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# A Glimpse into Statistical Relational AI

## Indistinguishability for the Win!

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# Agenda

- Statistical Relational Artificial Intelligence
  - Probabilistic relational models
  - Grounding semantics
  - Context
- Indistinguishability for Inference
  - Lifted query answering and tractability
  - Keeping indistinguishability over time
  - Indistinguishability in decision making
- Summary

# Statistical Relational Artificial Intelligence (StaRAI)

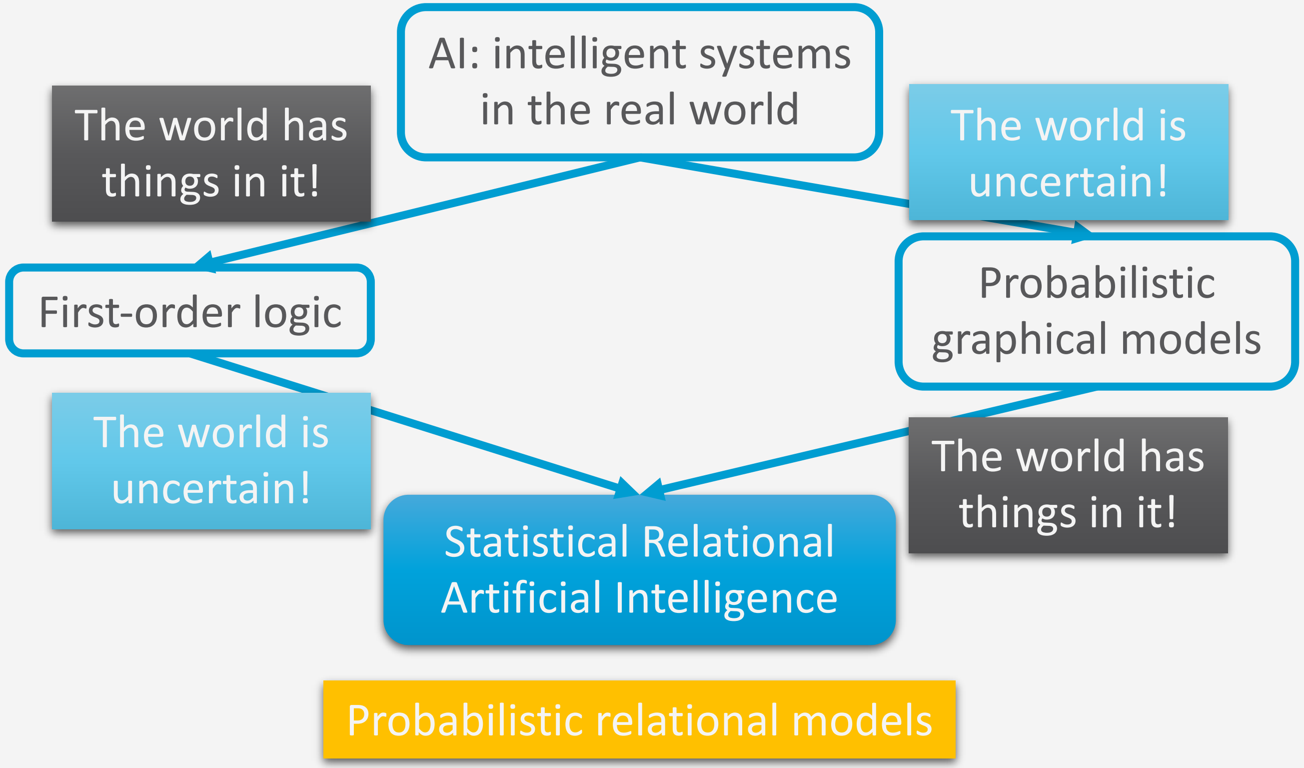
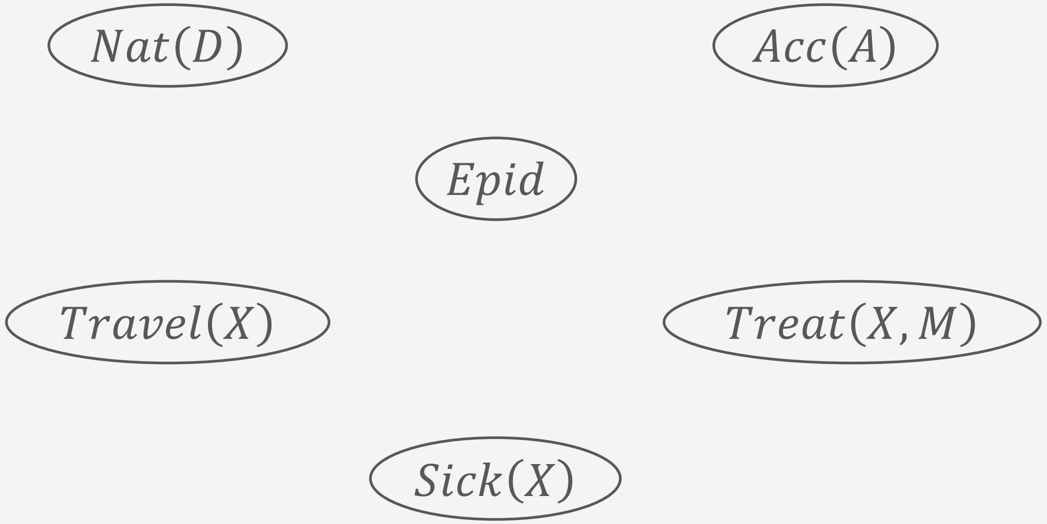



Figure based on Stuart Russell

# Application: Epidemics

- Atoms: Parameterised random variables = PRVs
  - With logical variables
    - E.g., $X, M$
    - Possible values (domain):
      $$dom(X) = \{alice, eve, bob\}$$
      $$dom(M) = \{injection, tablet\}$$
  - With range
    - E.g., Boolean
    - $ran\big(Travel(X)\big) = \{true, false\}$
  - Represent sets of *indistinguishable* random variables

$$Nat(D) = natural\ disaster\ D$$
$$Acc(A) = accident\ A$$

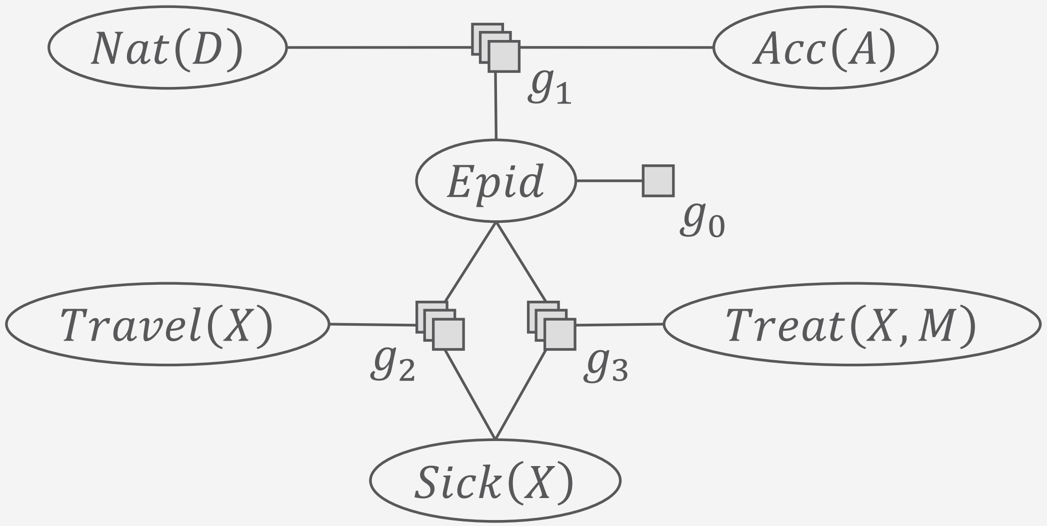


# Encoding the Joint Distribution: Factorisation

- Factors with PRVs = parfactors
  - E.g., $g_2$

| $Travel(X)$ | $Epid$ | $Sick(X)$ | $g_2$ |
|---|---|---|---|
| $false$ | $false$ | $false$ | 5 |
| $false$ | $false$ | $true$ | 0 |
| $false$ | $true$ | $false$ | 4 |
| $false$ | $true$ | $true$ | 6 |
| $true$ | $false$ | $false$ | 4 |
| $true$ | $false$ | $true$ | 6 |
| $true$ | $true$ | $false$ | 2 |
| $true$ | $true$ | $true$ | 9 |

*Potentials*
- In parfactors, just like in factors, no probability distribution as factors required



# Factors

- Grounding
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

| $Travel(X)$ | $Epid$ | $Sick(X)$ | $g_2$ |
|---|---|---|---|
| $false$ | $false$ | $false$ | 5 |
| $false$ | $false$ | $true$ | 0 |
| $false$ | $true$ | $false$ | 4 |
| $false$ | $true$ | $true$ | 6 |
| $true$ | $false$ | $false$ | 4 |
| $true$ | $false$ | $true$ | 6 |
| $true$ | $true$ | $false$ | 2 |
| $true$ | $true$ | $true$ | 9 |

| $Travel(eve)$ | $Epid$ | $Sick(eve)$ | $g_2$ |
|---|---|---|---|
| $false$ | $false$ | $false$ | 5 |
| $false$ | $false$ | $true$ | 0 |
| $false$ | $true$ | $false$ | 4 |
| $false$ | $true$ | $true$ | 6 |
| $true$ | $false$ | $false$ | 4 |
| $true$ | $fals$ | | |
| $true$ | $tru$ | | |
| $true$ | $tru$ | | |

| $Travel(bob)$ | $Epid$ | $Sick(bob)$ | $g_2$ |
|---|---|---|---|
| $false$ | $false$ | $false$ | 5 |
| $false$ | $false$ | $true$ | 0 |
| $false$ | $true$ | $false$ | 4 |
| | | $true$ | 6 |
| | | $false$ | 4 |
| | | $true$ | 6 |
| | | $false$ | 2 |
| | | $true$ | 9 |

| $Travel(alice)$ | $Epid$ | $Sick(alice)$ | $g_2$ |
|---|---|---|---|
| $false$ | $false$ | $false$ | 5 |
| $false$ | $false$ | $true$ | 0 |
| $false$ | $true$ | $false$ | 4 |
| $false$ | $true$ | $true$ | 6 |
| $true$ | $false$ | $false$ | 4 |
| $true$ | $false$ | $true$ | 6 |
| $true$ | $true$ | $false$ | 2 |
| $true$ | $true$ | $true$ | 9 |

$reat(X, M)$

# Encoding the Joint Distribution

- Set of parfactors = model
  - E.g., $G = \{g_1, g_2, g_3\}$
  - Semantics: Joint probability distribution $P_G$
    - Build by grounding, multiplying all grounded factors, and normalising the result
    - Grounding semantics [Sato 95, Fuhr 95]

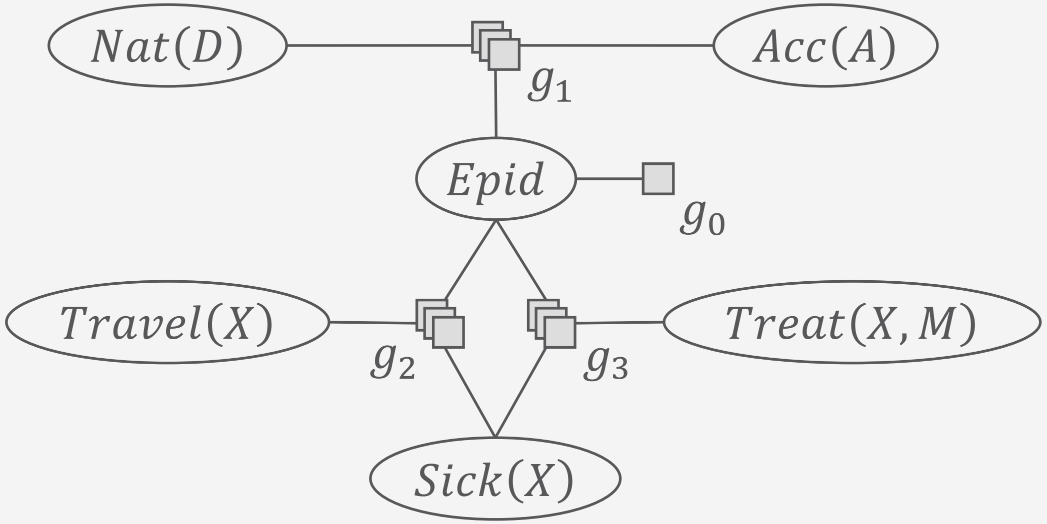
$$P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$$

$$Z = \sum_{v \in r(rv(gr(G)))} \prod_{f \in gr(G)} f_i(\pi_{rv(f_i)}(v))$$

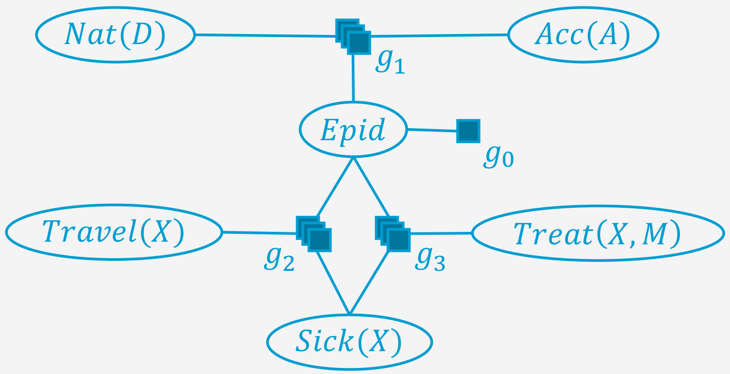
$\pi_{variables}(v)$ = projection of $v$ onto $variables$

**Sparse encoding of joint distribution**

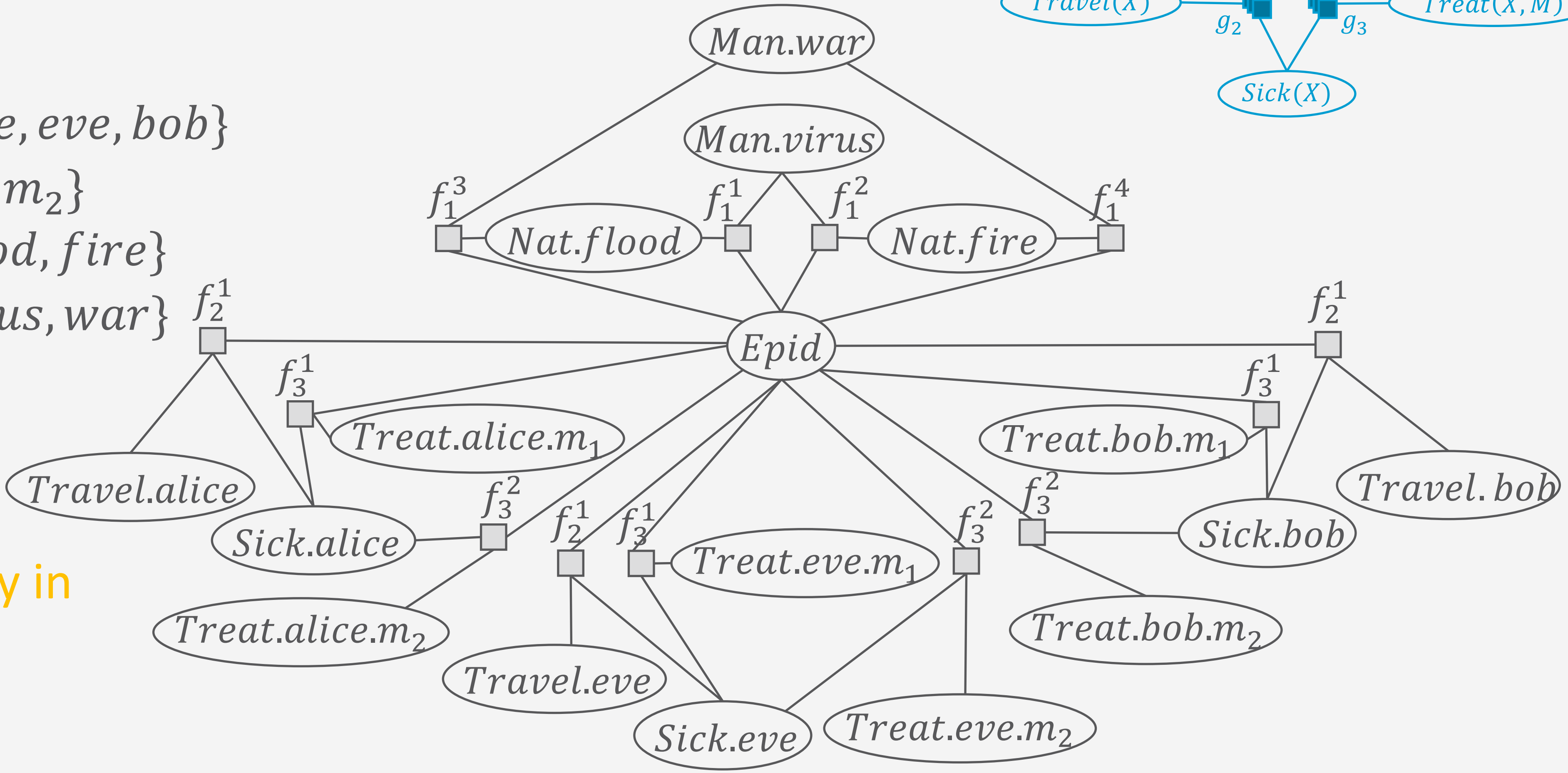
**$3 \cdot 2^3 = 24$ entries in 3 parfactors, 6 PRVs**



# Grounded Model




- Given domains
  - $dom(X) = \{alice, eve, bob\}$
  - $dom(M) = \{m_1, m_2\}$
  - $dom(D) = \{flood, fire\}$
  - $dom(W) = \{virus, war\}$




- Indistinguishability in
  - Graph structure
  - Factors

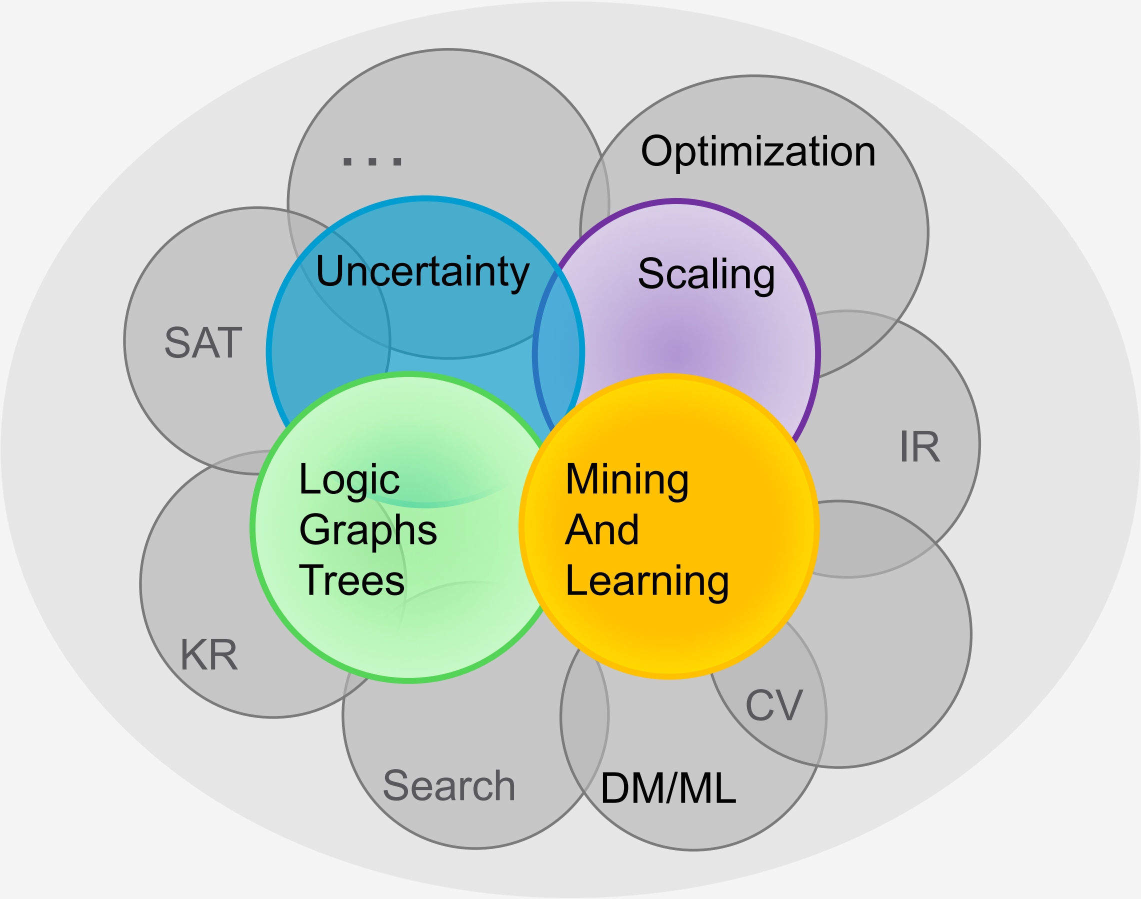
# Probabilistic Relational Models and Variants

- Parfactors Models
  [Poole 03, Taghipour et al. 13, B & Möller 16-19, Gehrke, B & Möller 18-19]
- Markov Logic Networks (MLNs) [Richardson & Domingos 06]
  - Use logical formulas to specify potential functions
- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Use density functions to specify potential functions
- Based on grounding semantics [Sato 95, Fuhr 95]

# The Larger Scope

## Statistical Relational Learning & AI

- Study and design
  - intelligent agents
  - that reason about and
  - act in noisy worlds
  - composed of objects and relations among the objects



# Agenda

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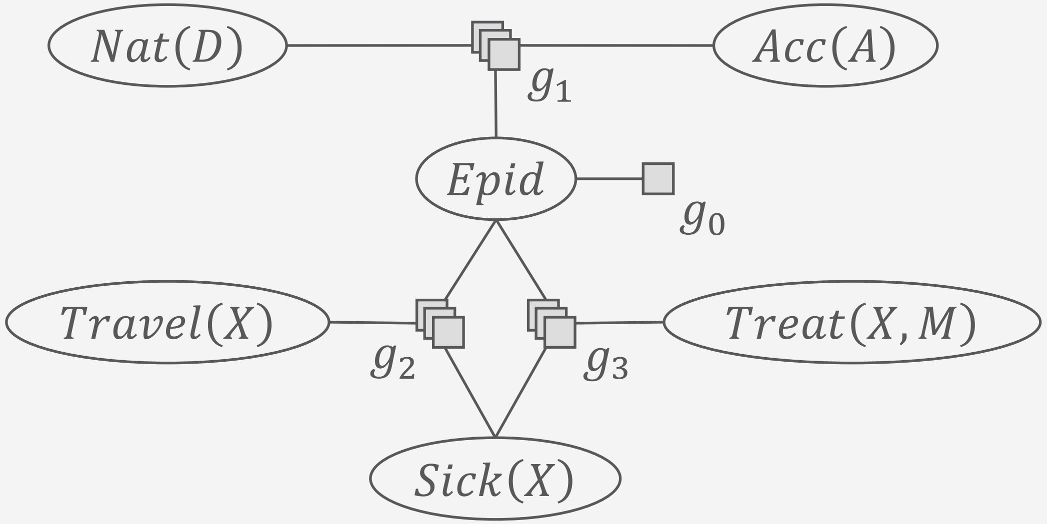
# Lifted Query Answering and Tractability

Indistinguishability for Inference

# Reasoning on Probabilistic Relational Models

- Inference task: query answering (QA)
- Queries:
  - Marginal distribution
    - $P(Sick(eve))$
    - $P(Travel(eve,)\ Treat(eve, m_1))$
  - Conditional distribution
    - $P(Sick(eve) | Epid)$
    - $P(Epid | Sick(eve) = true)$
  - Assignment queries: $\arg\max_{\boldsymbol{a} \in ran(\boldsymbol{A})} P(\boldsymbol{a} | \boldsymbol{e})$
    - MPE: $\boldsymbol{A} = rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
    - MAP: $\boldsymbol{A} \subseteq rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
      - What is not in $\boldsymbol{A}$ needs to be summed out

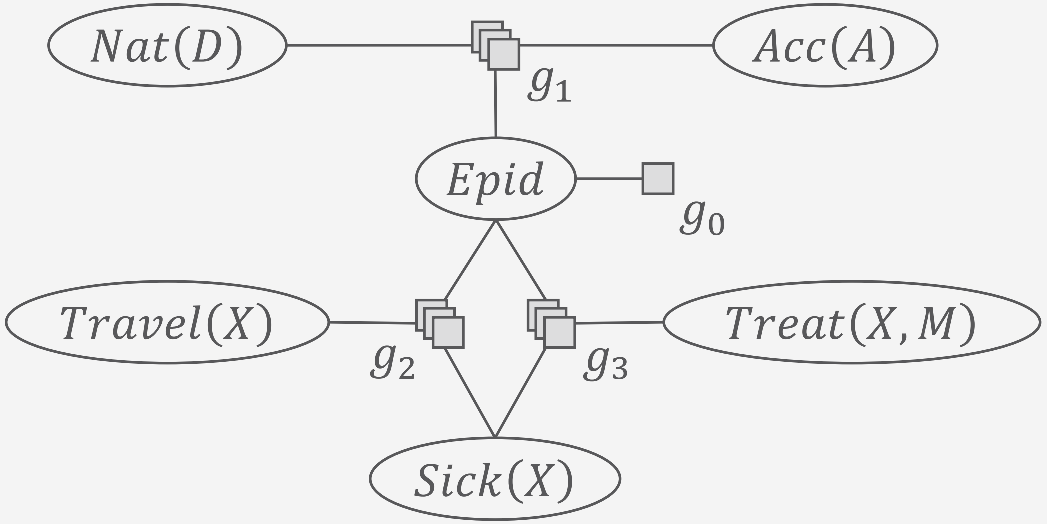
**Goal: Avoid groundings!**
**→ *lifted* inference**



# QA: Lifted Variable Elimination (LVE)

- Eliminate all variables not appearing in query
- Lifted summing out
  - Sum out *representative* instance as in propositional variable elimination
  - Exponentiate result for indistinguishable instances

- Correctness: Equivalent ground operation
  - Each instance is summed out
  - Result: factor $f$ that is identical for all instance
  - Multiplying indistinguishable results
    → exponentiation of one representative $f$



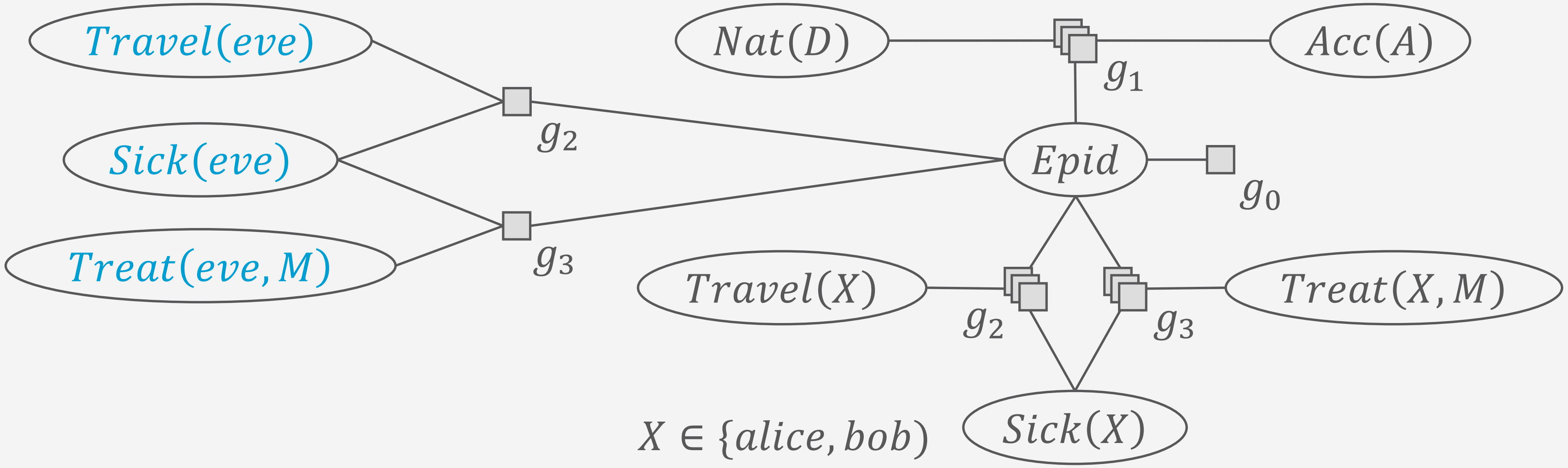



[Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a]

# QA: LVE in Detail

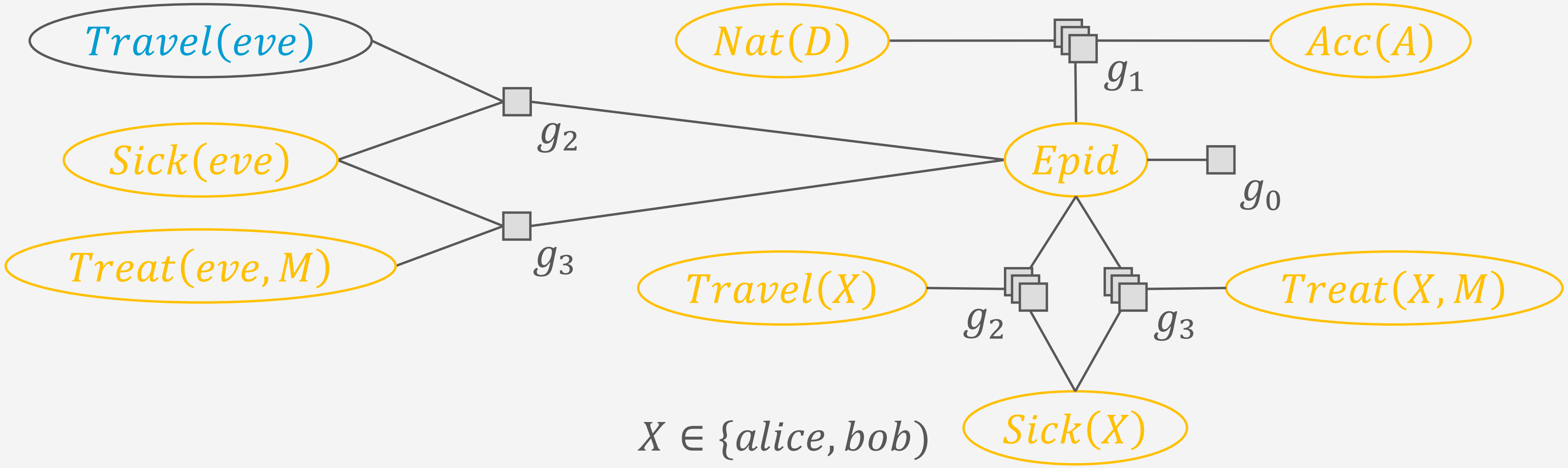
- E.g., marginal
  - $P(Travel(eve))$
  - Split atoms $R(\dots, X, \dots)$ w.r.t. $eve$ if $eve$ in $dom(X)$

Shattering



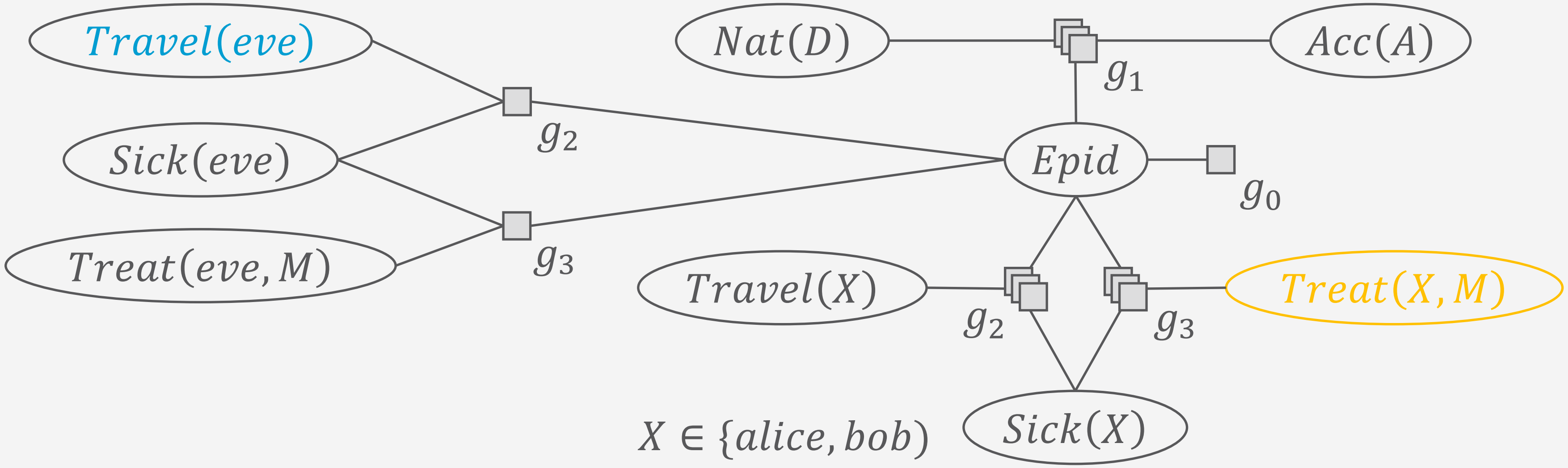
# QA: LVE in Detail

- E.g., marginal
  - $P(Travel(eve))$
  - Split atoms $R(\dots, X, \dots)$ w.r.t. $eve$ if $eve$ in $dom(X)$
  - Eliminate all non-query variables



# QA: LVE in Detail

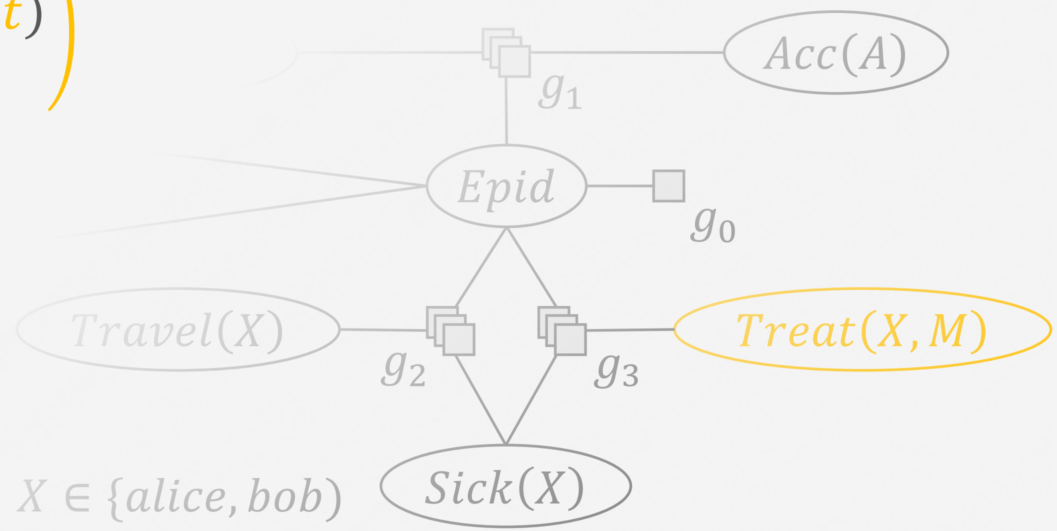
- Eliminate $Treat(X, M)$
  - Appears in only one $g$: $g_3$
  - Contains all logical variables of $g_3$: $X, M$
  - For each $X$ constant: the same number of $M$ constants
  - ✓ Preconditions of lifted summing out fulfilled, lifted summing out possible



# LVE in Detail: Lifted Summing Out

- Eliminate $Treat(X, M)$ by lifted summing out
  1. Sum out representative
  2. Exponentiate for indistinguishable objects

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t)\right)^{\#M|X}$$



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# LVE in Detail: Lifted Summing Out

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t)\right)^{\#M|X}$$

| $Epid$ | $Sick(X)$ | $Treat(X,M)$ | $g_3$ |
|---|---|---|---|
| $false$ | $false$ | $false$ | 9 |
| $false$ | $false$ | $true$ | 1 |
| $false$ | $true$ | $false$ | 6 |
| $false$ | $true$ | $true$ | 3 |
| $true$ | $false$ | $false$ | 7 |
| $true$ | $false$ | $true$ | 5 |
| $true$ | $true$ | $false$ | 4 |
| $true$ | $true$ | $true$ | 8 |

| $Epid$ | $Sick(X)$ | $\Sigma$ |
|---|---|---|
| $false$ | $false$ | 10 |
| $false$ | $true$ | 9 |
| $true$ | $false$ | 12 |
| $true$ | $true$ | 12 |

| $Epid$ | $Sick(X)$ | ^ |
|---|---|---|
| $false$ | $false$ | $10^2$ |
| $false$ | $true$ | $9^2$ |
| $true$ | $false$ | $12^2$ |
| $true$ | $true$ | $12^2$ |

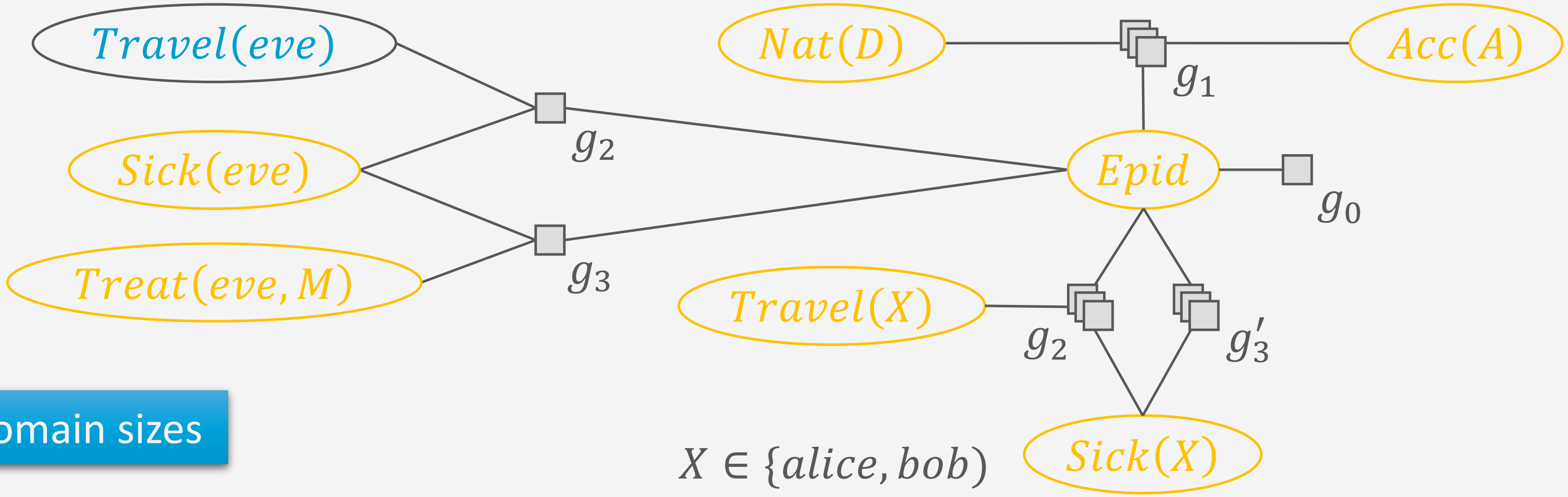
# LVE in Detail: Lifted Summing Out

Only here, domain size comes into play
→ no change in graph / parfactor if domain size changes

- Result after summing out $Treat(X, M)$

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t)\right)^{\#M|X}$$

| $Epid$ | $Sick(X)$ | $g_3'$ |
|---|---|---|
| $false$ | $false$ | 100 |
| $false$ | $true$ | 81 |
| $true$ | $false$ | 144 |
| $true$ | $true$ | 144 |




→ enables tractability w.r.t. domain sizes

# Tractability

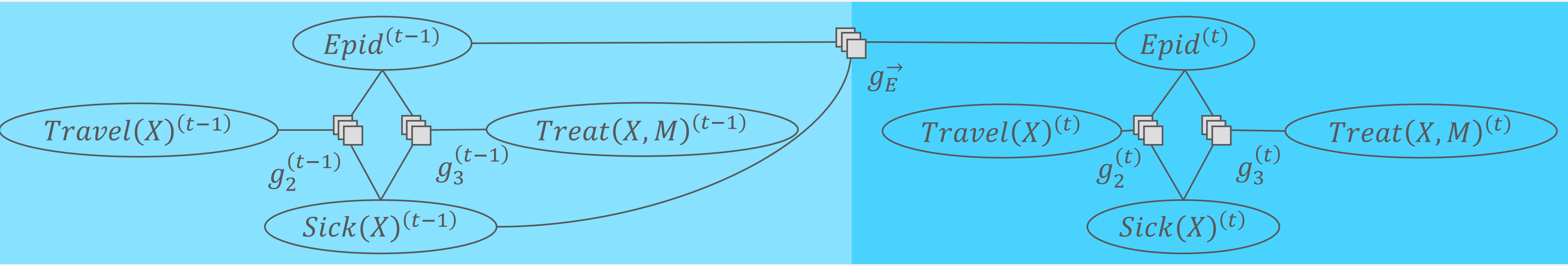
- Given a model that allows for lifted calculations
  - I.e., no groundings during solving an instance of the problem
- Solving an instance of the problem is possible in time polynomial in domain sizes

  → The query answering algorithm is domain-lifted
- An query answering problem is tractable
  - when it is solved by an efficient algorithm, running in time polynomial in the number of random variables
- Assume that the number of random variables is characterised by domain sizes
  - Then, solving a query answering problem is tractable under domain-liftability
    - Runtime might still be exponential in other terms
    - More general results by Niepert & Van den Broeck (2014)

# Keeping Indistinguishability over Time

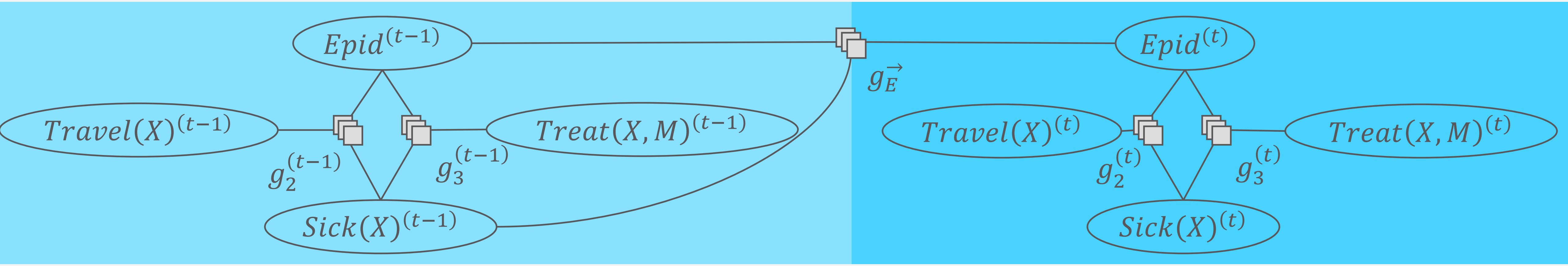
Indistinguishability for Inference

# Dynamic Probabilistic Relational Models & Temporal Queries




- Marginal distribution queries: $P\left(A_\pi^i \middle| E_{0:t}\right)$
  - Hindsight: $\pi < t$ (Was there an epidemic $t - \pi$ days ago?)
  - Filtering: $\pi = t$ (Is there currently an epidemic?)
  - Prediction: $\pi > t$ (Will there be an epidemic in $\pi - t$ days?)
- Assignment queries on temporal sequence

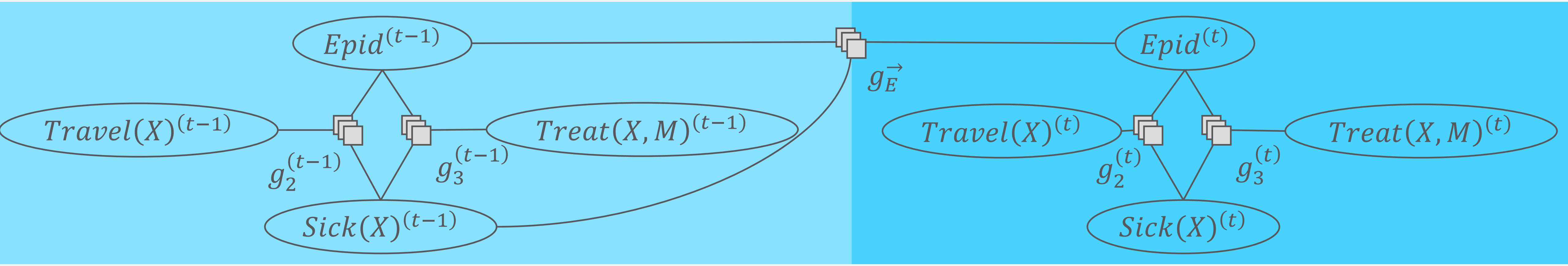
# Reasoning over Time: Interfaces




- Main idea: Use temporal conditional independences for efficient temporal QA
  - Normally only a subset of random variables influence next time step → interface variables
  - State description of interface from time slice $t-1$ suffices to perform inference on time slice $t$
  - → Makes present independent from past / future

Algorithms:
- Propositional: Interface Algorithm [Murphy, 2002]
- Lifted: Lifted Dynamic Junction Tree Algorithm [Gehrke et al, 2018]

# Taming Reasoning



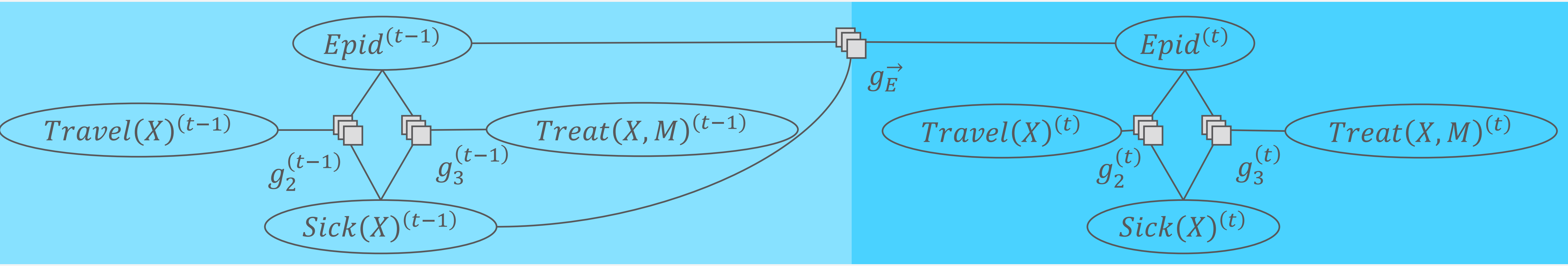

- Evidence can ground a model over time
- Non-symmetric evidence
  - Observe evidence for some instances in one time step
  - Observe evidence for a subset of these instances in another time step
  - Split the logical variable slowly over time

**Interface carries over splits, leading to slowly grounding a model over time**



Gehrke et al. (2020)

# Undoing Splits




- Need to undo splits to

  keep reasoning polynomial w.r.t. domain sizes
- Where can splits be undone efficiently?
  - When moving from one time step to the next, i.e., in the interface
- How to undo splits?
  - Find approximate symmetries → Clustering
  - Merge based on groundings
- Is it reasonable to undo splits?
  - Effect of slight differences in evidence limited due to temporal behaviour

# Indistinguishability in Decision Making

Indistinguishability for Inference

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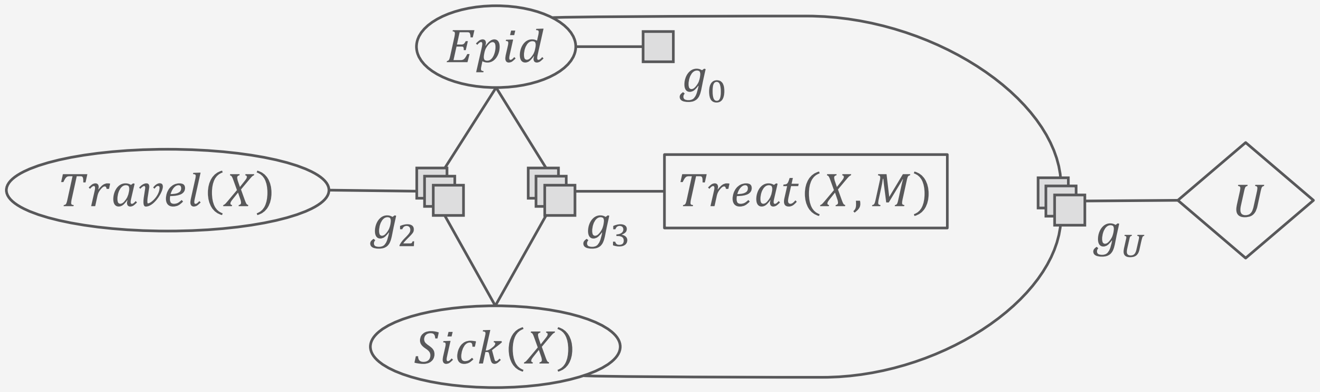
# Indistinguishability for Decision Making

- Online decision making: Graphical models extended by decision and utility nodes
  - Parameterise decisions to make decisions for whole groups of indistinguishable instances: $Treat(X, M)$ (box in graph)
  - PRVs in utility functions to denote identical share in contributed utility $U$ (diamond in graph) : $\phi_U\big(Epid, Sick(X)\big)$
  - (Dynamic) decision parfactor models, Markov logic decision networks



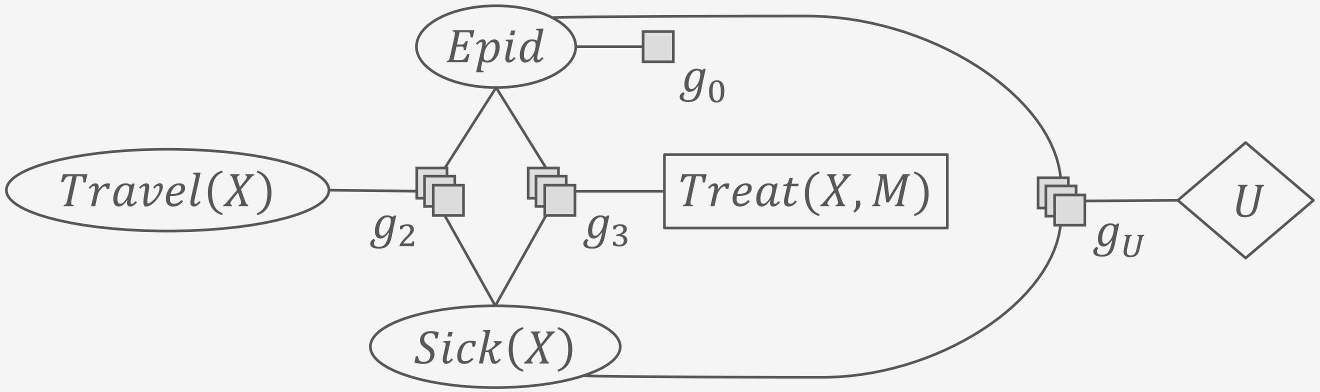



Gehrke et al. 2019b, Gehrke et al. 2019c; Nath & Domingos 2009

# Indistinguishability for Decision Making

- Inference task: maximum expected utility (MEU) query
  - *Which actions can be expected to lead to the maximum utility?*
  - Standard inference algorithms more or less still work
    - Iterate through all possible decisions, set decisions as evidence, calculate expected utility, store current maximum
    - Solve an MAP query with decision variables as query terms and the other variables in the model to eliminate

Assign same action to group of indistinguishable instances
- Fewer possible decisions to consider → *tractability*!



# Indistinguishability for Decision Making

- Offline decision making: solve a (partially observable) Markov decision problem (POMDP)
  - First-order / relational MDPs: indistinguishability in the environment [Sanner & Kersting 2012]
    - Based on situation calculus: work with representatives
      - E.g., it is important that a box with medical supplies arrives at a destination but not which one it is in particular (of a set of boxes with medical supplies)
    - Novel propositional situations worth exploring may be instances of a well-known context in the relational setting → *exploitation* promising
      - E.g., household robot learning water-taps
      - Having opened one or two water-taps in a kitchen, one can expect other water-taps in kitchens to work similarly

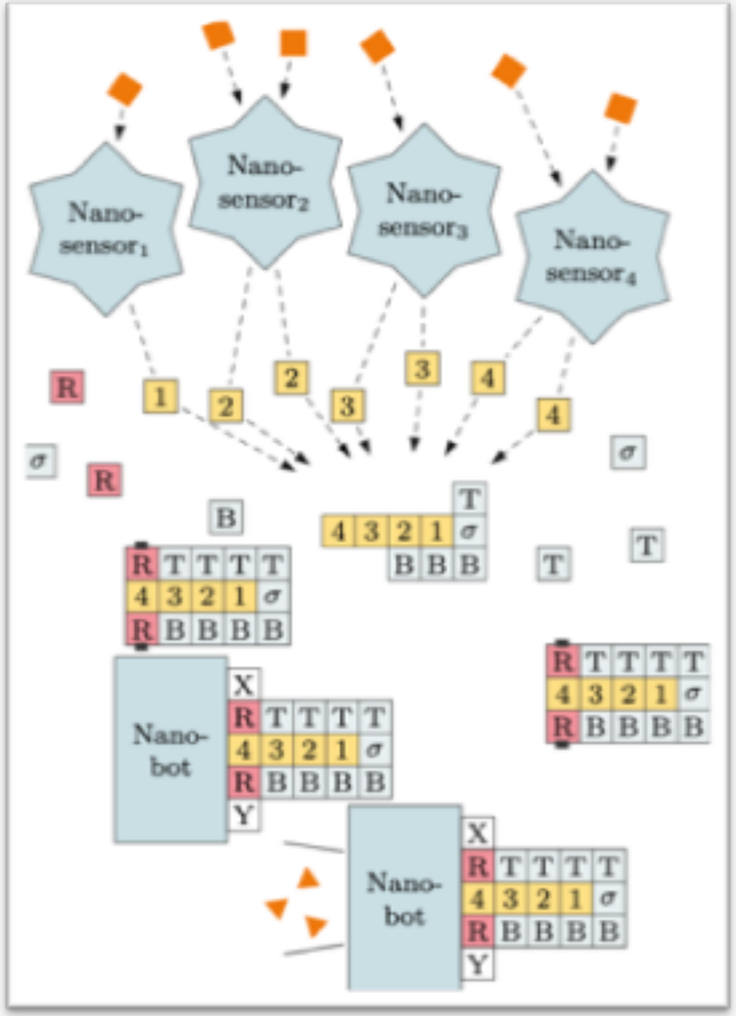
      ⇒ Priority for exploring water-taps in kitchens in general reduced

      ⇒ Information gathered likely to carry over to water-taps in other places

      ❖ Hard to model in propositional setting: each water-tap is novel

# Indistinguishability for Decision Making

- Multi-agent setting: decentralised POMDP [Oliehoek & Amato 2016]
  - Set of agents with
    - Own set of available actions, observations
    - *Shared* state and reward
- Lifting for agents [B et al. 2022]
  - Agents with indistinguishable behaviour → types
    - The same sets of actions, observations available
    - Same strategy / program applies if certain independences hold
  - Groups by types can be treated by representatives
    - Reduce exponential dependence on agent numbers
  - Application: Nanoagent network



# Outlook: Human-aware Decision Making

- *Human-aware agent*
  - Interaction with a human during acting (*human-in-the-loop*)
  - Example: *Urban Search and Rescue* with a robot
- Different models considered
  - Original agent model $\mathcal{M}^R$
  - Human model $\mathcal{M}^H$
  - Agent model $\widetilde{\mathcal{M}}_r^H$ of human model $\mathcal{M}^H$
  - Agent model $\widetilde{\mathcal{M}}_h^R$ of human model $\mathcal{M}_h^R$, which human has of $\mathcal{M}^R$

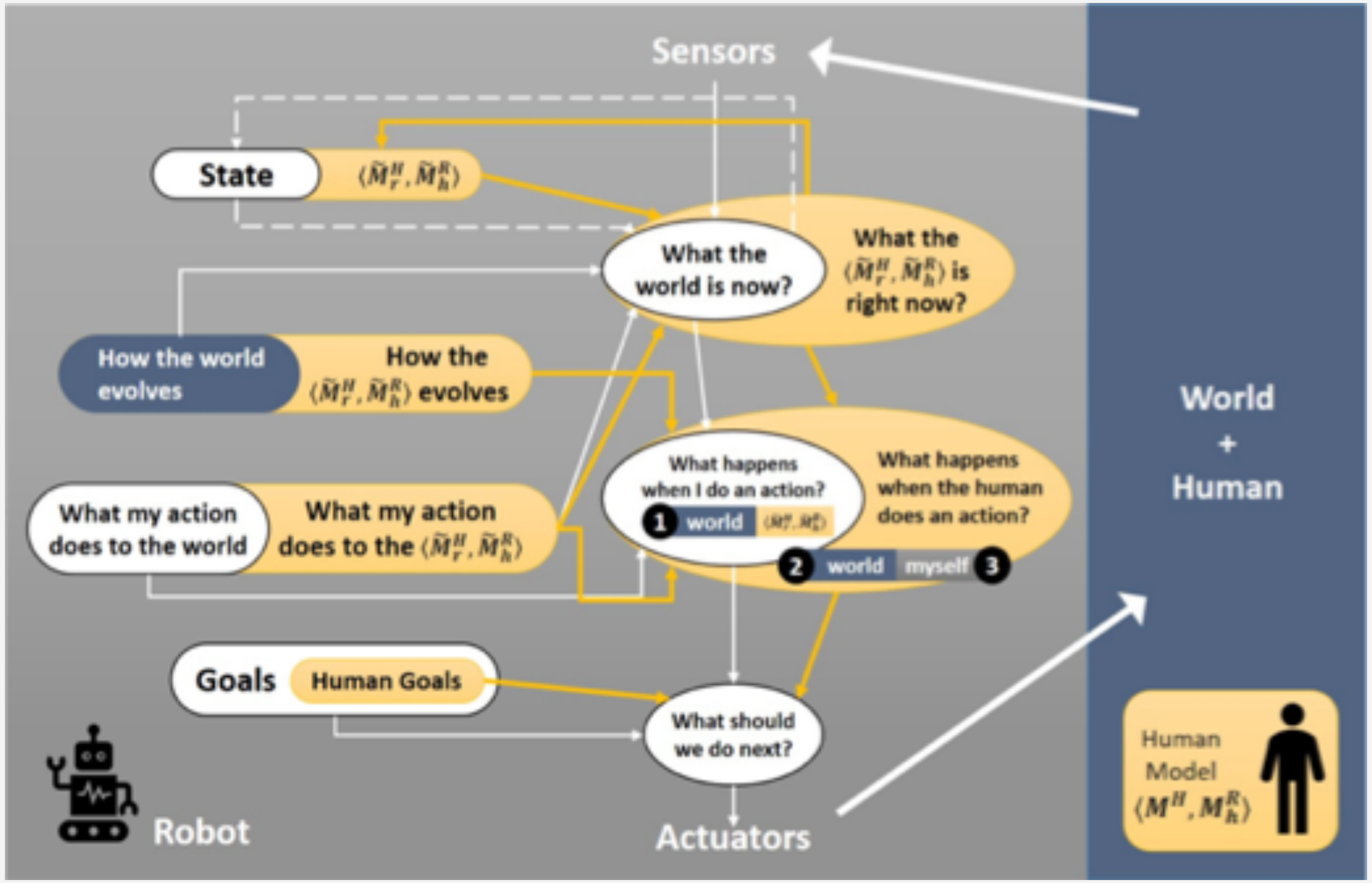



Figure: Subbarao (Rao) Kambhampati, http://bit.ly/3bno2io

# Outlook: Human-aware Decision Making

- Different models considered:
  - Original agent model $\mathcal{M}^R$
  - Human model $\mathcal{M}^H$
  - Agent model $\widetilde{\mathcal{M}}_r^H$ of human model $\mathcal{M}^H$
  - Agent model $\widetilde{\mathcal{M}}_h^R$ of human model $\mathcal{M}_h^R$, which human has of $\mathcal{M}^R$

- $\widetilde{\mathcal{M}}_r^H$ allows for anticipating human behaviour
- $\widetilde{\mathcal{M}}_h^R$ allows for conforming to human expectations

- *Goal: Use groups for explanations*

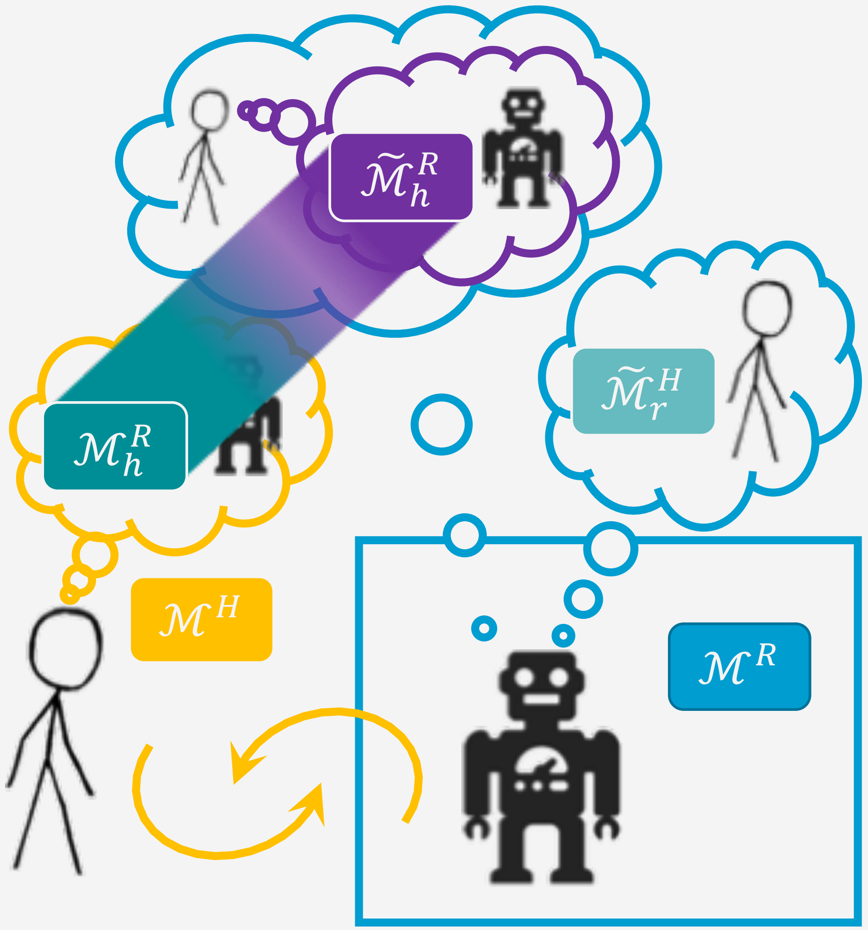



Figure based on figure by Subbarao (Rao) Kambhampati, http://bit.ly/3bno2io

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# The Finish Line: The Power of Indistinguishability

- Lifted query answering and tractability
  - Use information about indistinguishability to speed up inference
  - Tractability in terms of domain sizes through lifting
  - Handle evidence in groups of indistinguishable observations
  - Count values in histograms for lifted queries
- Keeping indistinguishability over time
  - Merge parfactors with bounded error
- Indistinguishability in decision making
  - Relational environment encoded
  - Agent types

What else is there to do? – Oh, so much…
- Approximating symmetries
- Generalising lifting operators
- More robust learning algorithms
- Privacy
- Ethical behaviour
- Explainability
- …

Thank you!

# Bibliography & Further Papers

Ordered topic-wise and then alphabetically

# Bibliography – General


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