

# Advanced Topics Data Science and AI

# Automated Planning and

# Acting

## Temporal Models

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# Content

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1. Planning and Acting with **Deterministic** Models
2. Planning and Acting with **Refinement** Methods
3. Planning and Acting with **Temporal** Models
  - a. Temporal Representation
  - b. Planning with Temporal Refinement Methods
  - c. Constraint Management
  - d. Acting with Temporal Models
4. Planning and Acting with **Nondeterministic** Models
5. Making Simple Decisions
6. Making Complex Decisions
7. Planning and Acting with **Probabilistic** Models
8. Provably Beneficial AI
  - Other: open world, perceiving, learning
    - If time permits

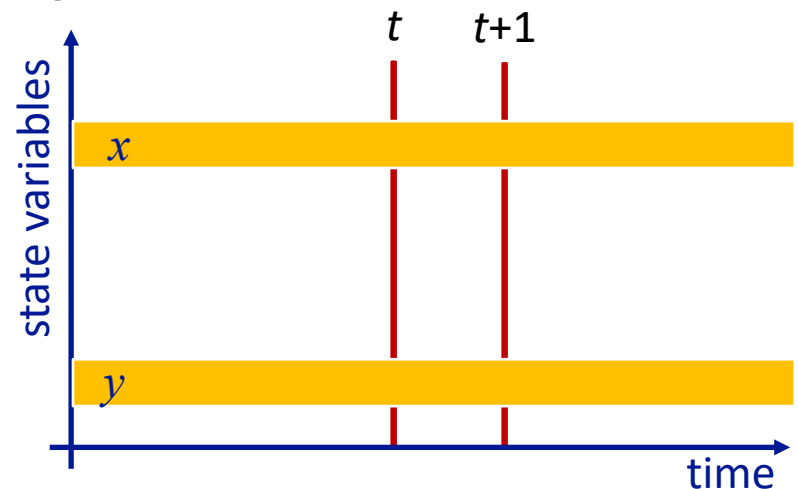
# Temporal Models

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- Durations of actions
- Delayed effects and preconditions
  - E.g., resources borrowed or consumed during an action
- Time constraints on goals
  - Relative or absolute
- Exogenous events expected to occur in the future
  - When?
- Maintenance actions:
  - Maintain a property ( $\neq$  changing a value)
  - E.g., track a moving target, keep a spring latch in position
- Concurrent actions
  - Interacting effects, joint effects
- Delayed commitment
  - Instantiation at acting time

# Timelines

- Up to now, “state-oriented view”
  - Time is a sequence of states  $s_0, s_1, s_2$
  - Instantaneous actions transform each state into the next one
  - No overlapping actions
- Switch to a “time-oriented view”
  - Sequence of integer time points
    - $t = 1, 2, 3, \dots$
  - For each state variable  $x$ , a **timeline**
    - values during different time intervals
  - State at time  $t = \{\text{state-variable values at time } t\}$



- Sets of constraints on state variables and events
  - Reflect predicted actions and events
- Planning is constraint-based



# Outline per the Book

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## **4.2 Representation**

- Timelines
- Actions and tasks
- Chronicles

## **4.3 Temporal planning**

- Resolvers and flaws
- Search space

## **4.4 Constraint management**

- Consistency of object constraints and time constraints
- Controlling the actions when we don't know how long they'll take

## **4.5 Acting with temporal models**

- Acting with atemporal refinement
- Dispatching
- Observation actions

# Representation

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- Quantitative model of time
  - Discrete: time points are integers
- Expressions:
  - time-point variables
    - $t, t', t_2, t_j, \dots$
  - simple constraints
    - $d \leq t' - t \leq d'$
- Temporal assertion:
  - Value of a state variable during a time interval
  - Persistence:  $[t_1, t_2]x = v$  entails  $t_1 < t_2$
  - Change:  $[t_1, t_2]x : (v_1, v_2)$  entails  $v_1 \neq v_2$

# Timeline

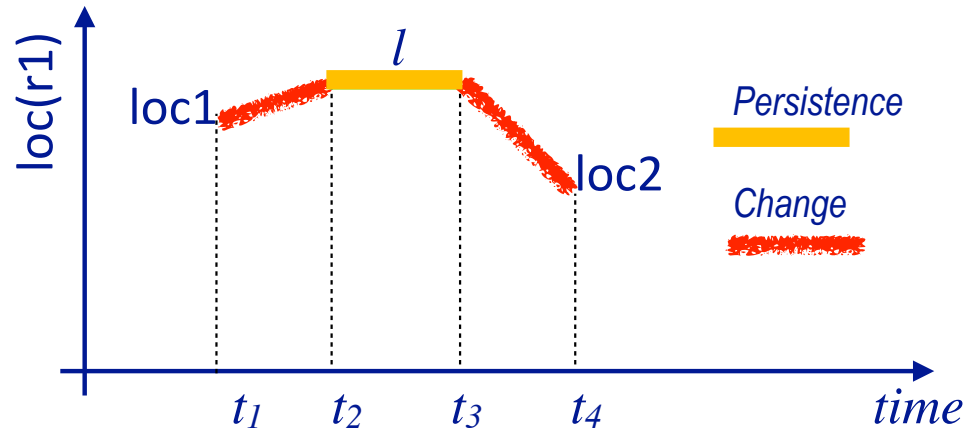
- **Timeline**: pair  $(\mathcal{T}, \mathcal{C})$ , partially predicted evolution of one state variable
  - Instance of  $(\mathcal{T}, \mathcal{C})$  = temporal and object variables instantiated

- $\mathcal{T}$  : temporal assertions

- $[t_1, t_2]loc(r1) : (loc1, l)$
- $[t_2, t_3]loc(r1) = l$
- $[t_3, t_4]loc(r1) : (l, loc2)$

- $\mathcal{C}$  : constraints

- $t_1 < t_2 < t_3 < t_4$
- $l \neq loc1$
- $l \neq loc2$
- If we want to restrict  $loc(r1)$  during  $[t_1, t_2]$ 
  - $[t_1, t_1 + 1]loc(r1) : (loc1, route)$
  - $[t_2 - 1, t_2]loc(r1) : (route, l)$
  - $[t_1 + 1, t_2 - 1]loc(r1) = route$



- An instance is **consistent** if it satisfies all constraints in  $\mathcal{C}$  and does not specify two different values for a state variable at the same time
- A timeline is **secure** if its set of consistent instances is not empty

# Actions

- Preliminaries:
  - Timelines  $(\mathcal{T}_1, \mathcal{C}_1), \dots, (\mathcal{T}_k, \mathcal{C}_k)$  for  $k$  different state variables
  - Their **union**:
    - $(\mathcal{T}_1, \mathcal{C}_1) \cup \dots \cup (\mathcal{T}_k, \mathcal{C}_k) = (\mathcal{T}_1 \cup \dots \cup \mathcal{T}_k, \mathcal{C}_1 \cup \dots \cup \mathcal{C}_k)$
  - If
    - every  $(\mathcal{T}_i, \mathcal{C}_i)$  is secure, and
    - no pair of timelines  $(\mathcal{T}_i, \mathcal{C}_i)$  and  $(\mathcal{T}_j, \mathcal{C}_j)$  have any unground variables in common
  - then
    - $(\mathcal{T}_1 \cup \dots \cup \mathcal{T}_k, \mathcal{C}_1 \cup \dots \cup \mathcal{C}_k)$  is also secure
- **Action** or **primitive task** (or just *primitive*):
  - a triple  $(head, \mathcal{T}, \mathcal{C})$ 
    - *head* is the name and arguments
    - $(\mathcal{T}, \mathcal{C})$  is the union of a set of timelines

# Actions

- $leave(r, d, w)$ 
  - Robot  $r$  leaves dock  $d$ , goes to adjacent waypoint  $w$

$leave(r, d, w)$

assertions:

$[t_s, t_e] \text{ loc}(r): (d, w)$

$[t_s, t_e] \text{ occupant}(d): (r, \text{empty})$

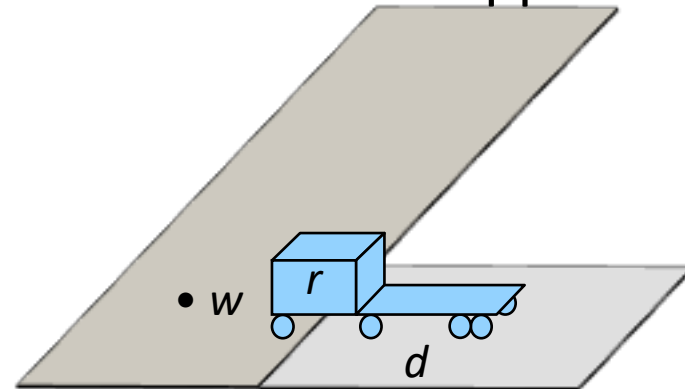
constraints:

$t_e \leq t_s + \delta_1$

$\text{adj}(d, w)$

- $\text{loc}(r)$  changes to  $w$  with delay  $\leq \delta_1$
- Dock  $d$  becomes empty

- Two additional parameters
  - Starting time  $t_s$
  - Ending time  $t_e$
- No separate preconditions and effects
  - Preconditions  $\Leftrightarrow$  need for causal support



# Actions

- $enter(r, d, w)$ 
  - $r$  enters  $d$  from an adjacent waypoint  $w$

$enter(r, d, w)$

assertions:

$[t_s, t_e] \text{ loc}(r): (w, d)$

$[t_s, t_e] \text{ occupant}(d): (\text{empty}, r)$

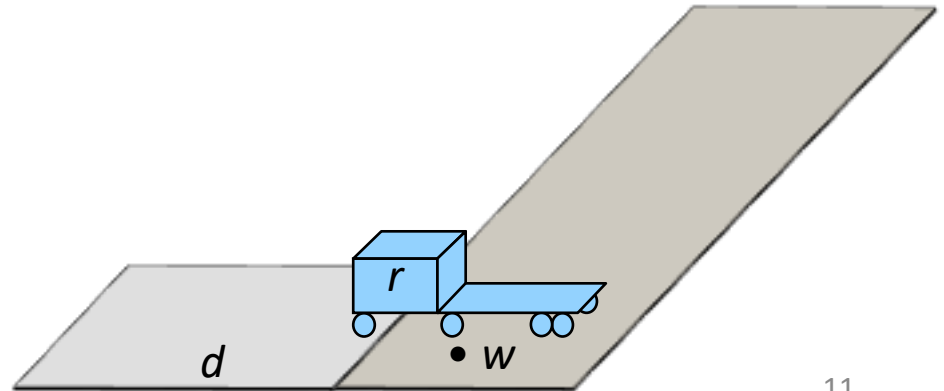
constraints:

$t_e \leq t_s + \delta_2$

$\text{adj}(d, w)$

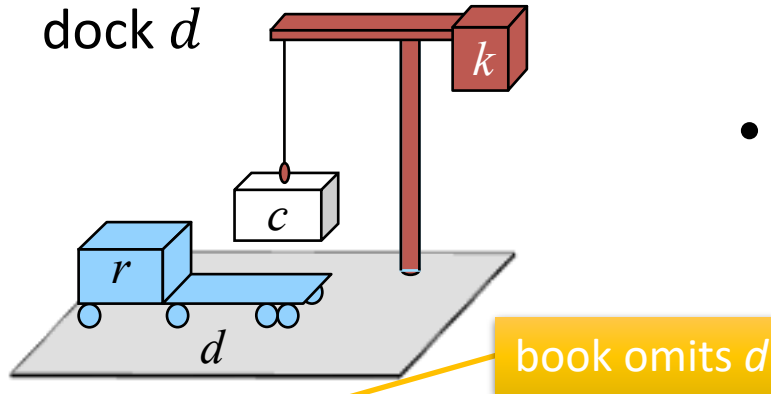
- $\text{loc}(r)$  changes to  $d$  with delay  $\leq \delta_2$
- Dock  $d$  becomes occupied by  $r$

- Two additional parameters
  - Starting time  $t_s$
  - Ending time  $t_e$
- No separate preconditions and effects
  - Preconditions  $\Leftrightarrow$  need for causal support



# Actions

- $take(k, c, r, d)$ 
  - Action: crane  $k$  takes container  $c$  from  $r$  on dock  $d$



- Two additional parameters
  - Starting time  $t_s$
  - Ending time  $t_e$
- No separate preconditions and effects
  - Preconditions  $\Leftrightarrow$  need for causal support

$take(k, c, r, d)$

assertions:

$[t_s, t_e]$  pos( $c$ ): ( $r, k$ )

// where container  $c$  is

$[t_s, t_e]$  grip( $k$ ): (empty,  $c$ )

// what crane  $k$ 's gripper is holding

$[t_s, t_e]$  freight( $r$ ): ( $c$ , empty)

// what  $r$  is carrying

$[t_s, t_e]$  loc( $r$ ) =  $d$

// where  $r$  is

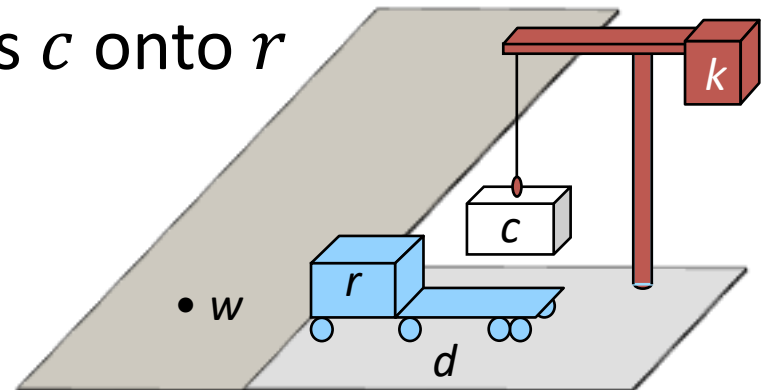
constraints:

attached( $k, d$ )

# Actions

- $leave(r, d, w)$  robot  $r$  leaves dock  $d$  to an adjacent waypoint  $w$
- $enter(r, d, w)$   $r$  enters  $d$  from an adjacent  $w$
- $take(k, c, r)$  crane  $k$  takes container  $c$  from  $r$
- $navigate(r, w, w')$   $r$  navigates from  $w$  to  $w'$
- $stack(k, c, p)$   $k$  stacks  $c$  on top of pile  $p$
- $unstack(k, c, p)$   $k$  takes  $c$  from top of  $p$
- $put(k, c, r)$   $k$  puts  $c$  onto  $r$

book omits  $r$

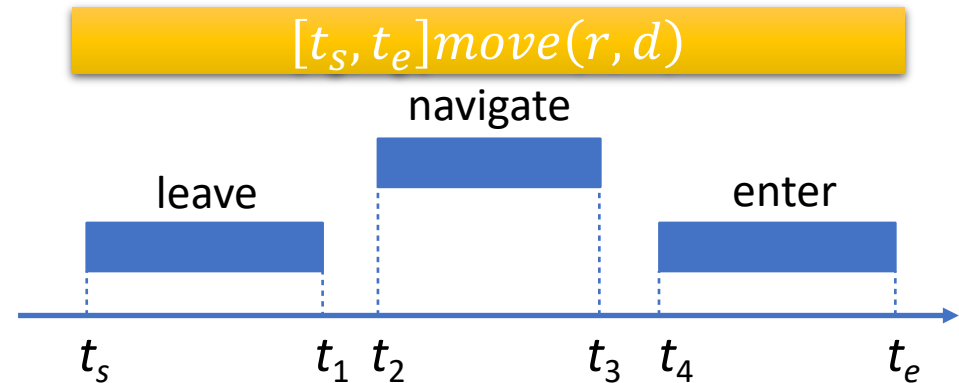


# Tasks and Methods

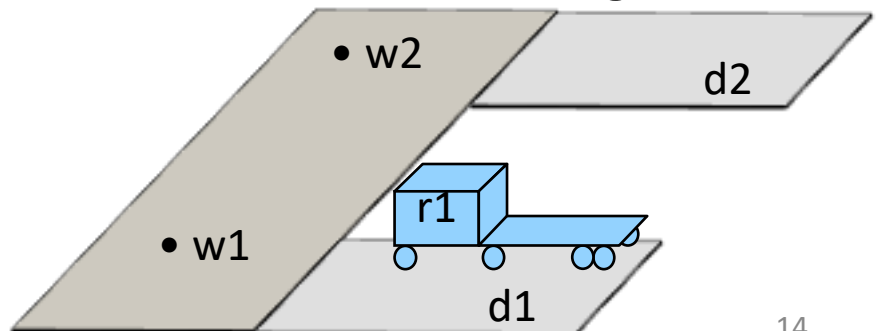
- Task: move robot  $r$  to dock  $d$ 
  - $[t_s, t_e] \text{move}(r, d)$
- Method:

```

m-move1( $r, d, d', w, w'$ )
  task:   move( $r, d$ )
  refinement:
     $[t_s, t_1]$  leave( $r, d', w'$ )
     $[t_2, t_3]$  navigate( $r, w', w$ )
     $[t_4, t_e]$  enter( $r, d, w$ )
  assertions:
     $[t_s, t_s+1]$  loc( $r$ ) =  $d'$ 
  constraints:
    adj( $d, w$ ),
    adj( $d', w'$ ),  $d \neq d'$ ,
    connected( $w, w'$ ),
     $t_1 \leq t_2, t_3 \leq t_4$ 
    
```



- $d'$  becomes empty during  $[t_s, t_1]$ 
  - another robot may enter it after  $t_1$
- $d$  doesn't need to be empty until  $t_4$ 
  - when  $r$  starts entering it

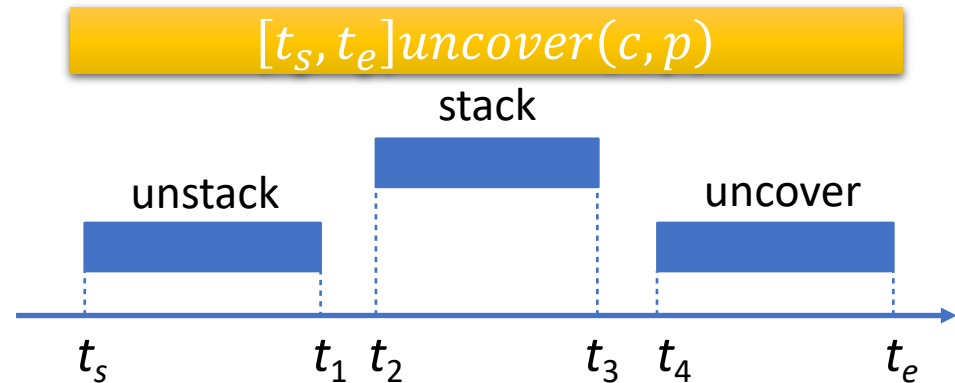


# Tasks and Methods

- Task: remove everything above container  $c$  in pile  $p$

- $[t_s, t_e] \text{uncover}(c, p)$

- Method:



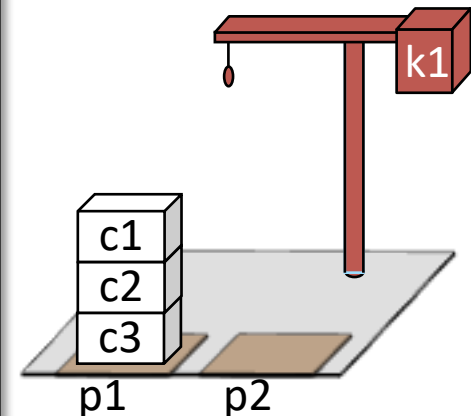
m-uncover( $c, p, k, d, p'$ )

task: uncover( $c, p$ )

refinement:  $[t_s, t_1]$  unstack( $k, c', p$ ) // action  
 $[t_2, t_3]$  stack( $k, c', p'$ ) // action  
 $[t_4, t_e]$  uncover( $c, p$ ) // recursive uncover

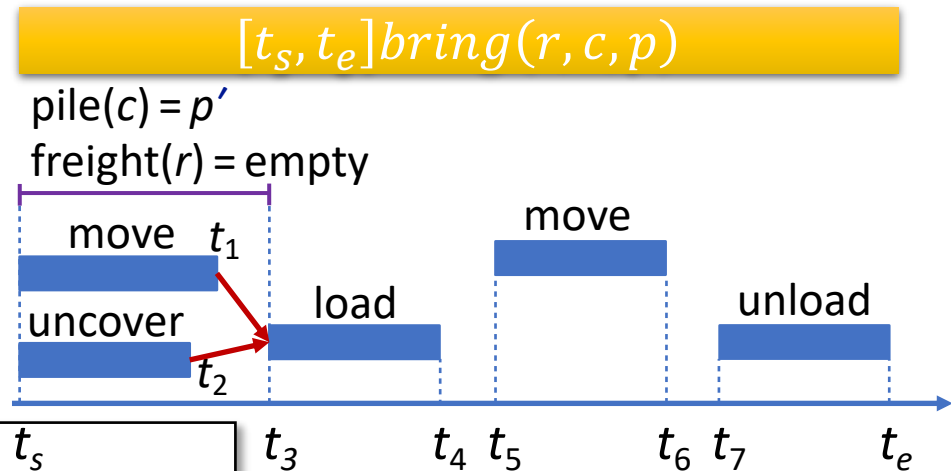
assertions:  $[t_s, t_s+1]$  pile( $c$ ) =  $p$   
 $[t_s, t_s+1]$  top( $p$ ) =  $c'$   
 $[t_s, t_s+1]$  grip( $k$ ) = empty

constraints: attached( $k, d$ ), attached( $p, d$ ),  
 attached( $p', d$ ),  
 $p \neq p', c' \neq c$ ,  
 $t_1 \leq t_2, t_3 \leq t_4$



# Tasks and Methods

- Task: robot  $r$  brings container  $c$  to pile  $p$ 
  - $[t_s, t_e]bring(r, c, p)$
- Method:



m-bring( $r, c, p, p', d, d'$ )

task: bring( $r, c, p$ )

refinement:  $[t_s, t_1] move(r, d')$

$[t_s, t_2] uncover(c, p')$

$[t_3, t_4] load(k', r, c, p')$

$[t_5, t_6] move(r, d)$

$[t_7, t_e] unload(k, r, c, p)$

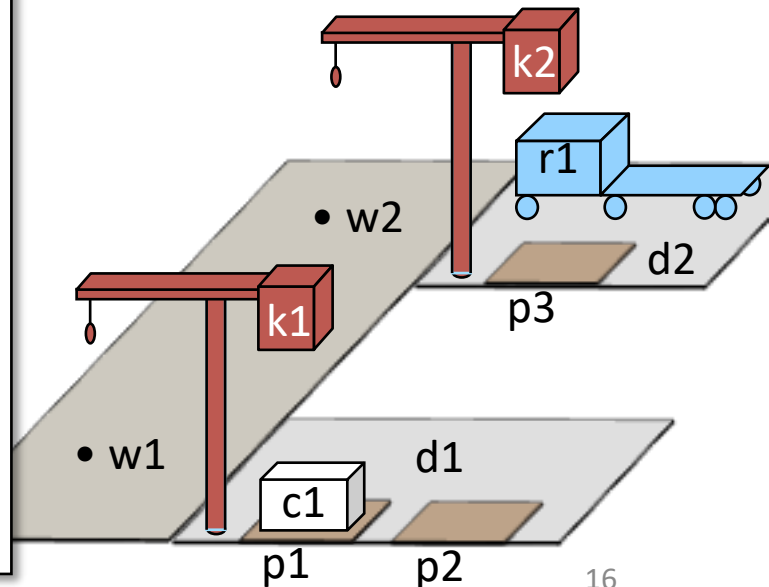
assertions:  $[t_s, t_3] pile(c) = p'$

$[t_s, t_3] freight(r) = empty$

constraints: attached( $p', d'$ ), attached( $p, d$ ),  $d \neq d'$

attached( $k', d'$ ), attached( $k, d$ ),  $k \neq k'$

$t_1 \leq t_3$ ,  $t_2 \leq t_3$ ,  $t_4 \leq t_5$ ,  $t_6 \leq t_7$



# Chronicles: Unions of Timelines

- Chronicle  $\phi = (\mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{C})$

- $\mathcal{A}$  : temporally qualified actions and tasks
- $\mathcal{S}$  : *a priori* supported assertions
- $\mathcal{T}$  : temporally qualified assertions
- $\mathcal{C}$  : constraints

- $\phi$  can include

- Current state, future predicted events
- Tasks to perform
- Assertions and constraints to satisfy

- Can represent

- Planning problem
- Plan or partial plan

$\phi_0$ :

tasks:  $[t, t']$  bring( $r, c1, d4$ )

supported:  $[t_s]$  loc( $r1$ )= $d1$

$[t_s]$  loc( $r2$ )= $d2$

$[t_s+10, t_s+\delta]$  docked(ship1)= $d3$

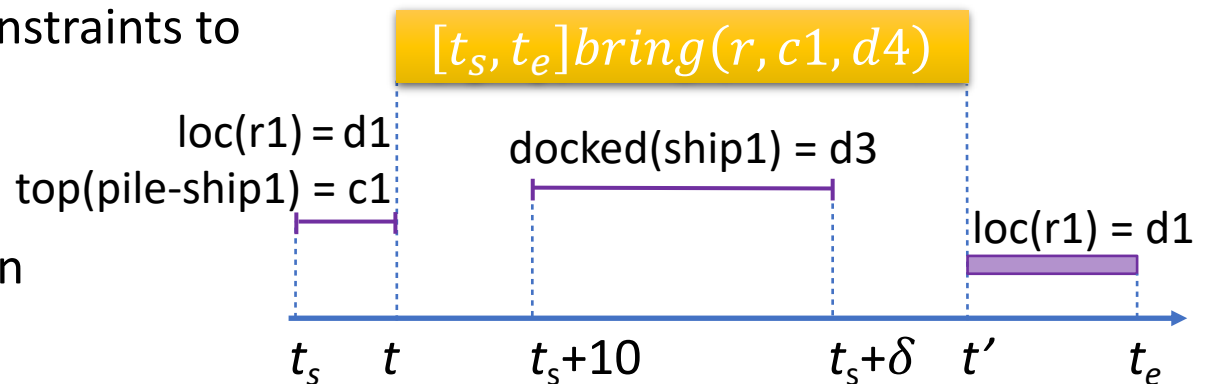
$[t_s]$  top(pile-ship1)= $c1$

$[t_s]$  pos( $c1$ )=pallet

assertions:  $[t_e]$  loc( $r1$ )= $d1$

$[t_e]$  loc( $r2$ )= $d2$

constraints:  $t_s = 0 < t < t' < t_e$ ,  $20 \leq \delta \leq 30$



# Intermediate Summary

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- Timelines
  - Temporal assertions (change, persistence), constraints
  - Conflicts, consistency, security, causal support
- Chronicle: union of several timelines
  - Consistency, security, causal support
- Actions represented by chronicles
  - No separate preconditions and effects
    - Preconditions  $\Leftrightarrow$  need for causal support

# Outline per the Book

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## *4.2 Representation*

- Timelines
- Actions and tasks
- Chronicles

## *4.3 Temporal planning*

- Resolvers and flaws
- Search space

## *4.4 Constraint management*

- Consistency of object constraints and time constraints
- Controlling the actions when we don't know how long they'll take

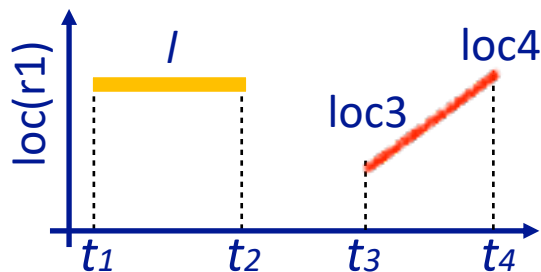
## *4.5 Acting with temporal models*

- Acting with atemporal refinement
- Dispatching
- Observation actions

# Planning

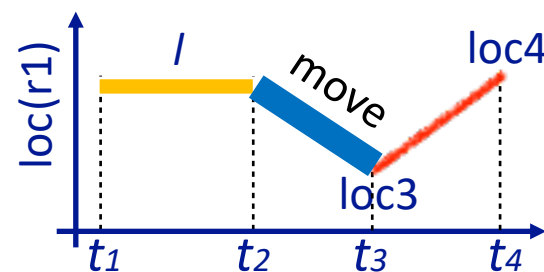
- Planning problem:
  - Chronicle  $\phi_0$  that has some flaws
  - Analogous to flaws in PSP

$\phi_0$ : tasks: *(none)*  
 supported: *(none)*  
 assertions:      $[t_1, t_2] \text{ loc}(r1) = l$   
                    $[t_3, t_4] \text{ loc}(r1) : (\text{loc3}, \text{loc4})$   
 constraints:     $\text{adj}(\text{loc3}, w1)$   
                    $\text{adj}(w1, \text{loc3})$   
                    $\text{adj}(\text{loc4}, w2)$   
                    $\text{adj}(w2, \text{loc4})$   
                    $\text{connected}(w1, w2)$



- Add new assertions, constraints, actions to resolve the flaws

$\phi_0$ : tasks:  $[t_2, t_3] \text{ move}(r1, \text{loc3})$   
 supported: *(none)*  
 assertions:      $[t_1, t_2] \text{ loc}(r1) = l$   
                    $[t_3, t_4] \text{ loc}(r1) : (\text{loc3}, \text{loc4})$   
 constraints:     $\text{adj}(\text{loc3}, w1)$   
                    $\text{adj}(w1, \text{loc3})$   
                    $\text{adj}(\text{loc4}, w2)$   
                    $\text{adj}(w2, \text{loc4})$   
                    $\text{connected}(w1, w2)$



# Flaws (1)

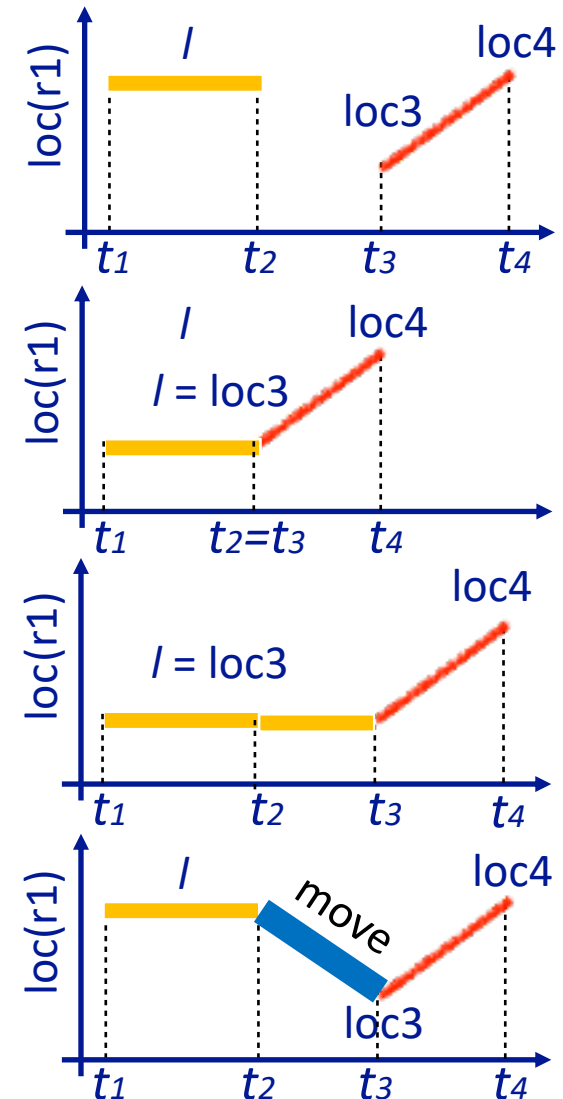
## 1. Temporal assertion $\alpha$ that isn't *causally supported*

- What causes  $r1$  to be at  $loc3$  at time  $t_3$ ?

Like an open goal in PSP

### • *Resolvers:*

- Add constraints to support  $\alpha$  from an assertion in  $\phi$ 
  - $l = loc3, t_2 = t_3$
- Add a new persistence assertion to support  $\alpha$ 
  - $l = loc3, [t_2, t_3] loc(r1) = loc3$
- Add a new task or action to support  $\alpha$ 
  - $[t_2, t_3] move(r1, loc3)$ 
    - Refining it will produce support for  $\alpha$

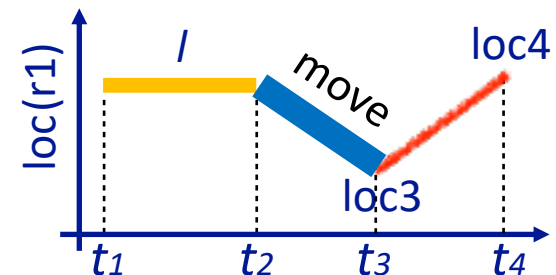


# Flaws (2)

## 2. Non-refined task

Like a task in SeRPE

- *Resolver*: refinement method  $m$ 
  - Applicable if it matches the task and its constraints are consistent with  $\phi$ 's
- Applying the resolver:
  - Modify  $\phi$  by replacing the task with  $m$
- Example:  $[t_2, t_3] \text{move}(r1, \text{loc3})$ 
  - Refinement will replace it with something like
    - $[t_2, t_5] \text{leave}(r1, l, w)$
    - $[t_5, t_6] \text{navigate}(r1, w, w')$
    - $[t_6, t_3] \text{enter}(r1, \text{loc3}, w')$
    - plus constraints



# Flaws (3)

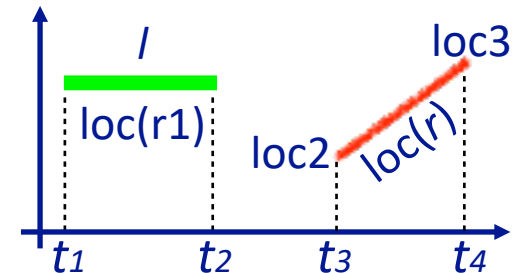
## 3. A pair of possibly-conflicting temporal assertions

Like a threat in PSP

- temporal assertions  $\alpha$  and  $\beta$  **possibly conflict** if they can have inconsistent instances

- Example

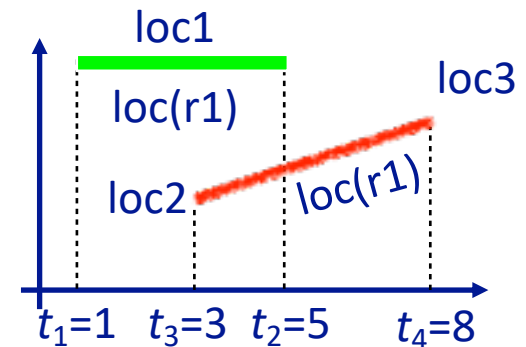
- $[t_1, t_2]loc(r1) = loc1, [t_3, t_4]loc(r) : (l, l')$



instance:  $[1, 5]loc(r1) = loc1, [3, 8]loc(r1) : (loc2, loc3)$

- Resolvers:** **separation** constraints

- $r \neq r1$
- $t_2 < t_3$
- $t_4 < t_1$
- $t_2 = t_3, r = r1, l = loc1$ 
  - Also provides causal support for  $[t_3, t_4]loc(r) : (l, l')$
- $t_4 = t_1, r = r1, l = loc1$ 
  - Also provides causal support for  $[t_1, t_2]loc(r1) = loc1$



# Planning Algorithm

- Like PSP in Chapter 2
  - Repeatedly selects flaws and chooses resolvers
- In the book, TemPlan uses recursion
  - Can be rewritten to use a loop
  - Just programming style, equivalent either way
- In a deterministic implementation
  - Selecting a resolver  $\rho$  is a backtracking point
  - Selecting a flaw isn't
- If it is possible to resolve all flaws, at least one of the nondeterministic execution traces will do so

**TemPlan** ( $\phi, \Sigma$ )

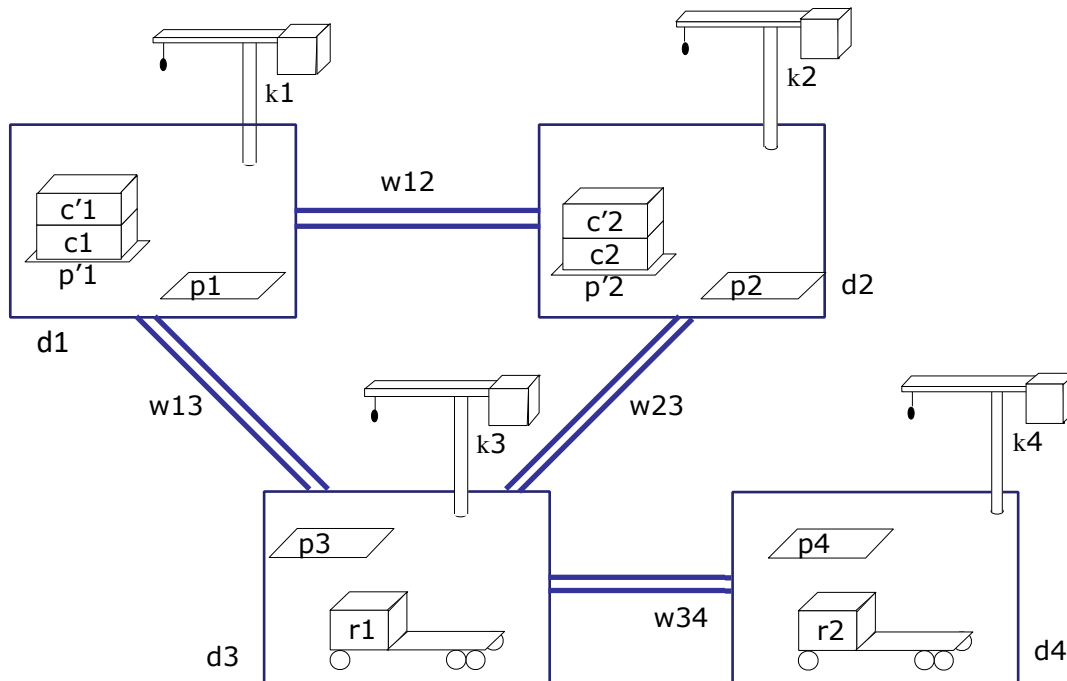
```
Flaws  $\leftarrow$  set of flaws of  $\phi$ 
if Flaws =  $\emptyset$  then
    return  $\phi$ 
arbitrarily select  $f \in$  Flaws
Resolvers  $\leftarrow$  set of resolvers of  $f$ 
if Resolvers =  $\emptyset$  then
    return failure
nondeterministically choose  $\rho \in$  Resolvers
 $\phi \leftarrow$  Transform( $\phi, \rho$ )
TemPlan ( $\phi, \Sigma$ )
```

**TemPlan** ( $\phi, \Sigma$ )

```
loop
    Flaws  $\leftarrow$  set of flaws of  $\phi$ 
    if Flaws =  $\emptyset$  then
        return  $\phi$ 
    arbitrarily select  $f \in$  Flaws
    Resolvers  $\leftarrow$  set of resolvers of  $f$ 
    if Resolvers =  $\emptyset$  then
        return failure
    nondeterministically choose  $\rho \in$  Resolvers
     $\phi \leftarrow$  Transform( $\phi, \rho$ )
```

# Example

- $\phi = (\mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{C})$ 
  - Establishes state-variable values at time  $t = 0$
  - Flaws: two unrefined tasks
    - $\text{bring}(r, c1, p3)$ ,  $\text{bring}(r', c2, p4)$



$\phi_0$ : tasks:  $\text{bring}(r, c1, p3)$   
 $\text{bring}(r', c2, p4)$

supported: [0]  $\text{loc}(r1) = d3$

[0]  $\text{freight}(r1) = \text{empty}$

[0]  $\text{pile}(c1) = p'1$

[0]  $\text{pile}(c'1) = p'1$

[0]  $\text{pos}(c1) = \text{pallet}$

[0]  $\text{pos}(c'1) = c1$

...

assertions: *(none)*

constraints:

$\text{adj}(d1, w12)$

$\text{adj}(d1, w13)$

...

# Example

- Flaws: two unrefined tasks
  - $\text{bring}(r, c1, p3)$ ,  $\text{bring}(r', c2, p4)$
- Refinement for both:

$m\text{-bring}(r, c, p, p', d, d', k, k')$

task:  $\text{bring}(r, c, p)$

refinement:  $[t_s, t_1]$   $\text{move}(r, d')$

$[t_s, t_2]$   $\text{uncover}(c, p')$

$[t_3, t_4]$   $\text{load}(k', r, c, p')$

$[t_5, t_6]$   $\text{move}(r, d)$

$[t_7, t_e]$   $\text{unload}(k, r, c, p)$

assertions:  $[t_s, t_3]$   $\text{pile}(c) = p'$

$[t_s, t_3]$   $\text{freight}(r) = \text{empty}$

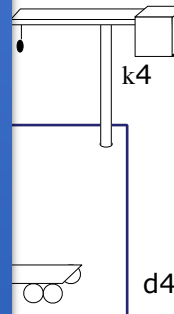
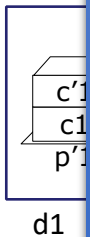
constraints:  $\text{attached}(p', d')$ ,

$\text{attached}(p, d), d \neq d'$

$\text{attached}(k', d')$ ,

$\text{attached}(k, d), k \neq k'$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$



$\phi_0$ : tasks:  $\text{bring}(r, c1, p3)$

$\text{bring}(r', c2, p4)$

supported:  $[0]$   $\text{loc}(r1) = d3$

$[0]$   $\text{freight}(r1) = \text{empty}$

$[0]$   $\text{pile}(c1) = p'1$

$[0]$   $\text{pile}(c'1) = p'1$

$[0]$   $\text{pos}(c1) = \text{pallet}$

$[0]$   $\text{pos}(c'1) = c1$

...

assertions:  $(\text{none})$

constraints:

$\text{adj}(d1, w12)$

$\text{adj}(d1, w13)$

...

# Method Instance

- Instantiate  $c = c1$  and  $p = p3$  to match *bring(r, c1, p3)*
  - $p', d, d', k, k'$  instantiated to match book
    - Needed later to satisfy action preconditions

m-bring( $r, c, p, p', d, d', k, k'$ )

refine

m-bring( $r, c1, p3, p'1, d3, d1, k3, k1$ )

task: bring( $r, c1, p3$ )

refinement:  $[t_s, t_1]$  move( $r, d1$ )

$[t_s, t_2]$  uncover( $c1, p'1$ )

$[t_3, t_4]$  load( $k1, r, c1, p'1$ )

$[t_5, t_6]$  move( $r, d3$ )

$[t_7, t_e]$  unload( $k3, r, c1, p3$ )

assertions:  $[t_s, t_3]$  pile( $c1$ ) =  $p'1$

$[t_s, t_3]$  freight( $r$ ) = empty

constraints: attached( $p'1, d1$ ),  
 attached( $p3, d3$ ),  $d3 \neq d1$   
 attached( $k1, d1$ ),  
 attached( $k3, d3$ ),  $k3 \neq k1$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$

asser

constr

$\phi_0$ : tasks: bring( $r, c1, p3$ )

bring( $r', c2, p4$ )

supported: [0] loc( $r1$ )= $d3$

[0] freight( $r1$ )=empty

[0] pile( $c1$ )= $p'1$

[0] pile( $c'1$ )= $p'1$

[0] pos( $c1$ )=pallet

[0] pos( $c'1$ )= $c1$

...

assertions: (*none*)

constraints:

adj( $d1, w12$ )

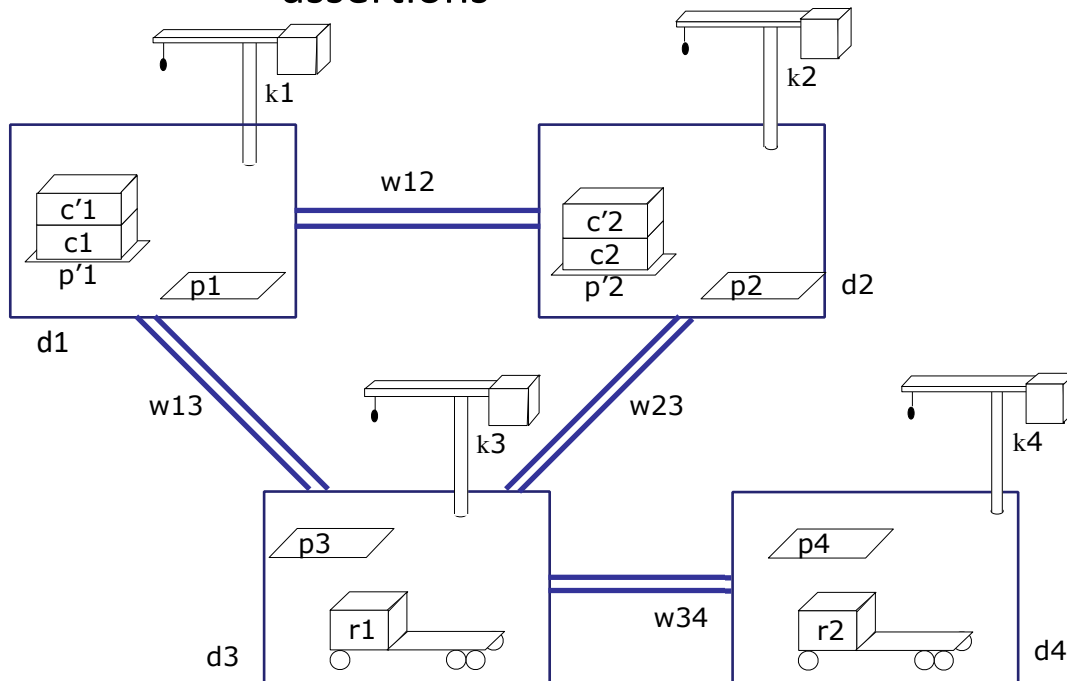
adj( $d1, w13$ )

...

d1

# Modified Chronicle

- Changes to  $\phi_0$ 
  - Removed *bring*( $r, c1, p3$ )
  - Added 5 tasks, 2 assertions, 4 constraints
- Flaws
  - 6 unrefined tasks, 2 unsupported assertions



$\phi_1$ : tasks:  $[t_s, t_1]$  move( $r, d1$ )  
 $[t_s, t_2]$  uncover( $c1, p'1$ )  
 $[t_3, t_4]$  load( $k1, r, c1, p'1$ )  
 $[t_5, t_6]$  move( $r, d3$ )  
 $[t_7, t_e]$  unload( $k3, r, c1, p3$ )  
 bring( $r', c2, p4$ )

supported:  $[0]$  loc( $r1$ )= $d3$   
 $[0]$  freight( $r1$ )=empty  
 $[0]$  pile( $c1$ )= $p'1$   
 $[0]$  pile( $c'1$ )= $p'1$   
 $[0]$  pos( $c1$ )=pallet  
 $[0]$  pos( $c'1$ )= $c1$

...

assertions:  $[t_s, t_3]$  pile( $c1$ ) =  $p'1$   
 $[t_s, t_3]$  freight( $r$ ) = empty

constraints:  $t_s < t_1 \leq t_3$ ,  $t_s < t_2 \leq t_3$ ,  $t_4 \leq t_5$ ,  $t_6 \leq t_7$ ,  
 adj( $d1, w12$ ),  
 adj( $d1, w13$ ),

...

# Method Instance

- Instantiate  $r = r', c = c2, p = p4$  to match  $\text{bring}(r', c2, p4)$ 
  - $p', d, d', k, k'$  instantiated to match book
  - Variables renamed to avoid name conflicts

m-bring( $r, c, p, p', d, d', k, k'$ )

task: bring( $r, c, p$ )

refinement:  $[t_s, t_1]$  move( $r, d'$ )

$[t_s, t_2]$  uncover( $c, p'$ )

$[t_3, t_4]$  load( $k', r, c, p'$ )

$[t_5, t_6]$  move( $r, d$ )

$[t_7, t_e]$  unload( $k, r, c, p$ )

assertions:  $[t_s, t_3]$  pile( $c$ ) =  $p'$

$[t_s, t_3]$  freight( $r$ ) = empty

constraints: attached( $p', d'$ ),

attached( $p, d$ ),  $d \neq d'$

attached( $k', d'$ ),

attached( $k, d$ ),  $k \neq k'$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7, t_e \neq k1$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$

$\phi_1$ : tasks:  $[t_s, t_1]$  move( $r, d1$ )  
 $[t_s, t_2]$  uncover( $c1, p'1$ )  
 $[t_3, t_4]$  load( $k1, r, c1, p'1$ )  
 $[t_5, t_6]$  move( $r, d3$ )  
 $[t_7, t_e]$  unload( $k3, r, c1, p3$ )  
bring( $r', c2, p4$ )

supported: [0] loc( $r1$ )= $d3$

[0] freight( $r1$ )=empty

[0] pile( $c1$ )= $p'1$

[0] pile( $c'1$ )= $p'1$

[0] pos( $c1$ )=pallet

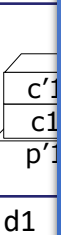
[0] pos( $c'1$ )= $c1$

...

assertions:  $[t_s, t_3]$  pile( $c1$ ) =  $p'1$

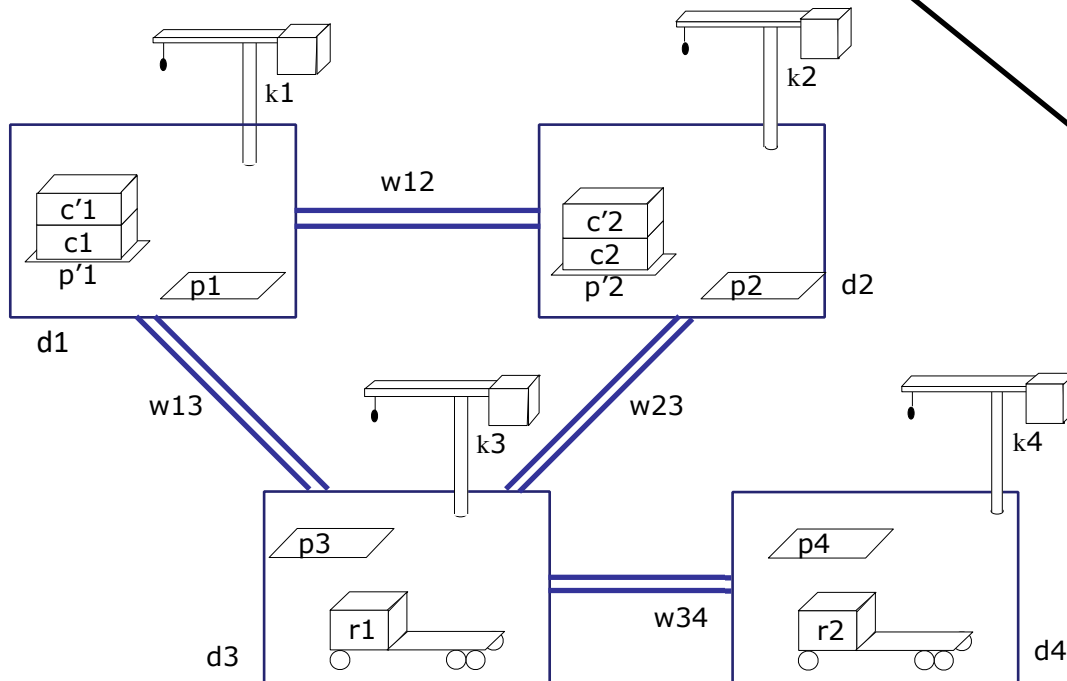
$[t_s, t_3]$  freight( $r$ ) = empty

constraints:  $t_s < t_1 \leq t_3, t_s < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$   
adj( $d1, w12$ ),  
adj( $d1, w13$ ),  
...



# Modified Chronicle

- Changes
  - Removed *bring(r', c2, p4)*
  - Added 5 tasks, 2 assertions, 4 constraints
- Flaws
  - 10 unrefined tasks, 4 unsupported assertions
- Next, work on these two assertions



$\phi_2$ : tasks:  $[t_s, t_1]$  move( $r, d1$ )  
 $[t_s, t_2]$  uncover( $c1, p'1$ )  
 $[t_3, t_4]$  load( $k1, r, c1, p'1$ )  
 $[t_5, t_6]$  move( $r, d3$ )  
 $[t_7, t_e]$  unload( $k3, r, c1, p3$ )  
 $[t'_s, t'_1]$  move( $r', d2$ )  
 $[t'_s, t'_2]$  uncover( $c2, p'2$ )  
 $[t'_3, t'_4]$  load( $k4, r', c2, p'2$ )  
 $[t'_5, t'_6]$  move( $r', d4$ )  
 $[t'_7, t'_e]$  unload( $k2, r', c2, p'2$ )

supported:  $[0]$  loc( $r1$ )= $d3$   
 $[0]$  freight( $r1$ )=empty  
 $[0]$  pile( $c1$ )= $p'1$

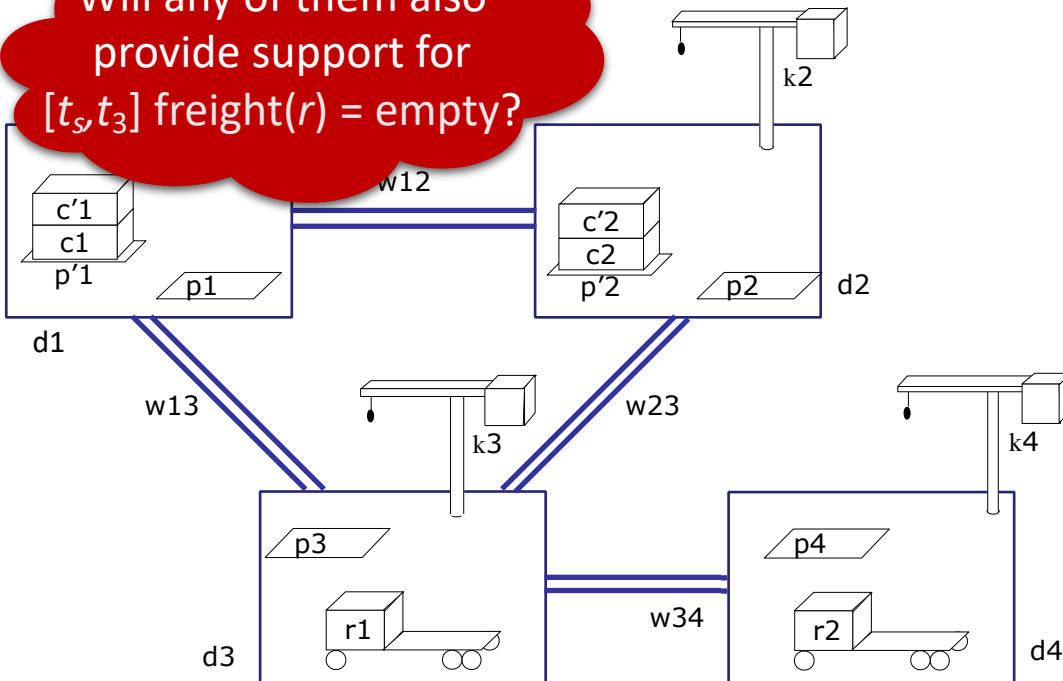
...  
 assertions:  $[t_s, t_3]$  pile( $c1$ ) =  $p'1$   
 $[t_s, t_3]$  freight( $r$ ) = empty  
 $[t'_s, t'_3]$  pile( $c2$ ) =  $p'2$   
 $[t'_s, t'_1]$  freight( $r'$ ) = empty

constraints:  $t_s < t_1 \leq t_3, t_s < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$   
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$   
 adj( $d1, w12$ ),  
 adj( $d1, w13$ ), ...

# Supporting the Assertions

- 3 ways to support  $[t_s, t_3] \text{pile}(c1) = p'1$ 
  1. Constrain  $t_s = 0$ , use  $[0] \text{pile}(c1) = p'1$
  2. Add persistence  $[0, t_s] \text{pile}(c1) = p'1$
  3. Add new action  $[t_8, t_s] \text{stack}(k1, c1, p'1)$

Will any of them also provide support for  $[t_s, t_3] \text{freight}(r) = \text{empty}$ ?



$\phi_2$ : tasks:  $[t_s, t_1] \text{move}(r, d1)$   
 $[t_s, t_2] \text{uncover}(c1, p'1)$   
 $[t_3, t_4] \text{load}(k1, r, c1, p'1)$   
 $[t_5, t_6] \text{move}(r, d3)$   
 $[t_7, t_e] \text{unload}(k3, r, c1, p3)$   
 $[t'_s, t'_1] \text{move}(r', d2)$   
 $[t'_s, t'_2] \text{uncover}(c2, p'2)$   
 $[t'_3, t'_4] \text{load}(k4, r', c2, p'2)$   
 $[t'_5, t'_6] \text{move}(r', d4)$   
 $[t'_7, t'_e] \text{unload}(k2, r', c2, p'2)$

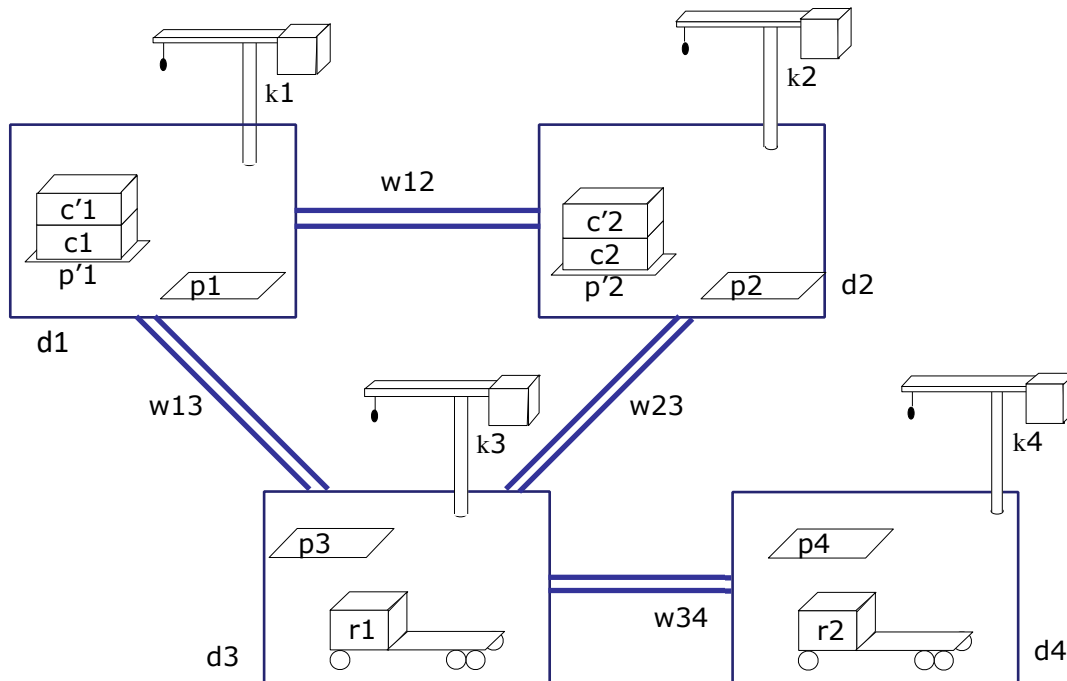
supported:  $[0] \text{loc}(r1) = d3$   
 $[0] \text{freight}(r1) = \text{empty}$   
 $[0] \text{pile}(c1) = p'1$   
 ...

assertions:  $[t_s, t_3] \text{pile}(c1) = p'1$   
 $[t_s, t_3] \text{freight}(r) = \text{empty}$   
 $[t'_s, t'_3] \text{pile}(c2) = p'2$   
 $[t'_s, t'_1] \text{freight}(r') = \text{empty}$

constraints:  $t_s < t_1 \leq t_3, t_s < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$   
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$   
 $\text{adj}(d1, w12),$   
 $\text{adj}(d1, w13), \dots$

# Supporting the Assertions

- To support  $[t_s, t_3] \text{pile}(c1) = p'1$ 
  - Constrain  $t_s = 0$ , use  $[0] \text{pile}(c1) = p'1$
- To support  $[0, t_3] \text{freight}(r) = \text{empty}$ 
  - Constrain  $r = r1$



$\phi_2$ : tasks:  $[0, t_1] \text{move}(r1, d1)$   
 $[0, t_2] \text{uncover}(c1, p'1)$   
 $[t_3, t_4] \text{load}(k1, r1, c1, p'1)$   
 $[t_5, t_6] \text{move}(r1, d3)$   
 $[t_7, t_e] \text{unload}(k3, r1, c1, p3)$   
 $[t'_s, t'_1] \text{move}(r', d2)$   
 $[t'_s, t'_2] \text{uncover}(c2, p'2)$   
 $[t'_3, t'_4] \text{load}(k4, r', c2, p'2)$   
 $[t'_5, t'_6] \text{move}(r', d4)$   
 $[t'_7, t'_e] \text{unload}(k2, r', c2, p'2)$

supported:  $[0] \text{loc}(r1) = d3$   
 $[0] \text{freight}(r1) = \text{empty}$   
 $[0] \text{pile}(c1) = p'1$

...

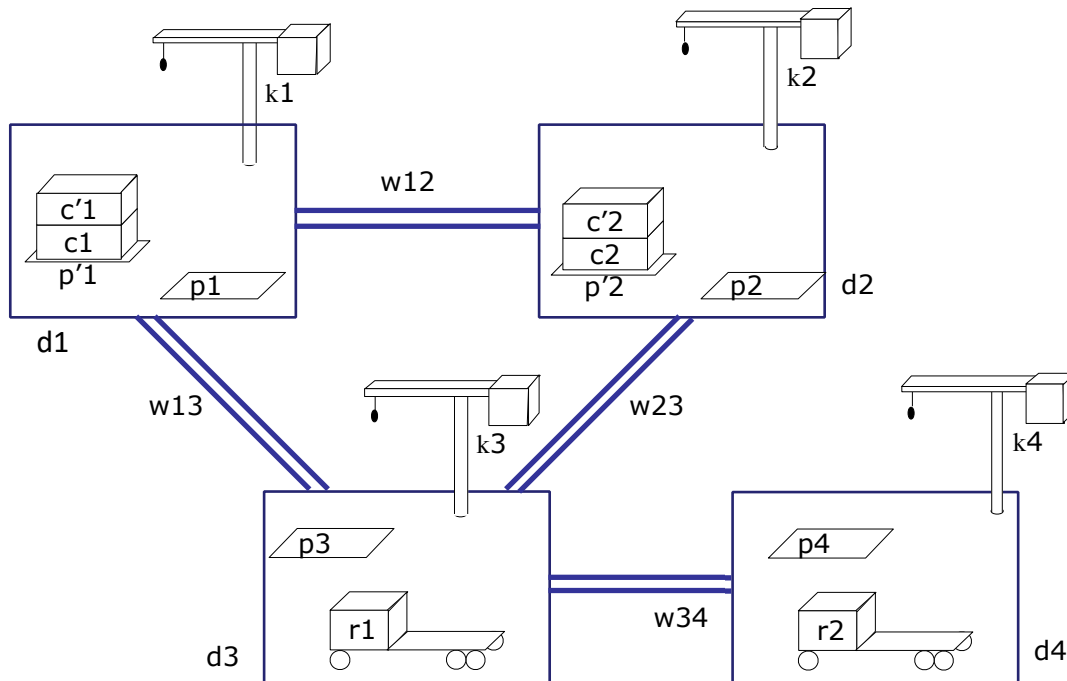
$[0, t_3] \text{pile}(c1) = p'1$   
 $[0, t_3] \text{freight}(r1) = \text{empty}$

assertions:  $[t'_s, t'_3] \text{pile}(c2) = p'2$   
 $[t'_s, t'_1] \text{freight}(r') = \text{empty}$

constraints:  $[0, t_1] \leq t_3, [0, t_2] \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$   
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$   
 $\text{adj}(d1, w12),$   
 $\text{adj}(d1, w13), \dots$

# Supporting the Assertions

- To support  $[t'_s, t'_3] \text{pile}(c2) = p'2$ 
  - Add persistence condition  $[0, t'_s] \text{pile}(c2) = p'2$ 
    - Alternatives: constrain  $t'_s = 0$  or add new action  $\text{stack}(k2, c2, p'2)$
- To support  $[t'_s, t'_1] \text{freight}(r') = \text{empty}$ 
  - Constrain  $r = r2$ , add persistence condition  $[0, t'_s] \text{freight}(r2) = \text{empty}$



$\phi_2$ : tasks:  $[0, t_1]$  move(r1, d1)  
 $[0, t_2]$  uncover(c1, p'1)  
 $[t_3, t_4]$  load(k1, r1, c1, p'1)  
 $[t_5, t_6]$  move(r1, d3)  
 $[t_7, t_e]$  unload(k3, r1, c1, p3)  
 $[t'_s, t'_1]$  move(r2, d2)  
 $[t'_s, t'_2]$  uncover(c2, p'2)  
 $[t'_3, t'_4]$  load(k4, r2, c2, p'2)  
 $[t'_5, t'_6]$  move(r2, d4)  
 $[t'_7, t'_e]$  unload(k2, r2, c2, p'2)

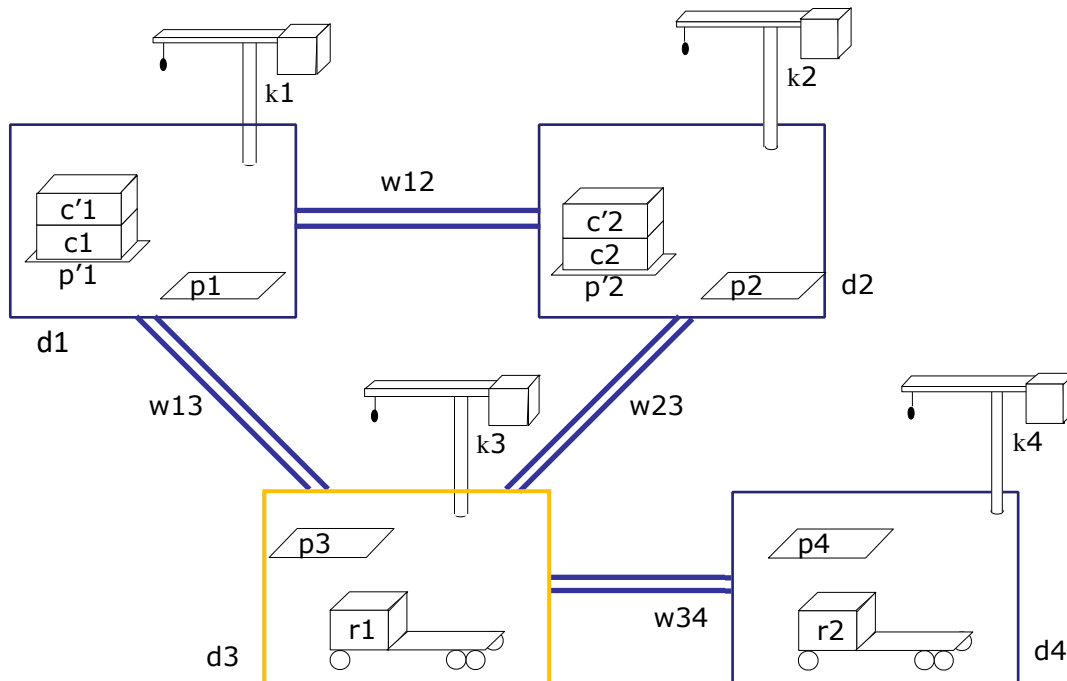
supported:  $[0] \text{loc}(r1) = d3$   
 $[0] \text{freight}(r1) = \text{empty}$   
 $[0] \text{pile}(c1) = p'1 \quad \dots$   
 $[0, t_3] \text{pile}(c1) = p'1$   
 $[0, t_3] \text{freight}(r1) = \text{empty}$   
 $[0, t'_s] \text{pile}(c2) = p'2$   
 $[t'_s, t'_3] \text{pile}(c2) = p'2$   
 $[0, t'_s] \text{freight}(r2) = \text{empty}$   
 $[t'_s, t'_1] \text{freight}(r2) = \text{empty}$

assertions: (none)

constraints:  $0 < t_1 \leq t_3, 0 < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$   
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$   
 $\text{adj}(d1, w12), \text{adj}(d1, w13), \dots$

# Example of Conflicts

- Refining tasks into actions will produce possibly-conflicting assertions
  - $\text{move}(r2, d4)$  must go through  $d3$
  - Conflict:  $\text{occupant}(d3)=r1, \text{occupant}(d3)=r2$
- Resolvers:
  - Separation constraints to ensure  $r2$  only goes through  $d3$  while  $r1$  away from  $d3$



$\phi_2$ : tasks:  $[0, t_1]$   $\text{move}(r1, d1)$   
 $[0, t_2]$   $\text{uncover}(c1, p'1)$   
 $[t_3, t_4]$   $\text{load}(k1, r1, c1, p'1)$   
 $[t_5, t_6]$   $\text{move}(r1, d3)$   
 $[t_7, t_e]$   $\text{unload}(k3, r1, c1, p3)$   
 $[t'_s, t'_1]$   $\text{move}(r2, d2)$   
 $[t'_s, t'_2]$   $\text{uncover}(c2, p'2)$   
 $[t'_3, t'_4]$   $\text{load}(k4, r2, c2, p'2)$   
 $[t'_5, t'_6]$   $\text{move}(r2, d4)$   
 $[t'_7, t'_e]$   $\text{unload}(k2, r2, c2, p'2)$

supported:  $[0]$   $\text{loc}(r1)=d3$   
 $[0]$   $\text{freight}(r1)=\text{empty}$   
 $[0]$   $\text{pile}(c1)=p'1 \quad \dots$   
 $[0, t_3]$   $\text{pile}(c1) = p'1$   
 $[0, t_3]$   $\text{freight}(r1) = \text{empty}$   
 $[0, t'_s]$   $\text{pile}(c2)=p'2$   
 $[t'_s, t'_3]$   $\text{pile}(c2) = p'2$   
 $[0, t'_s]$   $\text{freight}(r2)=\text{empty}$   
 $[t'_s, t'_1]$   $\text{freight}(r2) = \text{empty}$

assertions: *(none)*

constraints:  $0 < t_1 \leq t_3, 0 < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$   
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$   
 $\text{adj}(d1, w12), \text{adj}(d1, w13), \dots$

# Heuristics for Guiding TemPlan

- Flaw selection, resolver selection heuristics similar to those in PSP
  - Select the flaw with the smallest number of resolvers
  - Choose the resolver that rules out the fewest resolvers for the other flaws
- There is also a problem with constraint management
  - We ignored it when discussing PSP
  - Discuss it next

**TemPlan** ( $\phi, \Sigma$ )

```
Flaws  $\leftarrow$  set of flaws of  $\phi$ 
if Flaws =  $\emptyset$  then
    return  $\phi$ 
arbitrarily select  $f \in \textit{Flaws}$ 
Resolvers  $\leftarrow$  set of resolvers of  $f$ 
if Resolvers =  $\emptyset$  then
    return failure
nondeterministically choose  $\rho \in \textit{Resolvers}$ 
 $\phi \leftarrow \text{Transform}(\phi, \rho)$ 
TemPlan ( $\phi, \Sigma$ )
```

**PSP** ( $\Sigma, \pi$ )

loop

```
if Flaws( $\pi$ ) =  $\emptyset$  then
    return  $\pi$ 
arbitrarily select  $f \in \textit{Flaws}(\pi)$ 
 $R \leftarrow \{\text{all feasible resolvers for } f\}$ 
if  $R = \emptyset$  then
    return failure
nondeterministically choose  $\rho \in R$ 
 $\pi \leftarrow \rho(\pi)$ 
return  $\pi$ 
```

# Intermediate Summary

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- Planning problems
  - Three kinds of flaws and their resolvers:
    - tasks, causal support, security
  - Partial plans, solution plans
- Planning: TemPlan
  - Like PSP but with tasks, temporal assertions, temporal constraints

# Outline per the Book

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## *4.2 Representation*

- Timelines
- Actions and tasks
- Chronicles

## *4.3 Temporal planning*

- Resolvers and flaws
- Search space

## ***4.4 Constraint management***

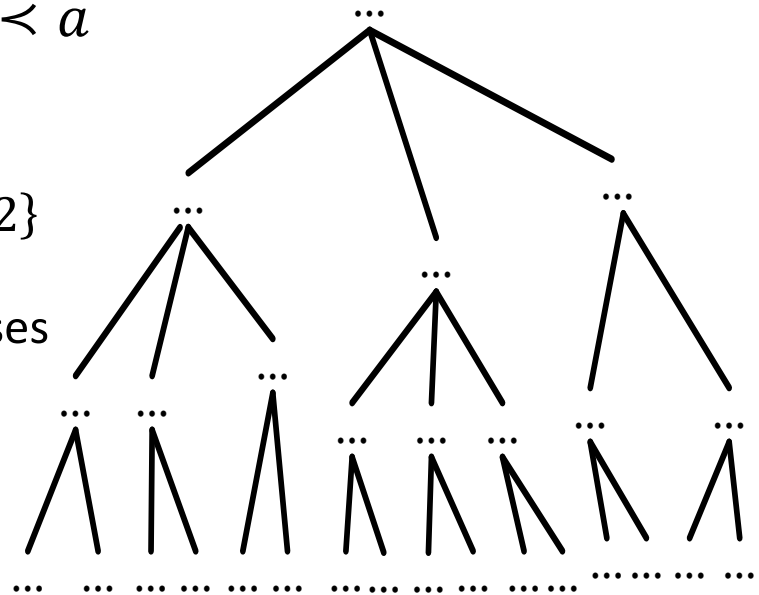
- Consistency of object constraints and time constraints
- Controlling the actions when we don't know how long they'll take

## *4.5 Acting with temporal models*

- Acting with atemporal refinement
- Dispatching
- Observation actions

# Constraint Management

- Each time TemPlan applies a resolver, it modifies  $(\mathcal{T}, \mathcal{C})$ 
  - Some resolvers will make  $(\mathcal{T}, \mathcal{C})$  inconsistent
    - No solution in this part of the search space
    - Detect inconsistency  $\Rightarrow$  prune this part of the search space
    - Do not detect it  $\Rightarrow$  waste time looking for a solution
- Analogy: PSP checked simple cases of inconsistency
  - E.g., cannot create a constraint  $a < b$  if there is already a constraint  $b < a$
  - Ignored more complicated cases
  - Example:
    - $c_1, c_2, c_3 \in \text{Containers} = \{c1, c2\}$
    - Threats involving  $c_1, c_2, c_3$
    - For resolvers, suppose PSP chooses
      - $c_1 \neq c_2, c_2 \neq c_3, c_1 \neq c_3$
    - No solutions in this part of the search space, but PSP searches it anyway



# Constraint Management in TemPlan

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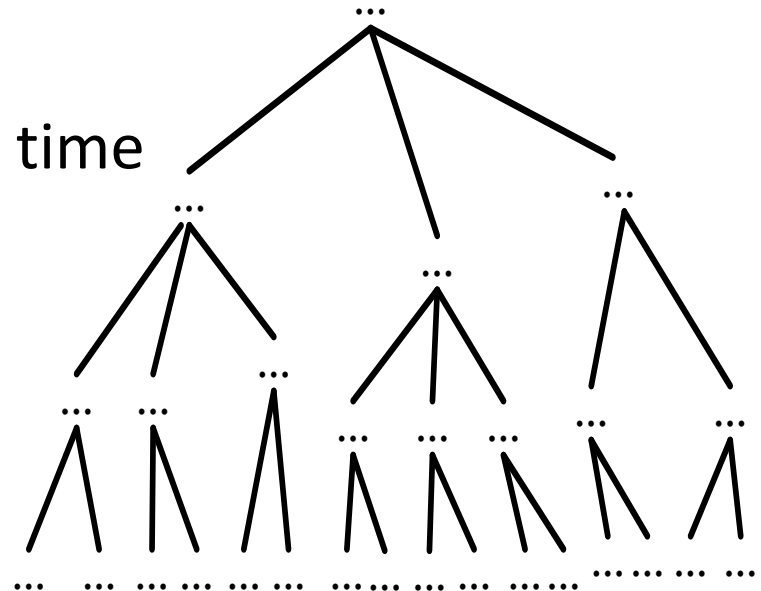
- At various points, check consistency of  $\mathcal{C}$ 
  - If  $\mathcal{C}$  is inconsistent, then  $(\mathcal{T}, \mathcal{C})$  is inconsistent
  - Can prune this part of the search space
- If  $\mathcal{C}$  is consistent, then  $(\mathcal{T}, \mathcal{C})$  may or may not be consistent
  - Example:
    - $\mathcal{T} = \{[t_1, t_2]loc(r1) = loc1, [t_3, t_4]loc(r1) = loc2\}$
    - $\mathcal{C} = (t_1 < t_3 < t_4 < t_2)$
  - Gives  $loc(r1)$  two values during  $[t_3, t_4]$

# Consistency of $\mathcal{C}$

- $\mathcal{C}$  contains two kinds of constraints
  - **Object** constraints
    - $loc(r) \neq l_2, \quad l \in \{loc3, loc4\}, \quad r = r1, \quad o \neq o'$
  - **Temporal** constraints
    - $t_1 < t_3, \quad a < t, \quad t < t', \quad a \leq t' - t \leq b$
- Assume object constraints are independent of temporal constraints and vice versa
  - Exclude things like  $t < f(l, r)$
- Then two separate subproblems
  - (1) check consistency of object constraints
  - (2) check consistency of temporal constraints
  - $\mathcal{C}$  is consistent iff both are consistent

# Object Constraints

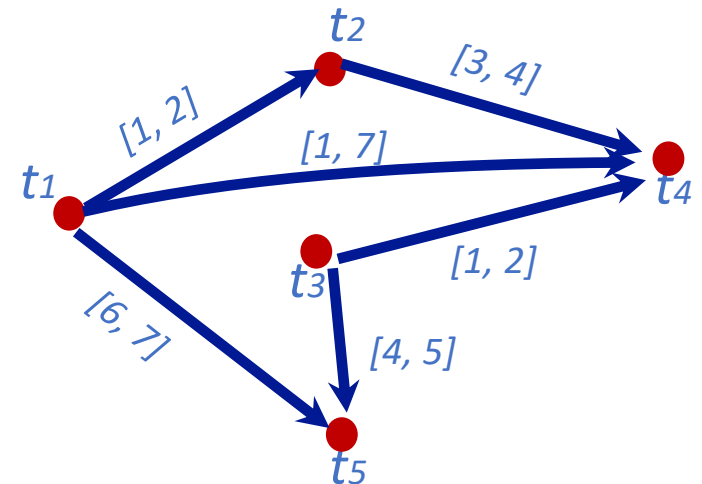
- Constraint-satisfaction problem (CSP) – NP-hard
- Can write an algorithm that is **complete** but runs in **exponential** time
  - If there is an inconsistency, always finds it
  - Might do a lot of pruning, but spend lots of time at each node
- Instead, use a technique that is **incomplete** but takes **polynomial** time
  - Edge consistency, path consistency
- Detects some inconsistencies but not others
  - Runs much faster, but prunes fewer nodes



# Time Constraints: Representation

- Simple Temporal Networks (STNs)
  - Networks of constraints on time points

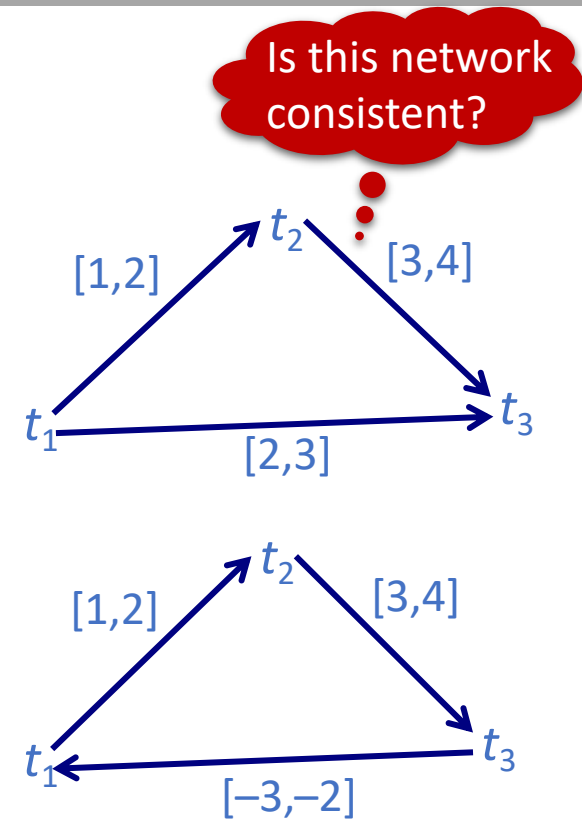
- Synthesise them incrementally starting from  $\phi_0$ 
  - TemPlan can check time constraints in time  $O(n^3)$



- Incrementally instantiated at acting time
- Kept consistent throughout planning and acting

# Simple Temporal Networks

- STN: a pair  $(\mathcal{V}, \mathcal{E})$ , where
  - $\mathcal{V} = \{\text{a set of temporal variables } t_1, \dots, t_n\}$
  - $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  is a set of edges
- Each edge  $(t_i, t_j)$  is labelled with an interval  $[a, b]$ 
  - Represents constraint  $a \leq t_j - t_i \leq b$
  - Equivalently,  $-b \leq t_i - t_j \leq -a$
- Representing unary constraints
  - Dummy variable  $t_0 = 0$
  - Edge  $r_{0i} = (t_0, t_i)$  labelled with  $[a, b]$  represents  $a \leq t_i - 0 \leq b$
- Shorthand: instead of  $a \leq t_j - t_i \leq b$ , write  $r_{ij} = [a_{ij}, b_{ij}]$
- **Solution** to an STN
  - Integer value for each  $t_i$
  - All constraints satisfied
- **Consistent** STN
  - Has a solution



Book says:

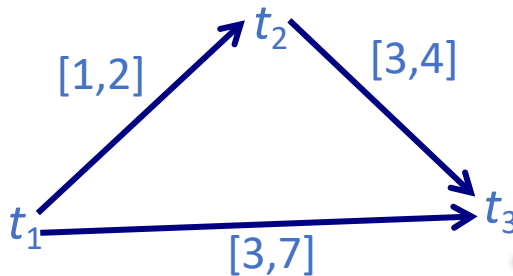
- Solution
  - Integer value for each  $t_i$
- Consistent:
  - Has a solution
  - All constraints satisfied



# Time Constraints

- Minimal STN:

- For every edge  $(t_i, t_j)$  with label  $[a, b]$ 
  - For every  $t \in [a, b]$ 
    - There is at least one solution such that  $t_j - t_i = t$
- Cannot make any of the time intervals shorter without excluding some solutions



Is this network minimal?

# Operations on STNs

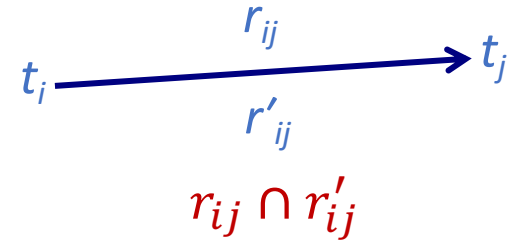
- Intersection,  $\cap$

- $t_j - t_i \in r_{ij} = [a_{ij}, b_{ij}]$

- $t_j - t_i \in r'_{ij} = [a'_{ij}, b'_{ij}]$

- Infer

$$t_j - t_i \in r_{ij} \cap r'_{ij} = [\max(a_{ij}, a'_{ij}), \min(b_{ij}, b'_{ij})]$$



- Composition,  $\circ$

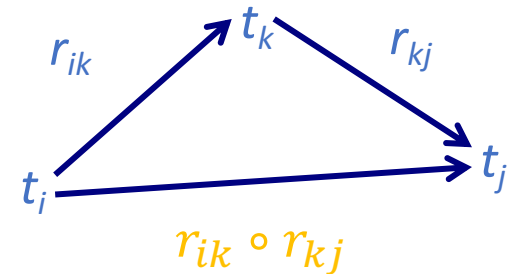
- $t_k - t_i \in r_{ik} = [a_{ik}, b_{ik}]$

- $t_j - t_k \in r_{kj} = [a_{kj}, b_{kj}]$

- Infer

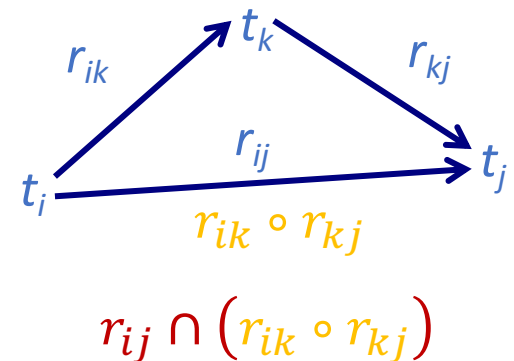
$$t_j - t_i \in r_{ik} \circ r_{kj} = [a_{ik} + a_{kj}, b_{ik} + b_{kj}]$$

- Reason: shortest and longest times for the two intervals

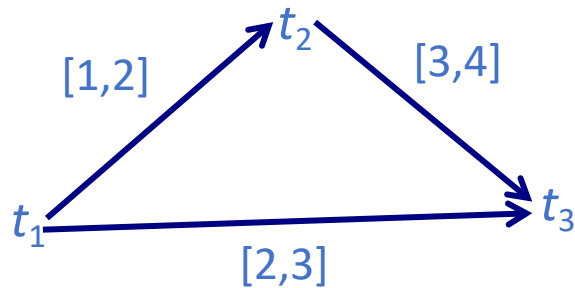


- Consistency checking

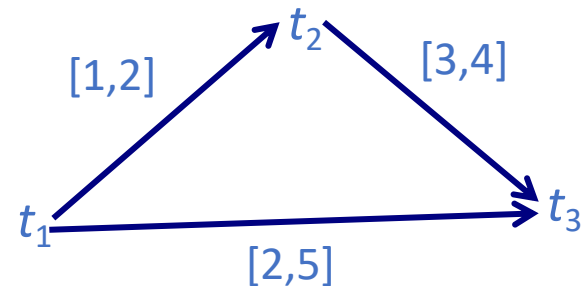
- Three constraints  $r_{ik}, r_{kj}, r_{ij}$  are consistent only if  $r_{ij} \cap (r_{ik} \circ r_{kj}) \neq \emptyset$



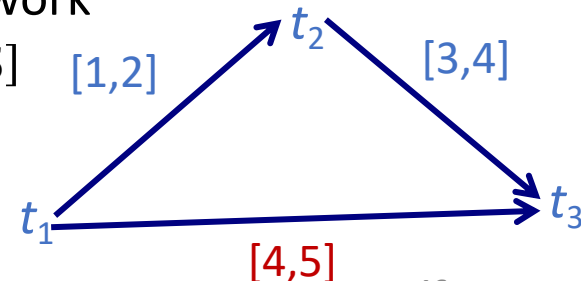
# Two Examples



- STN  $(\mathcal{V}, \mathcal{E})$ , where
  - $\mathcal{V} = \{t_1, t_2, t_3\}$
  - $\mathcal{E} = \{r_{12} = [1,2], r_{23} = [3,4], r_{13} = [2,3]\}$
- Composition
  - $r'_{13} = r_{12} \circ r_{23} = [4,6]$
- Cannot satisfy both  $r_{13}$  and  $r'_{13}$ 
  - $r_{13} \cap r'_{13} = [2,3] \cap [4,6] = \emptyset$
- $(\mathcal{V}, \mathcal{E})$  is inconsistent



- STN  $(\mathcal{V}, \mathcal{E})$ , where
  - $\mathcal{V} = \{t_1, t_2, t_3\}$
  - $\mathcal{E} = \{r_{12} = [1,2], r_{23} = [3,4], r_{13} = [2,5]\}$
- Composition (as before)
  - $r'_{13} = r_{12} \circ r_{23} = [4,6]$
- $(\mathcal{V}, \mathcal{E})$  is consistent
  - $r_{13} \cap r'_{13} = [2,5] \cap [4,6] = [4,5]$
- Minimal network
  - $r_{13} = [4,5]$

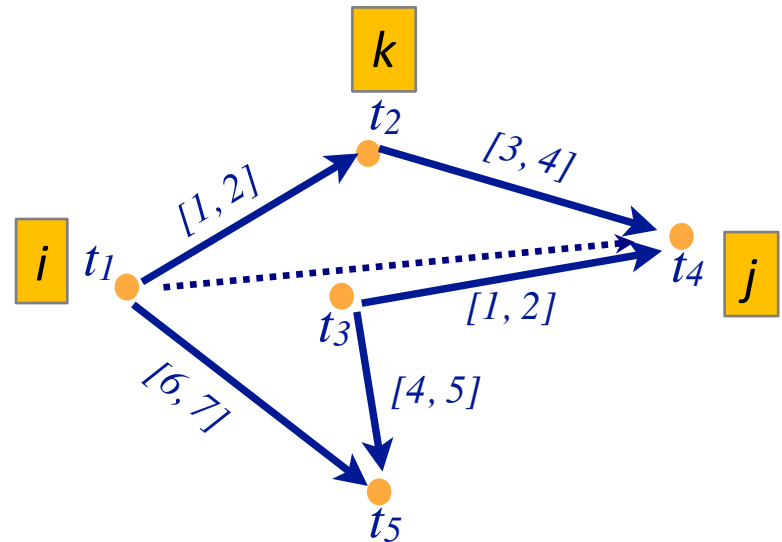


# Operations on STNs

- PC (*Path Consistency*) algorithm:
  - Consistency checking on all triples
  - If an edge has no constraint, use  $[-\infty, +\infty]$
  - $n$  constraints  
 $\Rightarrow n^3$  triples  
 $\Rightarrow$  time  $O(n^3)$

```
PC( $\mathcal{V}, \mathcal{E}$ )  
  for  $1 \leq k \leq n$  do  
    for  $1 \leq i < j \leq n, i \neq j, j \neq k$  do  
       $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \circ r_{kj}]$   
      if  $r_{ij} = \emptyset$  then  
        return inconsistent
```

- Example:
  - $k = 2, i = 1, j = 2$
  - $r_{12} = [1, 2]$
  - $r_{24} = [3, 4]$
  - $r_{14} = [-\infty, \infty]$
  - $r_{12} \circ r_{24} = [1 + 3, 2 + 4] = [4, 6]$
  - $r_{14} \leftarrow [\max(-\infty, 4), \min(\infty, 6)] = [4, 6]$



# Operations on STNs

- PC makes network minimal
  - Shrinks each  $r_{ij}$  to exclude values that are not in any solution
- Also detects inconsistent networks
  - $r_{ij} = [a_{ij}, b_{ij}]$  empty  $\Rightarrow$  inconsistent
- Graph: dashed lines
  - Constraints that were shrunk
- Can modify PC to make it incremental
  - Input
    - A consistent, minimal STN
    - A new constraint  $r'_{ij}$
  - Incorporate  $r'_{ij}$  in time  $O(n^2)$

PC( $\mathcal{V}, \mathcal{E}$ )

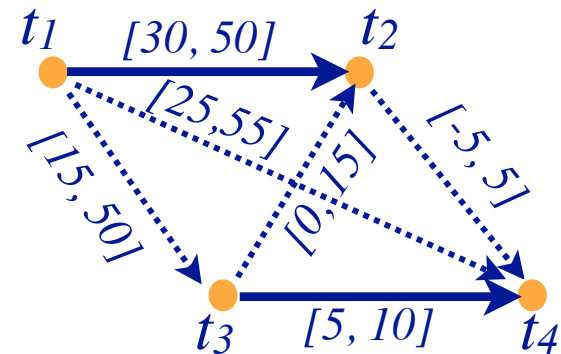
for  $1 \leq k \leq n$  do

for  $1 \leq i < j \leq n, i \neq j, j \neq k$  do

$r_{ij} \leftarrow r_{ij} \cap [r_{ik} \circ r_{kj}]$

if  $r_{ij} = \emptyset$  then

return inconsistent



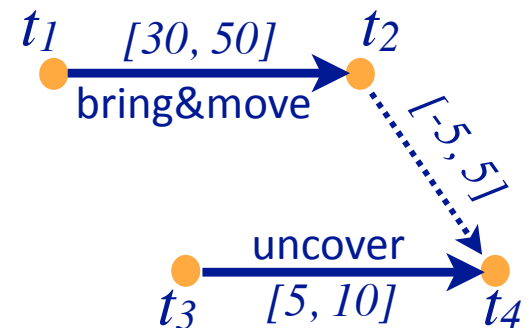
# Pruning TemPlan's search space

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- Take the time constraints in  $\mathcal{C}$ 
  - Write them as an STN
  - Use Path Consistency to check whether STN is consistent
  - If it is inconsistent, TemPlan can backtrack

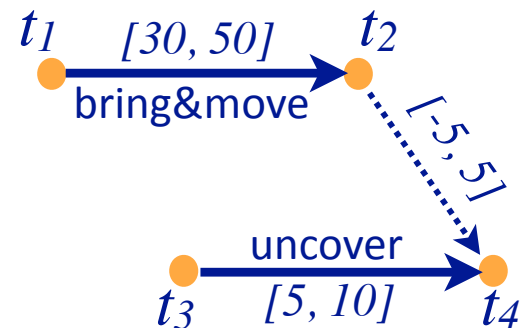
# Controllability

- Suppose TemPlan gives you a chronicle and you want to execute it
  - Constraints on time points
  - Need to reason about these in order to decide when to start each action



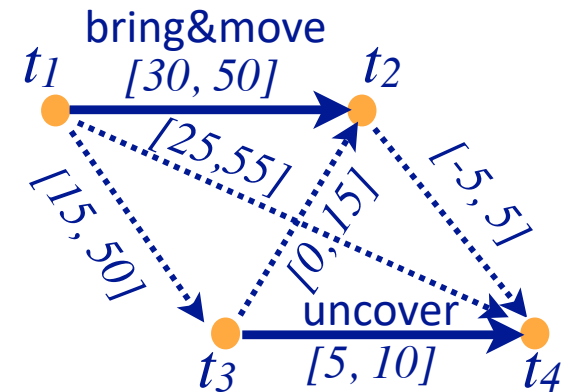
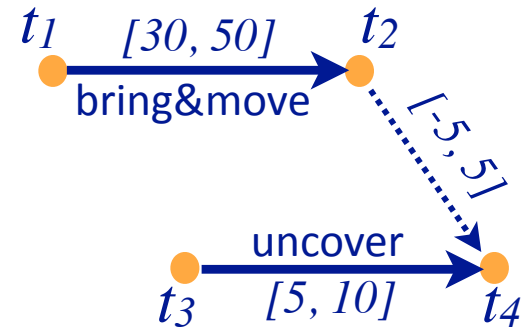
# Controllability

- Solid lines: **duration constraints**
  - Robot will do bring&move, will take 30 to 50 time units
  - Crane will do uncover, will take 5 to 10 time units
- Dashed line: **synchronization constraint**
  - Do not want either the crane or robot to wait long
  - At most 5 seconds between the two ending times
- Objective
  - Choose time points that will satisfy all the constraints



# Controllability

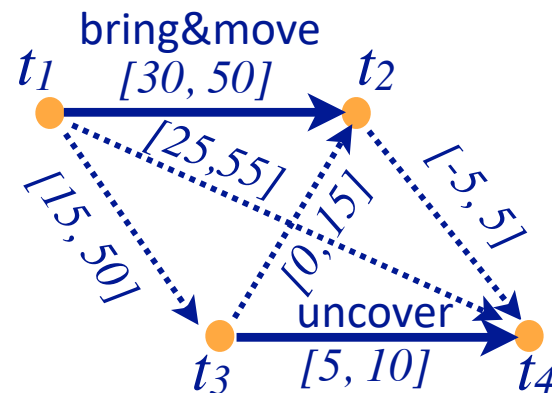
- Suppose we run PC
- PC returns a minimal and consistent network
- There *exist* time points that satisfy all the constraints
- Would work if we could choose all four time points
  - But we cannot choose  $t_2$  and  $t_4$
- $t_1$  and  $t_3$  are **controllable**
  - Actor can control when each action starts
- $t_2$  and  $t_4$  are **contingent**
  - Cannot control how long the actions take
  - Random variables that are known to satisfy the duration constraints
    - $t_2 \in [t_1 + 30, t_1 + 50]$
    - $t_4 \in [t_3 + 5, t_3 + 10]$



# Controllability

- Cannot guarantee that all constraints will be satisfied
- Start bring&move at time  $t_1 = 0$
- Suppose the durations are
  - bring&move 30, uncover 10
  - $t_2 = t_1 + 30 = 30$
  - $t_4 = t_3 + 10$
  - $t_4 - t_2 = t_3 - 20$
- Constraint  $r_{24}$ :
  - $-5 \leq t_4 - t_2 \leq 5$
  - $-5 \leq t_3 - 20 \leq 5$
  - $15 \leq t_3 \leq 25$
- Must start uncover at  $t_3 \leq 25$

- But if we start uncover at  $t_3 \leq 25$ , neither action has finished yet
  - We do not yet know how long they will take
- Durations might instead be
  - bring&move 50, uncover 5
  - $t_2 = t_1 + 50 = 50$
  - $t_4 = t_3 + 5 \leq 25 + 5 = 30$
  - $t_4 - t_2 \leq 30 - 50 = -20$ 
    - Violates  $r_{34}$



# STNUs

- **STNU (Simple Temporal Network with Uncertainty):**
  - A 4-tuple  $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ 
    - $\mathcal{V} = \{\text{controllable time points}\}$ 
      - E.g., starting times of actions
    - $\tilde{\mathcal{V}} = \{\text{contingent time points}\}$ 
      - E.g., ending times of actions
    - $\mathcal{E} = \{\text{controllable constraints}\}$
    - $\tilde{\mathcal{E}} = \{\text{contingent constraints}\}$
  - **Controllable and contingent constraints:**
    - Synchronization between two **starting** times: *controllable*
    - **Duration** of an action: *contingent*
    - Synchronization between **ending** points of two actions: *contingent*
    - Synchronization between end of one action, start of another:
      - *Controllable* if the new action starts after the old one ends
      - *Contingent* if the new action starts before the old one ends
  - Want a way for the actor to choose time points in  $\mathcal{V}$  (starting times) that guarantee that constraints are satisfied

# Three kinds of controllability

- $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  is **strongly controllable** if the actor can choose values for  $\mathcal{V}$  such that success will occur for all values of  $\tilde{\mathcal{V}}$  that satisfy  $\tilde{\mathcal{E}}$ 
  - Actor can choose the values for  $\mathcal{V}$  offline
  - The right choice will work regardless of  $\tilde{\mathcal{V}}$
- $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  is **weakly controllable** if the actor can choose values for  $\mathcal{V}$  such that success will occur for *at least one* combination of values for  $\tilde{\mathcal{V}}$ 
  - Actor can choose the values for  $\mathcal{V}$  only if the actor knows in advance what the values of  $\tilde{\mathcal{V}}$  will be
- **Dynamic controllability:**
  - Game-theoretic model: actor vs. environment
  - A player's **strategy**: a function  $\sigma$  telling what to do in every situation
    - Choices may differ depending on what has happened so far
  - $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  is **dynamically controllable** if  $\exists$  strategy for an actor that will guarantee success regardless of the environment's strategy

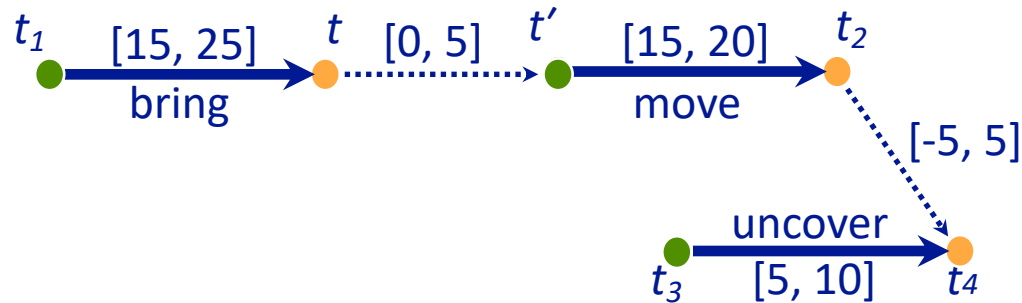
# Dynamic Execution

- For  $t = 0, 1, 2, \dots$ 
  1. Actor chooses an unassigned set of variables  $\mathcal{V}_t \subseteq \mathcal{V}$  that all can be assigned the value  $t$  without violating any constraints in  $\mathcal{E}$ 
    - $\approx$  actions the actor chooses to start at time  $t$
  2. Simultaneously, environment chooses an unassigned set of variables  $\tilde{\mathcal{V}}_t \subseteq \tilde{\mathcal{V}}$  that all can be assigned the value  $t$  without violating any constraints in  $\tilde{\mathcal{E}}$ 
    - $\approx$  actions that finish at time  $t$
  3. Each chosen time point  $v$  is assigned  $v \leftarrow t$
  4. Failure if any of the constraints in  $\mathcal{E} \cup \tilde{\mathcal{E}}$  are violated
    - There might be violations that neither  $\mathcal{V}_t$  nor  $\tilde{\mathcal{V}}_t$  caused individually
  5. Success if all variables in  $\mathcal{V} \cup \tilde{\mathcal{V}}$  have values and no constraints are violated
- **Dynamic execution strategies**  $\sigma_A$  for actor,  $\sigma_E$  for environment
  - $\sigma_A(h_{t-1}) = \{\text{what events in } \mathcal{V} \text{ to trigger at time } t, \text{ given } h_{t-1}\}$
  - $\sigma_E(h_{t-1}) = \{\text{what events in } \tilde{\mathcal{V}} \text{ to trigger at time } t, \text{ given } h_{t-1}\}$ 
    - $h_t = h_{t-1} \cdot (\sigma_A(h_{t-1}) \cup \sigma_E(h_{t-1}))$
- $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  is **dynamically controllable** if  $\exists \sigma_A$  that will guarantee success  $\forall \sigma_E$

$r_{ij} = [l, u]$  is **violated**  
if  $t_i$  and  $t_j$  have values  
and  $t_j - t_i \notin [l, u]$

# Example

- Instead of a single bring&move task, two separate bring and move tasks



- Actor's dynamic execution strategy
  - Trigger  $t_1$  at whatever time you want
  - Wait and observe  $t$
  - Trigger  $t'$  at any time from  $t$  to  $t + 5$
  - Trigger  $t_3 = t' + 10$
  - For every  $t_2 \in [t' + 15, t' + 20]$  and  $t_4 \in [t_3 + 5, t_3 + 10]$ 
    - $t_4 \in [t' + 15, t' + 20]$
    - So  $t_4 - t_3 \in [-5, 5]$
  - So all constraints are satisfied


# Dynamic Controllability Checking

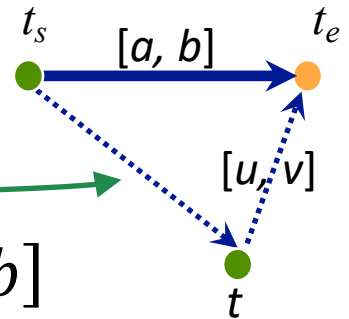
- For a chronicle  $\phi = (\mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{C})$ 
  - Temporal constraints in  $\mathcal{C}$  correspond to an STNU
  - Adapt TemPlan to test not only consistency but also dynamic controllability (\*) of the STNU
  - If we detect cases where it is not dynamically controllable, then backtrack

## \* Use PC as well

- If  $\text{PC}(\mathcal{V} \cup \tilde{\mathcal{V}}, \mathcal{E} \cup \tilde{\mathcal{E}})$  reduces a contingent constraint, then  $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  is not dynamically controllable
  - $\Rightarrow$  Can prune this branch
- If it *does not* reduce any contingent constraints, we don't know whether  $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  is dynamically controllable
  - Only **necessary**, **not sufficient** condition
- Two options
  - Either continue down this branch and backtrack later if necessary, or
  - Extend PC to detect more cases where  $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  isn't dynamically controllable
    - Additional constraint propagation rules

# Additional Constraint Propagation Rules

- Case 1:  $u \geq 0$ 
  - $t$  must come before  $t_e$
- Add a composition constraint  $[a', b']$  
- Find  $[a', b']$  such that  $[a', b'] \circ [u, v] = [a, b]$ 
  - $[a' + u, b' + v] = [a, b]$
  - $a' = a - u, b' = b - v$



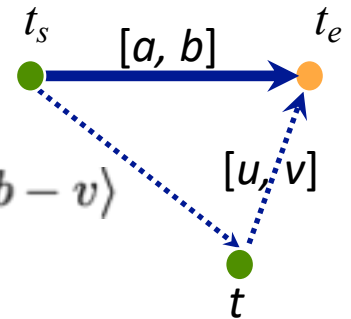
Conditions	Propagated constraint
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u < 0, v \geq 0$	$t_s \xrightarrow{\langle t_e, b' \rangle} t$
$t_s \xrightarrow{[a,b]} t_e, t_s \xrightarrow{\langle t_e, u \rangle} t$	$t_s \xrightarrow{[\min\{a, u\}, \infty]} t$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e, b' \rangle} t'$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t, t_e \neq t$	$t_s \xrightarrow{\langle t_e, b-u \rangle} t'$

$\Rightarrow$  contingent  $\rightarrow$  controllable  $a' = a - u, b' = b - v$

# Additional Constraint Propagation Rules

- Case 2:  $u < 0$  and  $v \geq 0$ 
  - $t$  may be before or after  $t_e$

- Add a **wait** constraint  $\langle t_e, \alpha \rangle$ 
  - $\alpha$  defined w.r.t. some controllable time point  $t_s$
  - Wait until either  $t_e$  occurs or current time is  $t_s + \alpha$ , whichever comes first



Conditions	Propagated constraint
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u < 0, v \geq 0$	$t_s \xrightarrow{\langle t_e, b' \rangle} t$
$t_s \xrightarrow{[a,b]} t_e, t_s \xrightarrow{\langle t_e, u \rangle} t$	$t_s \xrightarrow{[\min\{a,u\}, \infty]} t$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e, b' \rangle} t'$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t, t_e \neq t$	$t_s \xrightarrow{\langle t_e, b-u \rangle} t'$

$\Rightarrow$  contingent  $\rightarrow$  controllable  $a' = a - u, b' = b - v$

# Extended Version of PC

- We want a fast algorithm that TemPlan can run at each node, to decide whether to backtrack
- There is an extended version of PC that runs in polynomial time, but it has high overhead
- Possible compromise: use ordinary PC most of the time
  - Run extended version occasionally, or at end of search before returning plan

Conditions	Propagated constraint
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u < 0, v \geq 0$	$t_s \xrightarrow{\langle t_e, b' \rangle} t$
$t_s \xrightarrow{[a,b]} t_e, t_s \xrightarrow{\langle t_e, u \rangle} t$	$t_s \xrightarrow{[\min\{a,u\}, \infty]} t$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e, b' \rangle} t'$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t, t_e \neq t$	$t_s \xrightarrow{\langle t_e, b-u \rangle} t'$

# Intermediate Summary

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- Constraint management
  - Consistency of object constraints
    - Constraint-satisfaction problem
  - Consistency of time constraints
    - STN, solution, minimality, consistency
    - PC
- Controllability
  - STNU, controllable, contingent
  - Dynamic controllability

# Outline per the Book

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## *4.2 Representation*

- Timelines
- Actions and tasks
- Chronicles

## *4.3 Temporal planning*

- Resolvers and flaws
- Search space

## *4.4 Constraint management*

- Consistency of object constraints and time constraints
- Controlling the actions when we don't know how long they'll take

## ***4.5 Acting with temporal models***

- Acting with atemporal refinement
- Dispatching
- Observation actions

# Atemporal Refinement of Primitive Actions

- Templan's action templates may correspond to compound tasks
  - In RAE, refine into commands with refinement methods
  - Templan's action template (descriptive model)
- RAE's refinement method (operational model)

```
leave( $r, d, w$ )  
  assertions:   $[t_s, t_e]$  loc( $r$ ): ( $d, w$ )  
                $[t_s, t_e]$  occupant( $d$ ): ( $r, \text{empty}$ )  
  constraints:  $t_e \leq t_s + \delta_1$   
               adj( $d, w$ )
```

```
m-leave( $r, d, w, e$ )  
  task:  leave( $r, d, w$ )  
  pre:   loc( $r$ )= $d$ , adj( $d, w$ ), exit( $e, d, w$ )  
  body:  until empty( $e$ )  
         wait(1)  
         goto( $r, e$ )
```

# Discussion

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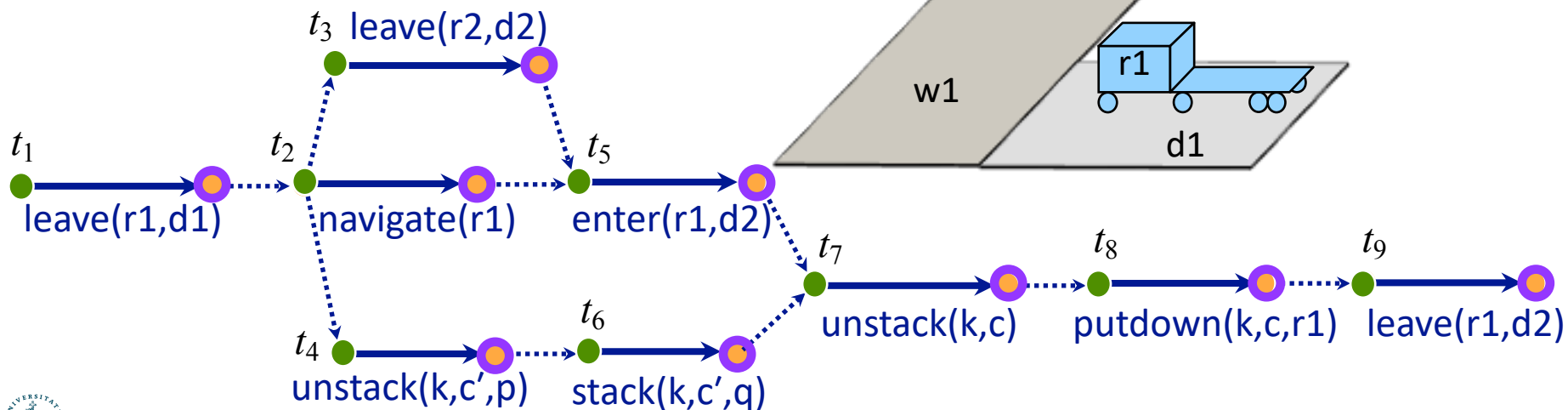
- Pros
  - Simple online refinement with RAE
  - Avoids breaking down uncertainty of contingent duration
  - Can be augmented with temporal monitoring functions in RAE
    - E.g., watchdogs, methods with duration preferences
- Cons
  - Does not handle temporal requirements at the command level,
    - e.g., synchronise two robots that must act concurrently
- Can augment RAE to include temporal reasoning
  - Call it eRAE
  - One essential component: a **dispatching** function

# Acting With Temporal Models

- Dispatching procedure: a dynamic execution strategy
  - Controls when to start each action
  - Given a dynamically controllable plan with executable primitives, triggers corresponding commands from online observations

- Example

- robot  $r2$  needs to leave dock  $d2$  before robot  $r1$  can enter  $d2$
- crane  $k$  needs to uncover  $c$  then put  $c$  onto  $r1$



# Dispatching

- Let  $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$  be a controllable STNU that is **grounded**
  - Different from a grounded expression in logic
  - At least one time point  $t$  is instantiated
  - This bounds each time point  $t$  within an interval  $[l_t, u_t]$

**Dispatch**  $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$

initialise the network

**while** there are time points in  $\mathcal{V}$  that have not been triggered **do**

update *now*

update the time points in  $\tilde{\mathcal{V}}$  that have been newly observed

update *enabled*

trigger every  $t \in \text{enabled}$  s.t.  $\text{now} = u_t$

arbitrarily choose other time points in *enabled* and trigger them

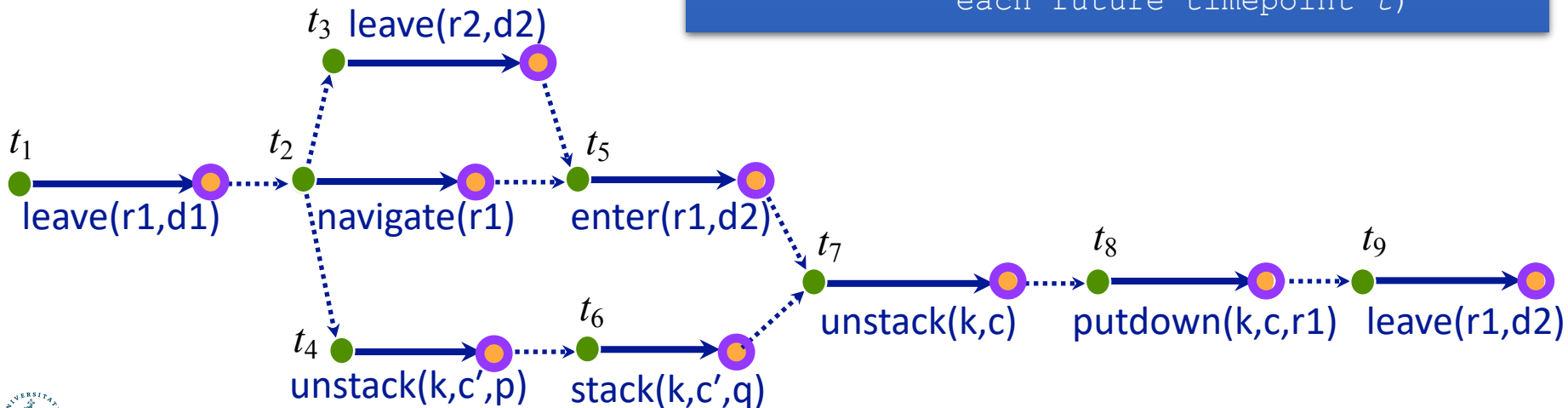
propagate values of triggered

timepoints (change  $[l_t, u_t]$  for each future timepoint  $t$ )

- Controllable time point  $t$  in the future:
  - $t$  is **alive** if current time  $\text{now} \in [l_t, u_t]$
  - $t$  is **enabled** if
    - It is alive
    - For every precedence constraint  $t' < t$ ,  $t'$  has occurred
    - For every wait constraint  $\langle t_e, \alpha \rangle$ ,  $t_e$  has occurred or  $\alpha$  has expired
      - $\alpha$  has expired if  $t_s$  has occurred and  $t_s + \alpha \leq \text{now}$

# Example

- Trigger  $t_1$ , observe leave finish
- Enable and trigger  $t_2$ , this enables  $t_3, t_4$
- Trigger  $t_3$  soon enough to allow  $enter(r1, d2)$  at time  $t_5$
- Trigger  $t_4$  soon enough to allow  $stack(k, c')$  at time  $t_6$
- Rest of plan is linear:
  - Choose each  $t_i$  after the previous action ends



**Dispatch** ( $\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}}$ )

initialise the network

**while** there are time points in  $\mathcal{V}$  that have not been triggered **do**

update *now*

update the time points in  $\tilde{\mathcal{V}}$  that have been newly observed

update *enabled*

trigger every  $t \in enabled$  s.t.  $now = u_t$

arbitrarily choose other time points in *enabled* and trigger them

propagate values of triggered timepoints (change  $[l_t, u_t]$  for each future timepoint  $t$ )

# Example from Slide 57

- Trigger  $t_1$  at time 0
- Wait and observe  $t$ ; this enables  $t'$
- Trigger  $t'$  at any time from  $t$  to  $t + 5$
- Trigger  $t_3$  at time  $t' + 10$ 
  - $t_2 \in [t' + 15, t' + 20]$
  - $t_4 \in [t_3 + 5, t_3 + 10] = [t' + 15, t' + 20]$
  - so  $t_4 - t_3 \in [-5, 5]$

**Dispatch** ( $\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}}$ )

initialise the network

**while** there are time points in  $\mathcal{V}$  that have not been triggered **do**

  update *now*

  update the time points in  $\tilde{\mathcal{V}}$  that have been newly observed

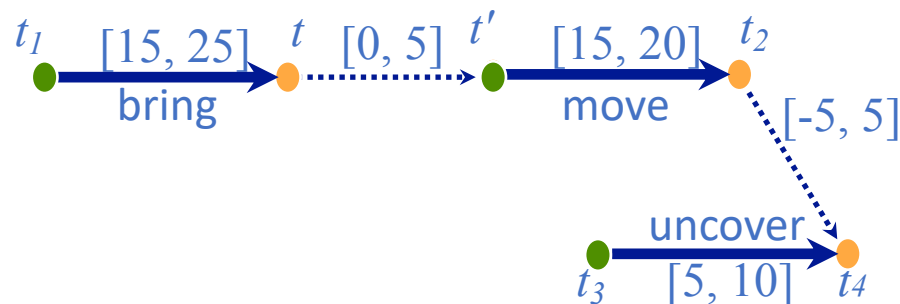
  update *enabled*

  trigger every  $t \in \text{enabled}$  s.t.  $\text{now} = u_t$

  arbitrarily choose other time points in *enabled* and trigger them

  propagate values of triggered

  timepoints (change  $[l_t, u_t]$  for each future timepoint  $t$ )



# Dispatching

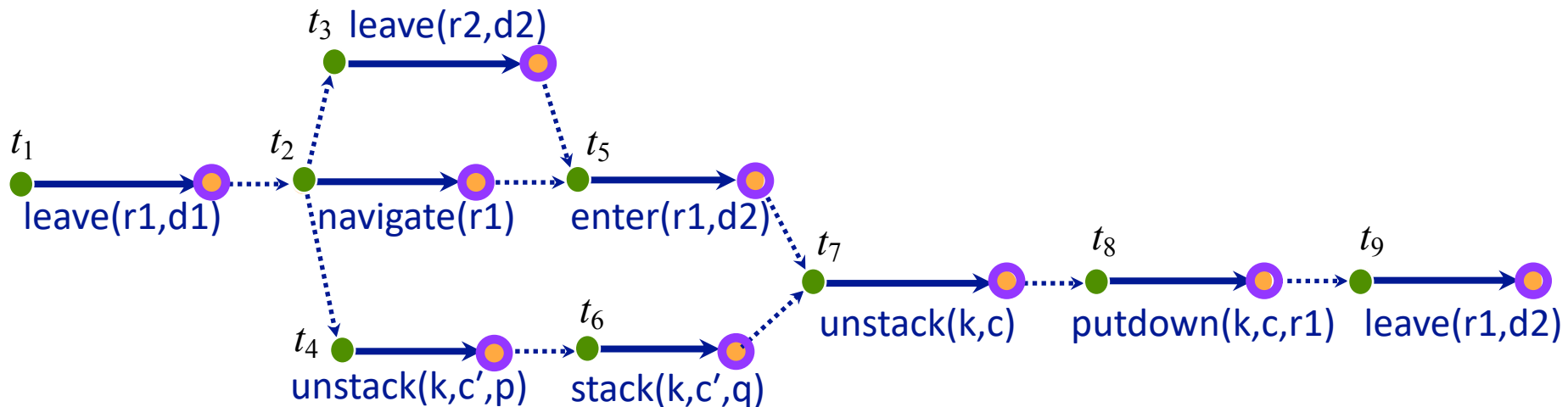
- Propagation step most costly one
  - $O(n^3)$
  - $n$  the number of remaining future time points in network

```
Dispatch( $\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}}$ )  
  initialise the network  
  while there are time points in  $\mathcal{V}$  that  
    have not been triggered do  
    update now  
    update the time points in  $\tilde{\mathcal{V}}$  that have  
      been newly observed  
    update enabled  
    trigger every  $t \in \text{enabled}$  s.t.  $\text{now} = u_t$   
    arbitrarily choose other time points  
      in enabled and trigger them  
    propagate values of triggered  
      timepoints (change  $[l_t, u_t]$  for  
        each future timepoint  $t$ )
```

- Ideally propagation fast enough to allow iterations and updates of *now* consistent with temporal granularity of plan

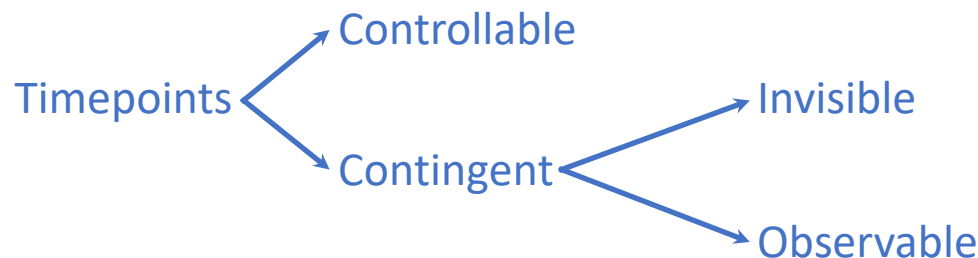
# Deadline Failures

- Suppose something makes it impossible to start an action on time
- Do one of the following:
  - Stop the delayed action, and look for new plan
  - Let the delayed action finish, try to repair the plan by resolving violated constraints at the STNU propagation level
    - E.g., accommodate a delay in navigate by delaying the whole plan
  - Let the delayed action finish, try to repair the plan some other way



# Partial Observability

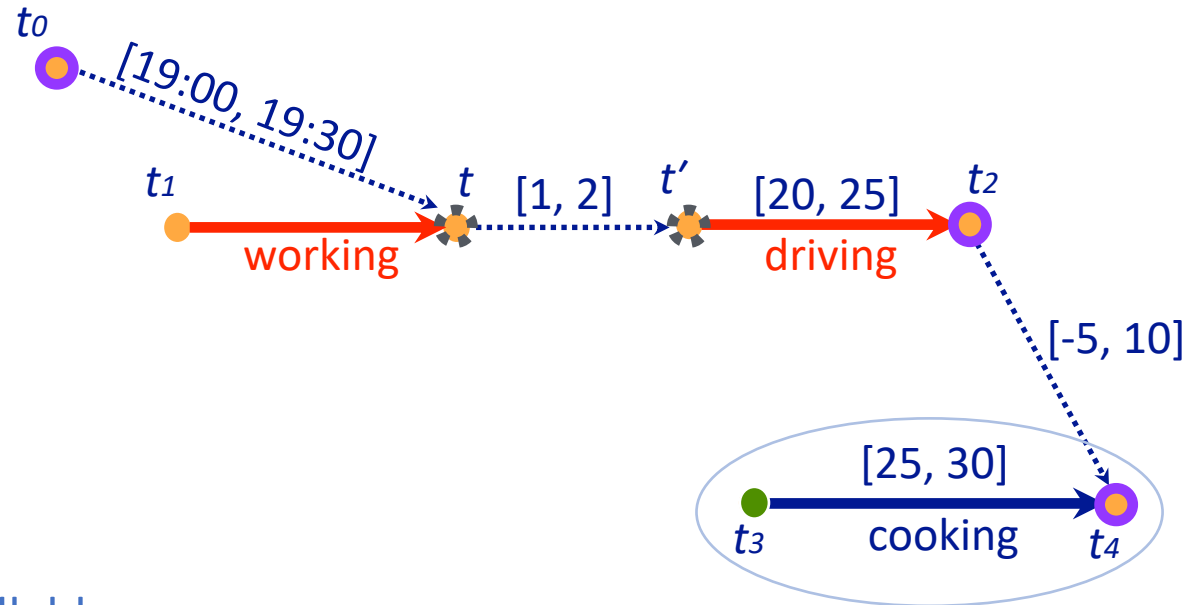
- Tacit assumption: All occurrences of contingent events are observable
  - Observation needed for dynamic controllability
- In general, not all events are observable
- POSTNU (Partially Observable STNU)



- Dynamically controllable?

# Observation Actions

- Example



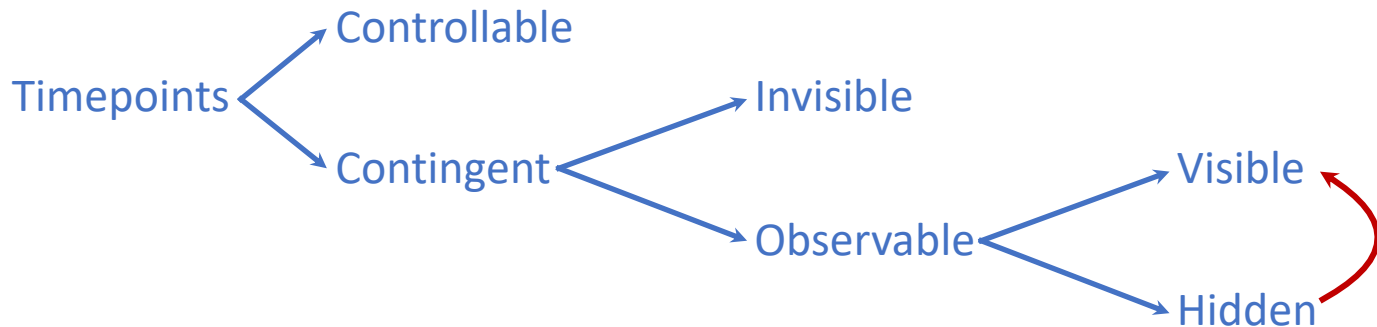
- Controllable

- Contingent

{		Invisible
		observable

# Dynamic Controllability

- A POSTNU is dynamically controllable if
  - there exists an execution strategy that chooses future controllable points to meet all the constraints, given the observation of past *visible* points
- Observable  $\neq$  visible
- Observable means it will be known **when observed**
  - It can be temporarily **hidden**



# Intermediate Summary

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- Acting
  - Atemporal refinement
    - eRAE
    - Dispatching
      - Alive, enabled
  - Deadline failures
  - Partial observability
    - Invisible, observable (hidden/visible)

# Outline per the Book

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## *4.2 Representation*

- Timelines
- Actions and tasks
- Chronicles

## *4.3 Temporal planning*

- Resolvers and flaws
- Search space

## *4.4 Constraint management*

- Consistency of object constraints and time constraints
- Controlling the actions when we don't know how long they'll take

## *4.5 Acting with temporal models*

- Acting with atemporal refinement
- Dispatching
- Observation actions

⇒ Next: Planning and Acting with Nondeterministic Models