

Advanced Topics Data Science and AI

Automated Planning and

Acting

Temporal Models

Tanya Braun



UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR INFORMATIONSSYSTEME

Content

1. Planning and Acting with **Deterministic** Models
2. Planning and Acting with **Refinement** Methods
3. Planning and Acting with **Temporal** Models
 - a. Temporal Representation
 - b. Planning with Temporal Refinement Methods
 - c. Constraint Management
 - d. Acting with Temporal Models
4. Planning and Acting with **Nondeterministic** Models
5. **Standard** Decision Making
6. Planning and Acting with **Probabilistic** Models
7. **Advanced** Decision Making
8. **Human-aware** Planning

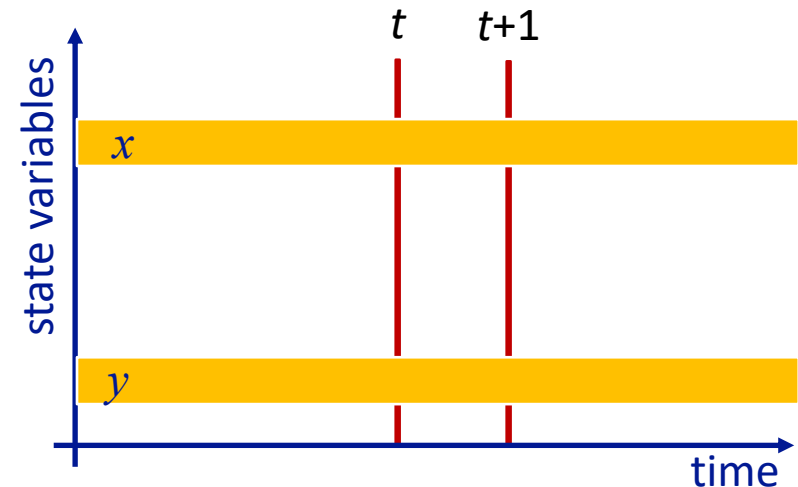
Temporal Models

- Durations of actions
- Delayed effects and preconditions
 - E.g., resources borrowed or consumed during an action
- Time constraints on goals
 - Relative or absolute
- Exogenous events expected to occur in the future
 - When?
- Maintenance actions:
 - Maintain a property (\neq changing a value)
 - E.g., track a moving target, keep a spring latch in position
- Concurrent actions
 - Interacting effects, joint effects
- Delayed commitment
 - Instantiation at acting time

Timelines

- Up to now, “state-oriented view”
 - Time is a sequence of states s_0, s_1, s_2
 - Instantaneous actions transform each state into the next one
 - No overlapping actions
- Switch to a “time-oriented view”

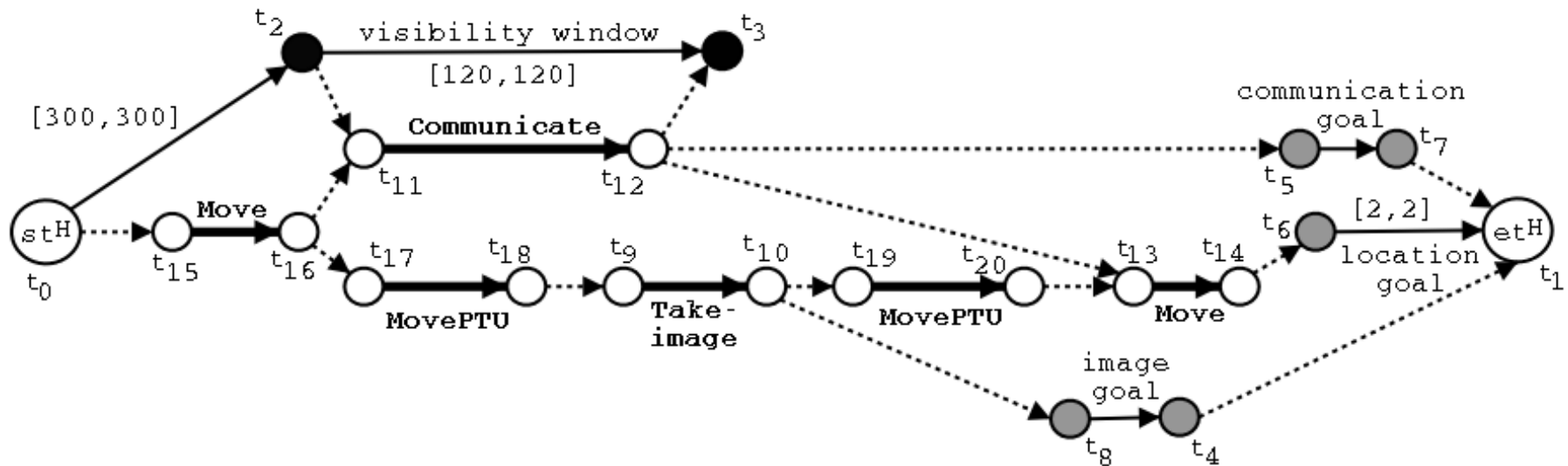
- Sequence of integer time points
 - $t = 1, 2, 3, \dots$
- For each state variable x , a **timeline**
 - values during different time intervals



- State at time $t = \{\text{state-variable values at time } t\}$

Timelines

- Sets of constraints on state variables and events
 - Reflect predicted actions and events
- Planning is constraint-based



Outline per the Book

4.2 Representation

- Timelines
- Actions and tasks
- Chronicles

4.3 Temporal Planning

- Resolvers and flaws
- Search space

4.4 Constraint Management

- Consistency of object constraints and time constraints
- Controlling the actions when we do not know how long they will take

4.5 Acting with Temporal Models

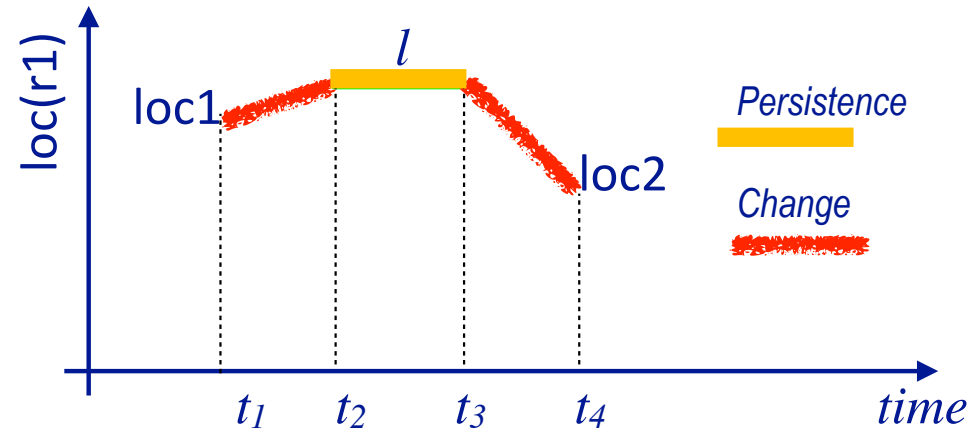
- Acting with atemporal refinement
- Dispatching
- Observation actions

Representation

- Quantitative model of time
 - Discrete: time points are integers
- Expressions:
 - time-point variables
 - t, t', t_2, t_j, \dots
 - simple constraints
 - $d \leq t' - t \leq d'$
- Temporal assertion:
 - Value of a state variable during a time interval
 - Persistence: $[t_1, t_2]x = v$ entails $t_1 < t_2$
 - Change: $[t_1, t_2]x : (v_1, v_2)$ entails $v_1 \neq v_2$

Timeline

- **Timeline**: pair $(\mathcal{T}, \mathcal{C})$, partially predicted evolution of one state variable
 - Instance of $(\mathcal{T}, \mathcal{C})$ = temporal and object variables instantiated
- \mathcal{T} : temporal assertions
 - $[t_1, t_2]loc(r1) : (loc1, l)$
 - $[t_2, t_3]loc(r1) = l$
 - $[t_3, t_4]loc(r1) : (l, loc2)$
- \mathcal{C} : constraints
 - $t_1 < t_2 < t_3 < t_4$
 - $l \neq loc1$
 - $l \neq loc2$
 - If we want to restrict $loc(r1)$ during $[t_1, t_2]$
 - $[t_1, t_1 + 1]loc(r1) : (loc1, route)$
 - $[t_2 - 1, t_2]loc(r1) : (route, l)$
 - $[t_1 + 1, t_2 - 1]loc(r1) = route$
- An instance is **consistent** if it satisfies all constraints in \mathcal{C} and does not specify two different values for a state variable at the same time
- A timeline is **secure** if its set of consistent instances is not empty



Actions

- Preliminaries:
 - Timelines $(\mathcal{T}_1, \mathcal{C}_1), \dots, (\mathcal{T}_k, \mathcal{C}_k)$ for k different state variables
 - Their **union**:
 - $(\mathcal{T}_1, \mathcal{C}_1) \cup \dots \cup (\mathcal{T}_k, \mathcal{C}_k) = (\mathcal{T}_1 \cup \dots \cup \mathcal{T}_k, \mathcal{C}_1 \cup \dots \cup \mathcal{C}_k)$
 - If
 - every $(\mathcal{T}_i, \mathcal{C}_i)$ is secure, and
 - no pair of timelines $(\mathcal{T}_i, \mathcal{C}_i)$ and $(\mathcal{T}_j, \mathcal{C}_j)$ has any unground variables in common
 - then
 - $(\mathcal{T}_1 \cup \dots \cup \mathcal{T}_k, \mathcal{C}_1 \cup \dots \cup \mathcal{C}_k)$ is also secure
- **Action** or **primitive task** (or just *primitive*):
 - a triple $(head, \mathcal{T}, \mathcal{C})$
 - *head* is the name and arguments
 - $(\mathcal{T}, \mathcal{C})$ is the union of a set of timelines

Actions

- $leave(r, d, w)$
 - Robot r leaves dock d , goes to adjacent waypoint w

$leave(r, d, w)$

assertions:

$[t_s, t_e] loc(r): (d, w)$

$[t_s, t_e] occupant(d): (r, empty)$

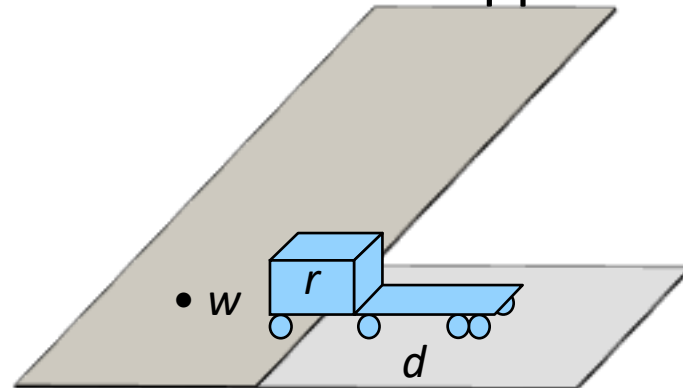
constraints:

$t_e \leq t_s + \delta_1$

$adj(d, w)$

- $loc(r)$ changes to w with delay $\leq \delta_1$
- Dock d becomes empty

- Two additional parameters
 - Starting time t_s
 - Ending time t_e
- No separate preconditions and effects
 - Preconditions \Leftrightarrow need for causal support



Actions

- $enter(r, d, w)$
 - r enters d from an adjacent waypoint w

$enter(r, d, w)$

assertions:

$[t_s, t_e] loc(r): (w, d)$

$[t_s, t_e] occupant(d): (empty, r)$

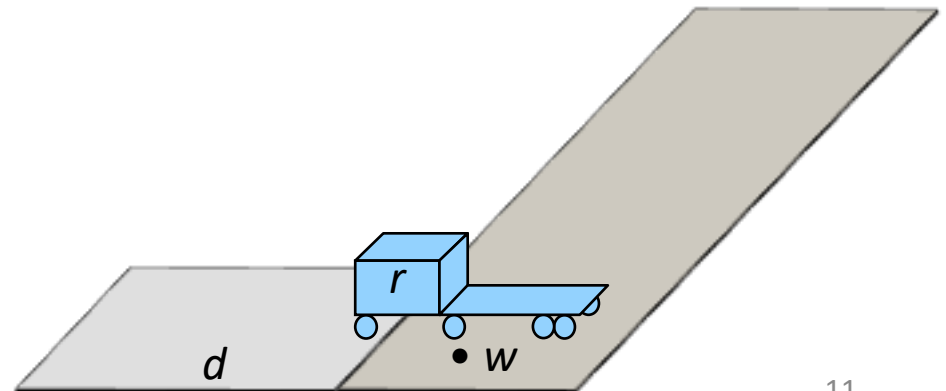
constraints:

$t_e \leq t_s + \delta_2$

$adj(d, w)$

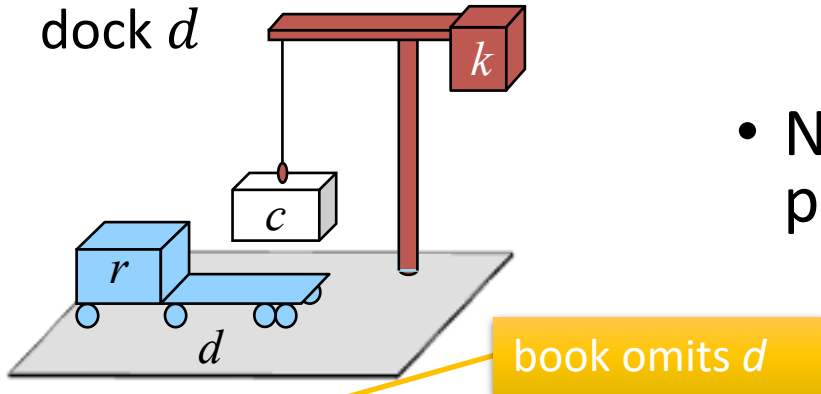
- $loc(r)$ changes to d with delay $\leq \delta_2$
- Dock d becomes occupied by r

- Two additional parameters
 - Starting time t_s
 - Ending time t_e
- No separate preconditions and effects
 - Preconditions \Leftrightarrow need for causal support



Actions

- $take(k, c, r, d)$
 - Action: crane k takes container c from r on dock d



- Two additional parameters
 - Starting time t_s
 - Ending time t_e
- No separate preconditions and effects
 - Preconditions \Leftrightarrow need for causal support

$take(k, c, r, d)$

assertions:

$[t_s, t_e]$ $pos(c): (r, k)$

// where container c is

$[t_s, t_e]$ $grip(k): (empty, c)$

// what crane k 's gripper is holding

$[t_s, t_e]$ $freight(r): (c, empty)$

// what r is carrying

$[t_s, t_e]$ $loc(r) = d$

// where r is

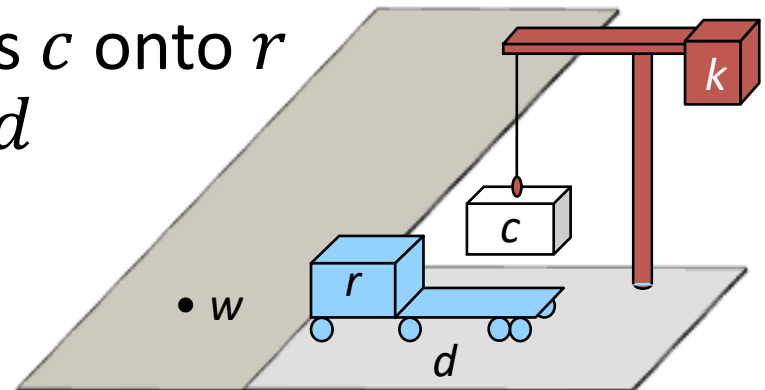
constraints:

$attached(k, d)$

Actions

- $leave(r, d, w)$ robot r leaves dock d to an adjacent waypoint w
- $enter(r, d, w)$ r enters d from an adjacent w
- $take(k, c, r, d)$ crane k takes cont. c from r at d
- $navigate(r, w, w')$ r navigates from w to w'
- $stack(k, c, p)$ k stacks c on top of pile p
- $unstack(k, c, p)$ k takes c from top of p
- $put(k, c, r, d)$ k puts c onto r at d

book omits d

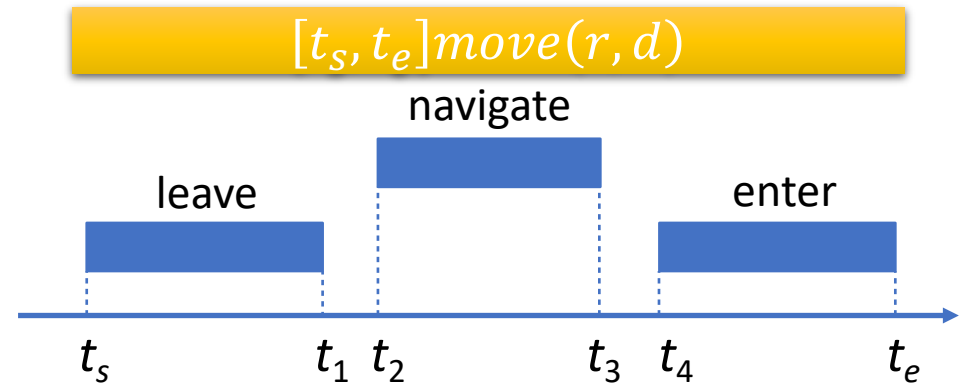


Tasks and Methods

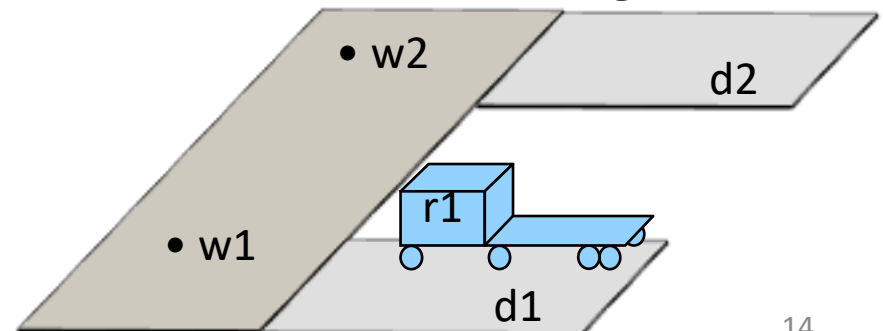
- Task: move robot r to dock d
 - $[t_s, t_e] \text{move}(r, d)$
- Method:

```

m-move1( $r, d, d', w, w'$ )
  task:    $\text{move}(r, d)$ 
  refinement:
    [ $t_s, t_1$ ]  $\text{leave}(r, d', w')$ 
    [ $t_2, t_3$ ]  $\text{navigate}(r, w', w)$ 
    [ $t_4, t_e$ ]  $\text{enter}(r, d, w)$ 
  assertions:
    [ $t_s, t_s+1$ ]  $\text{loc}(r) = d'$ 
  constraints:
     $\text{adj}(d, w)$ ,
     $\text{adj}(d', w')$ ,  $d \neq d'$ ,
     $\text{connected}(w, w')$ ,
     $t_1 \leq t_2$ ,  $t_3 \leq t_4$ 
    
```

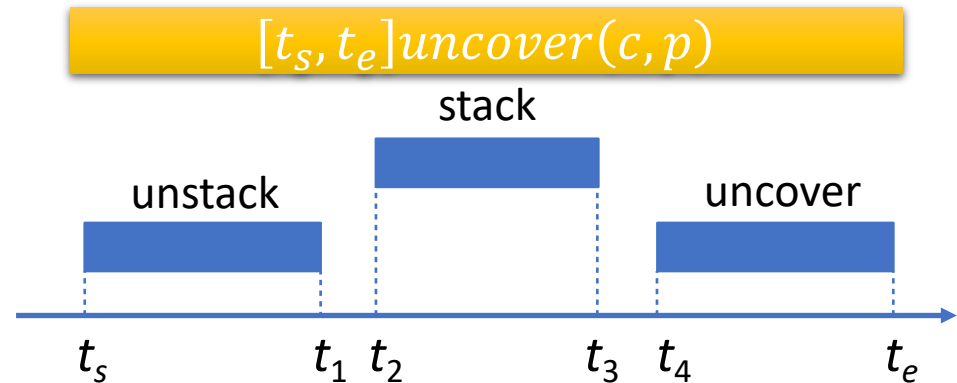


- d' becomes empty during $[t_s, t_1]$
 - another robot may enter it after t_1
- d doesn't need to be empty until t_4
 - when r starts entering it



Tasks and Methods

- Task: remove everything above container c in pile p
 - $[t_s, t_e] \text{uncover}(c, p)$
- Method:



m-uncover(c, p, k, d, p')

task: uncover(c, p)

refinement: $[t_s, t_1]$ unstack(k, c', p) // action

$[t_2, t_3]$ stack(k, c', p') // action

$[t_4, t_e]$ uncover(c, p) // recursive uncover

assertions: $[t_s, t_s+1]$ pile(c) = p

$[t_s, t_s+1]$ top(p) = c'

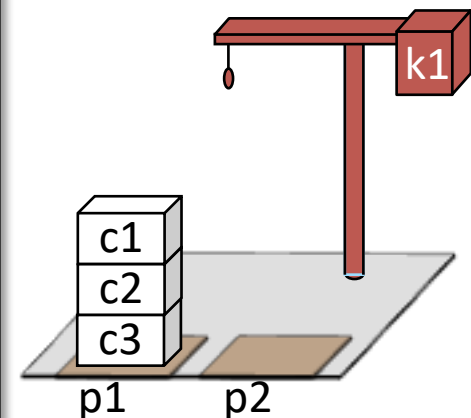
$[t_s, t_s+1]$ grip(k) = empty

constraints: attached(k, d), attached(p, d),

attached(p', d),

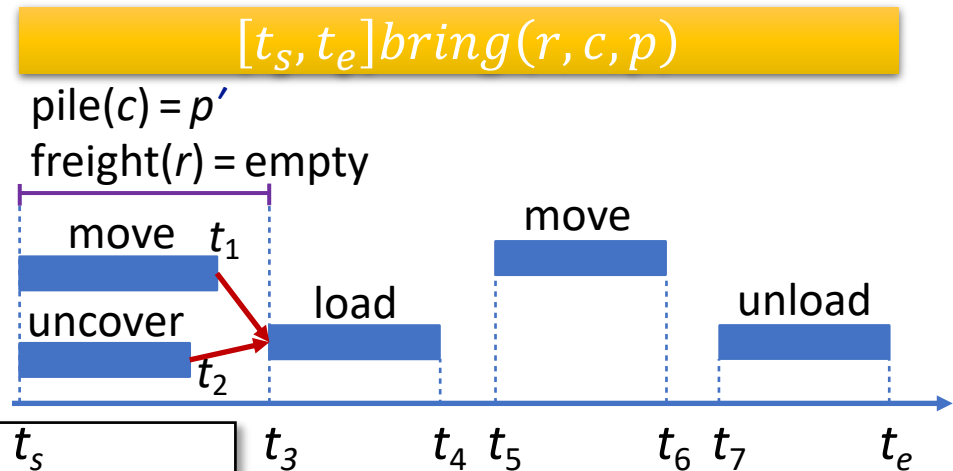
$p \neq p', c' \neq c$,

$t_1 \leq t_2, t_3 \leq t_4$



Tasks and Methods

- Task: robot r brings container c to pile p
 - $[t_s, t_e]bring(r, c, p)$
- Method:



m-bring(r, c, p, p', d, d')

task: bring(r, c, p)

refinement: $[t_s, t_1] move(r, d')$

$[t_s, t_2] uncover(c, p')$

$[t_3, t_4] load(k', r, c, p')$

$[t_5, t_6] move(r, d)$

$[t_7, t_e] unload(k, r, c, p)$

assertions: $[t_s, t_3] pile(c) = p'$

$[t_s, t_3] freight(r) = empty$

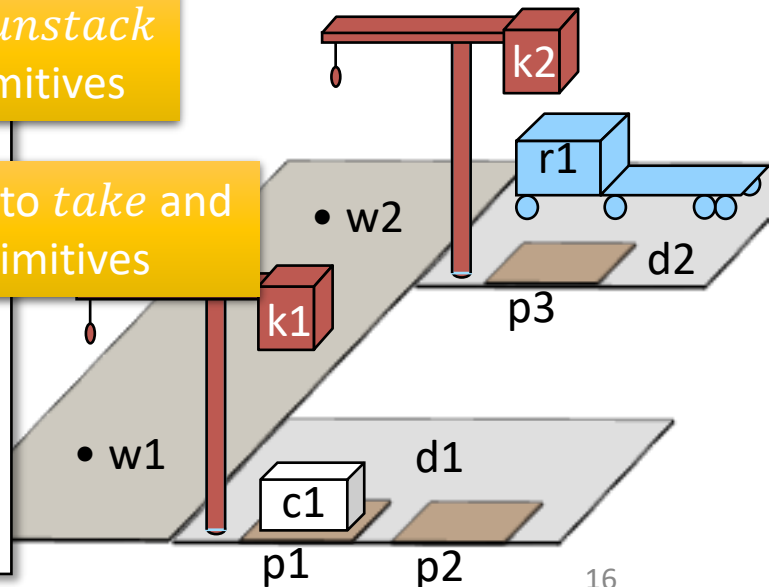
constraints: attached(p', d'), attached(p, d), $d \neq d'$

attached(k', d'), attached(k, d), $k \neq k'$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$

Refine into *unstack* and *put* primitives

Refine into *take* and *stack* primitives



Chronicles: Unions of Timelines

- Chronicle $\phi = (\mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{C})$
 - \mathcal{A} : temporally qualified actions and tasks
 - \mathcal{S} : *a priori* supported assertions
 - \mathcal{T} : temporally qualified assertions
 - \mathcal{C} : constraints
- ϕ can include
 - Current state, future predicted events
 - Tasks to perform
 - Assertions and constraints to satisfy
- Can represent
 - Planning problem
 - Plan or partial plan

ϕ_0 :

tasks: $[t, t']$ bring($r, c1, d4$)

supported: $[t_s]$ loc($r1$)= $d1$

$[t_s]$ loc($r2$)= $d2$

$[t_s+10, t_s+\delta]$ docked($ship1$)= $d3$

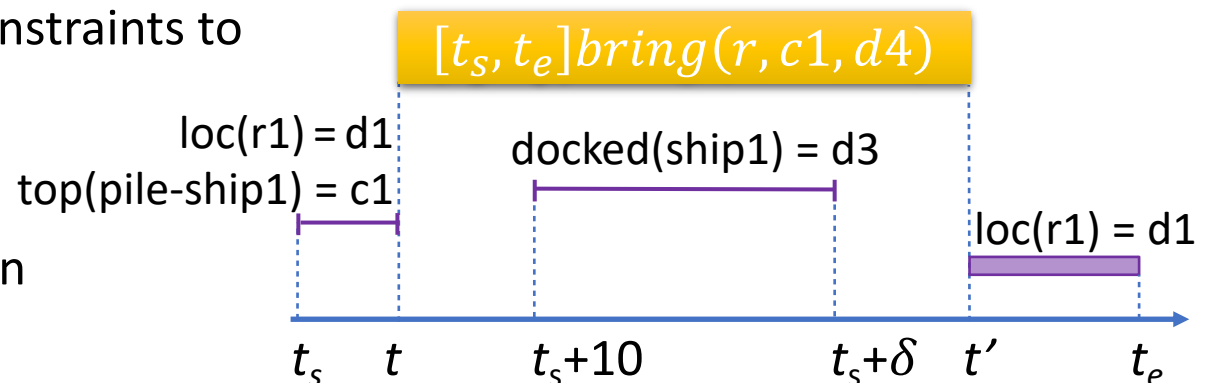
$[t_s]$ top($pile$ - $ship1$)= $c1$

$[t_s]$ pos($c1$)= $pallet$

assertions: $[t_e]$ loc($r1$)= $d1$

$[t_e]$ loc($r2$)= $d2$

constraints: $t_s = 0 < t < t' < t_e$, $20 \leq \delta \leq 30$



Intermediate Summary

- Timelines
 - Temporal assertions (change, persistence), constraints
 - Conflicts, consistency, security, causal support
- Chronicle: union of several timelines
 - Consistency, security, causal support
- Actions represented by chronicles
 - No separate preconditions and effects
 - Preconditions \Leftrightarrow need for causal support

Outline per the Book

4.2 Representation

- Timelines
- Actions and tasks
- Chronicles

4.3 Temporal Planning

- Resolvers and flaws
- Search space

4.4 Constraint Management

- Consistency of object constraints and time constraints
- Controlling the actions when we do not know how long they will take

4.5 Acting with Temporal Models

- Acting with atemporal refinement
- Dispatching
- Observation actions

Planning

- Planning problem:
 - Chronicle ϕ_0 that has some flaws
 - Analogous to flaws in PSP
- Add new assertions, constraints, actions to resolve the flaws

ϕ_0 : tasks: *(none)*
 supported: *(none)*
 assertions: $[t_1, t_2] \text{ loc}(r1) = l$
 $[t_3, t_4] \text{ loc}(r1) : (\text{loc3}, \text{loc4})$
 constraints: $\text{adj}(\text{loc3}, w1)$
 $\text{adj}(w1, \text{loc3})$
 $\text{adj}(\text{loc4}, w2)$
 $\text{adj}(w2, \text{loc4})$
 $\text{connected}(w1, w2)$

ϕ_0 : tasks: $[t_2, t_3] \text{ move}(r1, \text{loc3})$
 supported: *(none)*
 assertions: $[t_1, t_2] \text{ loc}(r1) = l$
 $[t_3, t_4] \text{ loc}(r1) : (\text{loc3}, \text{loc4})$
 constraints: $\text{adj}(\text{loc3}, w1)$
 $\text{adj}(w1, \text{loc3})$
 $\text{adj}(\text{loc4}, w2)$
 $\text{adj}(w2, \text{loc4})$
 $\text{connected}(w1, w2)$



Flaws (1)

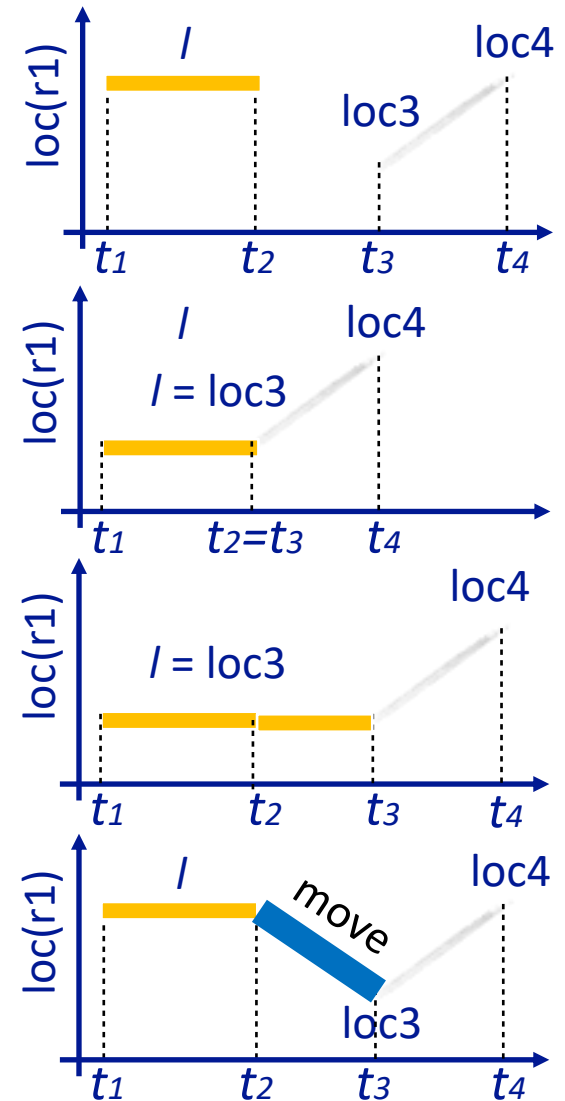
1. Temporal assertion α that is not *causally supported*

- What causes $r1$ to be at $loc3$ at time t_3 ?

Like an open goal in PSP

• *Resolvers:*

- Add constraints to support α from an assertion in ϕ
 - $l = loc3, t_2 = t_3$
- Add a new persistence assertion to support α
 - $l = loc3, [t_2, t_3]loc(r1) = loc3$
- Add a new task or action to support α
 - $[t_2, t_3]move(r1, loc3)$
 - Refining it will produce support for α

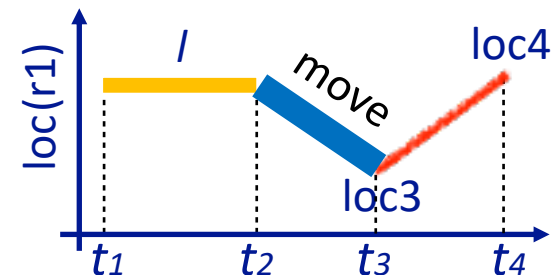


Flaws (2)

2. Non-refined task

Like a task in SeRPE

- *Resolver*: refinement method m
 - Applicable if it matches the task and its constraints are consistent with ϕ 's
- Applying the resolver:
 - Modify ϕ by replacing the task with m
- Example: $[t_2, t_3]move(r1, loc3)$
 - Refinement will replace it with something like
 - $[t_2, t_5]leave(r1, l, w)$
 - $[t_5, t_6]navigate(r1, w, w')$
 - $[t_6, t_3]enter(r1, loc3, w')$
 - plus constraints



Flaws (3)

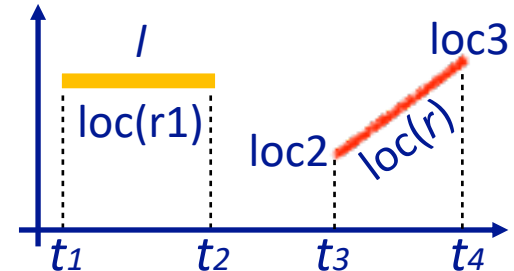
3. A pair of possibly-conflicting temporal assertions

Like a threat in PSP

- temporal assertions α and β **possibly conflict** if they can have inconsistent instances

- Example

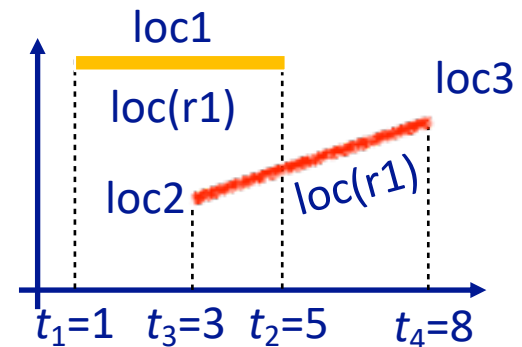
- $[t_1, t_2]loc(r1) = loc1, [t_3, t_4]loc(r) : (l, l')$



instance: $[1, 5]loc(r1) = loc1, [3, 8]loc(r) : (loc2, loc3)$

- Resolvers:** separation constraints

- $r \neq r1$
- $t_2 < t_3$
- $t_4 < t_1$
- $t_2 = t_3, r = r1, l = loc1$
 - Also provides causal support for $[t_3, t_4]loc(r) : (l, l')$
- $t_4 = t_1, r = r1, l' = loc1$
 - Also provides causal support for $[t_1, t_2]loc(r1) = loc1$



Planning Algorithm

- Like PSP in Ch. 2
 - Repeatedly selects flaws and chooses resolvers
- In the book, TemPlan uses recursion
 - Can be rewritten with a loop
- If resolving all flaws possible, at least one nondeterministic execution trace will do so
- In a deterministic implementation
 - Selecting a resolver ρ is a backtracking point
 - Selecting a flaw is not
 - (As in PSP)

TemPlan (ϕ, Σ)

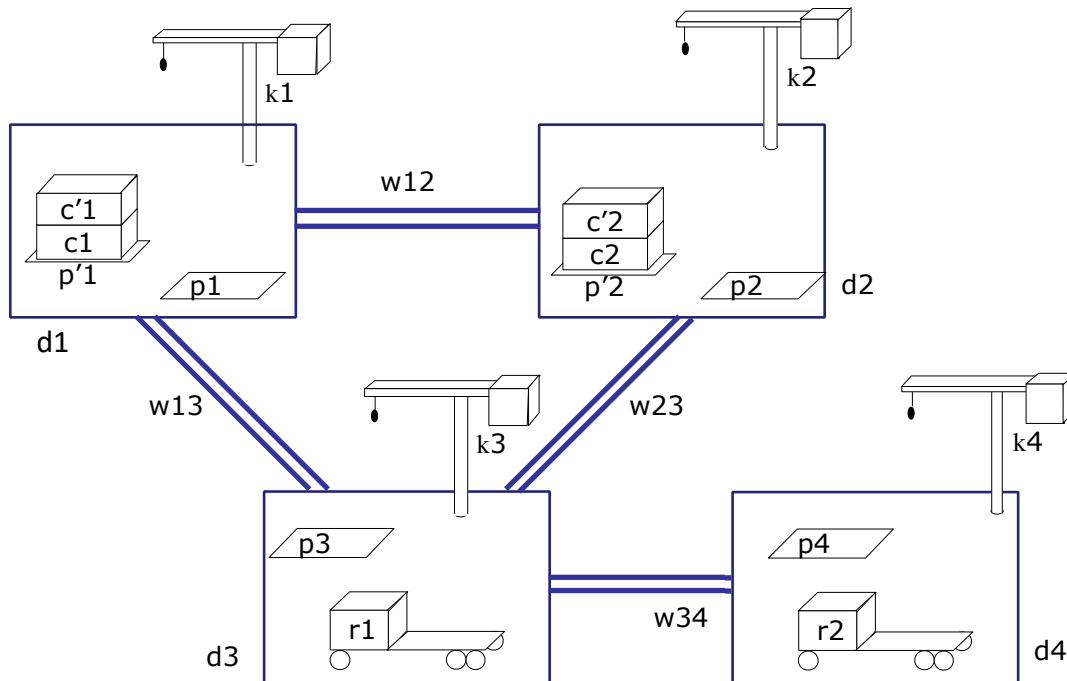
```
Flaws  $\leftarrow$  set of flaws of  $\phi$ 
if Flaws =  $\emptyset$  then
    return  $\phi$ 
arbitrarily select  $f \in$  Flaws
Resolvers  $\leftarrow$  set of resolvers of  $f$ 
if Resolvers =  $\emptyset$  then
    return failure
nondeterministically choose  $\rho \in$  Resolvers
 $\phi \leftarrow$  Transform( $\phi, \rho$ )
TemPlan ( $\phi, \Sigma$ )
```

TemPlan (ϕ, Σ)

```
loop
    Flaws  $\leftarrow$  set of flaws of  $\phi$ 
    if Flaws =  $\emptyset$  then
        return  $\phi$ 
    arbitrarily select  $f \in$  Flaws
    Resolvers  $\leftarrow$  set of resolvers of  $f$ 
    if Resolvers =  $\emptyset$  then
        return failure
    nondeterministically choose  $\rho \in$  Resolvers
     $\phi \leftarrow$  Transform( $\phi, \rho$ )
```


Example

- $\phi = (\mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{C})$
 - Establishes state-variable values at time $t = 0$
 - Flaws: two unrefined tasks
 - $\text{bring}(r, c1, p3)$, $\text{bring}(r', c2, p4)$



ϕ_0 : tasks: $\text{bring}(r, c1, p3)$
 $\text{bring}(r', c2, p4)$

supported: $[0] \text{loc}(r1)=d3$

$[0] \text{freight}(r1)=\text{empty}$

$[0] \text{pile}(c1)=p'1$

$[0] \text{pile}(c'1)=p'1$

$[0] \text{pos}(c1)=\text{pallet}$

$[0] \text{pos}(c'1)=c1$

...

assertions: (none)

constraints:

$\text{adj}(d1, w12)$

$\text{adj}(d1, w13)$

...

Example

- Flaws: two unrefined tasks
 - $\text{bring}(r,c1,p3)$, $\text{bring}(r',c2,p4)$
- Refinement for both:

$m\text{-bring}(r,c,p,p',d,d',k,k')$

task: $\text{bring}(r,c,p)$

refinement: $[t_s, t_1]$ $\text{move}(r,d')$

$[t_s, t_2]$ $\text{uncover}(c,p')$

$[t_3, t_4]$ $\text{load}(k',r,c,p')$

$[t_5, t_6]$ $\text{move}(r,d)$

$[t_7, t_e]$ $\text{unload}(k,r,c,p)$

assertions: $[t_s, t_3]$ $\text{pile}(c) = p'$

$[t_s, t_3]$ $\text{freight}(r) = \text{empty}$

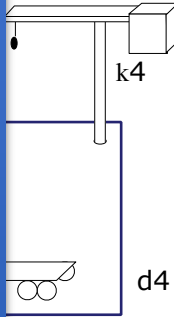
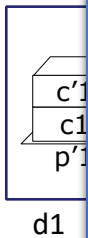
constraints: $\text{attached}(p',d')$,

$\text{attached}(p,d), d \neq d'$

$\text{attached}(k',d')$,

$\text{attached}(k,d), k \neq k'$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$



ϕ_0 : tasks: $\text{bring}(r,c1,p3)$
 $\text{bring}(r',c2,p4)$

supported: $[0]$ $\text{loc}(r1)=d3$
 $[0]$ $\text{freight}(r1)=\text{empty}$
 $[0]$ $\text{pile}(c1)=p'1$
 $[0]$ $\text{pile}(c'1)=p'1$
 $[0]$ $\text{pos}(c1)=\text{pallet}$
 $[0]$ $\text{pos}(c'1)=c1$
 ...

assertions: (none)

constraints:
 $\text{adj}(d1,w12)$
 $\text{adj}(d1,w13)$
 ...

Method Instance

- Instantiate $c = c1$ and $p = p3$ to match $bring(r, c1, p3)$
 - p', d, d', k, k' instantiated to match book
 - Needed later to satisfy action preconditions

m-bring($r, c, p, p', d, d', k, k'$)

refine

m-bring($r, c1, p3, p'1, d3, d1, k3, k1$)

task: bring($r, c1, p3$)

refinement: [t_s, t_1] move($r, d1$)

[t_s, t_2] uncover($c1, p'1$)

[t_3, t_4] load($k1, r, c1, p'1$)

[t_5, t_6] move($r, d3$)

[t_7, t_e] unload($k3, r, c1, p3$)

assertions: [t_s, t_3] pile($c1$) = $p'1$

[t_s, t_3] freight(r) = empty

constraints: attached($p'1, d1$),
attached($p3, d3$), $d3 \neq d1$
attached($k1, d1$),
attached($k3, d3$), $k3 \neq k1$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$

ϕ_0 : tasks: bring($r, c1, p3$)
bring($r', c2, p4$)

supported: [0] loc($r1$)= $d3$

[0] freight($r1$)=empty

[0] pile($c1$)= $p'1$

[0] pile($c'1$)= $p'1$

[0] pos($c1$)=pallet

[0] pos($c'1$)= $c1$

...

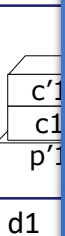
assertions: (*none*)

constraints:

adj($d1, w12$)

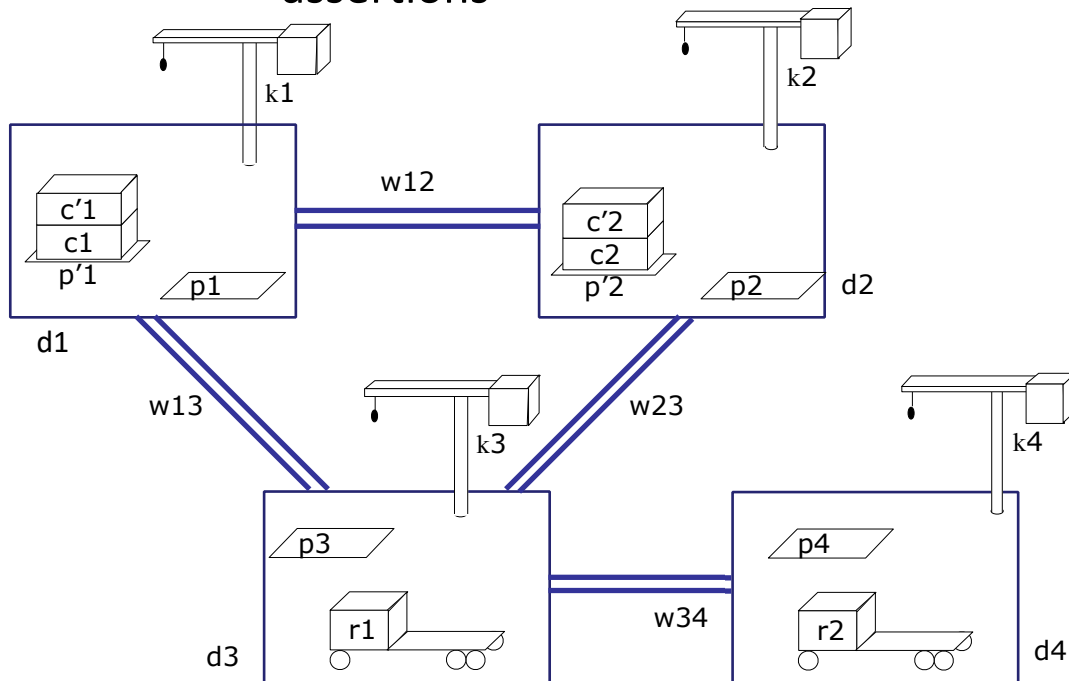
adj($d1, w13$)

...



Modified Chronicle

- Changes to ϕ_0
 - Removed *bring*($r, c1, p3$)
 - Added 5 tasks, 2 assertions, 4 constraints
- Flaws
 - 6 unrefined tasks, 2 unsupported assertions



ϕ_1 : tasks: $[t_s, t_1]$ *move*($r, d1$)
 $[t_s, t_2]$ *uncover*($c1, p'1$)
 $[t_3, t_4]$ *load*($k1, r, c1, p'1$)
 $[t_5, t_6]$ *move*($r, d3$)
 $[t_7, t_e]$ *unload*($k3, r, c1, p3$)
bring($r', c2, p4$)

supported: $[0]$ *loc*($r1$)= $d3$
 $[0]$ *freight*($r1$)=*empty*
 $[0]$ *pile*($c1$)= $p'1$
 $[0]$ *pile*($c'1$)= $p'1$
 $[0]$ *pos*($c1$)=*pallet*
 $[0]$ *pos*($c'1$)= $c1$

...

assertions: $[t_s, t_3]$ *pile*($c1$) = $p'1$
 $[t_s, t_3]$ *freight*(r) = *empty*

constraints: $t_s < t_1 \leq t_3$, $t_s < t_2 \leq t_3$, $t_4 \leq t_5$, $t_6 \leq t_7$,
adj($d1, w12$),
adj($d1, w13$),

...

Method Instance

- Instantiate $r = r', c = c2, p = p4$ to match $bring(r', c2, p4)$
 - p', d, d', k, k' instantiated to match book

m-bring($r, c, p, p', d, d', k, k'$)

refine

m-bring($r', c2, p4, p'2, d4, d2, k4, k2$)

task: bring($r', c2, p4$)

refinement: [t_s, t_1] move($r', d2$)

[t_s, t_2] uncover($c2, p'2$)

[t_3, t_4] load($k2, r', c2, p'2$)

[t_5, t_6] move($r', d4$)

[t_7, t_e] unload($k4, r', c2, p4$)

assertions: [t_s, t_3] pile($c2$) = $p'2$

[t_s, t_3] freight(r') = empty

constraints: attached($p'2, d2$),

attached($p4, d4$), $d4 \neq d2$

attached($k2, d2$),

attached($k4, d4$), $k4 \neq k2$

$t_1 \leq t_3, t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7$

asser

constr

ϕ_1 : tasks: [t_s, t_1] move($r, d1$)
 [t_s, t_2] uncover($c1, p'1$)
 [t_3, t_4] load($k1, r, c1, p'1$)
 [t_5, t_6] move($r, d3$)
 [t_7, t_e] unload($k3, r, c1, p3$)
 bring($r', c2, p4$)

supported: [0] loc($r1$)= $d3$

[0] freight($r1$)=empty

[0] pile($c1$)= $p'1$

[0] pile($c'1$)= $p'1$

[0] pos($c1$)=pallet

[0] pos($c'1$)= $c1$

...

assertions: [t_s, t_3] pile($c1$) = $p'1$

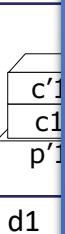
[t_s, t_3] freight(r) = empty

constraints: $t_s < t_1 \leq t_3, t_s < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$

adj($d1, w12$),

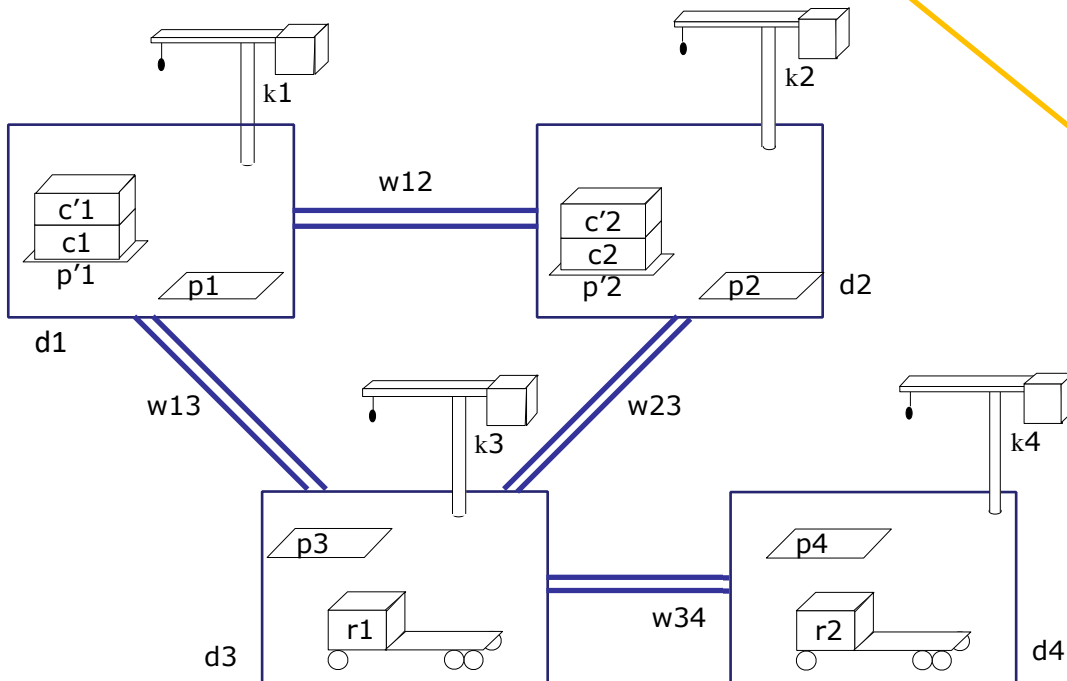
adj($d1, w13$),

...



Modified Chronicle

- Changes
 - Removed *bring(r', c2, p4)*
 - Added 5 tasks, 2 assertions, 4 constraints
- Flaws
 - 10 unrefined tasks, 4 unsupported assertions
- Next, work on these two assertions



ϕ_2 : tasks: $[t_s, t_1]$ move(*r*, d1)
 $[t_s, t_2]$ uncover(*c1*, p'1)
 $[t_3, t_4]$ load(*k1*, *r*, *c1*, p'1)
 $[t_5, t_6]$ move(*r*, d3)
 $[t_7, t_e]$ unload(*k3*, *r*, *c1*, p3)
 $[t'_s, t'_1]$ move(*r'*, d2)
 $[t'_s, t'_2]$ uncover(*c2*, p'2)
 $[t'_3, t'_4]$ load(*k4*, *r'*, *c2*, p'2)
 $[t'_5, t'_6]$ move(*r'*, d4)
 $[t'_7, t'_e]$ unload(*k2*, *r'*, *c2*, p'2)

supported: [0] loc(*r1*)=d3
 [0] freight(*r1*)=empty
 [0] pile(*c1*)=p'1
 ...

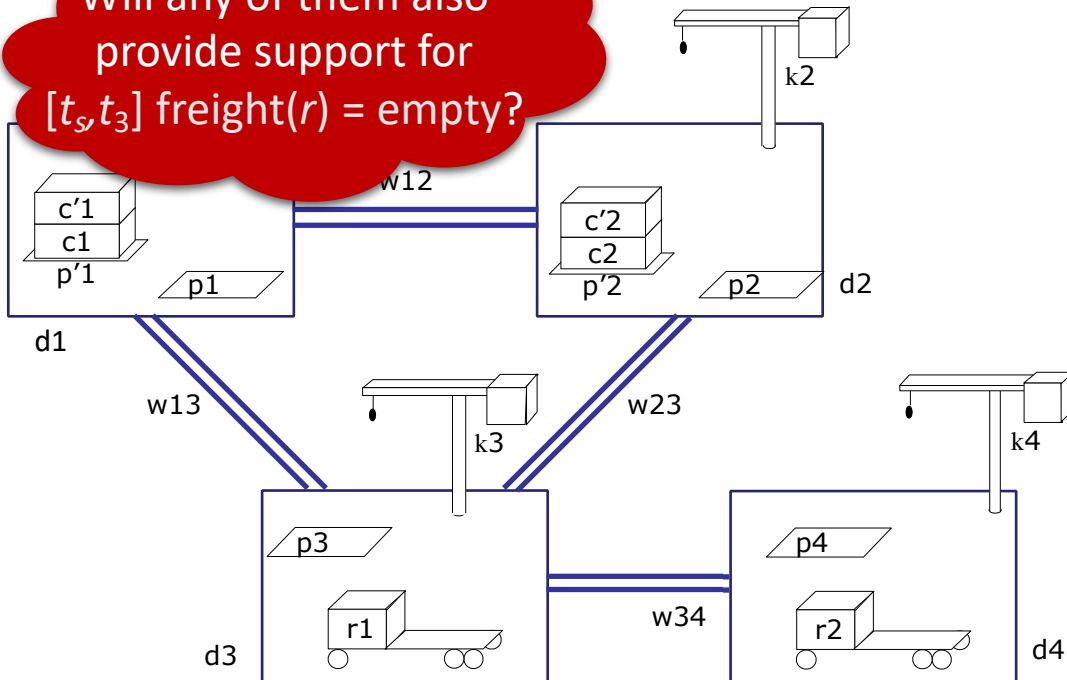
assertions: $[t_s, t_3]$ pile(*c1*) = p'1
 $[t_s, t_3]$ freight(*r*) = empty
 $[t'_s, t'_3]$ pile(*c2*) = p'2
 $[t'_s, t'_1]$ freight(*r'*) = empty

constraints: $t_s < t_1 \leq t_3$, $t_s < t_2 \leq t_3$, $t_4 \leq t_5$, $t_6 \leq t_7$,
 $t'_s < t'_1 \leq t'_3$, $t'_s < t'_2 \leq t'_3$, $t'_4 \leq t'_5$, $t'_6 \leq t'_7$,
 adj(d1, w12),
 adj(d1, w13), ...

Supporting the Assertions

- 3 ways to support $[t_s, t_3] \text{pile}(c1) = p'1$
 1. **Constrain $t_s = 0$** , use $[0] \text{pile}(c1) = p'1$
 2. Add persistence $[0, t_s] \text{pile}(c1) = p'1$
 3. Add new action $[t_8, t_s] \text{stack}(k1, c1, p'1)$

Will any of them also provide support for $[t_s, t_3] \text{freight}(r) = \text{empty}$?



ϕ_2 : tasks: $[t_s, t_1] \text{move}(r, d1)$
 $[t_s, t_2] \text{uncover}(c1, p'1)$
 $[t_3, t_4] \text{load}(k1, r, c1, p'1)$
 $[t_5, t_6] \text{move}(r, d3)$
 $[t_7, t_e] \text{unload}(k3, r, c1, p3)$
 $[t'_s, t'_1] \text{move}(r', d2)$
 $[t'_s, t'_2] \text{uncover}(c2, p'2)$
 $[t'_3, t'_4] \text{load}(k4, r', c2, p'2)$
 $[t'_5, t'_6] \text{move}(r', d4)$
 $[t'_7, t'_e] \text{unload}(k2, r', c2, p'2)$

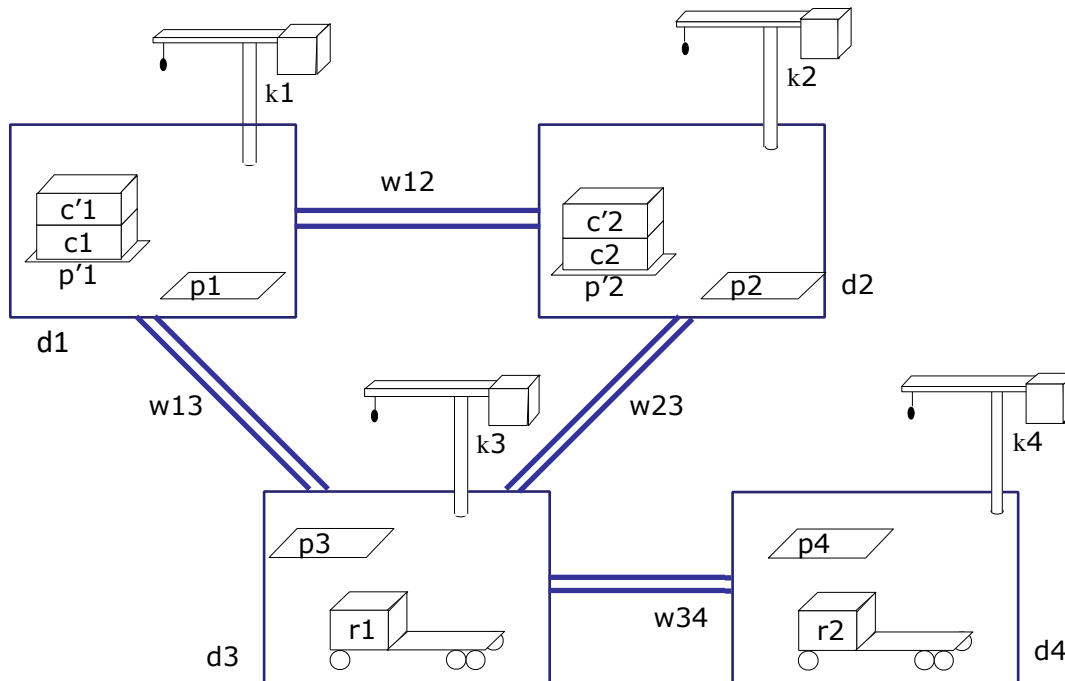
supported: $[0] \text{loc}(r1) = d3$
 $[0] \text{freight}(r1) = \text{empty}$
 $[0] \text{pile}(c1) = p'1$
 ...

assertions: $[t_s, t_3] \text{pile}(c1) = p'1$
 $[t_s, t_3] \text{freight}(r) = \text{empty}$
 $[t'_s, t'_3] \text{pile}(c2) = p'2$
 $[t'_s, t'_1] \text{freight}(r') = \text{empty}$

constraints: $t_s < t_1 \leq t_3, t_s < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$
 $\text{adj}(d1, w12),$
 $\text{adj}(d1, w13), \dots$

Supporting the Assertions

- 3 ways to support $[t_s, t_3] \text{pile}(c1) = p'1$
 1. Constrain $t_s = 0$, use $[0] \text{pile}(c1) = p'1$
- To support $[0, t_3] \text{freight}(r) = \text{empty}$
 1. Constrain $r = r1$



ϕ_2 : tasks: $[0, t_1]$ move($r, d1$)
 $[0, t_2]$ uncover($c1, p'1$)
 $[t_3, t_4]$ load($k1, r, c1, p'1$)
 $[t_5, t_6]$ move($r, d3$)
 $[t_7, t_e]$ unload($k3, r, c1, p3$)
 $[t'_s, t'_1]$ move($r', d2$)
 $[t'_s, t'_2]$ uncover($c2, p'2$)
 $[t'_3, t'_4]$ load($k4, r', c2, p'2$)
 $[t'_5, t'_6]$ move($r', d4$)
 $[t'_7, t'_e]$ unload($k2, r', c2, p'2$)

supported: $[0] \text{loc}(r1) = d3$
 $[0] \text{freight}(r1) = \text{empty}$
 $[0] \text{pile}(c1) = p'1$

...
 assertions: $[0, t_3] \text{pile}(c1) = p'1$
 $[0, t_3] \text{freight}(r) = \text{empty}$

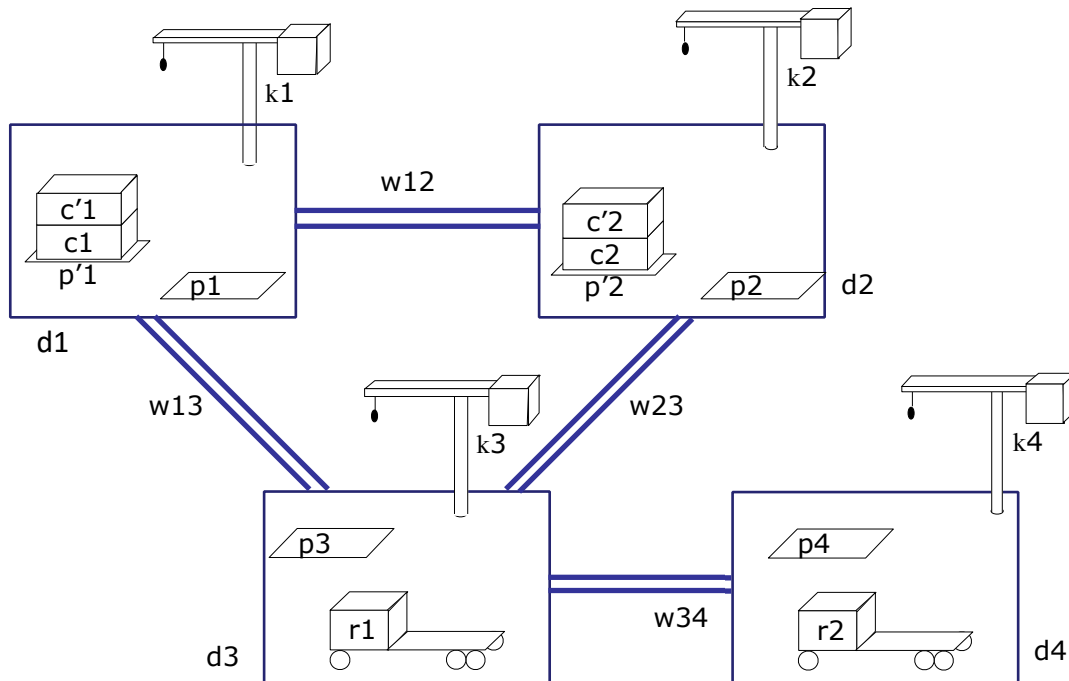
$[t'_s, t'_3] \text{pile}(c2) = p'2$
 $[t'_s, t'_1] \text{freight}(r') = \text{empty}$

constraints: $[0] < t_1 \leq t_3, [0] < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$

adj($d1, w12$),
 adj($d1, w13$), ...

Supporting the Assertions

- 3 ways to support $[t_s, t_3] \text{pile}(c1) = p'1$
 1. Constrain $t_s = 0$, use $[0] \text{pile}(c1) = p'1$
- To support $[0, t_3] \text{freight}(r) = \text{empty}$
 1. **Constrain $r = r1$**

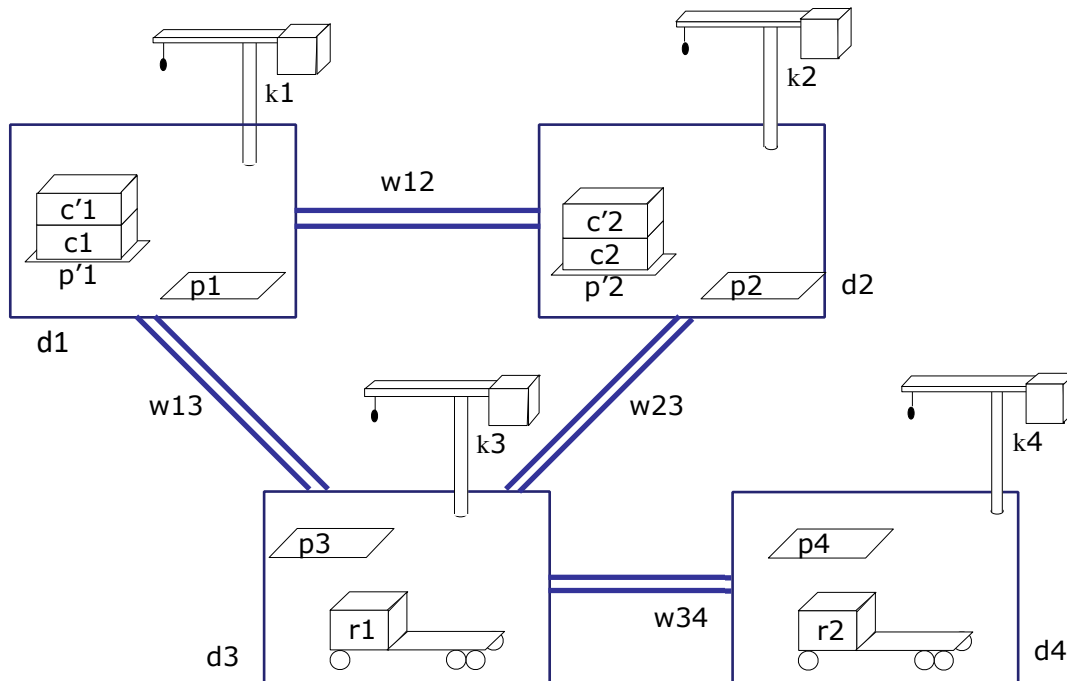


ϕ_2 : tasks: $[0, t_1]$ move(**r1**, d1)
 $[0, t_2]$ uncover(c1, p'1)
 $[t_3, t_4]$ load(k1, **r1**, c1, p'1)
 $[t_5, t_6]$ move(**r1**, d3)
 $[t_7, t_e]$ unload(k3, **r1**, c1, p3)
 $[t'_s, t'_1]$ move(r', d2)
 $[t'_s, t'_2]$ uncover(c2, p'2)
 $[t'_3, t'_4]$ load(k4, r', c2, p'2)
 $[t'_5, t'_6]$ move(r', d4)
 $[t'_7, t'_e]$ unload(k2, r', c2, p'2)

supported: $[0] \text{loc}(r1) = d3$
 $[0] \text{freight}(r1) = \text{empty}$
 $[0] \text{pile}(c1) = p'1$
 ...
 $[0, t_3] \text{pile}(c1) = p'1$
 $[0, t_3] \text{freight}(\mathbf{r1}) = \text{empty}$
 assertions: $[t'_s, t'_3] \text{pile}(c2) = p'2$
 $[t'_s, t'_1] \text{freight}(r') = \text{empty}$
 constraints: $0 < t_1 \leq t_3, 0 < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$
 $\text{adj}(d1, w12),$
 $\text{adj}(d1, w13), \dots$

Supporting the Assertions

- To support $[t'_s, t'_3] \text{pile}(c2) = p'2$
 - Add persistence condition
 - $[0, t'_s] \text{pile}(c2) = p'2$
 - Alternatives:
 - Constrain $t'_s = 0$ or
 - add new action $\text{stack}(k2, c2, p'2)$

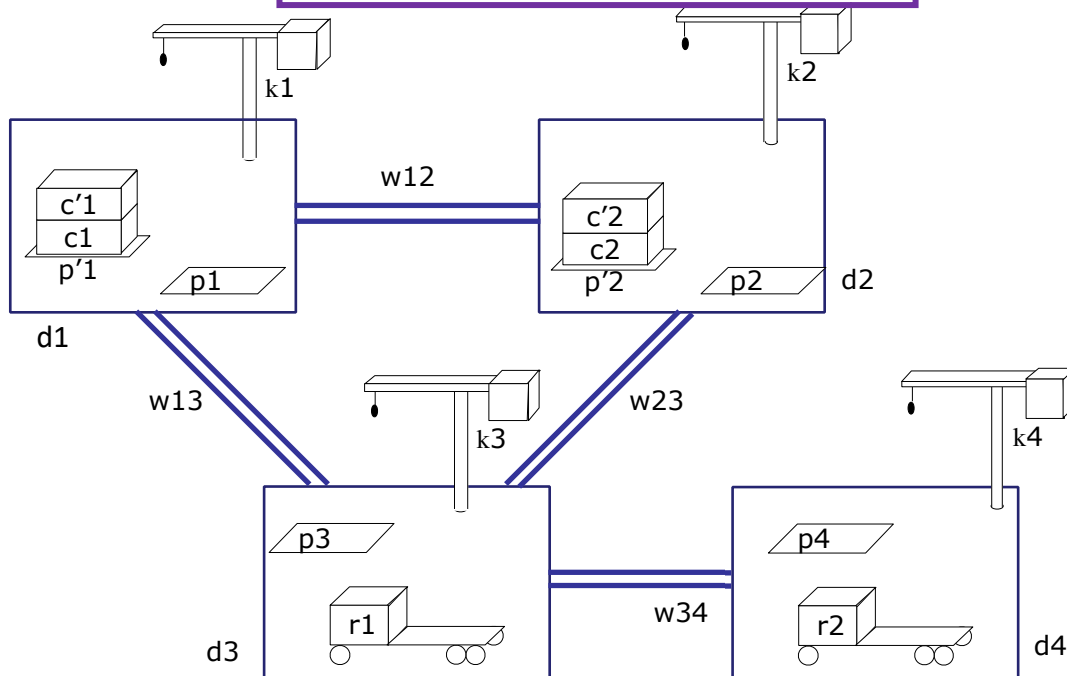


ϕ_2 : tasks: $[0, t_1]$ $\text{move}(r1, d1)$
 $[0, t_2]$ $\text{uncover}(c1, p'1)$
 $[t_3, t_4]$ $\text{load}(k1, r1, c1, p'1)$
 $[t_5, t_6]$ $\text{move}(r1, d3)$
 $[t_7, t_e]$ $\text{unload}(k3, r1, c1, p3)$
 $[t'_s, t'_1]$ $\text{move}(r', d2)$
 $[t'_s, t'_2]$ $\text{uncover}(c2, p'2)$
 $[t'_3, t'_4]$ $\text{load}(k4, r', c2, p'2)$
 $[t'_5, t'_6]$ $\text{move}(r', d4)$
 $[t'_7, t'_e]$ $\text{unload}(k2, r', c2, p'2)$

supported: $[0]$ $\text{loc}(r1) = d3$
 $[0]$ $\text{freight}(r1) = \text{empty}$
 $[0]$ $\text{pile}(c1) = p'1$
 \dots
 $[0, t_3]$ $\text{pile}(c1) = p'1$
 $[0, t_3]$ $\text{freight}(r1) = \text{empty}$
 assertions: $[t'_s, t'_3]$ $\text{pile}(c2) = p'2$
 $[t'_s, t'_1]$ $\text{freight}(r') = \text{empty}$
 constraints: $0 < t_1 \leq t_3$, $0 < t_2 \leq t_3$, $t_4 \leq t_5$, $t_6 \leq t_7$,
 $t'_s < t'_1 \leq t'_3$, $t'_s < t'_2 \leq t'_3$, $t'_4 \leq t'_5$, $t'_6 \leq t'_7$,
 $\text{adj}(d1, w12)$,
 $\text{adj}(d1, w13), \dots$

Supporting the Assertions

- To support $[t'_s, t'_3] \text{pile}(c2) = p'2$
 - Add $[0, t'_s] \text{pile}(c2) = p'2$
- To support $[t'_s, t'_1] \text{freight}(r') = \text{empty}$
 - Constrain $r' = r2$,
add persistence condition
 $[0, t'_s] \text{freight}(r2) = \text{empty}$



ϕ_2 : tasks: $[0, t_1]$ move(r1, d1)
 $[0, t_2]$ uncover(c1, p'1)
 $[t_3, t_4]$ load(k1, r1, c1, p'1)
 $[t_5, t_6]$ move(r1, d3)
 $[t_7, t_e]$ unload(k3, r1, c1, p3)
 $[t'_s, t'_1]$ move(r', d2)
 $[t'_s, t'_2]$ uncover(c2, p'2)
 $[t'_3, t'_4]$ load(k4, r', c2, p'2)
 $[t'_5, t'_6]$ move(r', d4)
 $[t'_7, t'_e]$ unload(k2, r', c2, p'2)

supported: $[0] \text{loc}(r1) = d3$
 $[0] \text{freight}(r1) = \text{empty}$
 $[0] \text{pile}(c1) = p'1 \dots$
 $[0, t_3] \text{pile}(c1) = p'1$
 $[0, t_3] \text{freight}(r1) = \text{empty}$

$[0, t'_s] \text{pile}(c2) = p'2$

$[t'_s, t'_3] \text{pile}(c2) = p'2$

assertions: $[t'_s, t'_1] \text{freight}(r') = \text{empty}$

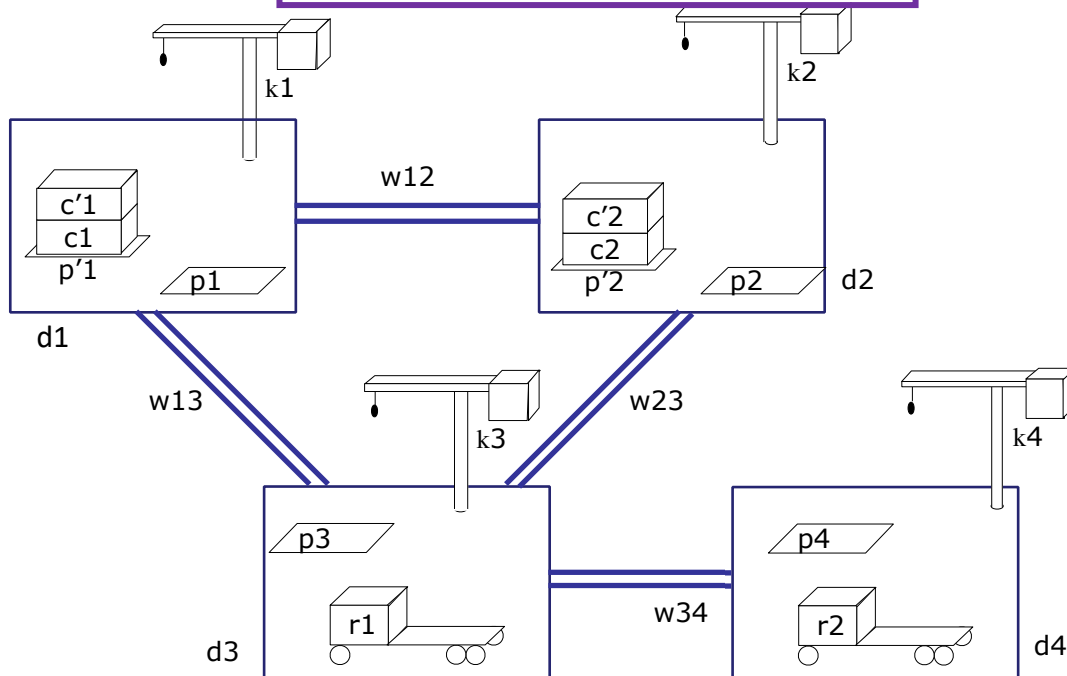
constraints: $0 < t_1 \leq t_3, 0 < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$

adj(d1, w12),

adj(d1, w13), ...

Supporting the Assertions

- To support $[t'_s, t'_3] \text{pile}(c2) = p'2$
 - Add $[0, t'_s] \text{pile}(c2) = p'2$
- To support $[t'_s, t'_1] \text{freight}(r') = \text{empty}$
 - Constrain $r' = r2$,
add persistence condition
 $[0, t'_s] \text{freight}(r2) = \text{empty}$



ϕ_2 : tasks: $[0, t_1]$ move(r1, d1)
 $[0, t_2]$ uncover(c1, p'1)
 $[t_3, t_4]$ load(k1, r1, c1, p'1)
 $[t_5, t_6]$ move(r1, d3)
 $[t_7, t_e]$ unload(k3, r1, c1, p3)
 $[t'_s, t'_1]$ move(r2, d2)
 $[t'_s, t'_2]$ uncover(c2, p'2)
 $[t'_3, t'_4]$ load(k4, r2, c2, p'2)
 $[t'_5, t'_6]$ move(r2, d4)
 $[t'_7, t'_e]$ unload(k2, r2, c2, p'2)

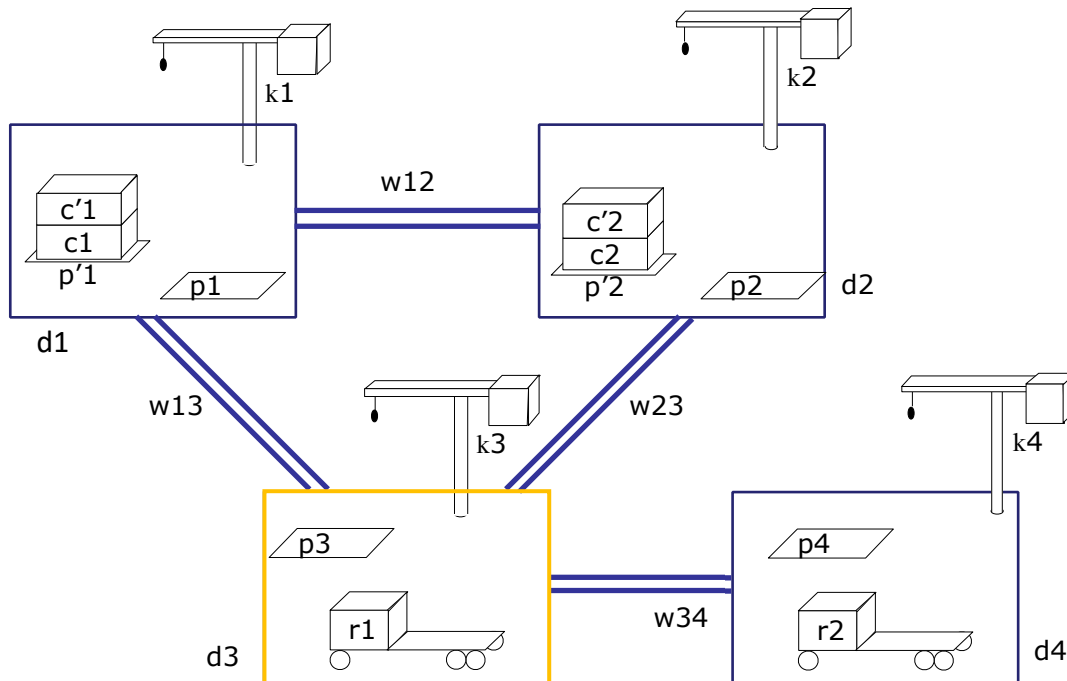
supported: $[0] \text{loc}(r1) = d3$
 $[0] \text{freight}(r1) = \text{empty}$
 $[0] \text{pile}(c1) = p'1 \dots$
 $[0, t_3] \text{pile}(c1) = p'1$
 $[0, t_3] \text{freight}(r1) = \text{empty}$
 $[0, t'_s] \text{pile}(c2) = p'2$
 $[t'_s, t'_3] \text{pile}(c2) = p'2$
 $[0, t'_s] \text{freight}(r2) = \text{empty}$
 $[t'_s, t'_1] \text{freight}(r2) = \text{empty}$

assertions: (none)

constraints: $0 < t_1 \leq t_3, 0 < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$
adj(d1, w12), adj(d1, w13), ...

Example of Conflicts

- Refining tasks into actions will produce possibly-conflicting assertions
 - $\text{move}(r2,d4)$ must go through $d3$
 - Conflict: $\text{occupant}(d3)=r1, \text{occupant}(d3)=r2$
- Resolvers:
 - Separation constraints to ensure $r2$ only goes through $d3$ while $r1$ away from $d3$



ϕ_2 : tasks: $[0, t_1]$ $\text{move}(r1, d1)$
 $[0, t_2]$ $\text{uncover}(c1, p'1)$
 $[t_3, t_4]$ $\text{load}(k1, r1, c1, p'1)$
 $[t_5, t_6]$ $\text{move}(r1, d3)$
 $[t_7, t_e]$ $\text{unload}(k3, r1, c1, p3)$
 $[t'_s, t'_1]$ $\text{move}(r2, d2)$
 $[t'_s, t'_2]$ $\text{uncover}(c2, p'2)$
 $[t'_3, t'_4]$ $\text{load}(k4, r2, c2, p'2)$
 $[t'_5, t'_6]$ $\text{move}(r2, d4)$
 $[t'_7, t'_e]$ $\text{unload}(k2, r2, c2, p'2)$

supported: $[0]$ $\text{loc}(r1)=d3$
 $[0]$ $\text{freight}(r1)=\text{empty}$
 $[0]$ $\text{pile}(c1)=p'1 \dots$
 $[0, t_3]$ $\text{pile}(c1) = p'1$
 $[0, t_3]$ $\text{freight}(r1) = \text{empty}$
 $[0, t'_s]$ $\text{pile}(c2)=p'2$
 $[t'_s, t'_3]$ $\text{pile}(c2) = p'2$
 $[0, t'_s]$ $\text{freight}(r2)=\text{empty}$
 $[t'_s, t'_1]$ $\text{freight}(r2) = \text{empty}$

assertions: (none)

constraints: $0 < t_1 \leq t_3, 0 < t_2 \leq t_3, t_4 \leq t_5, t_6 \leq t_7,$
 $t'_s < t'_1 \leq t'_3, t'_s < t'_2 \leq t'_3, t'_4 \leq t'_5, t'_6 \leq t'_7,$
 $\text{adj}(d1, w12), \text{adj}(d1, w13), \dots$

Heuristics for Guiding TemPlan

- Flaw selection, resolver selection heuristics similar to those in PSP
 - Select the flaw with the smallest number of resolvers
 - Choose the resolver that rules out the fewest resolvers for the other flaws
- There is also a problem with constraint management
 - We ignored it when discussing PSP
 - We discuss it next

TemPlan (ϕ, Σ)

```
Flaws  $\leftarrow$  set of flaws of  $\phi$ 
if Flaws =  $\emptyset$  then
    return  $\phi$ 
arbitrarily select  $f \in$  Flaws
Resolvers  $\leftarrow$  set of resolvers of  $f$ 
if Resolvers =  $\emptyset$  then
    return failure
nondeterministically choose  $\rho \in$  Resolvers
 $\phi \leftarrow$  Transform( $\phi, \rho$ )
TemPlan ( $\phi, \Sigma$ )
```

PSP (Σ, π)

loop

```
if Flaws( $\pi$ ) =  $\emptyset$  then
    return  $\pi$ 
arbitrarily select  $f \in$  Flaws( $\pi$ )
 $R \leftarrow$  {all feasible resolvers for  $f$ }
if  $R = \emptyset$  then
    return failure
nondeterministically choose  $\rho \in R$ 
 $\pi \leftarrow \rho(\pi)$ 
return  $\pi$ 
```

Intermediate Summary

- Planning problems
 - Three kinds of flaws and their resolvers:
 - tasks (that need to be refined),
 - causal support (for assertions),
 - security (of instantiations)
 - Partial plans, solution plans
- Planning: TemPlan
 - Like PSP but with tasks, temporal assertions, temporal constraints

Outline per the Book

4.2 Representation

- Timelines
- Actions and tasks
- Chronicles

4.3 Temporal Planning

- Resolvers and flaws
- Search space

4.4 Constraint Management

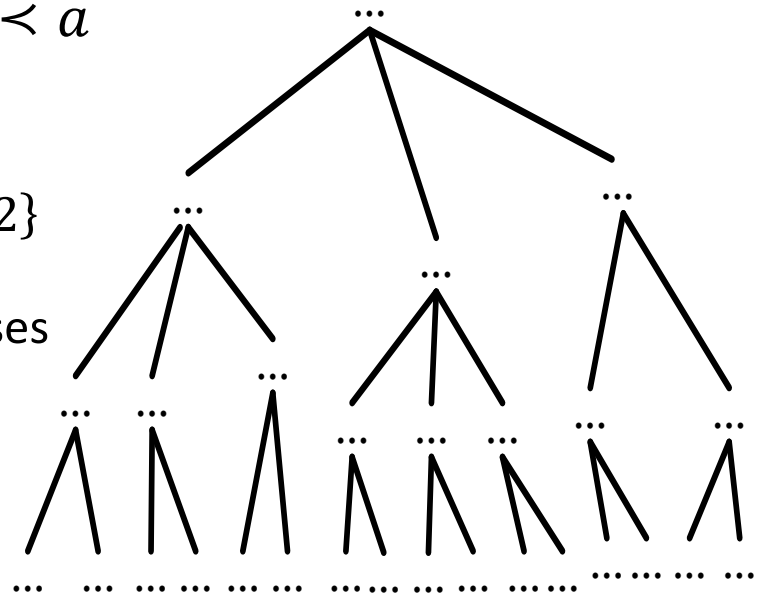
- Consistency of object constraints and time constraints
- Controlling the actions when we do not know how long they will take

4.5 Acting with Temporal Models

- Acting with atemporal refinement
- Dispatching
- Observation actions

Constraint Management

- Each time TemPlan applies a resolver, it modifies $(\mathcal{T}, \mathcal{C})$
 - Some resolvers will make $(\mathcal{T}, \mathcal{C})$ inconsistent
 - No solution in this part of the search space
 - Detect inconsistency \Rightarrow prune this part of the search space
 - Do not detect it \Rightarrow waste time looking for a solution
- Analogy: PSP checks simple cases of inconsistency
 - E.g., cannot create a constraint $a < b$ if there is already a constraint $b < a$
 - Ignores more complicated cases
 - Example:
 - $c_1, c_2, c_3 \in Containers = \{c1, c2\}$
 - Threats involving c_1, c_2, c_3
 - For resolvers, suppose PSP chooses
 - $c_1 \neq c_2, c_2 \neq c_3, c_1 \neq c_3$
 - No solutions in this part of the search space, but PSP searches it anyway



Constraint Management in TemPlan

- At various points, check consistency of \mathcal{C}
 - If \mathcal{C} is inconsistent, then $(\mathcal{T}, \mathcal{C})$ is inconsistent
 - Can prune this part of the search space
- If \mathcal{C} is consistent, then $(\mathcal{T}, \mathcal{C})$ may or may not be consistent
 - Example:
 - $\mathcal{T} = \{[t_1, t_2]loc(r1) = loc1, [t_3, t_4]loc(r1) = loc2\}$
 - $\mathcal{C} = (t_1 < t_3 < t_4 < t_2)$
 - Gives $loc(r1)$ two values during $[t_3, t_4]$

An instance is **consistent** if

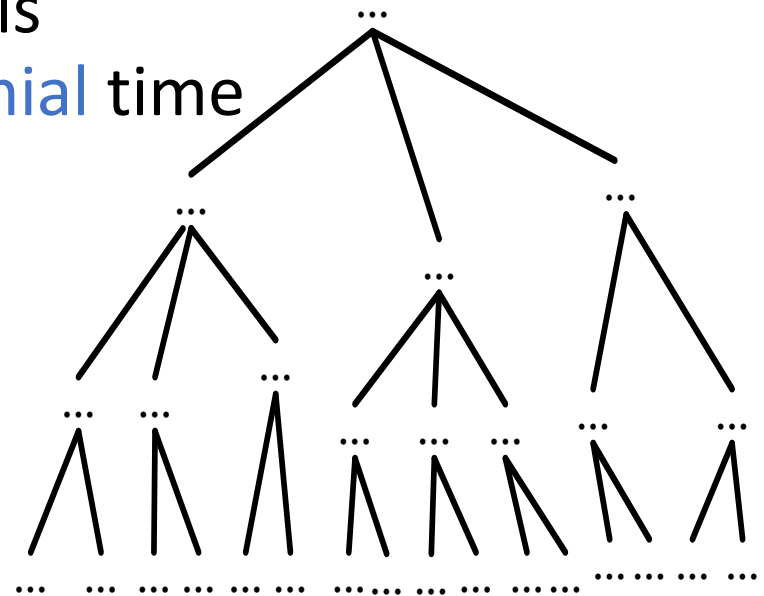
- it satisfies all constraints in \mathcal{C} and
- does not specify two different values for a state variable at the same time

Consistency of \mathcal{C}

- \mathcal{C} contains two kinds of constraints
 - **Object** constraints
 - $loc(r) \neq l_2, l \in \{loc3, loc4\}, r = r1, o \neq o'$
 - **Temporal** constraints
 - $t_1 < t_3, a < t, t < t', a \leq t' - t \leq b$
- Assume object constraints are independent of temporal constraints and vice versa
 - Exclude things like $t < f(l, r)$
- Then two separate subproblems:
 1. Check consistency of object constraints
 2. Check consistency of temporal constraints
 - \mathcal{C} is consistent iff both are consistent

Object Constraints

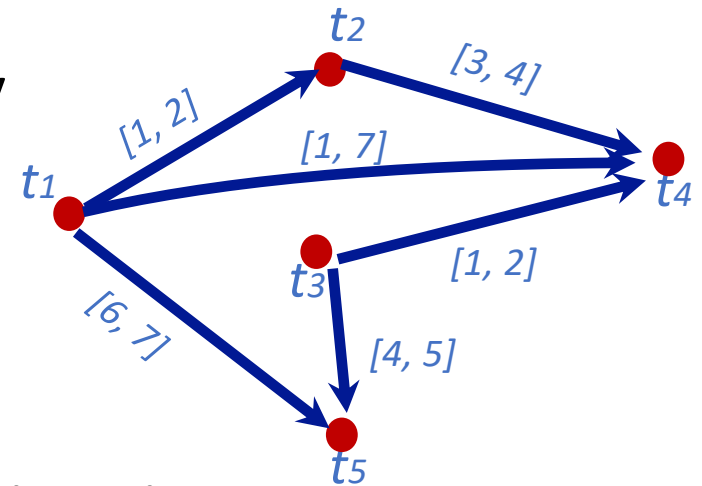
- Constraint-satisfaction problem – NP-complete
- Can write an algorithm that is **complete** but runs in **exponential** time
 - If there is an inconsistency, always finds it
 - Might prune a lot, but spends lots of time at each node
- Instead, use a technique that is **incomplete** but takes **polynomial** time
 - Detects some inconsistencies but not others
 - Runs much faster, but prunes fewer nodes



Time Constraints: Representation

- Simple Temporal Networks (STNs)
 - Networks of constraints on time points

- Synthesise an STN incrementally starting from ϕ_0
 - TemPlan can check time constraints in time $O(n^3)$

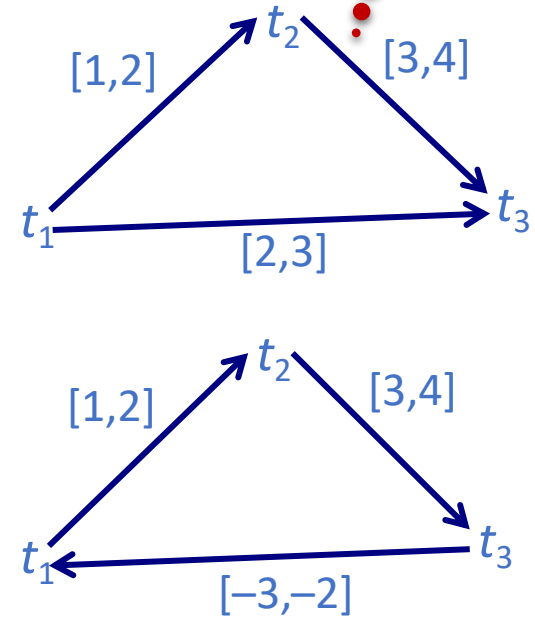


- Incrementally instantiated at acting time
- Kept consistent throughout planning and acting

Simple Temporal Networks

Is this network consistent?

- STN: a pair $(\mathcal{V}, \mathcal{E})$, where
 - $\mathcal{V} = \{\text{a set of temporal variables } t_1, \dots, t_n\}$
 - $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges
- Each edge (t_i, t_j) is labelled with an interval $[a, b]$
 - Shorthand: represents constraint $a \leq t_j - t_i \leq b$
 - Equivalently, $-b \leq t_i - t_j \leq -a$
- Representing unary constraints
 - Dummy variable $t_0 = 0$
 - Edge (t_0, t_i) labelled with $[a, b]$ represents $a \leq t_i - 0 \leq b$
- **Solution** to an STN
 - Integer value for each t_i
 - All constraints satisfied
- **Consistent** STN
 - Has a solution



Book says:

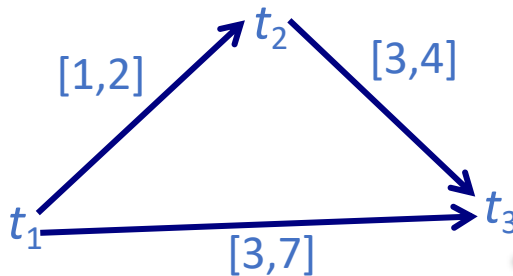
- Solution
 - Integer value for each t_i
- Consistent:
 - Has a solution
 - *All constraints satisfied*



Time Constraints

- Minimal STN:

- For every edge (t_i, t_j) with label $[a, b]$
 - For every $t \in [a, b]$
 - There is at least one solution such that $t_j - t_i = t$
- Cannot make any of the time intervals shorter without excluding some solutions



Is this network minimal?

Operations on STNs

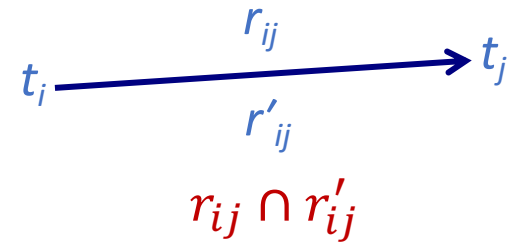
- Intersection, \cap

- $t_j - t_i \in r_{ij} = [a_{ij}, b_{ij}]$

- $t_j - t_i \in r'_{ij} = [a'_{ij}, b'_{ij}]$

- Infer

- $t_j - t_i \in r_{ij} \cap r'_{ij} = [\max(a_{ij}, a'_{ij}), \min(b_{ij}, b'_{ij})]$



- Composition, \circ

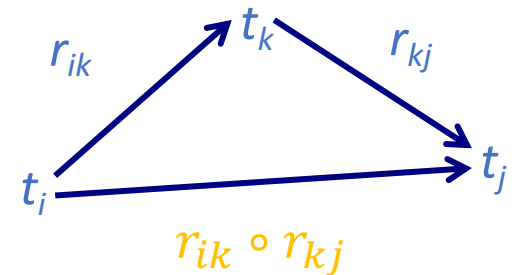
- $t_k - t_i \in r_{ik} = [a_{ik}, b_{ik}]$

- $t_j - t_k \in r_{kj} = [a_{kj}, b_{kj}]$

- Infer

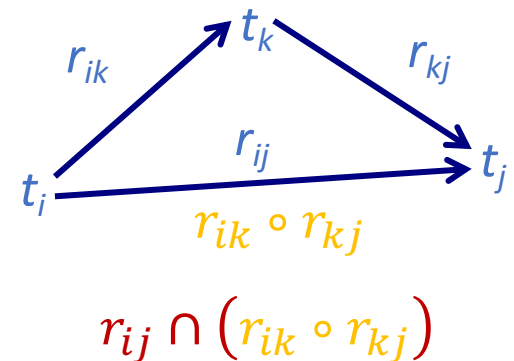
- $t_j - t_i \in r_{ik} \circ r_{kj} = [a_{ik} + a_{kj}, b_{ik} + b_{kj}]$

- Reasoning: shortest and longest times of the two intervals

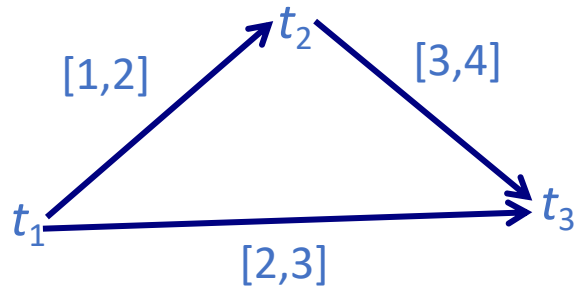


- Consistency checking

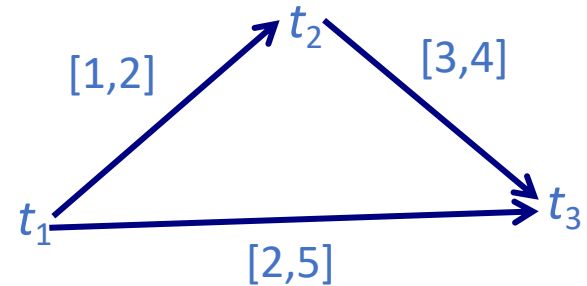
- Three constraints r_{ik}, r_{kj}, r_{ij} are consistent only if $r_{ij} \cap (r_{ik} \circ r_{kj}) \neq \emptyset$ (empty interval)



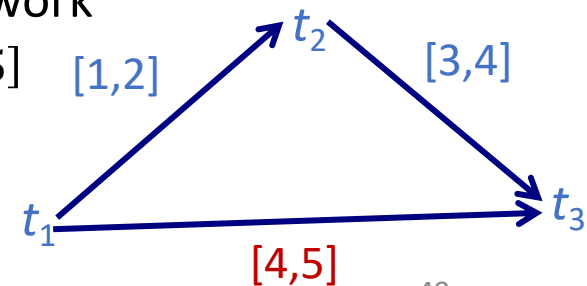
Two Examples



- STN $(\mathcal{V}, \mathcal{E})$, where
 - $\mathcal{V} = \{t_1, t_2, t_3\}$
 - $\mathcal{E} = \{r_{12} = [1,2], r_{23} = [3,4], r_{13} = [2,3]\}$
- Composition
 - $r'_{13} = r_{12} \circ r_{23} = [4,6]$
- Cannot satisfy both r_{13} and r'_{13}
 - $r_{13} \cap r'_{13} = [2,3] \cap [4,6] = \emptyset$
- $(\mathcal{V}, \mathcal{E})$ is inconsistent



- STN $(\mathcal{V}, \mathcal{E})$, where
 - $\mathcal{V} = \{t_1, t_2, t_3\}$
 - $\mathcal{E} = \{r_{12} = [1,2], r_{23} = [3,4], r_{13} = [2,5]\}$
- Composition (as before)
 - $r'_{13} = r_{12} \circ r_{23} = [4,6]$
- $(\mathcal{V}, \mathcal{E})$ is consistent
 - $r_{13} \cap r'_{13} = [2,5] \cap [4,6] = [4,5]$
- Minimal network
 - $r_{13} = [4,5]$



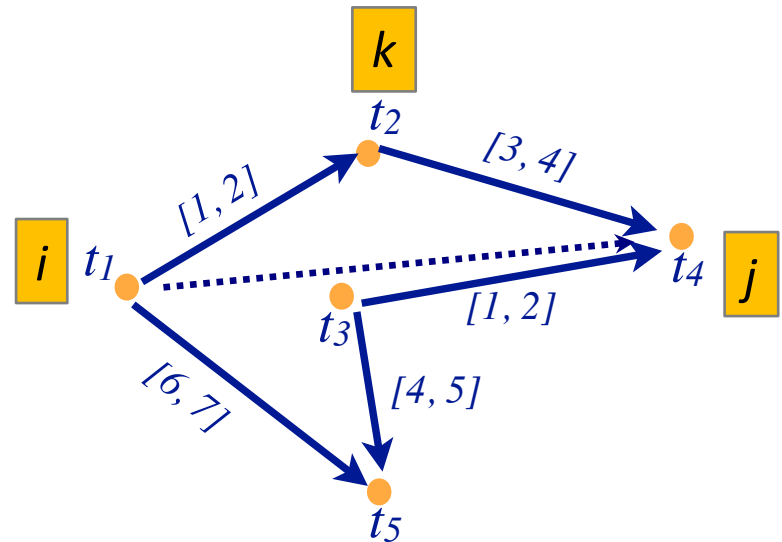
Operations on STNs

- PC (*Path Consistency*) algorithm:
 - Consistency checking on all triples
 - If an edge has no constraint, use $[-\infty, +\infty]$
 - n constraints $\Rightarrow n^3$ triples \Rightarrow time $O(n^3)$

```
PC( $\mathcal{V}, \mathcal{E}$ )
  for  $1 \leq k \leq n$  do
    for  $1 \leq i < j \leq n, i \neq j, j \neq k$  do
       $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \circ r_{kj}]$ 
      if  $r_{ij} = \emptyset$  then
        return inconsistent
  return consistent
```

- Example:

- $k = 2, i = 1, j = 4$
- $r_{12} = [1, 2]$
- $r_{24} = [3, 4]$
- $r_{14} = [-\infty, \infty]$
- $r_{12} \circ r_{24} = [1 + 3, 2 + 4] = [4, 6]$
- $r_{14} \leftarrow [\max(-\infty, 4), \min(\infty, 6)] = [4, 6]$



Operations on STNs

- PC makes network minimal

- Shrinks each r_{ij} to exclude values that are not in any solution
- Doing so, it detects inconsistent networks
 - $r_{ij} = [a_{ij}, b_{ij}]$ empty
 \Rightarrow inconsistent

PC(\mathcal{V}, \mathcal{E})

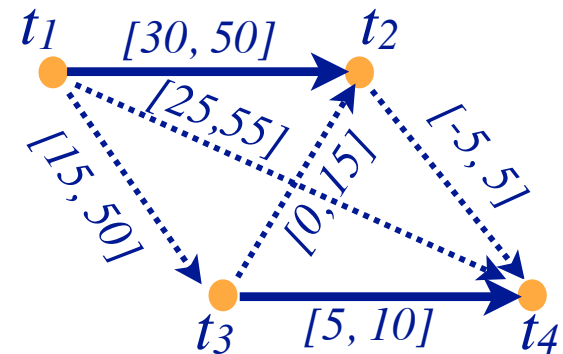
```
for 1 ≤ k ≤ n do
  for 1 ≤ i < j ≤ n, i ≠ j, j ≠ k do
     $r_{ij} \leftarrow r_{ij} \cap [r_{ik} \circ r_{kj}]$ 
  if  $r_{ij} = \emptyset$  then
    return inconsistent
return consistent
```

- Graph: dashed lines

- Constraints that were shrunk

- Can modify PC to make it incremental

- Input
 - A consistent, minimal STN
 - A new constraint r'_{ij}
- Incorporate r'_{ij} in time $O(n^2)$



Pruning TemPlan's search space

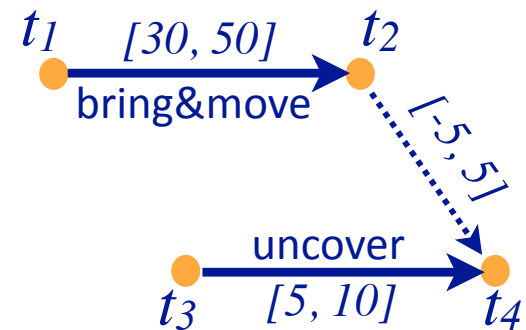
- Take the time constraints in \mathcal{C}
 - Write them as an STN
 - Use PC to check whether STN is consistent
 - If it is inconsistent, TemPlan can backtrack

Controllability

Constraint Management with Uncertain Durations

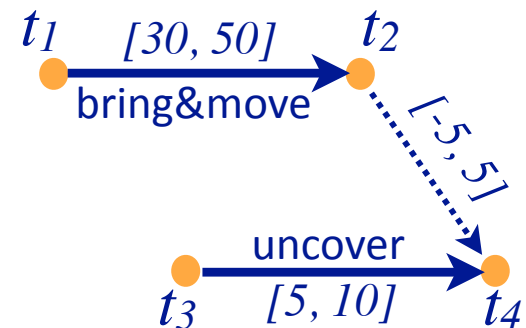
Controllability

- Suppose TemPlan gives you a chronicle and you want to execute it
 - Constraints on time points
 - Need to reason about these in order to decide when to start each action



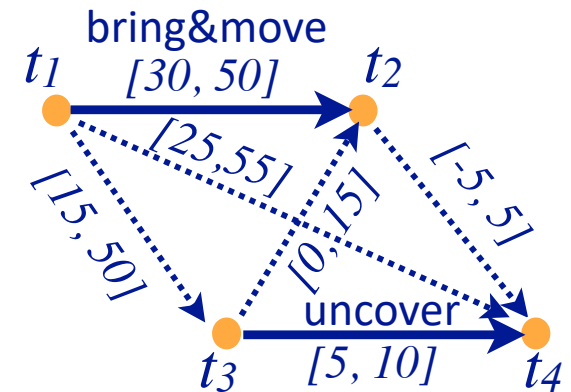
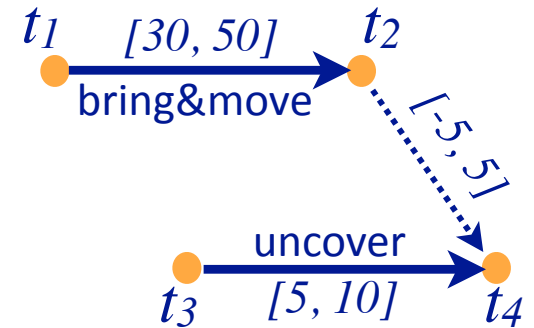
Controllability

- Solid lines: **duration constraints**
 - Robot will do bring&move, will take 30 to 50 time units
 - Crane will do uncover, will take 5 to 10 time units
- Dashed line: **synchronization constraint**
 - Do not want either the crane or robot to wait long
 - At most 5 seconds between the two ending times
- Objective
 - Choose time points that will satisfy all the constraints



Controllability

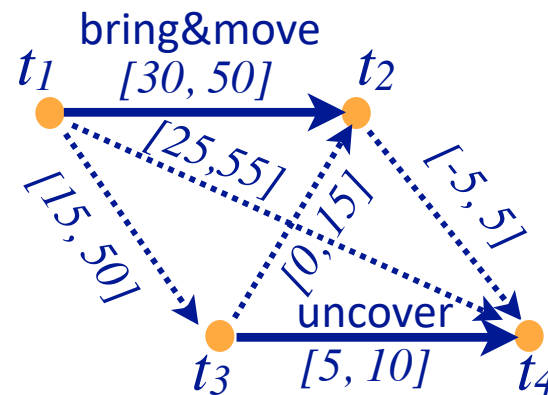
- Suppose we run PC
- PC returns a minimal and consistent network
- There *exist* time points that satisfy all the constraints
- Would work if we could choose all four time points
 - But we cannot choose t_2 and t_4
- t_1 and t_3 are **controllable**
 - Actor can control when each action starts
- t_2 and t_4 are **contingent**
 - Cannot control how long the actions take
 - Random variables that are known to satisfy the duration constraints
 - $t_2 \in [t_1 + 30, t_1 + 50]$
 - $t_4 \in [t_3 + 5, t_3 + 10]$



Controllability

- Cannot guarantee that all constraints will be satisfied
- Start bring&move at time $t_1 = 0$
- Suppose the durations are
 - bring&move 30, uncover 10
 - $t_2 = t_1 + 30 = 30$
 - $t_4 = t_3 + 10$
 - $t_4 - t_2 = t_3 - 20$
- Constraint r_{24} :
 - $-5 \leq t_4 - t_2 \leq 5$
 - $-5 \leq t_3 - 20 \leq 5$
 - $15 \leq t_3 \leq 25$
- Must start uncover at $t_3 \leq 25$

- But if we start uncover at $t_3 \leq 25$, neither action has finished yet
 - We do not yet know how long they will take
- Durations might instead be
 - bring&move 50, uncover 5
 - $t_2 = t_1 + 50 = 50$
 - $t_4 = t_3 + 5 \leq 25 + 5 = 30$
 - $t_4 - t_2 \leq 30 - 50 = -20$
 - Violates r_{34}



STNUs

- **STNU (Simple Temporal Network with Uncertainty):**
 - A 4-tuple $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$
 - $\mathcal{V} = \{\text{controllable time points}\}$
 - E.g., starting times of actions
 - $\tilde{\mathcal{V}} = \{\text{contingent time points}\}$
 - E.g., ending times of actions
 - $\mathcal{E} = \{\text{controllable constraints}\}$
 - $\tilde{\mathcal{E}} = \{\text{contingent constraints}\}$
- **Controllable and contingent constraints:**
 - Synchronization between two **starting** times: *controllable*
 - **Duration** of an action: *contingent*
 - Synchronization between **ending** points of two actions: *contingent*
 - Synchronization between end of one action, start of another:
 - *Controllable* if the new action starts after the old one ends
 - *Contingent* if the new action starts before the old one ends
- Want a way for the actor to choose time points in \mathcal{V} (starting times) that guarantee that constraints are satisfied

Three kinds of controllability

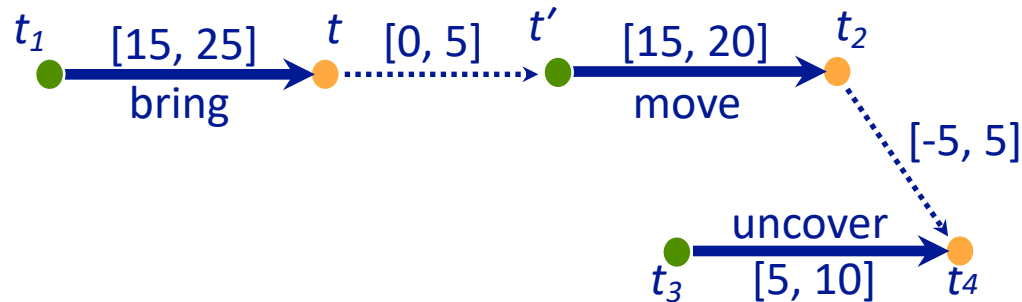
- $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is **strongly controllable** if the actor can choose values for \mathcal{V} such that success will occur for all values of $\tilde{\mathcal{V}}$ that satisfy $\tilde{\mathcal{E}}$
 - Actor can choose the values for \mathcal{V} offline
 - The right choice will work regardless of $\tilde{\mathcal{V}}$
- $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is **weakly controllable** if the actor can choose values for \mathcal{V} such that success will occur for *at least one* combination of values for $\tilde{\mathcal{V}}$
 - Actor can choose the values for \mathcal{V} only if the actor knows in advance what the values of $\tilde{\mathcal{V}}$ will be
- **Dynamic controllability:**
 - Game-theoretic model: actor vs. environment
 - A player's **strategy**: a function σ telling what to do in every situation
 - Choices may differ depending on what has happened so far
 - $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is **dynamically controllable** if \exists strategy for an actor that will guarantee success regardless of the environment's strategy

Dynamic Execution

- For $t = 0, 1, 2, \dots$
 1. Actor chooses an unassigned set of variables $\mathcal{V}_t \subseteq \mathcal{V}$ that all can be assigned the value t without violating any constraints in \mathcal{E}
 - \approx actions the actor chooses to start at time t
 2. Simultaneously, environment chooses an unassigned set of variables $\tilde{\mathcal{V}}_t \subseteq \tilde{\mathcal{V}}$ that all can be assigned the value t without violating any constraints in $\tilde{\mathcal{E}}$
 - \approx actions that finish at time t
 3. Each chosen time point v is assigned $v \leftarrow t$
 4. Failure if any of the constraints in $\mathcal{E} \cup \tilde{\mathcal{E}}$ are violated
 - There might be violations that neither \mathcal{V}_t nor $\tilde{\mathcal{V}}_t$ caused individually
 5. Success if all variables in $\mathcal{V} \cup \tilde{\mathcal{V}}$ have values and no constraints are violated
- $r_{ij} = [l, u]$ is **violated** if t_i and t_j have values and $t_j - t_i \notin [l, u]$
- **Dynamic execution strategies** σ_A for actor, σ_E for environment
 - $\sigma_A(h_{t-1}) = \{\text{what events in } \mathcal{V} \text{ to trigger at time } t, \text{ given } h_{t-1}\}$
 - $\sigma_E(h_{t-1}) = \{\text{what events in } \tilde{\mathcal{V}} \text{ to trigger at time } t, \text{ given } h_{t-1}\}$
 - $h_t = h_{t-1} \cdot (\sigma_A(h_{t-1}) \cup \sigma_E(h_{t-1}))$
 - $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is **dynamically controllable** if $\exists \sigma_A$ that will guarantee success $\forall \sigma_E$

Example

- Instead of a single bring&move task, two separate bring and move tasks



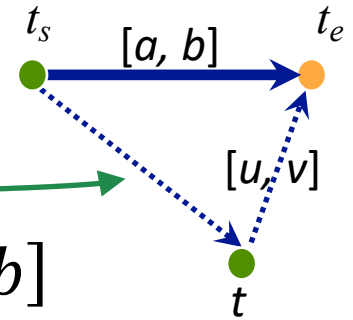
- Actor's dynamic execution strategy
 - Trigger t_1 at whatever time you want
 - Wait and observe t
 - Trigger t' at any time from t to $t + 5$
 - Trigger $t_3 = t' + 10$
 - For every $t_2 \in [t' + 15, t' + 20]$ and $t_4 \in [t_3 + 5, t_3 + 10]$
 - $t_4 \in [t' + 15, t' + 20]$
 - So, $t_4 - t_2 \in [-5, 5]$
 - Thus, all constraints are satisfied

Dynamic Controllability Checking

- For a chronicle $\phi = (\mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{C})$
 - Temporal constraints in \mathcal{C} correspond to an STNU
 - Adapt TemPlan to test not only consistency but also dynamic controllability (*) of the STNU
 - If we detect cases where it is not dynamically controllable, then backtrack
- * Use PC as well
 - If $\text{PC}(\mathcal{V} \cup \tilde{\mathcal{V}}, \mathcal{E} \cup \tilde{\mathcal{E}})$ reduces a contingent constraint, then $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is not dynamically controllable
 - ⇒ Can prune this branch
 - If it *does not* reduce any contingent constraints, we do not know whether $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is dynamically controllable
 - Only **necessary**, **not sufficient** condition
 - Two options
 - Either continue down this branch and backtrack later if necessary, or
 - Extend PC to detect more cases where $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ is not dynamically controllable
 - Additional constraint propagation rules

Additional Constraint Propagation Rules

- Case 1: $u \geq 0$
 - t must come before t_e
- Add a composition constraint $[a', b']$
- Find $[a', b']$ such that $[a', b'] \circ [u, v] = [a, b]$
 - $[a' + u, b' + v] = [a, b]$
 - $a' = a - u, b' = b - v$



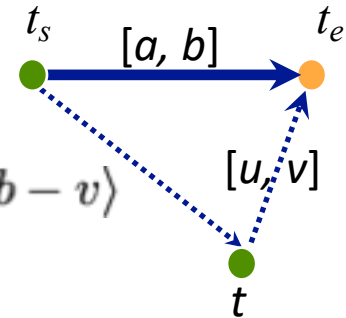
Conditions	Propagated constraint
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u < 0, v \geq 0$	$t_s \xrightarrow{\langle t_e, b' \rangle} t$
$t_s \xrightarrow{[a,b]} t_e, t_s \xrightarrow{\langle t_e, u \rangle} t$	$t_s \xrightarrow{[\min\{a, u\}, \infty]} t$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e, b' \rangle} t'$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t, t_e \neq t$	$t_s \xrightarrow{\langle t_e, b - u \rangle} t'$

\Rightarrow contingent \rightarrow controllable $a' = a - u, b' = b - v$

Additional Constraint Propagation Rules

- Case 2: $u < 0$ and $v \geq 0$
 - t may be before or after t_e

- Add a **wait** constraint $\langle t_e, \alpha \rangle$
 - α defined w.r.t. some controllable time point t_s
 - Wait until either t_e occurs or current time is $t_s + \alpha$, whichever comes first



Conditions	Propagated constraint
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xrightarrow{[a,b]} t_e, t \xrightarrow{[u,v]} t_e, u < 0, v \geq 0$	$t_s \xrightarrow{\langle t_e, b' \rangle} t$
$t_s \xrightarrow{[a,b]} t_e, t_s \xrightarrow{\langle t_e, u \rangle} t$	$t_s \xrightarrow{[\min\{a,u\}, \infty]} t$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e, b' \rangle} t'$
$t_s \xrightarrow{\langle t_e, b \rangle} t, t' \xrightarrow{[u,v]} t, t_e \neq t$	$t_s \xrightarrow{\langle t_e, b-u \rangle} t'$

\Rightarrow contingent \rightarrow controllable $a' = a - u, b' = b - v$

Extended Version of PC

- We want a fast algorithm that TemPlan can run at each node, to decide whether to backtrack
- There is an extended version of PC that runs in polynomial time, but it has high overhead
- Possible compromise: use ordinary PC most of the time
 - Run extended version occasionally, or at end of search before returning plan

Conditions	Propagated constraint
$t_s \xrightarrow{[a,b]} t_e , t \xrightarrow{[u,v]} t_e , u \geq 0$	$t_s \xrightarrow{[b',a']} t$
$t_s \xrightarrow{[a,b]} t_e , t \xrightarrow{[u,v]} t_e , u < 0 , v \geq 0$	$t_s \xrightarrow{\langle t_e, b' \rangle} t$
$t_s \xrightarrow{[a,b]} t_e , t_s \xrightarrow{\langle t_e, u \rangle} t$	$t_s \xrightarrow{[\min\{a,u\}, \infty]} t$
$t_s \xrightarrow{\langle t_e, b \rangle} t , t' \xrightarrow{[u,v]} t$	$t_s \xrightarrow{\langle t_e, b' \rangle} t'$
$t_s \xrightarrow{\langle t_e, b \rangle} t , t' \xrightarrow{[u,v]} t , t_e \neq t$	$t_s \xrightarrow{\langle t_e, b-u \rangle} t'$

Intermediate Summary

- Constraint management
 - Consistency of object constraints
 - Constraint-satisfaction problem
 - Consistency of time constraints
 - STN, solution, minimality, consistency
 - PC
- Controllability
 - STNU, controllable, contingent
 - Dynamic controllability

Outline per the Book

4.2 Representation

- Timelines
- Actions and tasks
- Chronicles

4.3 Temporal Planning

- Resolvers and flaws
- Search space

4.4 Constraint Management

- Consistency of object constraints and time constraints
- Controlling the actions when we do not know how long they will take

4.5 Acting with Temporal Models

- Acting with atemporal refinement
- Dispatching
- Observation actions

Atemporal Refinement of Primitive Actions

- TemPlan's action templates may correspond to compound tasks
 - In RAE, refine into commands with refinement methods

- TemPlan's action template (descriptive model)

```
leave(r,d,w)  
  assertions:  [ts,te] loc(r): (d,w)  
               [ts,te] occupant(d): (r,empty)  
  constraints:  $t_e \leq t_s + \delta_1$   
               adj(d,w)
```

- RAE's refinement method (operational model)

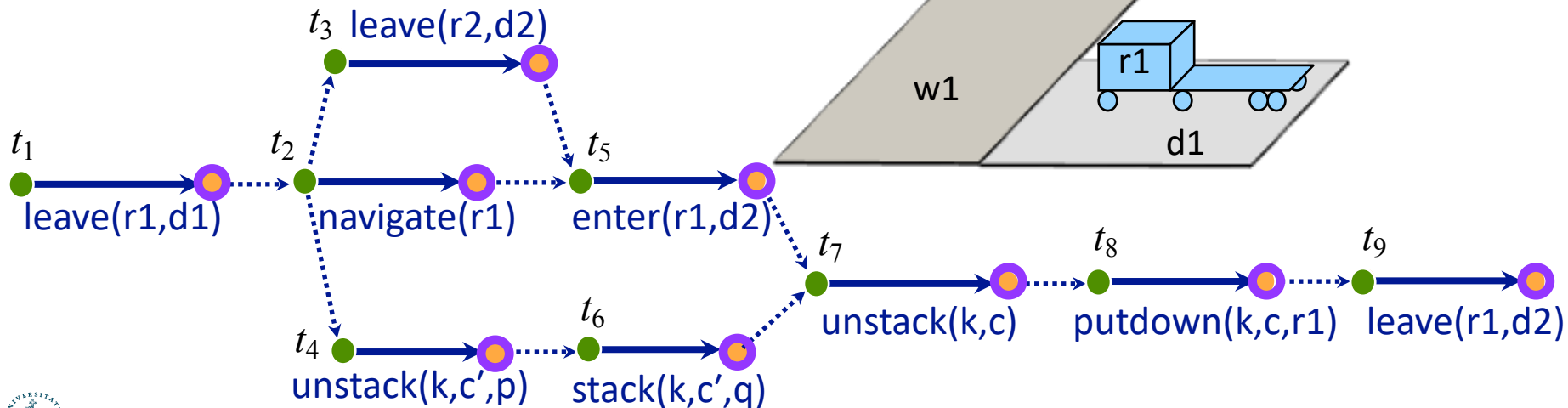
```
m-leave(r,d,w,e)  
  task:  leave(r,d,w)  
  pre:   loc(r)=d, adj(d,w), exit(e,d,w)  
  body:  until empty(e)  
         wait(1)  
         goto(r,e)
```

Discussion

- Pros
 - Simple online refinement with RAE
 - Avoids breaking down uncertainty of contingent duration
 - Can be augmented with temporal monitoring functions in RAE
 - E.g., watchdogs, methods with duration preferences
- Cons
 - Does not handle temporal requirements at the command level,
 - E.g., synchronise two robots that must act concurrently
- Can augment RAE to include temporal reasoning
 - Call it eRAE
 - One essential component: a **dispatching** function

Acting With Temporal Models

- Dispatching procedure: a dynamic execution strategy
 - Controls when to start each action
 - Given a dynamically controllable plan with executable primitives, it triggers corresponding commands from online observations
- Example
 - robot $r2$ needs to leave dock $d2$ before robot $r1$ can enter $d2$
 - crane k needs to uncover c then put c onto $r1$



Dispatching

- Let $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$ be a controllable STNU that is **grounded**
 - Different from a grounded expression in logic
 - At least one time point t^* is instantiated
 - Bounds each time point t within an interval $[l_t, u_t]$

Dispatch $(\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}})$

initialise the network

while there are time points in \mathcal{V} that have not been triggered **do**

update *now*

update the time points in $\tilde{\mathcal{V}}$ that have been newly observed

update *enabled*

trigger every $t \in \text{enabled}$ s.t. $\text{now} = u_t$

arbitrarily choose other time points in *enabled* and trigger them

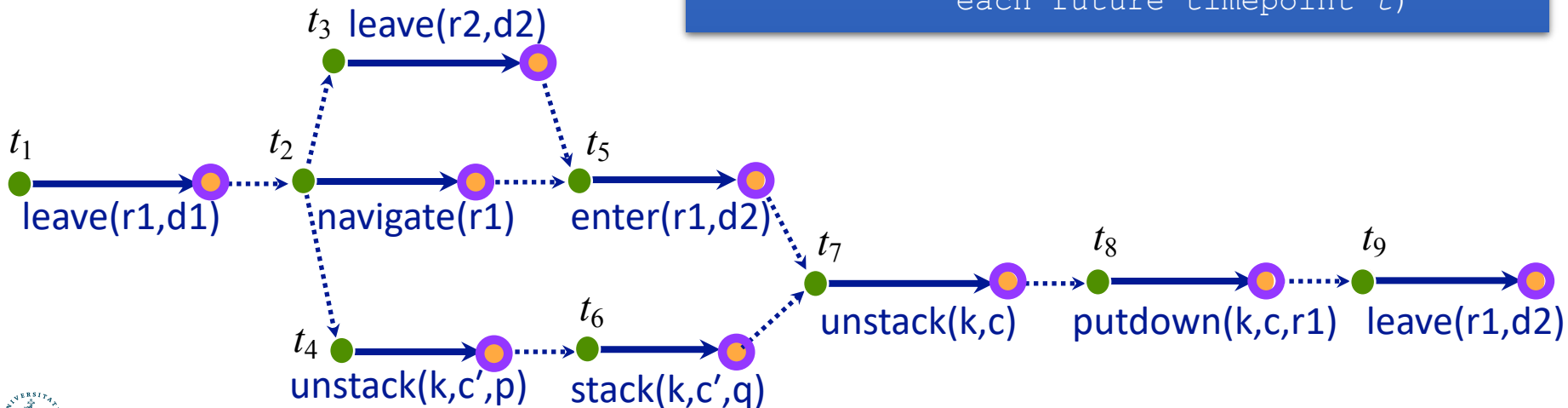
propagate values of triggered

timepoints (change $[l_t, u_t]$ for each future timepoint t)

- Controllable time point t in the future:
 - t is **alive** if current time $\text{now} \in [l_t, u_t]$
 - t is **enabled** if
 - It is alive
 - For every precedence constraint $t' < t$, t' has occurred
 - For every wait constraint $\langle t_e, \alpha \rangle$, t_e has occurred or α has expired
 - α has expired if t_s has occurred and $t_s + \alpha \leq \text{now}$

Example

- Trigger t_1 , observe leave finish
- Enable and trigger t_2 , this enables t_3, t_4
- Trigger t_3 soon enough to allow $enter(r1, d2)$ at time t_5
- Trigger t_4 soon enough to allow $stack(k, c')$ at time t_6
- Rest of plan is linear:
 - Choose each t_i after the previous action ends



Dispatch ($\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}}$)

initialise the network

while there are time points in \mathcal{V} that have not been triggered **do**

update *now*

update the time points in $\tilde{\mathcal{V}}$ that have been newly observed

update *enabled*

trigger every $t \in \text{enabled}$ s.t. $\text{now} = u_t$

arbitrarily choose other time points in *enabled* and trigger them

propagate values of triggered

timepoints (change $[l_t, u_t]$ for each future timepoint t)

Example from Slide 61

- Trigger t_1 at time 0
- Wait and observe t ; this enables t'
- Trigger t' at any time from t to $t + 5$
- Trigger t_3 at time $t' + 10$
 - $t_2 \in [t' + 15, t' + 20]$
 - $t_4 \in [t_3 + 5, t_3 + 10] = [t' + 15, t' + 20]$
 - so $t_4 - t_2 \in [-5, 5]$

Dispatch ($\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}}$)

initialise the network

while there are time points in \mathcal{V} that have not been triggered **do**

 update *now*

 update the time points in $\tilde{\mathcal{V}}$ that have been newly observed

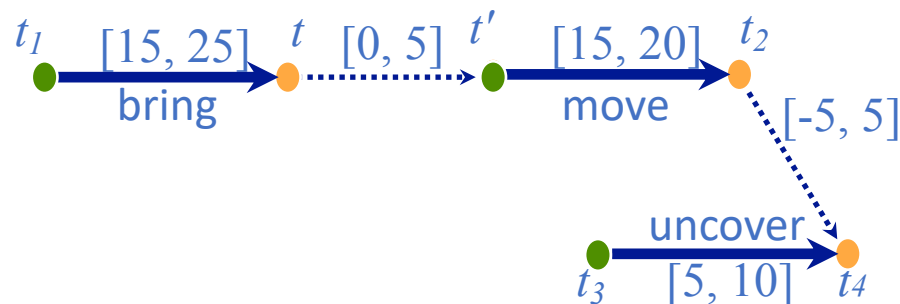
 update *enabled*

 trigger every $t \in \text{enabled}$ s.t. $\text{now} = u_t$

 arbitrarily choose other time points in *enabled* and trigger them

 propagate values of triggered

 timepoints (change $[l_t, u_t]$ for each future timepoint t)



Dispatching

- Propagation step most costly one
 - $O(n^3)$
 - n the number of remaining future time points in network

```
Dispatch( $\mathcal{V}, \tilde{\mathcal{V}}, \mathcal{E}, \tilde{\mathcal{E}}$ )
```

```
  initialise the network
```

```
  while there are time points in  $\mathcal{V}$  that  
    have not been triggered do
```

```
    update now
```

```
    update the time points in  $\tilde{\mathcal{V}}$  that have  
      been newly observed
```

```
    update enabled
```

```
    trigger every  $t \in \text{enabled}$  s.t.  $\text{now} = u_t$ 
```

```
    arbitrarily choose other time points  
      in enabled and trigger them
```

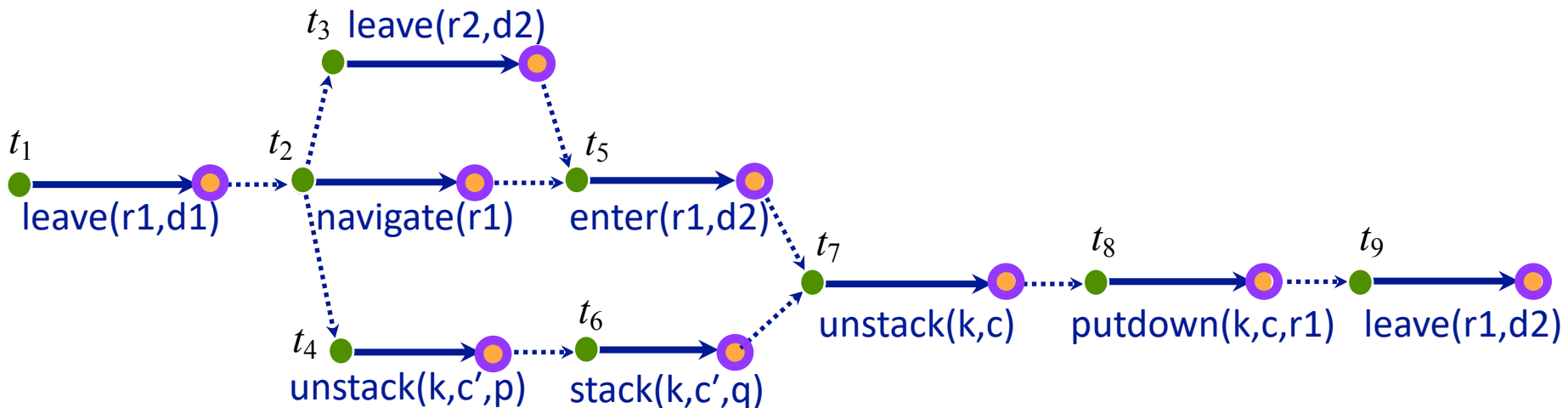
```
    propagate values of triggered
```

```
      timepoints (change  $[l_t, u_t]$  for  
        each future timepoint  $t$ )
```

- Ideally propagation fast enough to allow iterations and updates of *now* consistent with temporal granularity of plan

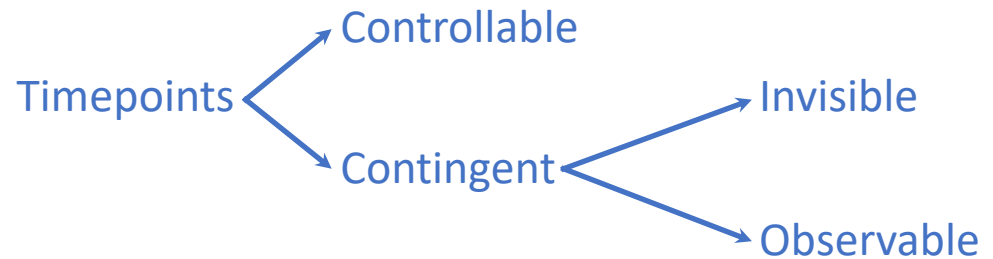
Deadline Failures

- Suppose something makes it impossible to start an action on time
- Do one of the following:
 - Stop the delayed action, and look for new plan
 - Let the delayed action finish, try to repair the plan by resolving violated constraints at the STNU propagation level
 - E.g., accommodate a delay in navigate by delaying the whole plan
 - Let the delayed action finish, try to repair the plan some other way



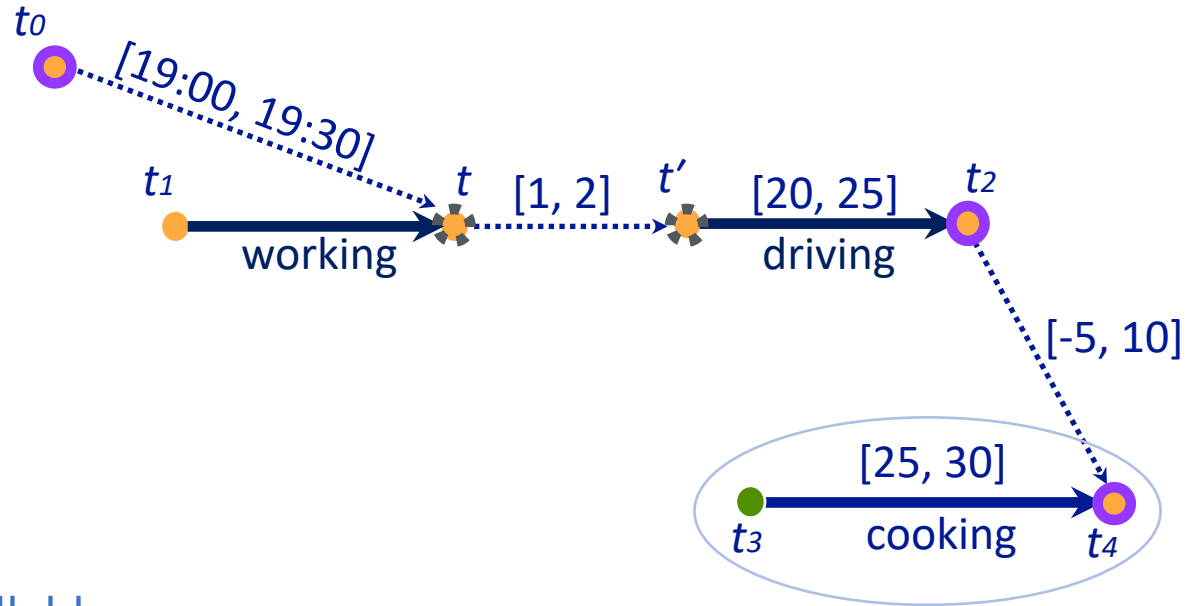
Partial Observability

- Tacit assumption: All occurrences of contingent events are observable
 - Observation needed for dynamic controllability
- In general, not all events are observable
- POSTNU (Partially Observable STNU)
 - STNU where the contingent time points are given by a set of invisible and a set of observable timepoints
 - POSTNU = STNU if Invisible = \emptyset
 - Dynamically controllable?



Observation Actions

- Example



● Controllable

● Contingent

- Invisible
- observable

Dynamic Controllability

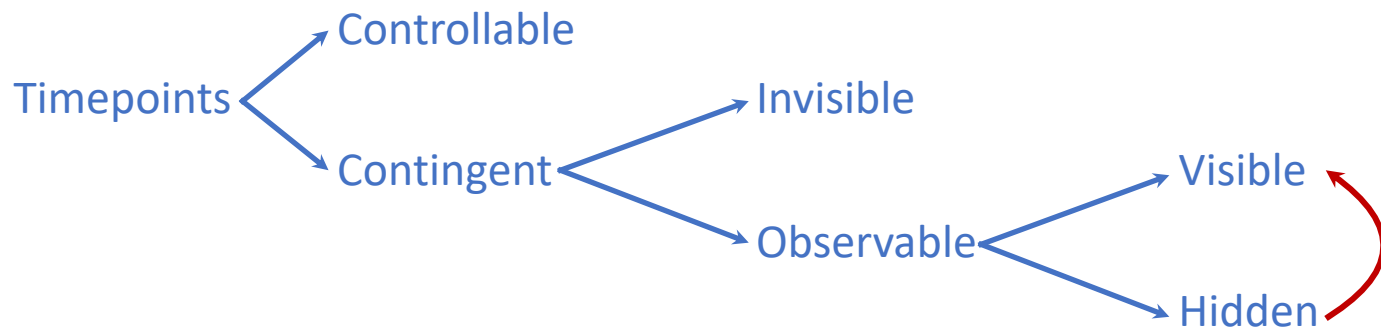
- A POSTNU is dynamically controllable if
 - there exists an execution strategy that chooses future controllable points to meet all the constraints, given the observation of past *visible* points
- Check dynamic controllability
 - Map an POSTNU to an STNU by deleting invisible time points and adding corresponding constraints on controllable and observable time points
 - Check dynamic controllability of the mapped STNU
 - E.g., using the extended PC algorithm
 - More details in the paper

Dynamic Controllability

- A POSTNU is dynamically controllable if
 - there exists an execution strategy that chooses future controllable points to meet all the constraints, given the observation of past *visible* points

- Observable \neq visible

- Observable means it will be known **when observed**
 - It can be temporarily **hidden**



- Aim: Find out which time points need to be observed for the plan to be dynamically controllable (details in paper)

Intermediate Summary

- Acting
 - Atemporal refinement
 - eRAE
 - Dispatching
 - Alive, enabled
 - Deadline failures
 - Partial observability
 - Invisible, observable (hidden/visible)

Outline per the Book

4.2 Representation

- Timelines
- Actions and tasks
- Chronicles

4.3 Temporal Planning

- Resolvers and flaws
- Search space

4.4 Constraint Management

- Consistency of object constraints and time constraints
- Controlling the actions when we do not know how long they will take

4.5 Acting with Temporal Models

- Acting with atemporal refinement
- Dispatching
- Observation actions

⇒ Next: Planning and Acting with Nondeterministic Models