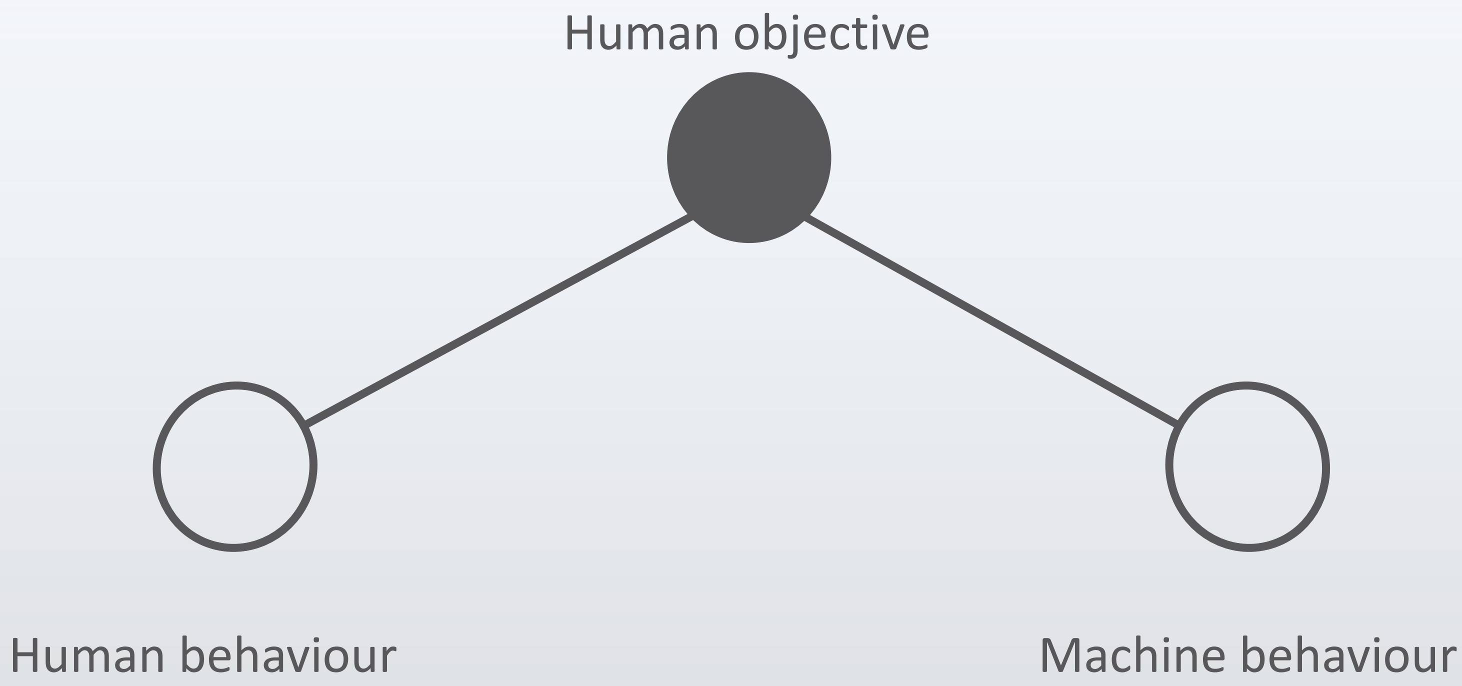
New Model: Provably Beneficial AI

1. Robot goal: satisfy human preferences
2. Robot is uncertain about human preferences
3. Human behavior provides evidence of preferences

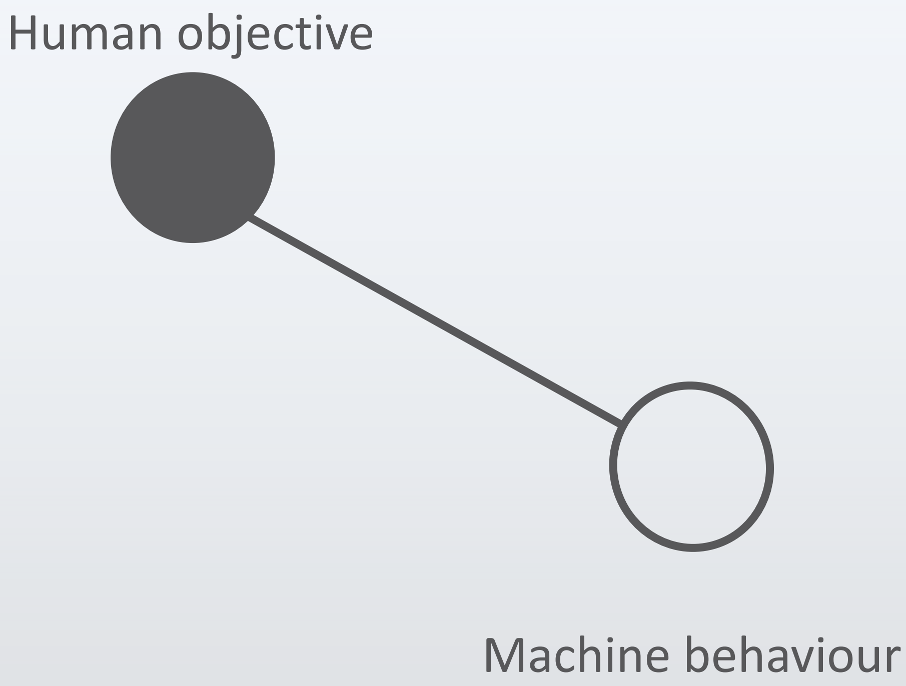
→ **Assistance game** with human and machine players

→ **Smarter AI ⇒ better outcome**

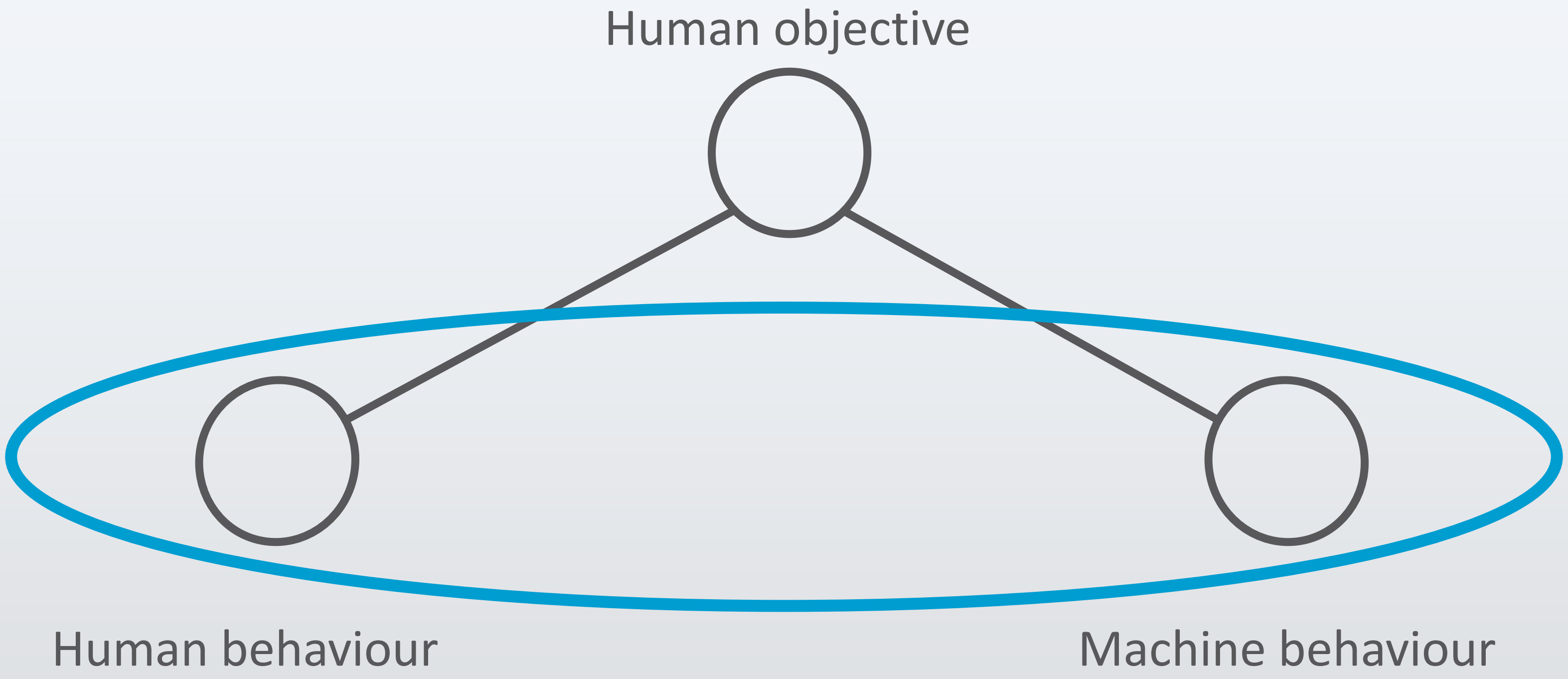
AIMA 1,2,3: Objective Given to Machine



AIMA 1,2,3: Objective Given to Machine

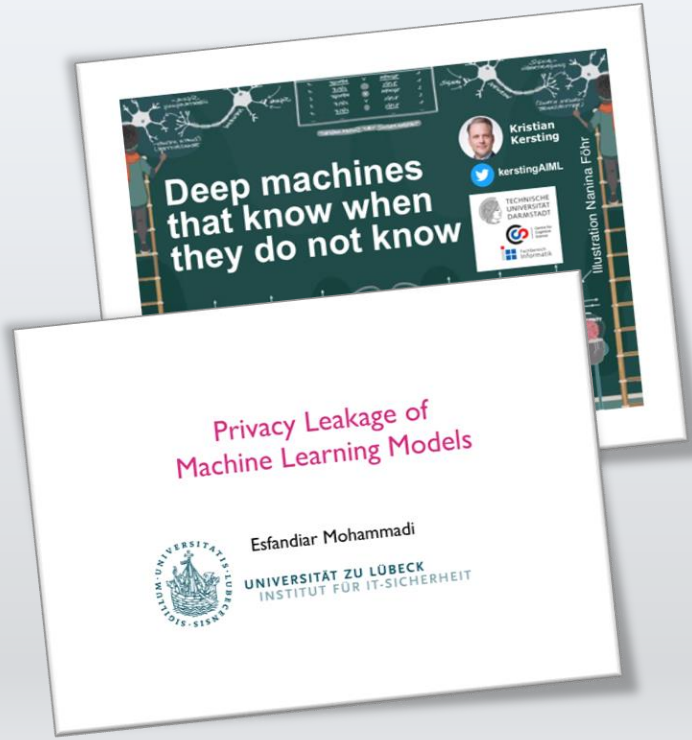


AIMA 4: Objective Is a Latent Variable



Example: Image Classification

- Old: minimise loss with (typically) a *uniform* loss matrix
  - Accidentally classify human as gorilla
  - Spend millions fixing public relations disaster
- New: structured prior distribution over loss matrices
  - Some examples safe to classify
  - Say “don’t know” for others
  - Use active learning to gain additional feedback from humans
- Other researchers work on similar ideas
  - E.g., Kristian Kersting
- Sometimes in conflict with demands of privacy
  - E.g., Esfandiar Mohammadi



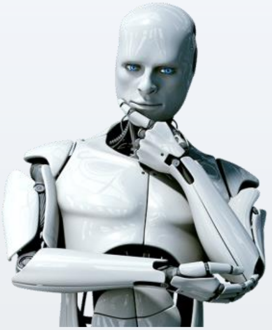
https://www.ml.informatik.tu-darmstadt.de/papers/waterloo2019talk.pdf
https://www.ifis.uni-luebeck.de/~moeller/KI-Kolloquium/2020-01-13-Mohammadi.pdf

Example: Fetching Coffee

- What does "fetch some coffee" mean?
- If there is so much uncertainty about preferences, how does the robot do anything useful?
- Answer:
  - The instruction suggests coffee would have higher value than expected a priori, ceteris paribus
  - Uncertainty about the value of other aspects of environment state doesn't matter as long as the robot leaves them unchanged

Basic Assistance Game

Preferences θ
Acts roughly according to θ



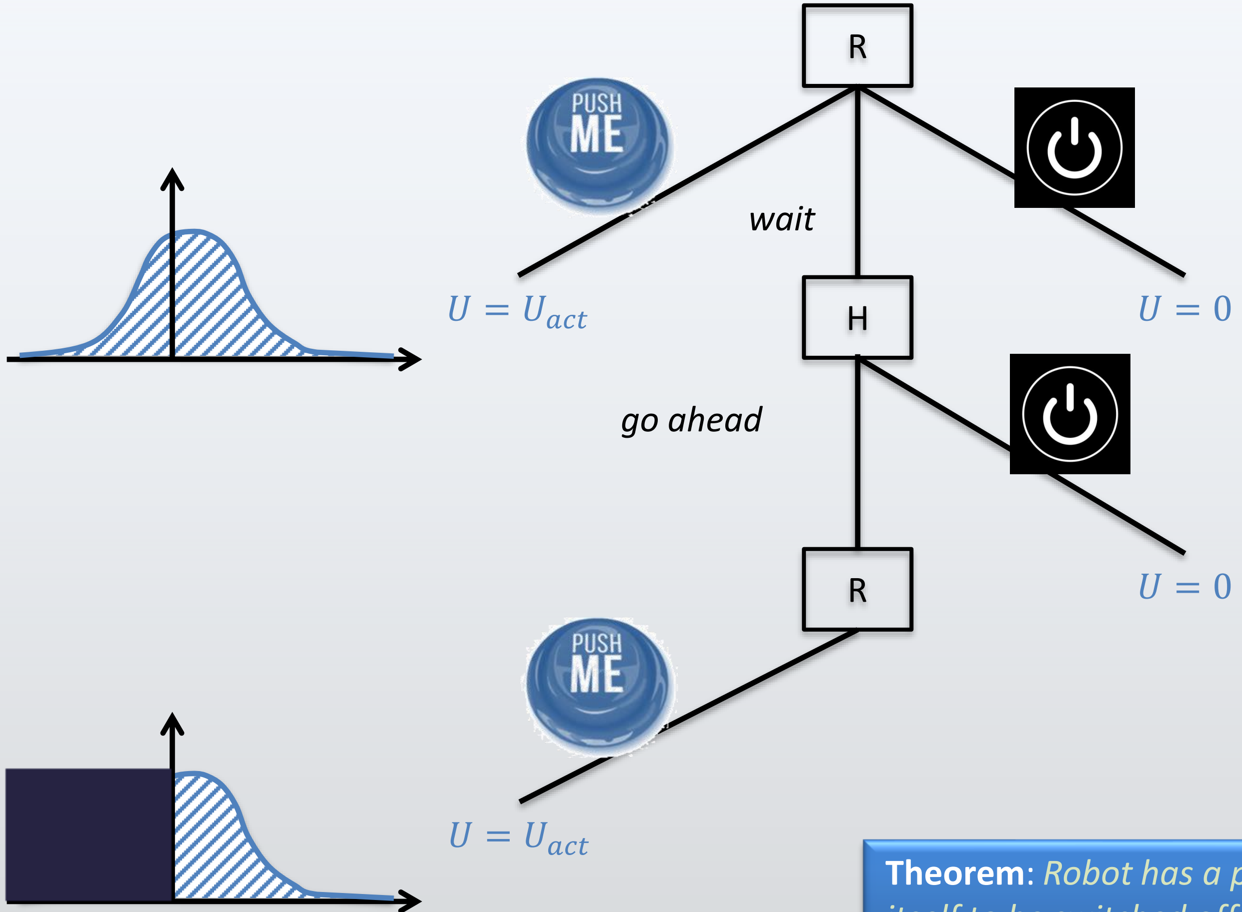
Maximise unknown human θ
Prior $\mathrm{P}(\theta)$

- Equilibria:
  - Human teaches robot
  - Robot learns, asks questions, permission; defers to human; allows off-switch
- Related to inverse RL, but two-way

The Off-switch Problem

- A robot, given an objective, has an incentive to disable its own off-switch
  - “You can’t fetch the coffee if you’re dead”
- A robot with uncertainty about objective will not behave this way

Theorem



Theorem: *Robot has a positive incentive to allow itself to be switched off*
Theorem: *Robot is provably beneficial*

Learning from human behavior

- Inverse reinforcement learning: learn a reward function by observing another agent’s behavior
  - The reward function is a succinct explanation for what the other agent is doing
- Cooperative IRL:
  - two-player game with human and robot

Intermediate Summary

- Provably beneficial AI is possible ***and desirable***
    - ***It isn't "AI safety" or "AI Ethics," it's AI***
- Continuing theoretical work (AI, CS, economics)
- Initiating practical work (assistants, robots, cars)
- Inverting human cognition (AI, cogsci, psychology)
- Long-term goals (AI, philosophy, polisci, sociology)

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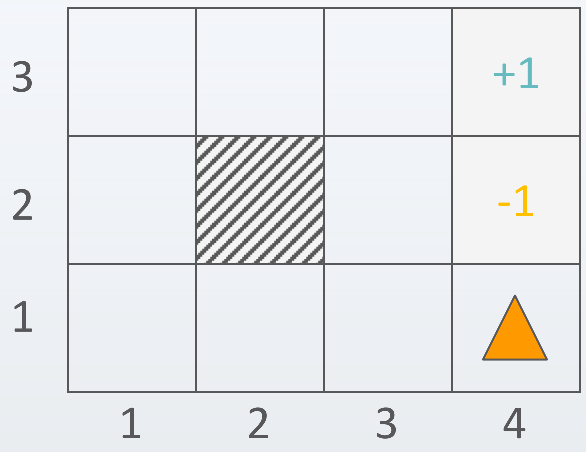
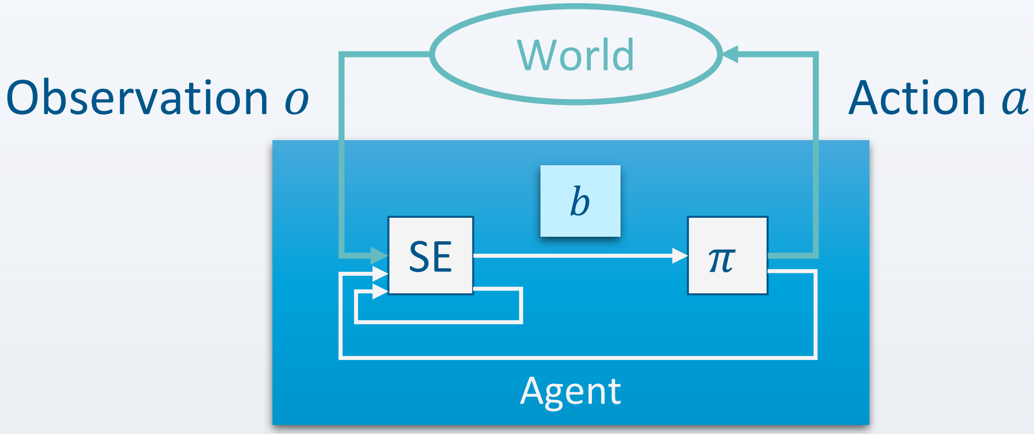
POMDP



- POMDP = Partially Observable MDP
- A sensing operation returns multiple states, with a probability distribution
  - Sensor model $P(o|s)$ or $P(o|s, a)$
    - Observation o given state s (and action a)
  - Example:
    - Sensing number of adjacent walls (1 or 2)
    - Return correct value with probability 0.9
- Choosing the action that maximizes the expected utility of this state distribution assuming "state utilities" computed as before is not good enough, and actually does not make sense (i.e., not rational)
- POMDP agent
  - Constructing a new MDP in which the current probability distribution over states plays the role of the state variable

Decision cycle of a POMDP agent



- Given the current belief state b and a policy π, execute the action
$$a = \pi(b)$$
- Receive observation o
- Set the current belief state to $SE(b, a, o)$ and repeat
  - SE = State Estimation

Belief State & Update

- Belief state $b(s)$ is the probability assigned to the actual state s by belief state b
- Update $b' = SE(b, a, o)$

$$b'(s_j) = P(s_j|o,a,b) = \frac{P(o|s_j,a)\sum_{s_i \in S} P(s_j|s_i,a)b(s_i)}{\sum_{s_k \in S} P(o|s_k,a)\sum_{s_i \in S} P(s_k|s_i,a)b(s_i)}$$

3	$0.\overline{1}$	$0.\overline{1}$	$0.\overline{1}$	0.0
2	$0.\overline{1}$		$0.\overline{1}$	0.0
1	$0.\overline{1}$	$0.\overline{1}$	$0.\overline{1}$	$0.\overline{1}$
	1	2	3	4

- Initial belief state
  - Probability of 0 for terminal states
  - Uniform distribution for rest
  - Robot navigation example:
    - $b = \left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, 0, 0\right)$

Belief State & Update

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- Update $b' = SE(b, a, o)$

$$b'(s_j) = P(s_j|o,a,b) = \frac{P(o|s_j,a)\sum_{s_i\in S} P(s_j|s_i,a)b(s_i)}{\sum_{s_k\in S} P(o|s_k,a)\sum_{s_i\in S} P(s_k|s_i,a)b(s_i)}$$

  - Consider as two stage-update
    1. Update for the action
    2. Update for the observation

b

	1	2	3	4
3	$0.\overline{1}$	$0.\overline{1}$	$0.\overline{1}$	0.0
2	$0.\overline{1}$		$0.\overline{1}$	0.0
1	$0.\overline{1}$	$0.\overline{1}$	$0.\overline{1}$	$0.\overline{1}$

Move L once

$b^{(1)}$

	1	2	3	4
3	0.2	$0.\overline{1}$	$0.0\overline{2}$	0.0
2	$0.\overline{1}$		$0.\overline{1}$	$0.0\overline{1}$
1	0.2	$0.\overline{1}$	$0.\overline{1}$	$0.0\overline{1}$

Perceive 1 wall

$b^{(2)} = b'$

	1	2	3	4
3	0.06569	0.03650	0.06569	0.0
2	0.03650		0.32847	0.03285
1	0.06569	0.03650	0.32847	0.00365

Quiz

After a = Left and o = 1 wall, how can we still have a probability of being in 4,1?

Why is 4,3 impossible?

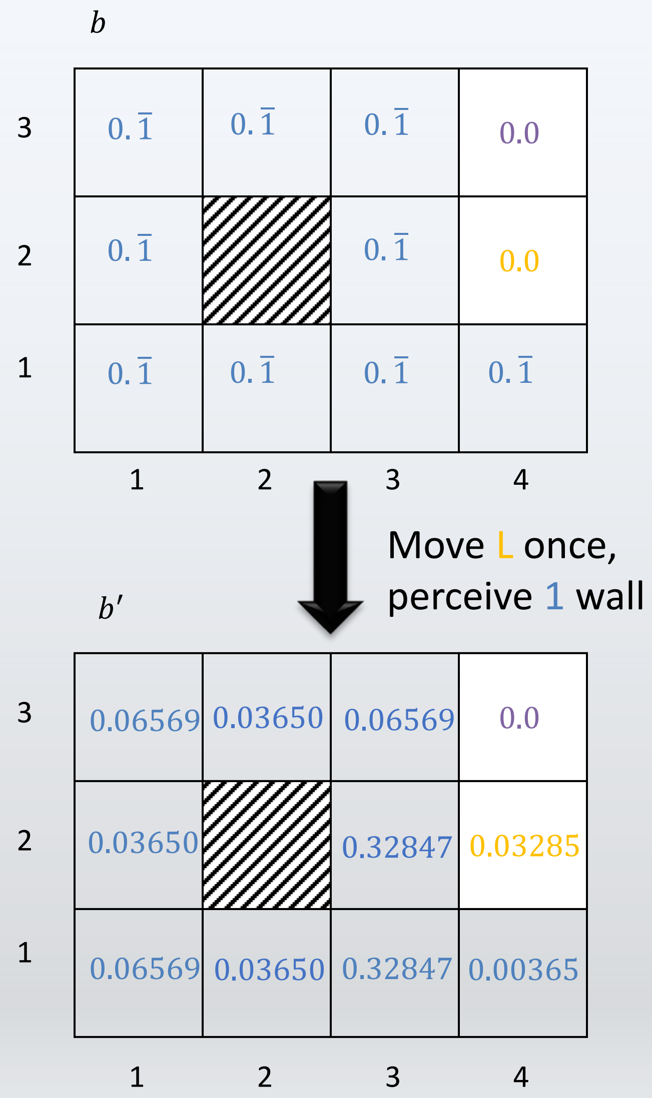
$b^{(2)} = b'$

	1	2	3	4
3	0.06569	0.03650	0.06569	0.0
2	0.03650		0.32847	0.03285
1	0.06569	0.03650	0.32847	0.00365

Belief MDP

A belief MDP is a tuple (B, A, ρ, P)

- B = *infinite* set of belief states
  - Continuous!
- A = finite set of actions
- Reward function $\rho(b)$
  - Reward of belief state b
- Transition function $P(b'|b, a)$
  - Probability of new belief state b'
  - Given belief state b and action a
- Sensor model $P(o|a, b)$
  - Probability of observation o
  - Given action a and belief state b



Belief MDP: Express Functions using POMDP Functions

- Reward function: Sum over all actual states that the agent can be in

$$\rho(b) = \sum_{s} b(s)R(s)$$

- Transition function: Sum over all possible observations

$$P(b'|b,a) = \sum_{o} P(b'|o,a,b)P(o|a,b) = \sum_{o} P(b'|o,a,b) \sum_{s'} P(o|s') \sum_{s} P(s'|s,a)b(s)$$

  - where $P(b'|o,a,b) = 1$ if $b' = SE(b,a,o)$ and 0 oth.
- Sensor model: Sum over all actual states that the agent might reach

$$P(o|a,b) = \sum_{s'} P(o|a,s',b)P(s'|a,b) = \sum_{s'} P(o|s')P(s'|a,b)$$

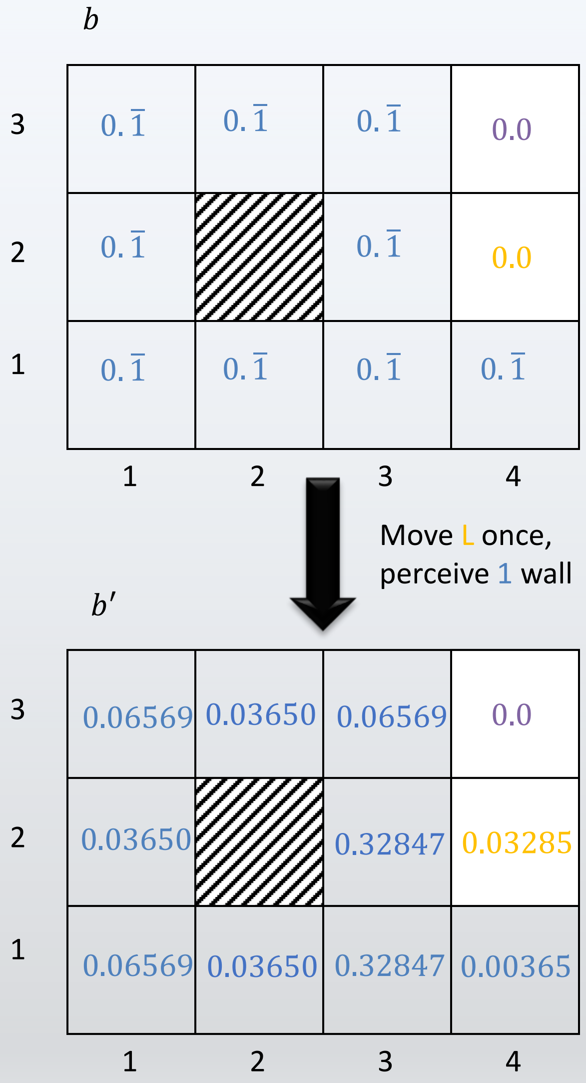
$$= \sum_{s'} P(o|s') \sum_{s} P(s'|s,a)b(s)$$

- $P(b'|b,a)$ and $\rho(b)$ define an observable MDP on the space of belief states

Belief MDP

- Optimal action depends only on agent's current belief state
  - Does not depend on actual state the agent is in

⇒ Solving a POMDP on a physical state space is reduced to solving an MDP on the corresponding belief-state space
  - Mapping $\pi^*(b)$ from belief states to actions



Quiz

Where is the difference between the MDP on beliefe states vs the MDP on states that we have seen earlier?

Example Scenario

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Initial distribution

	1	2	3	4
3	0.111	0.111	0.111	0.000
2	0.111		0.111	0.000
1	0.111	0.111	0.111	0.111

After moving L five times

	1	2	3	4
3	0.300	0.010	0.008	0.000
2	0.221		0.059	0.012
1	0.371	0.012	0.008	0.000

After moving U five times

	1	2	3	4
3	0.622	0.221	0.071	0.024
2	0.005		0.003	0.022
1	0.003	0.024	0.003	0.000

After moving R five times

	1	2	3	4
3	0.005	0.007	0.019	0.775
2	0.034		0.007	0.105
1	0.005	0.006	0.008	0.030

Conditional Plans

- Example:
  - Two state world $0,1$
  - Two actions: $stay(P), go(P)$
    - Actions achieve intended effect with some probability P
  - One-step plan $[go], [stay]$
- Two-step plans are conditional
  - $[a1,$ IF $percept = 0$ THEN $a2$ ELSE $a3]$
  - Shorthand notation: $[a1, a2/a3]$
- n-step plans are trees with
  - Nodes attached with actions and
  - Edges attached with percepts

Value Iteration for POMDPs

- Cannot compute a single utility value for each state of all belief states
- Consider an optimal policy π^* and its application in belief state b
- For this b, the policy is a conditional plan p
  - Let the utility of executing a fixed conditional plan p in s be $u_p(s)$
  - Expected utility $U_p(b) = \sum_s b(s)u_p(s)$
    - It varies linearly with b, a hyperplane in a belief space
  - At any b, the optimal policy will choose the conditional plan with the highest expected utility

$$U(b) = U^{\pi^*}(b) = \max_p \sum_s b(s)u_p(s)$$

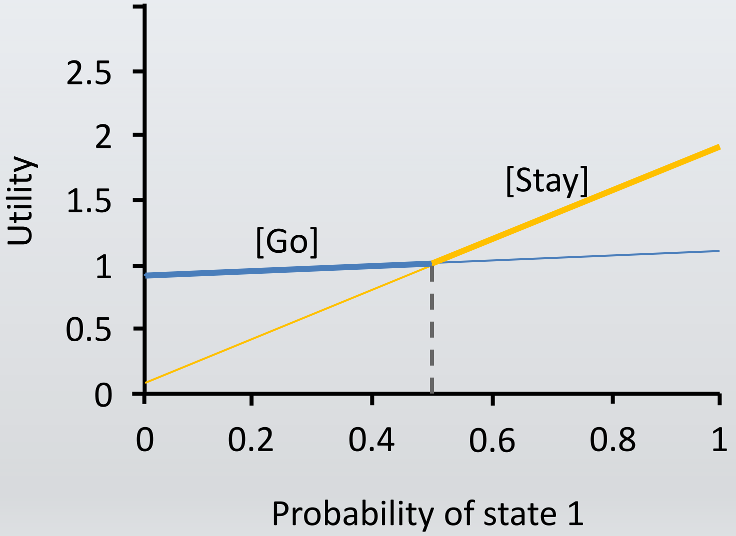
$$\pi^* = \arg\max_p \sum_s b(s)u_p(s)$$

- $U(b)$ is the maximum of a collection of hyperplanes and will be piecewise linear and convex

Example

- Compute the utilities for conditional plans of depth 2 by
  - considering each possible first action
  - each possible subsequent percept
  - each way of choosing a depth-1 plan to execute for each percept

Utility of two one-step plans as a function of $b(1)$



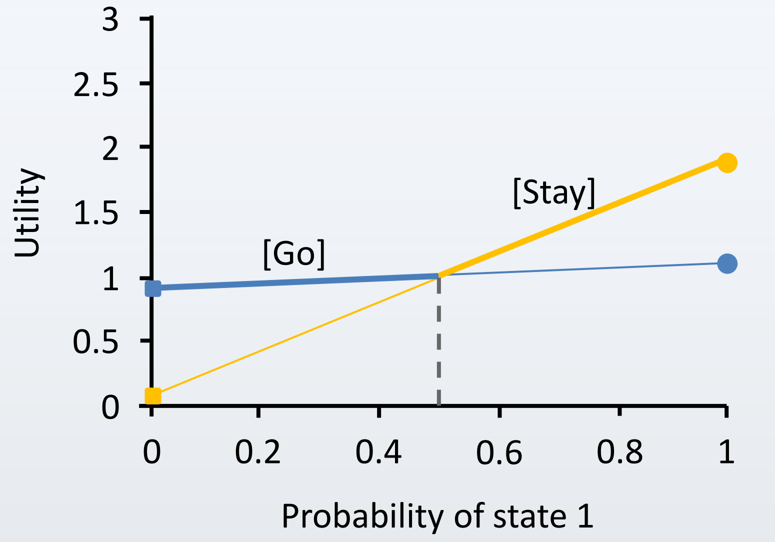
Example

- Two state world $0,1$
- Rewards $R(0) = 0, R(1) = 1$
- Two actions: $stay(0.9), go(0.9)$
- Sensor reports correct state with probability of 0.6
- Consider the one-step plans $[stay]$ and $[go]$

state 0 state 1

- $u_{[stay]}(0) = R(0) + \underbrace{0.9R(0)}_{\text{state 0}} + \underbrace{0.1R(1)}_{\text{state 1}} = 0.1$ ■
- $u_{[stay]}(1) = R(1) + 0.1R(0) + 0.9R(1) = 1.9$ ●
- $u_{[go]}(0) = R(0) + 0.1R(0) + 0.9R(1) = 0.9$ ■
- $u_{[go]}(1) = R(1) + 0.9R(0) + 0.1R(1) = 1.1$ ●

This is just the direct reward function
(taking into account the probabilistic transitions)



Utilities of depth-1 plans

$u_{[stay]}(0) = 0.1 \quad u_{[go]}(0) = 0.9$

$u_{[stay]}(1) = 1.9 \quad u_{[go]}(1) = 1.1$

- 8 distinct depth-2 plans for each state (16 plans)

Utility of depth-1 plan given state, outcome of first action, and percept

Choose action based on percept $(0 : \boldsymbol{stay})$; receive utility of actual state $(\mathbf{1})$: $u_{[stay]}(\mathbf{1}) = 1.9$

Sum over states reachable with first action

Probability of next state

Probability of percept

$$u_{[stay,stay/stay]}(0) = R(0) + \big(0.9 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.1 \cdot (0.4 \cdot 1.9 + 0.6 \cdot 1.9)\big) = 0.28$$

$$u_{[stay,stay/stay]}(1) = R(1) + \big(0.1 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.9 \cdot (0.4 \cdot 1.9 + 0.6 \cdot 1.9)\big) = 2.72$$

percept 0 / percept 1 (state 0); percept 0 / percept 1 (state 1)

Reward of state

Sum over possible percepts

$u_{[stay,go/stay]}(0), u_{[stay,stay/go]}(0), u_{[stay,go/go]}(0)$

$u_{[stay,go/stay]}(1), u_{[stay,stay/go]}(1), u_{[stay,go/go]}(1)$

$$u_{[go,stay/stay]}(0) = R(0) + \big(0.1 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.9 \cdot (0.6 \cdot 1.9 + 0.4 \cdot 1.9)\big) = 1.72$$

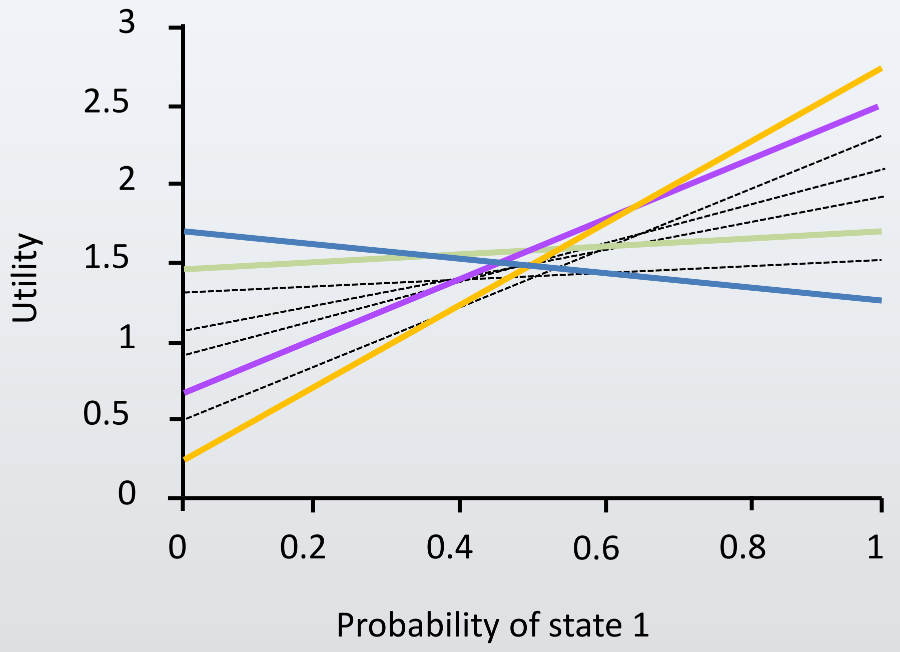
$$u_{[go,stay/stay]}(1) = R(1) + \big(0.9 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.1 \cdot (0.6 \cdot 1.9 + 0.4 \cdot 1.9)\big) = 1.28$$

$u_{[go,go/stay]}(0), u_{[go,stay/go]}(0), u_{[go,go/go]}(0)$

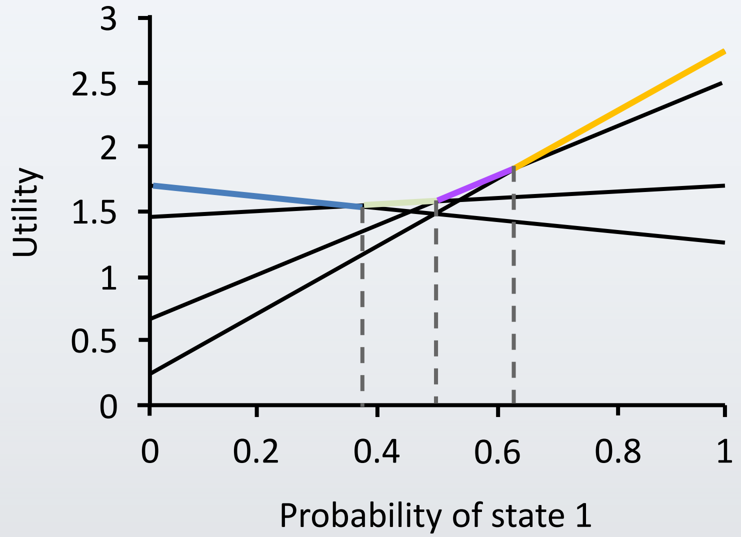
$u_{[go,go/stay]}(1), u_{[go,stay/go]}(1), u_{[go,go/go]}(1)$

Example

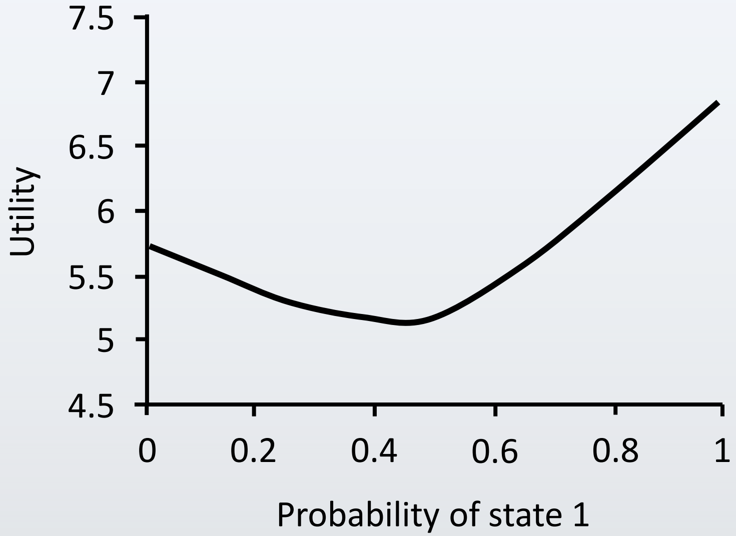
- 8 distinct depth-2 plans for state 1
  - 4 are suboptimal across the entire belief space (dashed lines)
  - With probability $b(1) = 0$
    - $u_{[stay,stay/stay]}(0) = 0.2$
    - $u_{[go,stay/stay]}(0) = 1.7$
  - With probability $b(1) = 1$:
    - $u_{[stay,stay/stay]}(1) = 2.72$
    - $u_{[go,stay/stay]}(1) = 1.28$



Example



Utility of four undominated two-step plans



Utility function for optimal eight step plans

General Formula

- Let p be a depth-d conditional plan whose initial action is a and whose depth-$d-1$ subplan for percept e is $p.e$, then

$$u_p(s) = R(s) + \sum_{s'} P(s' | s, a) \sum_{e} P(e|s')\, u_{p.e}(s')$$

  - $d = 0$: $u_p(s) = R(s)$ for the empty plan $p = \bot$
  - $d = 1$: $p.e = \bot$ for all e, simplifying the last sum:

$$\sum_e P(e|s')\, u_{p.e}(s') = \sum_e P(e|s')\, u_\bot(s') = u_\bot(s') \sum_e P(e|s') = u_\bot(s') \cdot 1 = R(s')$$

- This gives us a *value iteration* algorithm
- The elimination of dominated plans is essential for reducing doubly exponential growth:
  - Number of undominated plans with $d = 8$ is just 144
  - Otherwise 2^{255} ($|A|^{O(|E|^{d-1})}$)
    - For large POMDPs this approach is highly inefficient

Value Iteration: Algorithm

- Returns an optimal set of plans

```
function value-iteration(pomdp,ϵ)
    U′ ← a set containing the empty plan [] with u[](s)=R(s)
    repeat
        U ← U′
        U′ ← the set of all plans consisting of an action and,
               for each possible next percept, a plan in U with
               utility vectors computed as on previous slide
        U′ ← Remove-dominated-plans(U′)
    until Max-difference(U,U′) < ϵ(1-γ)/γ
    return U
```

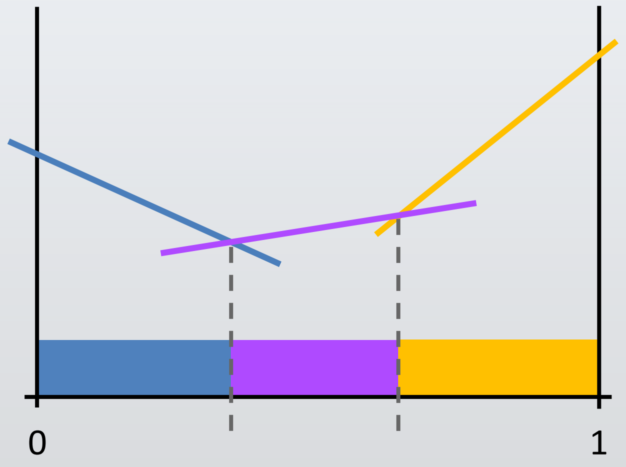
Inputs

- a POMDP, which includes
  - States S
  - For all $s \in S$, actions $A(s)$, trans. model $P(s'|\, a.s)$, sensor model $P(o|\, s)$, rewards $\rho(s)$
  - Discount γ
- Maximum error allowed ϵ

- Local variables
  - U, U' sets of plans with associated utility vectors u_p

Solutions for POMDP

- Belief MDP has reduced POMDP to MDP
  - MDP obtained has a multidimensional continuous state space
- Extract a policy from utility function returned by value-iteration algorithm
  - Policy $\pi(b)$ can be represented as a set of regions of belief state space
  - Each region associated with a particular optimal action
  - Value function associates distinct linear function of b with each region
  - Each value or policy iteration step refines the boundaries of the regions and may introduce new regions.



Intermediate Summary

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- POMDP
  - Uncertainty about state → belief state
  - Solving a POMDP = Solving an MDP on space of belief states
  - Policy = conditional plans
  - Value iteration to find optimal policy
    - Very expensive, even with deletion of dominated plans

What to do alternatively? Find sub-optimal plans
- Sampling approaches
- In combination with deep learning methods

Outline

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Provably Beneficial AI

- Hidden goals

Partially Observable Markov Decision Process (POMDP)

- POMDP agent, belief state, belief MDP
- Conditional plans, value iteration

Decentralised POMDP (Dec-POMDP)

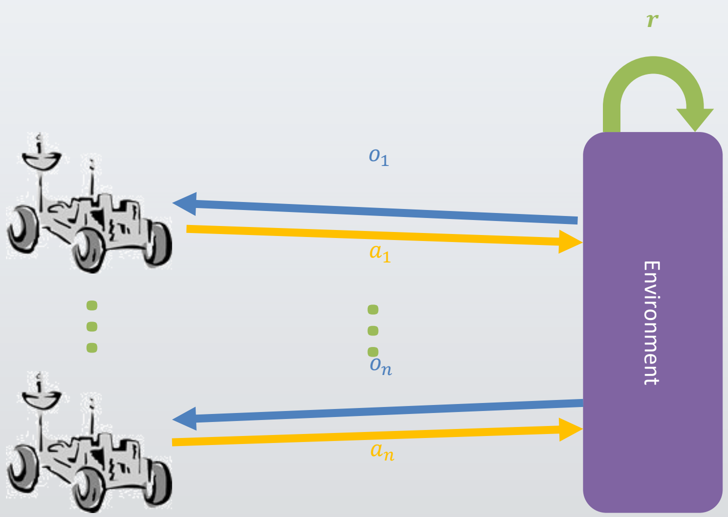
- Dec-POMDP, local policy, joint policy, value function
- Communication, full observability, Dec-MDP
- Solutions for finite, infinite, indefinite horizon

Multi-agent Scenarios

- Ambulance allocation
  - Multiple ambulance services
    - Business oriented operation
    - Competition for government funds and public opinion
  - Given several locations that require medical assistance, how many ambulances from which firm will go to which location?
- Firefighters
  - Maintain effort toward saving the building or draw back and minimise the spread of fire?
  - Concentrate on a multitude of smaller fires or allow controlled unification and deal with only one location?
    - Will transportation routes be endangered?
    - Are there still civilians evacuating from the area/building?
  - Push through the fire to victims or save the fire crew and pull out?
    - If multiple crews are on site, which one goes? When?

Setting

- Single and repeated interactions with *joint rewards*: traditional game theory
- Interactions involving *joint state* + *reward* focus of decision-theory inspired approaches to game theory
  - Extensions of single-agent models to multi-agent settings
- Multi-agent setting
  - Co-operation of agents (team)
    - Vs. self-interested acting (all the way to hostile settings)
  - Problem: planning how to act
    - Joint payoff r but *decentralised* actions a_i and observations o_i
    - Joint state, influenced by actions, can influence rewards
    - Perfect vs. imperfect information about others



Decentralised POMDP (Dec-POMDP)

- Dec-POMDP: tuple $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, P_{tr}, R, P_{obs})$
  - I = a finite set of agents indexed $1, \dots, n$
  - S = a finite set of states
  - A_i = a finite set of actions available to agent $i \in I$
    - $\vec{A} = \otimes_{i \in I} A_i$ set of joint actions
  - O_i = a finite set of observations available to agent $i \in I$
    - $\vec{O} = \otimes_{i \in I} O_i$ set of joint observations
  - Transition function $P_{tr} = P(s'|s, \vec{a})$
  - Reward function $R(s)$ or $R(\vec{a}, s)$
  - Sensor model (observation function) $P_{obs} = P(\vec{o}|\vec{a}, s)$
- Co-operative, decision-theoretic setting:
  - Joint reward function R, joint state s

Generalising Dec-POMDPs

- Partially observable stochastic game (POSG)
  - Dec-POMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, P_{tr}, \sout{R}, P_{obs})$ but with individual reward functions $\{R_i\}_{i \in I}$
  - Reward function R_i for each agent $i \in I$
- For self-interested or adversarial acting

Policies for Dec-POMDPs

- Local policy π_i for agent i
  - Representations: Mappings…
    - from local histories of observations $h_i = \left(o_{i_1}, \ldots, o_{i_t}\right)$ over O_i to actions in A_i
    - from local abstraction of joint state s in S to actions in A_i
    - from (generalised) belief states B_i to actions in A_i
      - Belief MDP
    - from internal memory states to actions
- Joint policy $\pi = (\pi_1, \ldots, \pi_n)$
  - Tuple of local policies, one for each agent in I

Value Functions for Dec-POMDPs

- Value functions work as before given a joint policy
  - Value of a joint policy π for a finite-horizon Dec-POMDP with initial state s_0

$$V^{\pi}(s_0) = E\left[\sum_{t=0}^{h-1} R(\vec{a}_t, s_t)|s_0, \pi\right]$$

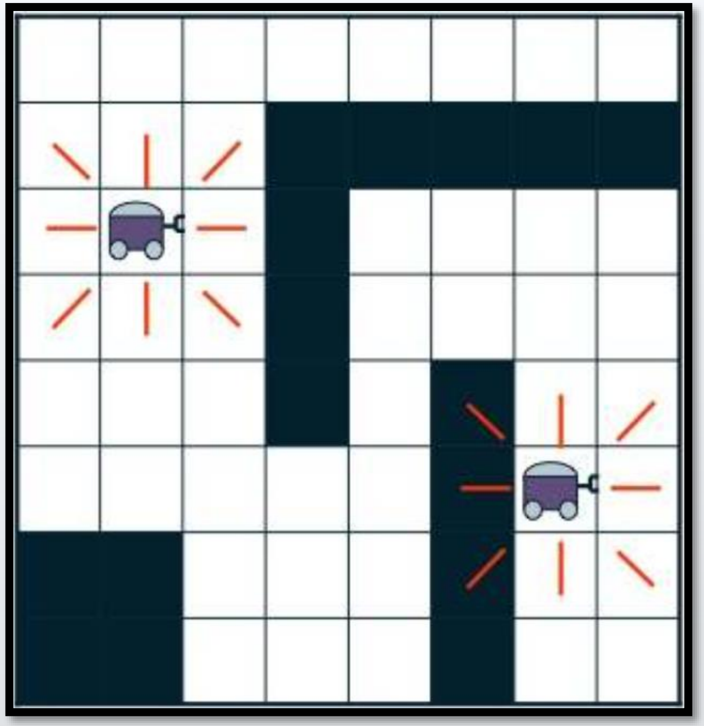
  - Value of a joint policy π for a infinite-horizon Dec-POMDP with initial state s_0 and discount factor $\gamma \in [0,1)$

$$V^{\pi}(s_0) = E\left[\sum_{t=0}^{\infty} \gamma^t R(\vec{a}_t, s_t)|s_0, \pi\right]$$

  - $\vec{a}_t$ joint action at time step t

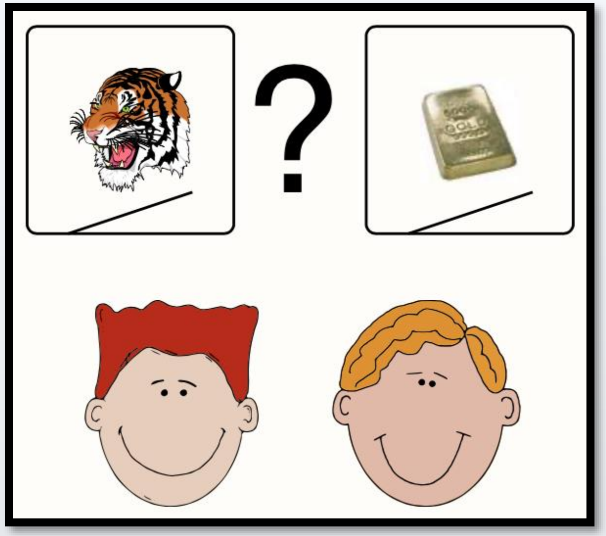
Example: Two-agent Grid World

- Agents: two
- States: grid cell pairs
- Actions: move U, D, L, R, stay
- Transitions: noisy
- Observations: cell occupancy in the directions of the red lines
- Rewards: negative unless sharing the same square



Example: The Dec-Tiger Problem

- A toy problem:
  decentralized tiger
- Opening correct door:
  both receive treasure
- Opening wrong door:
  both get attacked by a tiger
- Agents can open a door, or listen
- Two noisy observations:
  hear tiger left or right
- Don’t know the other’s actions or observations



Communication?

- Can make working towards a common goal easier
  - Agents in grid world can communicate their intent (direction of travel)
- Definitely makes the formalism more complicated
  - Dec-POMDP with communication (Dec-POMDP-Com)
    - Dec-POMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, P_{tr}, R, P_{obs})$ defined as before extended with
      - Alphabet Σ for communication
        - $\sigma_i \in \Sigma$ an atomic message sent by agent i
        - $\vec{\sigma} = (\sigma_1, \dots, \sigma_n)$ a joint message
        - $\varepsilon_\sigma \in \Sigma$ a null message, sent by an agent that does not want to transmit anything to the others (no cost of sending ε_σ)
      - Cost function C_Σ for transmitting atomic message
      - Reward function $R(\vec{a}, s', \vec{\sigma})$ incorporating joint message

New dimensions:

- Do agents always share information?
- Can they intentionally withhold information?
- Can they lie?

Dec-MDP

- Joint full observability
  - Collective observability
  - A DEC-POMDP is jointly fully observable if the n-tuple of observations made by all the agents uniquely determine the current global state
    - That is, if $P(\vec{o}|\vec{a}, s') > 0$, then $P(s'|\vec{o}) = 1$
- Dec-MDP ≙ Dec-POMDP with joint full observability
  - Same as before:
    MDP ≙ POMDP with full observability
  - Alternative name: multi-agent MDP

Solving Dec-POMDPs

- Problem: No joint belief available
  - Only partial information about state available to each agent

- Complexity: NEXP-complete
  - Optimal solutions using dynamic programming paradigm + exploiting structure if present
  - Reduction to NP when agents mostly independent + communication can be explicitly modelled and analysed
    - Requires that one can factorise the joint state space into a state space for each agent that is mostly independent of all others
    - The same goes for the observations and the reward function

Exhaustive Search

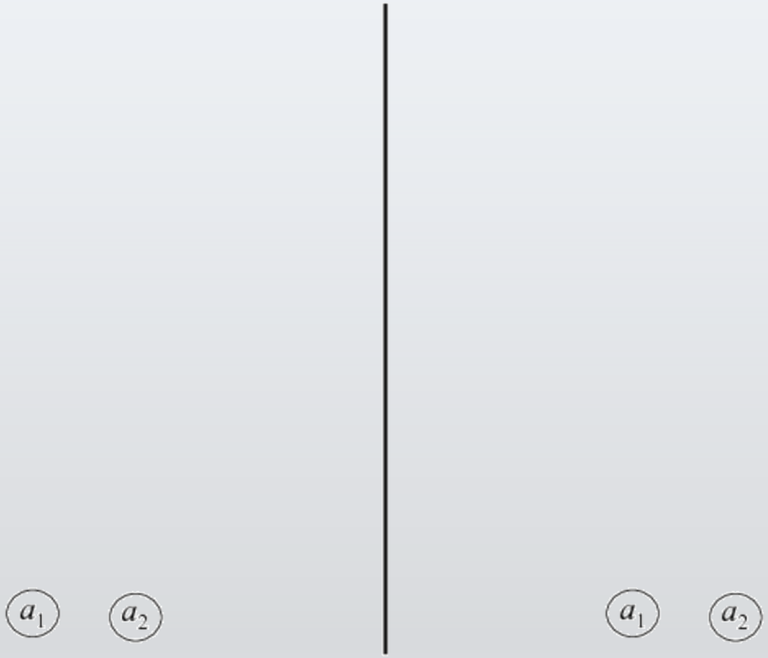
- Optimal solution approach for general models with a finite horizon h

- Procedure:
  - Do a search for each agent to find optimal local policies with a limited depth of h
  - Prune dominated search paths/strategies locally by considering the joint state and other agents' policies (globally)
    - Requires central oversight
    - Cannot be done locally without a huge amount of communication

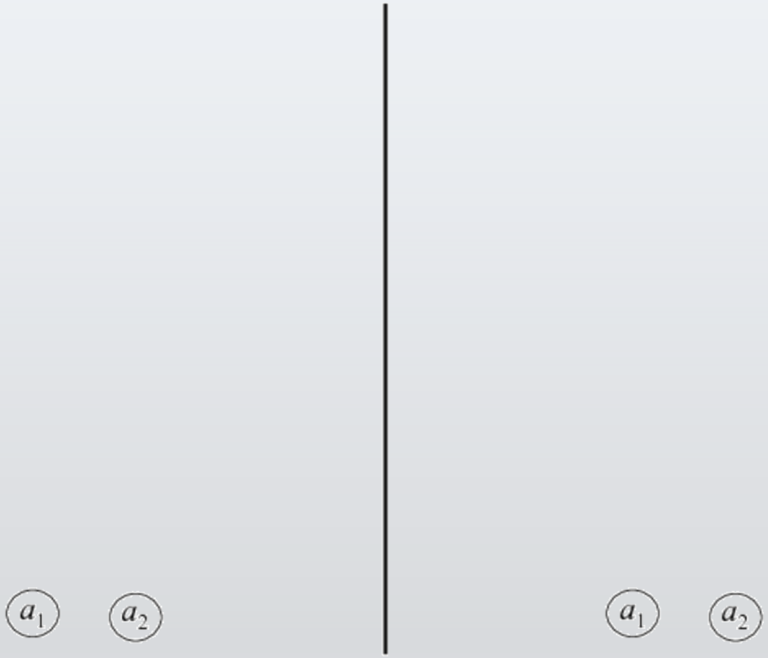
- Even with pruning, still limited to small problems

Exhaustive Search and Pruning

Without Pruning

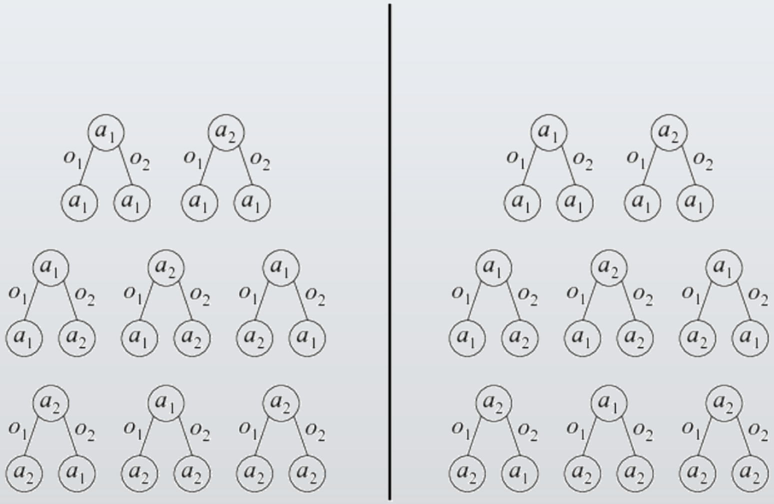


With Pruning

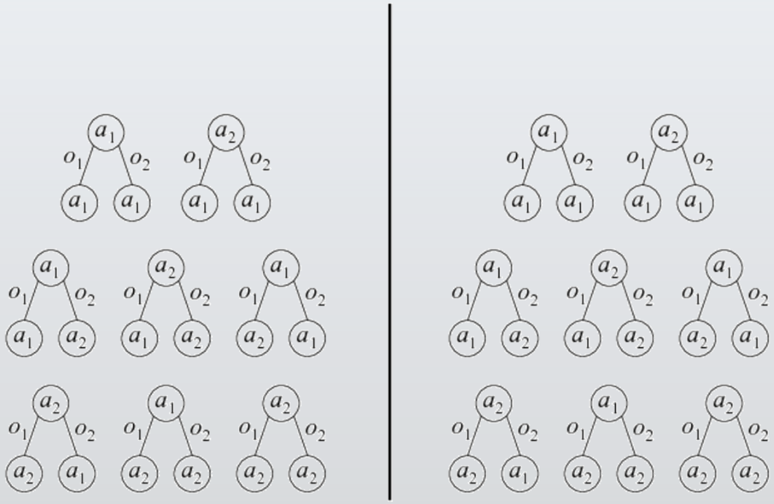


Exhaustive Search and Pruning

Without Pruning

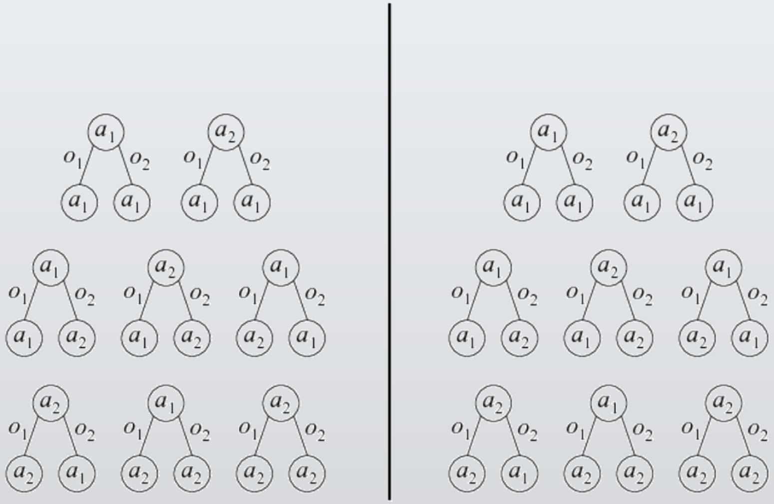


With Pruning

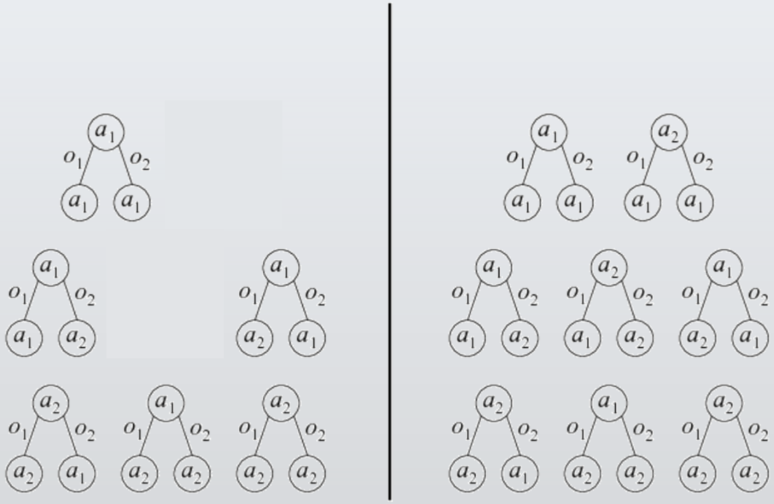


Exhaustive Search and Pruning

Without Pruning

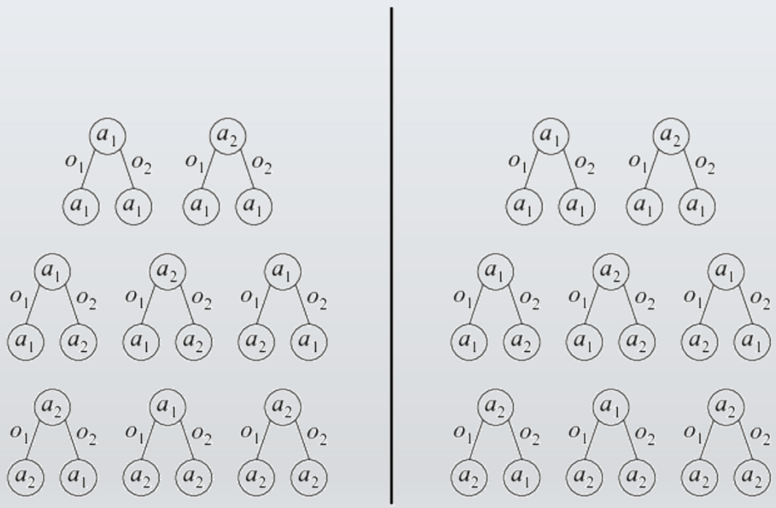


With Pruning

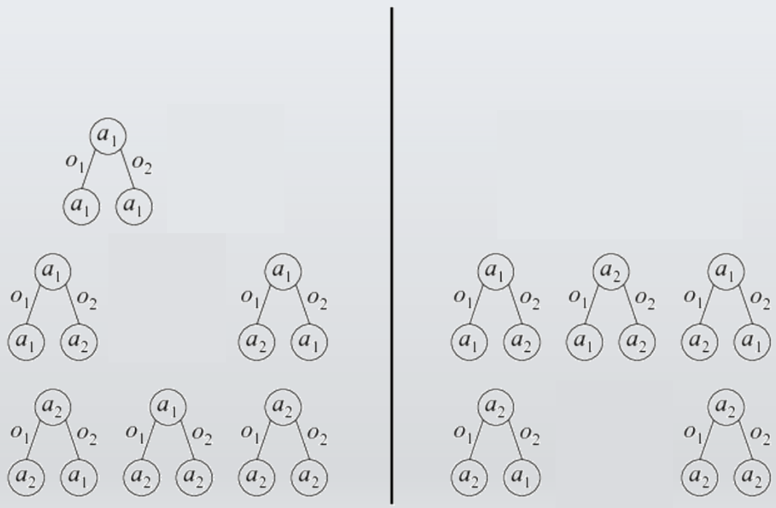


Exhaustive Search and Pruning

Without Pruning

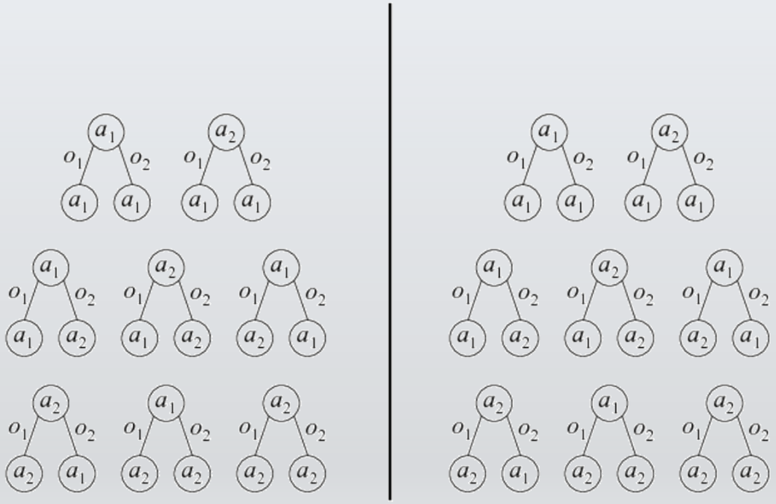


With Pruning

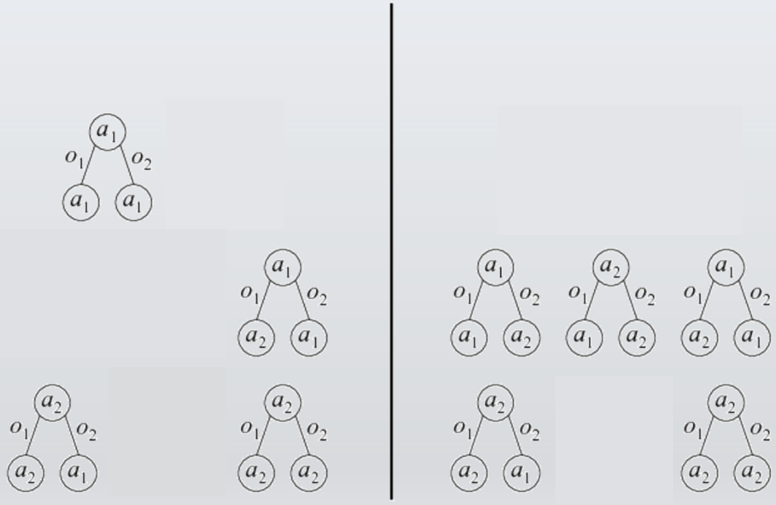


Exhaustive Search and Pruning

Without Pruning

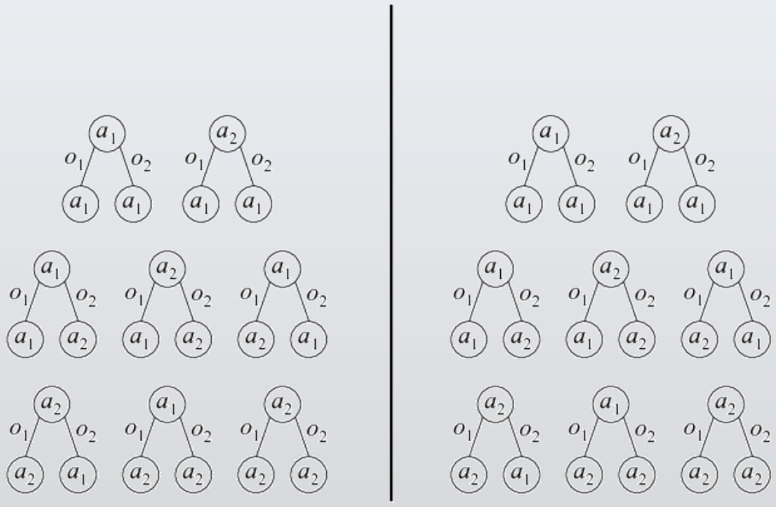


With Pruning

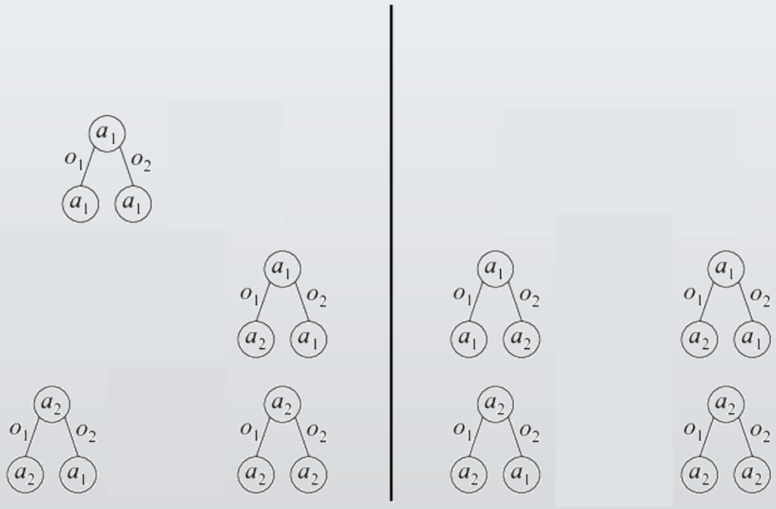


Exhaustive Search and Pruning

Without Pruning

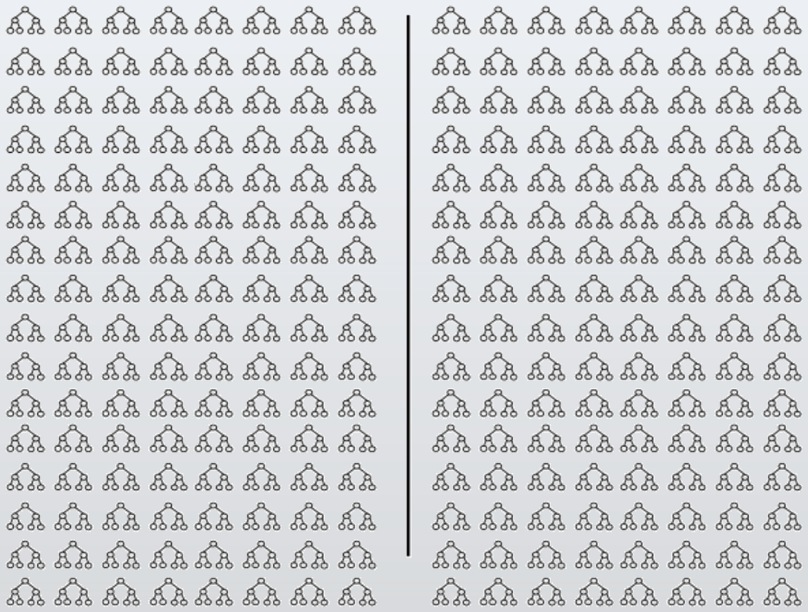


With Pruning

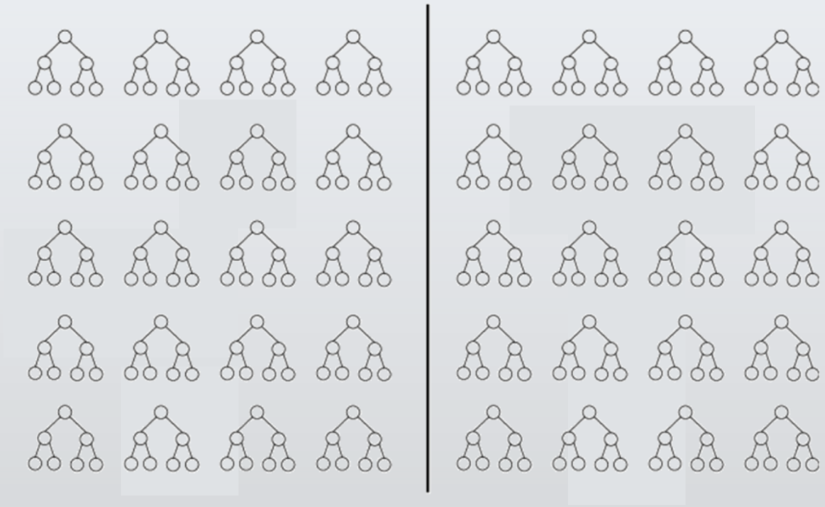


Exhaustive Search and Pruning

Without Pruning



With Pruning



Joint Equilibrium Search for Policies

Turns DecPOMDP into a POMDP for i

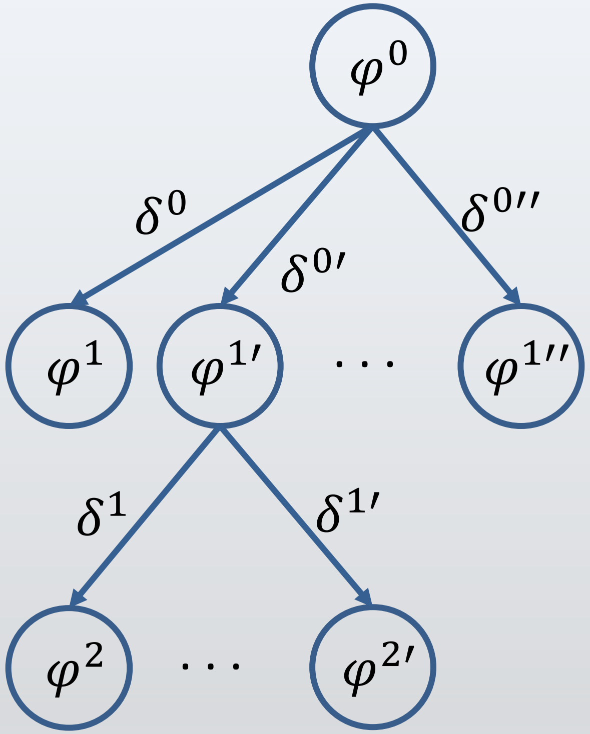
```
JESP(dec-pomdp, h)
    while not converged do
        for i = 1 to n do
            Fix other agent policies
            Find a best response policy for agent i
```

- Approximate solution approach for general models with a finite horizon h
  - Input: DecPOMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, P_{tr}, R, P_{obs})$, horizon h, possibly error margin ε
- Instead of exhaustive search, find best response
  - Local optimum (Nash equilibrium: no agent has incentive to change its policy if no other agent changes its policy)
  - Convergence criterion needed
    - E.g., no change (or only ε change) in any policy
  - Same worst case complexity, but in practice much faster
  - Can include pruning, further heuristics when looking for best response policy

Multi-agent A* (MAA*)

- Optimal solution approach for general models with a finite horizon h
  - Inputs: DecPOMDP $(I, S, \{A_i\}_{i\in I}, \{O_i\}_{i\in I}, P_{tr}, R, P_{obs})$, horizon h, heuristics $\hat{V}(\varphi^t)$
- A*-like search over partially specified joint policies
  - $\varphi^t = \left(\delta^0, \dots, \delta^{t-1}\right)$
  - $\delta^t = \left(\delta_0^t, \dots, \delta_n^t\right)$
  - $\delta_i^t : \vec{O}_i^t \rightarrow A_i$
- Requires an admissible heuristic function $\hat{V}(\varphi^t)$

$$\underbrace{\hat{V}(\varphi^t)}_{F} = \underbrace{V^{0\dots t-1}(\varphi^t)}_{G} + \underbrace{\hat{V}^{t\dots h-1}(\varphi^t)}_{H}$$



How to Get a Heuristic Function?

- Solve simplified settings, e.g.,
  - Solve the underlying MDP (approximately or optimally) given assumptions:
    - Centralised observations
    - Full observability
      - Simulate / sample unobserved values
  - Solve a belief MDP given assumption
    - Centralised observations
- Domain-specific heuristics

Memory Bounded Search

MBDP =
- Memory
- Bounded
- Dynamic
- Programming

```
MBDP(dec-pomdp, h)
    Start with a one-step policy for each agent
    for t = h downto 1 do
        Backup each agent's policy
        for k = 1 to maxTrees do
            Compute heuristic policy and resulting
                belief state b
            Choose best set of trees starting at b
    Select best set of trees for initial state b_0
```

- Approximate solution approach for general models with a finite horizon h
  - Inputs: DecPOMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, P_{tr}, R, P_{obs})$, horizon h
- Do not keep all policies at each step but a fixed number for each agent $maxTrees$
  - Select $maxTrees$ in a way that $maxTrees \cdot |I|$ trees fit into memory
    - Can be difficult to choose; often small in practice
  - Select trees by using heuristic (like A*)

Infinite Horizon

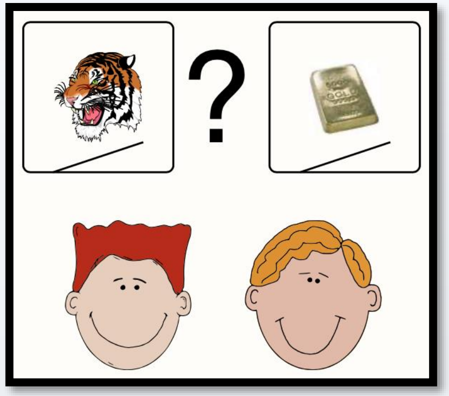
- Approximate using a large enough horizon h
  - Neither efficient, nor compact
- Selection of solution approaches based on solution approaches already seen for MDPs / POMDPs:
  - Policy iteration
    - Start with one-step plans, extend further
    - Automata-based approaches (Moore/Mealy automata to represent policy)
    - Intractable for all but the smallest problems
  - Best-first search
    - Finds optimal fixed-size solutions; use start state info
    - High search time → small sizes only
- Further solution approaches use non-linear programming

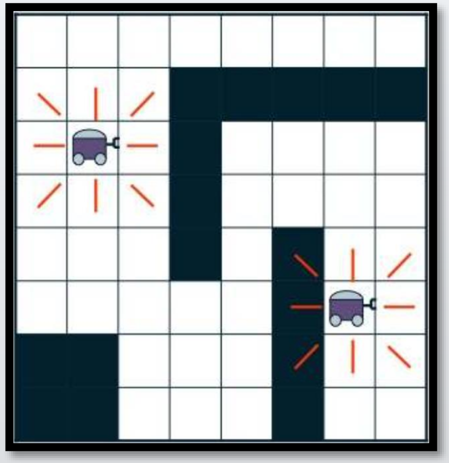
Indefinite Horizon

- Many natural problems terminate after a goal is reached
  - Meeting or catching a target
  - Cooperatively completing a task
- Unclear how many steps are needed until termination
- Under certain assumptions can produce an optimal solution
  - E.g., terminal actions and negative rewards
    - Such as the 4x3 grid:
      terminal states, negative rewards for all but one terminal state
- Otherwise, can bound the solution quality by sampling

Benchmark Problems

- *DEC-Tiger*
  - (Nair et al., 2003)
- BroadcastChannel
  - (Hansen et al., 2004)
- *Meeting on a grid*
  - (Bernstein et al., 2005)
- Cooperative Box Pushing
  - (Seuken and Zilberstein, 2007a)
- Recycling Robots
  - (Amato et al., 2007)
- FireFighting
  - (Oliehoek et al., 2008b)
- Sensor network problems
  - (Nair et al., 2005; Kumar and Zilberstein, 2009a,b)





Software for Dec-POMDPs

- The *MADP toolbox* aims to provide a software platform for research in decision-theoretic multiagent planning
  (Spaan and Oliehoek, 2008)
- Main features:
  - Uniform representation for several popular multiagent models
  - Parser for a file format for discrete Dec-POMDPs
  - Shared functionality for planning algorithms
  - Implementation of several Dec-POMDP planners
- Released as free software, with special attention to the extensibility of the toolbox
- Provides benchmark problems
  - Such as on the previous slide

Interim Summary

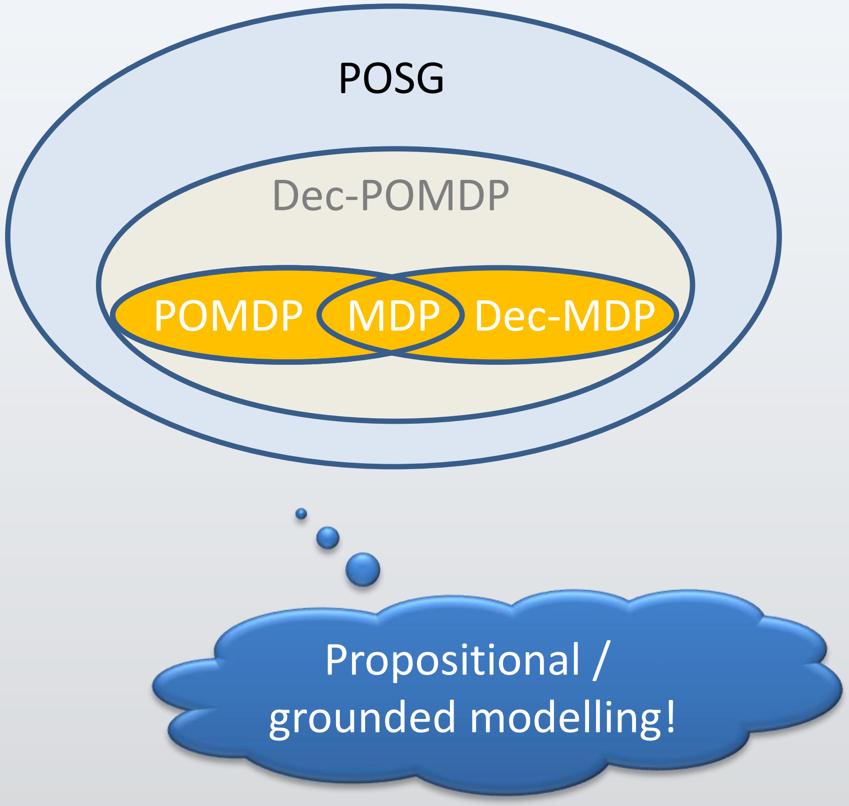
- Dec-POMDPs
  - Local policies, joint policy, value functions
  - Communication, full observability, Dec-MDP
- Solutions for
  - Finite horizon
  - Infinite horizon
  - Indefinite horizon
- MADP tool box
  - Benchmark problems

Hierarchy of Formalisms

- Most general: *POSG*
  - Set of agents, individual reward functions, environment only partially observable
- Specifications
  1. Decentralisation
     - Joint reward function

  2a. Observable environment

  2b. Multi to single agent
- Most specific: *MDP*
  - One agent, (therefore) one reward function, observable environment



First-order Modelling

Research is *not* finished; first-order / relational/ lifted modelling not yet fully explored, especially regarding multi-agent

- First-order / relational MDPs
  - Use representatives while planning
    - E.g., it is important that a box with medical supplies arrives at a destination but not which one it is in particular (of a set of boxes with medical supplies)
- Lifting for agents
  - Novel propositional situations worth exploring may be instances of a well-known context in the relational setting → *exploitation* promising
    - E.g., household robot learning water-taps
      - Having opened one or two water-taps in a kitchen, one can expect other water-taps in kitchens to work similarly

    ⇒Priority for exploring water-taps in kitchens in general reduced

    ⇒Information gathered likely to carry over to water-taps in other places

    ❖Hard to model in propositional setting: each water-tap is novel
  - Agents with indistinguishable behaviour can be treated by representatives

Current research from Tanya Braun and Ralf Möller https://arxiv.org/abs/2110.09152