Inference Techniques for Resilience

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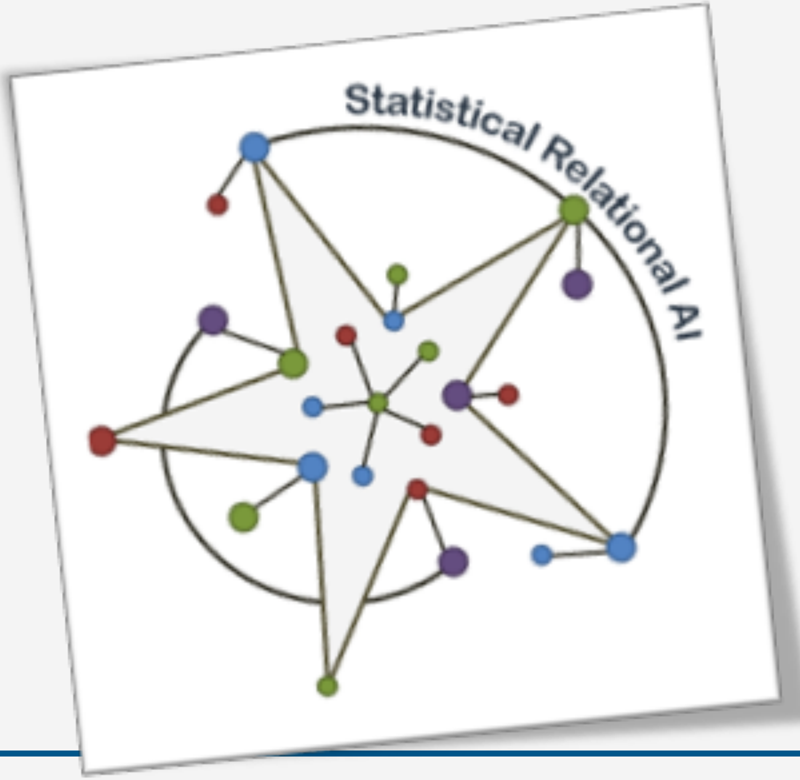

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Resilience, Inference & Resilient Inference

- Avizienis et al. (2004)

  A system's ability to remain operational – although at potentially lower operational levels – when exposed to stressors and to adapt its functioning if those stressors persist

- What has inference to do with this?
  - Use formal model + probabilistic inference (reasoning) to
    - Predict disruptions or identify stressors through queries given observations
    - Compute necessary adaptations to formal model
  - System's resilience includes an algorithmic technical side regarding inference:
    - Keep inference going even under an influx of queries or observations
    - Resume inference as soon as possible after adapting a model to the environment under duress

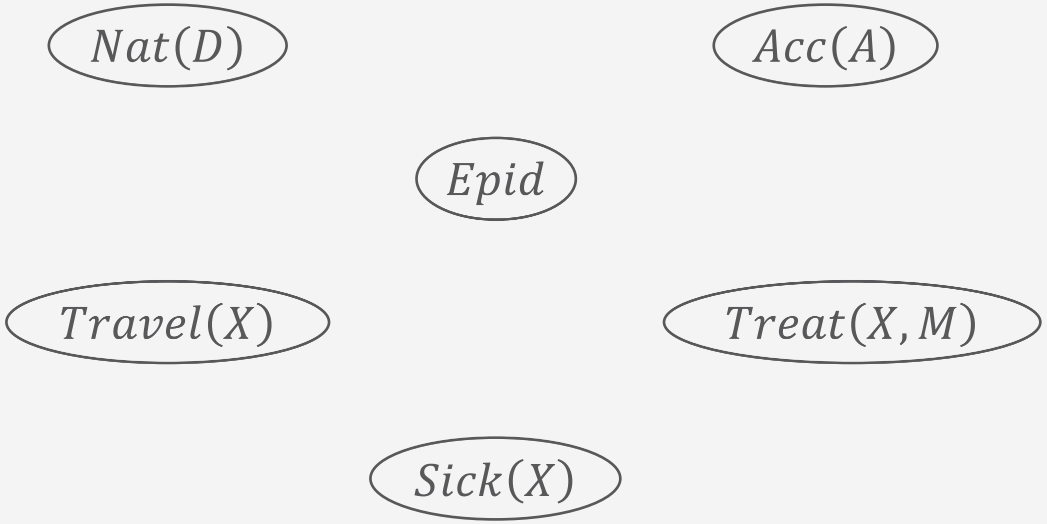
Agenda

- Probabilistic Relational Models (PRMs)
  - Application example
  - Semantics
  - Query answering / basic inference
- Lifting Algorithms for More Resilient Inference
  - Cluster trees for efficient multi-query inference
  - Adaptive inference in cluster trees
  - Changing domains
  - Keeping inference going over time
- Summary

Application: Epidemics

- Atoms: Parameterised random variables = PRVs
  - With logical variables
    - E.g., X, M
    - Possible values (domain):
      $$dom(X) = \{alice, eve, bob\}$$
      $$dom(M) = \{injection, tablet\}$$
  - With range
    - E.g., Boolean
    - $ran\big(Travel(X)\big) = \{true, false\}$
  - Represent sets of *indistinguishable* random variables

$$Nat(D) = natural\ disaster\ D$$
$$Acc(A) = accident\ A$$

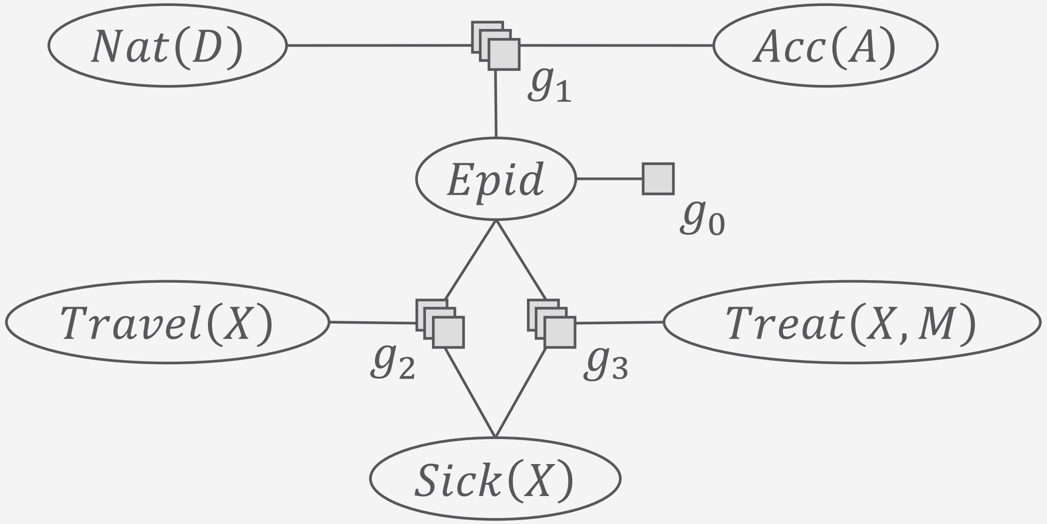


Encoding the Joint Distribution: Factorisation

- Factors with PRVs = parfactors
  - E.g., g_2

$Travel(X)$	$Epid$	$Sick(X)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

Potentials
- In parfactors, just like in factors, no probability distribution as factors required



Factors

- Grounding
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

$Travel(X)$	$Epid$	$Sick(X)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$Travel(eve)$	$Epid$	$Sick(eve)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	fals		
$true$	tru		
$true$	tru		

$Travel(bob)$	$Epid$	$Sick(bob)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
		$true$	6
		$false$	4
		$true$	6
		$false$	2
		$true$	9

$Travel(alice)$	$Epid$	$Sick(alice)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$reat(X,M)$

Encoding the Joint Distribution

- Set of parfactors = model
  - E.g., $G = \{g_1, g_2, g_3\}$
  - Semantics: Joint probability distribution P_G
    - Build by grounding, multiplying all grounded factors, and normalising the result
    - Grounding semantics [Sato 95, Fuhr 95]

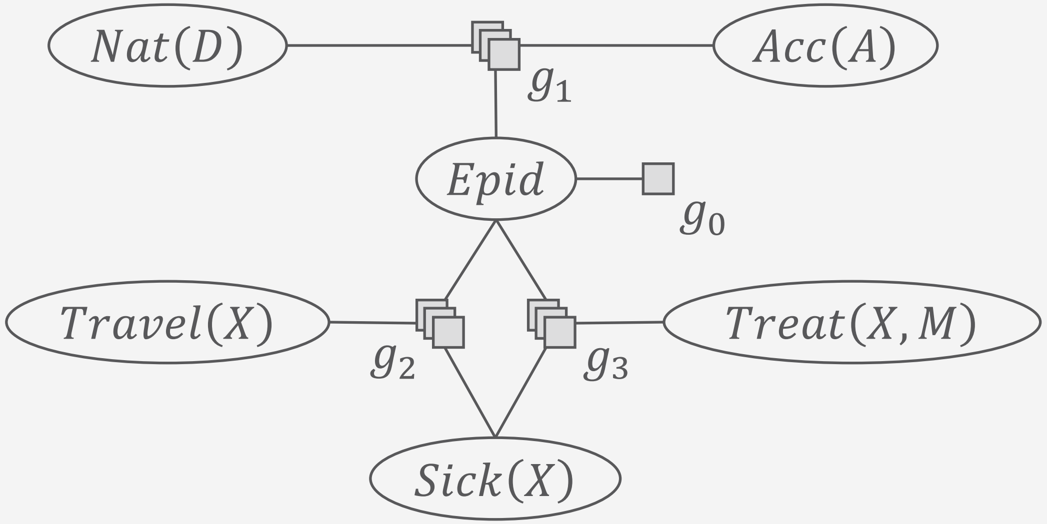
$$P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$$

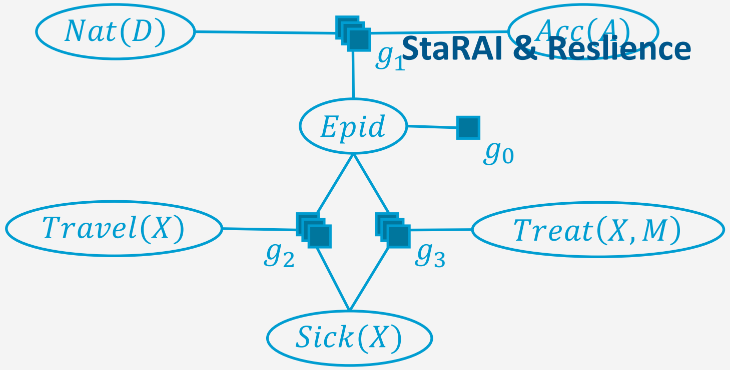
$$Z = \sum_{v \in r(rv(gr(G)))} \prod_{f \in gr(G)} f_i(\pi_{rv(f_i)}(v))$$

$\pi_{variables}(v)$ = projection of v onto $variables$

Sparse encoding of joint distribution

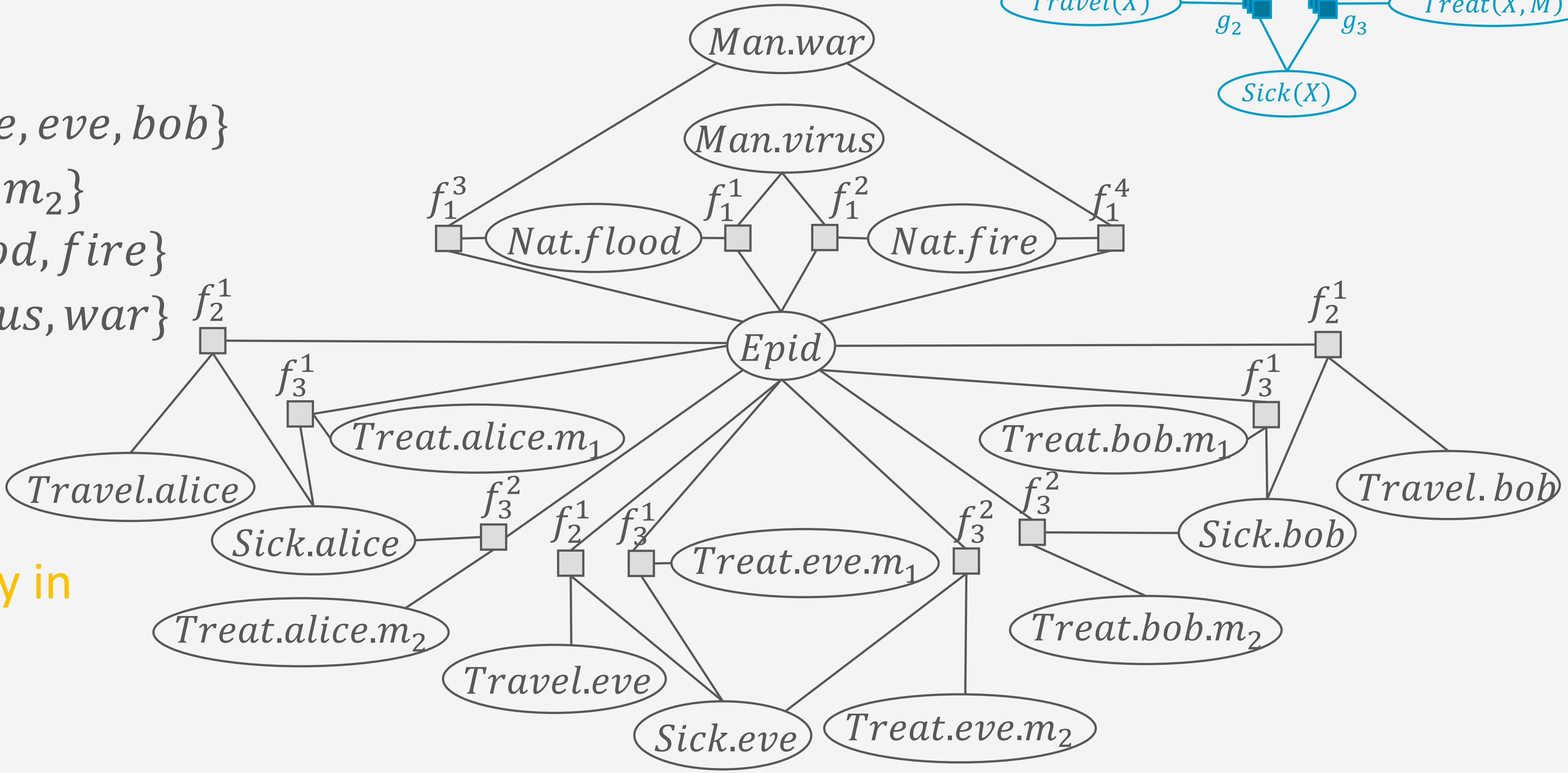
$3 \cdot 2^3 = 24$ entries in 3 parfactors, 6 PRVs





Grounded Model

- Given domains
  - $dom(X) = \{alice, eve, bob\}$
  - $dom(M) = \{m_1, m_2\}$
  - $dom(D) = \{flood, fire\}$
  - $dom(W) = \{virus, war\}$
- Indistinguishability in
  - Graph structure
  - Factors



Probabilistic Relational Models and Variants

- Parfactors Models
  [Poole 03, Taghipour et al. 13, B & Möller 16-19, Gehrke, B & Möller 18-19]

- Markov Logic Networks (MLNs) [Richardson & Domingos 06]
  - Use logical formulas to specify potential functions

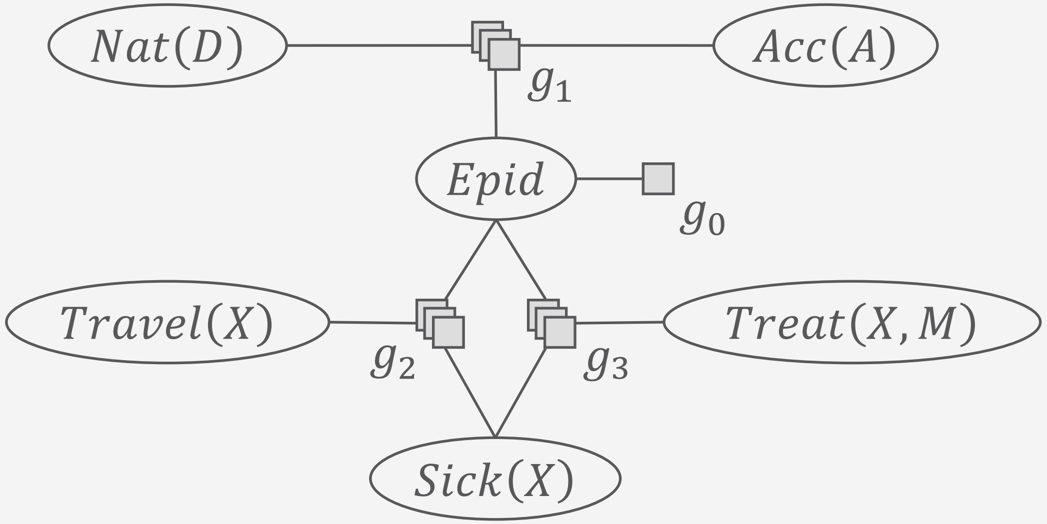
- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Use density functions to specify potential functions

- Based on grounding semantics [Sato 95, Fuhr 95]

Reasoning on Probabilistic Relational Models

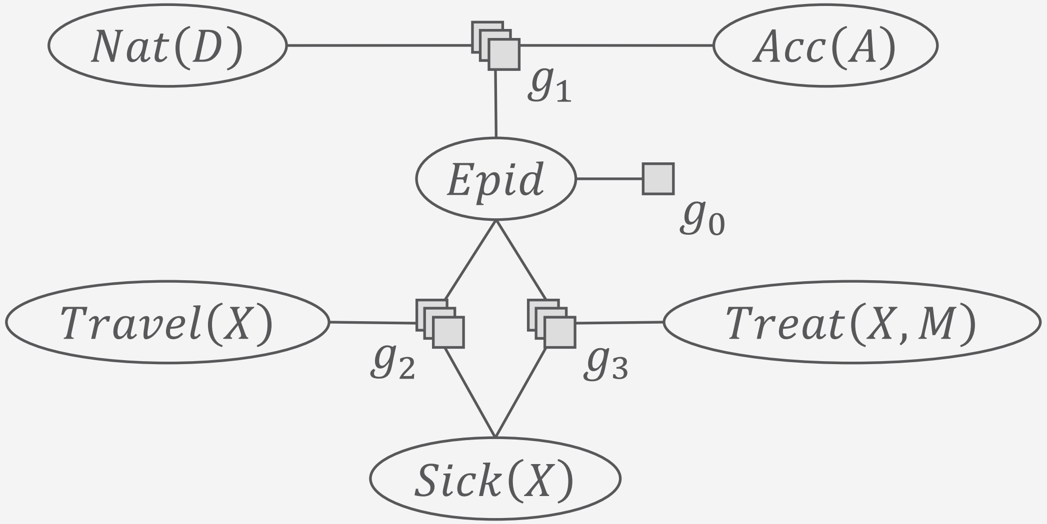
- Inference task: query answering (QA)
- Queries:
  - Marginal distribution
    - $P(Sick(eve))$
    - $P(Travel(eve,)\ Treat(eve, m_1))$
  - Conditional distribution
    - $P(Sick(eve)|Epid)$
    - $P(Epid|Sick(eve) = true)$
  - Assignment queries: $\arg\max_{\boldsymbol{a} \in ran(\boldsymbol{A})} P(\boldsymbol{a}|\boldsymbol{e})$
    - MPE: $\boldsymbol{A} = rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
    - MAP: $\boldsymbol{A} \subseteq rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
      - What is not in $\boldsymbol{A}$ needs to be summed out

Goal: Avoid groundings!
→ *lifted* inference



QA: Lifted Variable Elimination (LVE)

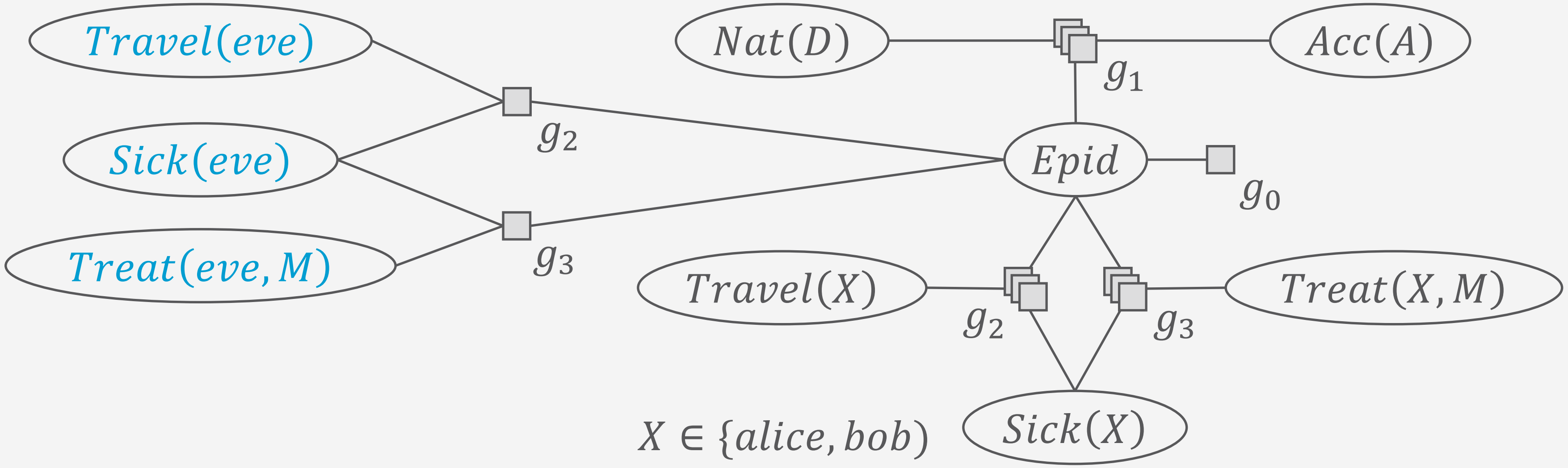
- Eliminate all variables not appearing in query
- Lifted summing out
  - Sum out *representative* instance as in propositional variable elimination
  - Exponentiate result for indistinguishable instances
- Correctness: Equivalent ground operation
  - Each instance is summed out
  - Result: factor f that is identical for all instance
  - Multiplying indistinguishable results
    → exponentiation of one representative f



QA: LVE in Detail

- E.g., marginal
  - $P(Travel(eve))$
  - Split atoms $R(\dots, X, \dots)$ w.r.t. eve if eve in $dom(X)$

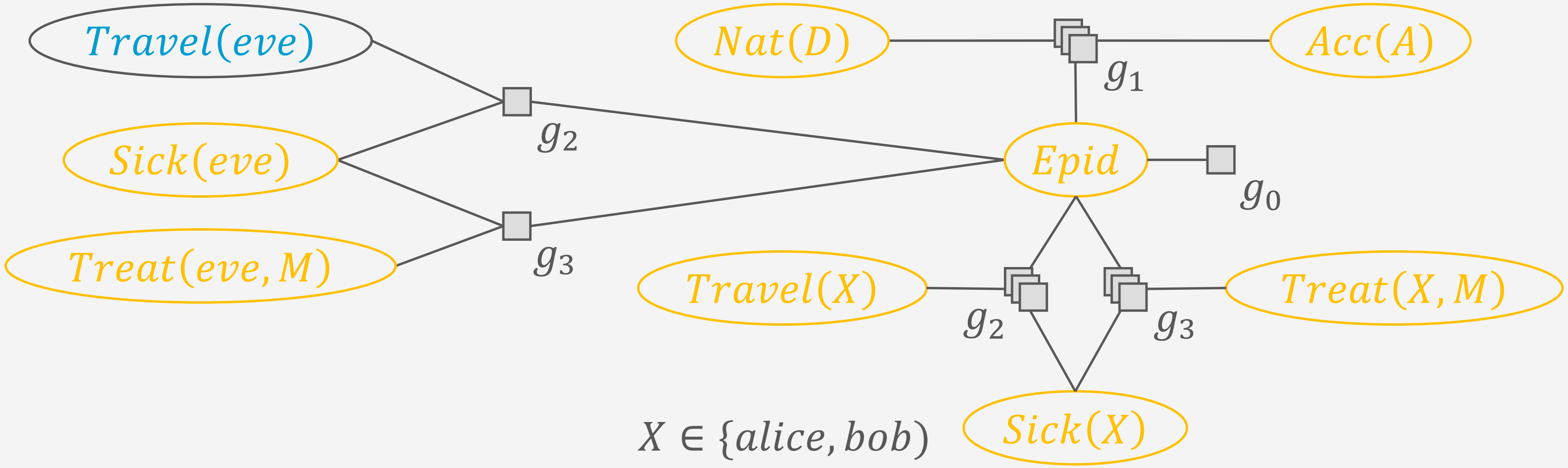
Shattering



$X \in \{alice, bob\}$

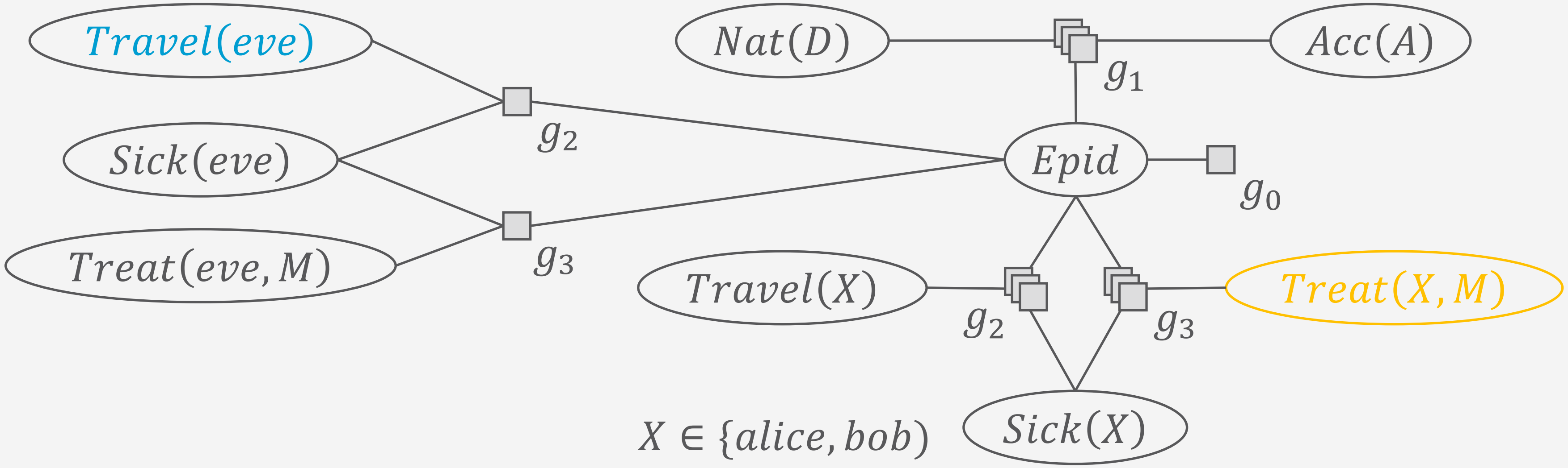
QA: LVE in Detail

- E.g., marginal
  - $P\big(Travel(eve)\big)$
  - Split atoms $R(\dots, X, \dots)$ w.r.t. eve if eve in $dom(X)$
  - Eliminate all non-query variables



QA: LVE in Detail

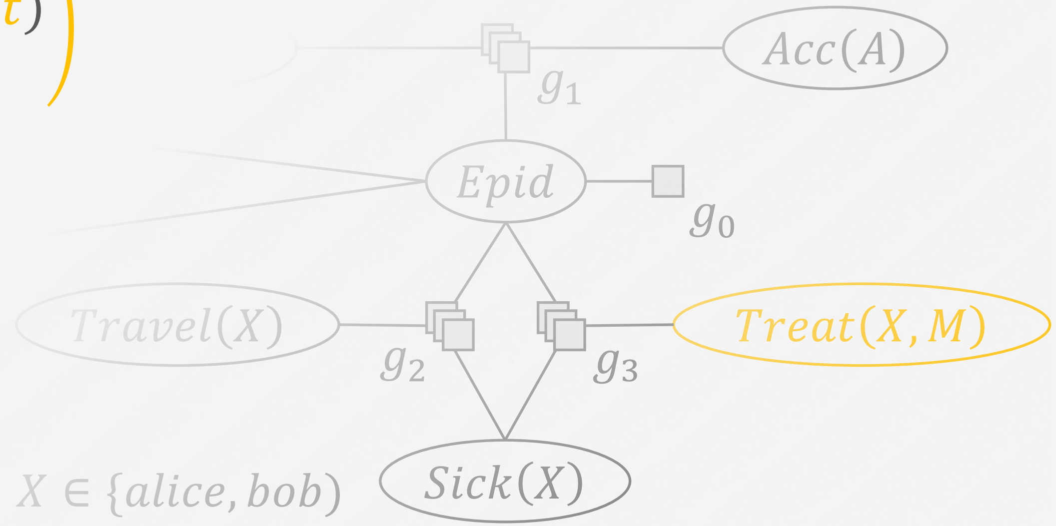
- Eliminate $Treat(X, M)$
  - Appears in only one g: g_3
  - Contains all logical variables of g_3: X, M
  - For each X constant: the same number of M constants
  - ✓ Preconditions of lifted summing out fulfilled, lifted summing out possible



LVE in Detail: Lifted Summing Out

- Eliminate $Treat(X, M)$ by lifted summing out
  1. Sum out representative
  2. Exponentiate for indistinguishable objects

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t) \right)^{\#M|X}$$



LVE in Detail: Lifted Summing Out

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t)\right)^{\#M|X}$$

$Epid$	$Sick(X)$	$Treat(X,M)$	g_3
$false$	$false$	$false$	9
$false$	$false$	$true$	1
$false$	$true$	$false$	6
$false$	$true$	$true$	3
$true$	$false$	$false$	7
$true$	$false$	$true$	5
$true$	$true$	$false$	4
$true$	$true$	$true$	8

+

$Epid$	$Sick(X)$	Σ
$false$	$false$	10
$false$	$true$	9
$true$	$false$	12
$true$	$true$	12

^

$Epid$	$Sick(X)$	^
$false$	$false$	10^2
$false$	$true$	9^2
$true$	$false$	12^2
$true$	$true$	12^2

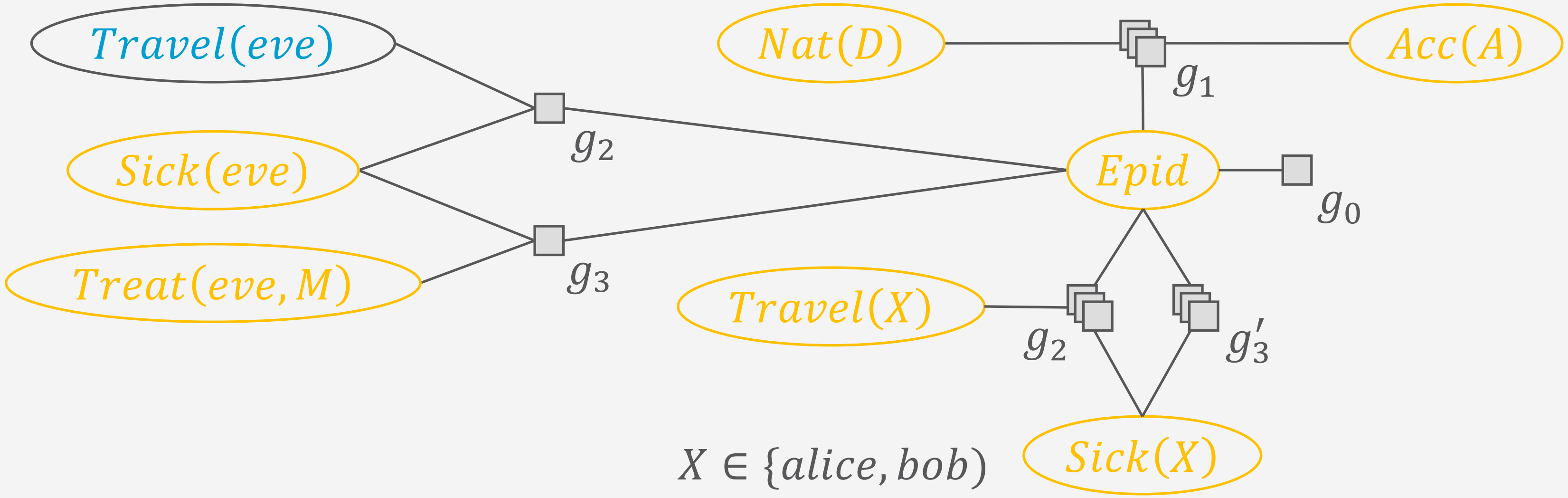
LVE in Detail: Lifted Summing Out

Only here, domain size comes into play
→ no change in graph / parfactor if domain size changes

- Result after summing out $Treat(X,M)$

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t)\right)^{\#M|X}$$

$Epid$	$Sick(X)$	g_3'
$false$	$false$	100
$false$	$true$	81
$true$	$false$	144
$true$	$true$	144



Tractability

- Given a model that allows for lifted calculations
  - I.e., no groundings during solving an instance of the problem
- Solving an instance of the problem is possible in time polynomial in domain sizes

  → The query answering algorithm is domain-lifted
- An query answering problem is tractable
  - when it is solved by an efficient algorithm, running in time polynomial in the number of random variables
- Assume that the number of random variables is characterised by domain sizes
  - Then, solving a query answering problem is tractable under domain-liftability
    - Runtime might still be exponential in other terms
    - More general results by Niepert & Van den Broeck (2014)

Agenda

- Probabilistic Relational Models (PRMs)
  - Application example
  - Semantics, static vs. dynamic behavior
  - Query answering / basic inference
- Algorithms for More Resilient Inference
  - Cluster trees for efficient multi-query inference
  - Adaptive inference in cluster trees
  - Changing domains
  - Keeping inference going over time
- Summary

Cluster Trees for Efficient Multi-query Inference

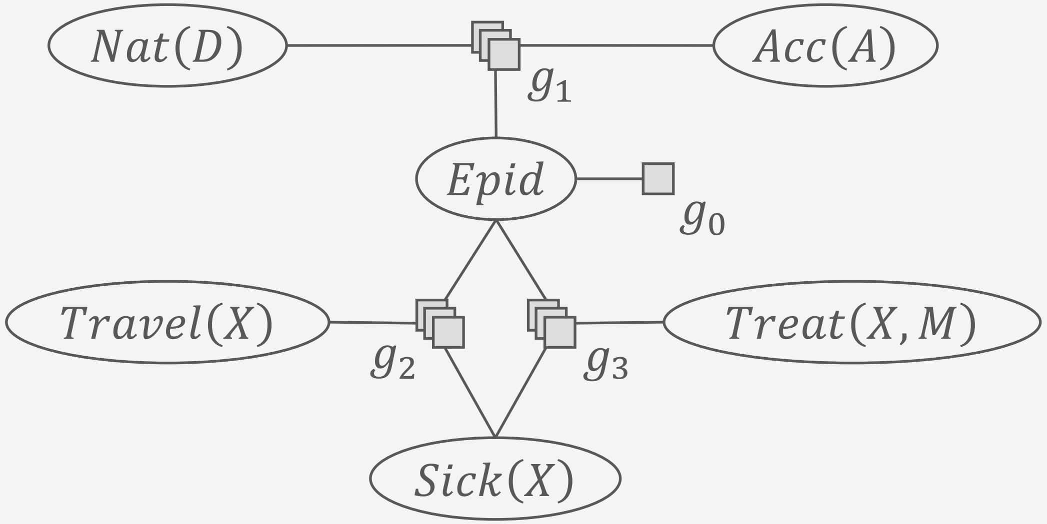
Algorithms for More Resilient Inference

Many Queries: LJT

Cluster tree based on (conditional) independences

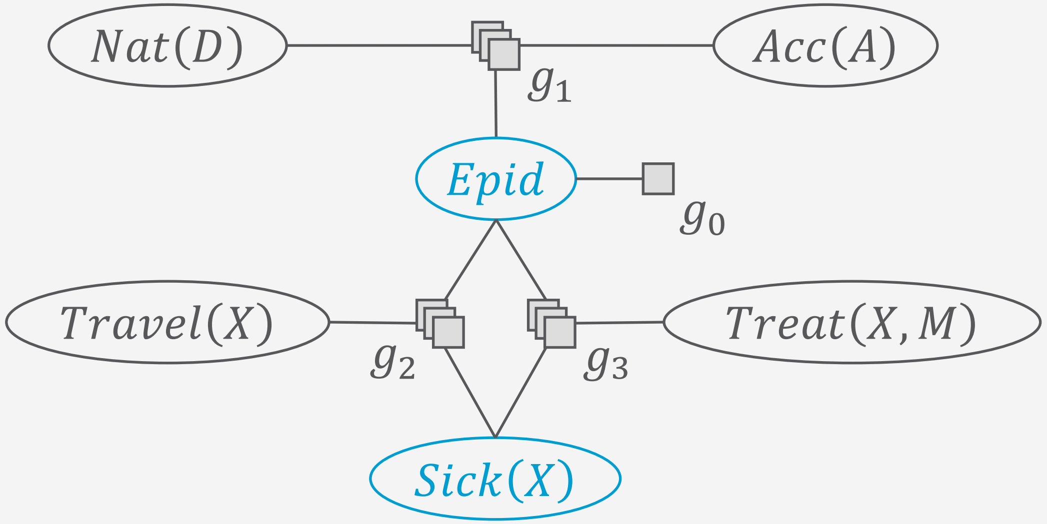
- Set of queries
  - $P(Travel(eve))$
  - $P(Sick(bob))$
  - $P(Treat(eve, m_1))$
  - $P(Epid)$
  - $P(Nat(flood))$
  - $P(Man(virus))$
  - Combinations of variables
- Under evidence
  - $Sick(X') = true$
  - $X' \in \{alice, eve\}$

- Challenges:
  - Do not start from scratch for every query
  - Support QA on subset of atoms
  - Avoid groundings



Solution: Submodels

- Identify submodel sufficient for query
  - Find PRVs that make submodel independent from remaining model
    - Separator
  - “Query” over separator collects all influences of remaining model on PRVs in submodel
    - PRVs of submodel = parcluster

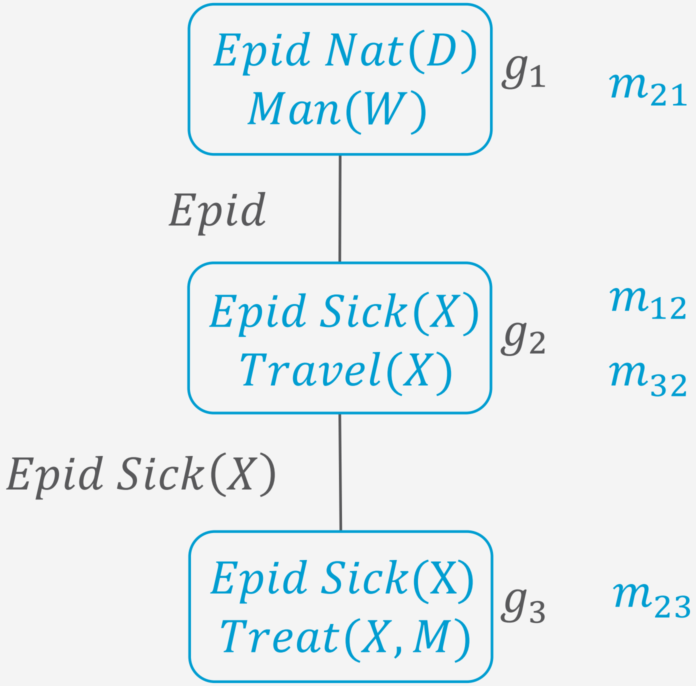


Lauritzen and Spiegelhalter (1988), Shafer and Shenoy (1989), Jensen et al. (1990), B and Möller (2016)

Solution: Submodels

- Network of submodels with separators
  - Recursive “queries” to make submodels independent from each other
  - (First-order) Junction tree
    - Acyclic, running intersection property
- Recursive queries from each node
  - Arrange queries using dynamic programming
    - Also known as message passing

Answer queries on subtree over the query terms
- Use middle cluster for $P(Sick(eve))$



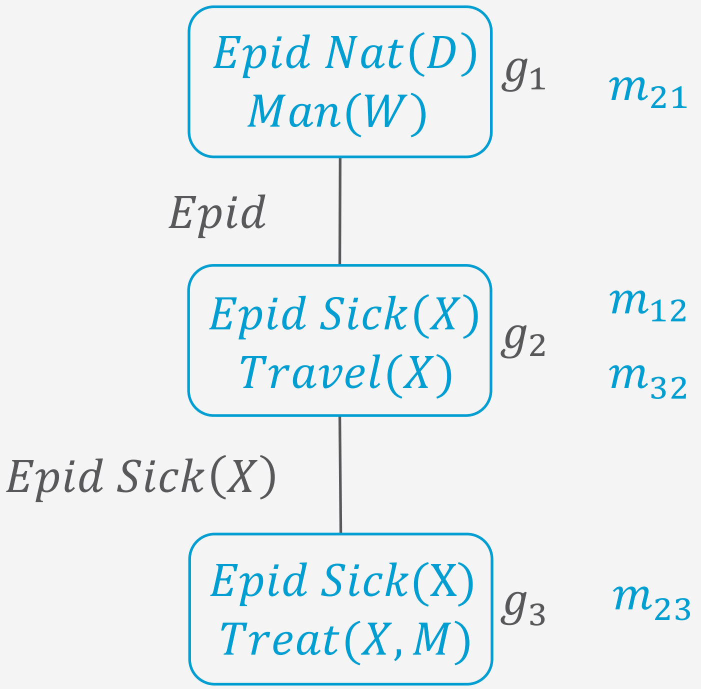
Lauritzen and Spiegelhalter (1988), Shafer and Shenoy (1989), Jensen et al. (1990), B and Möller (2016)

Adaptive Inference in Cluster Trees

Algorithms for More Resilient Inference

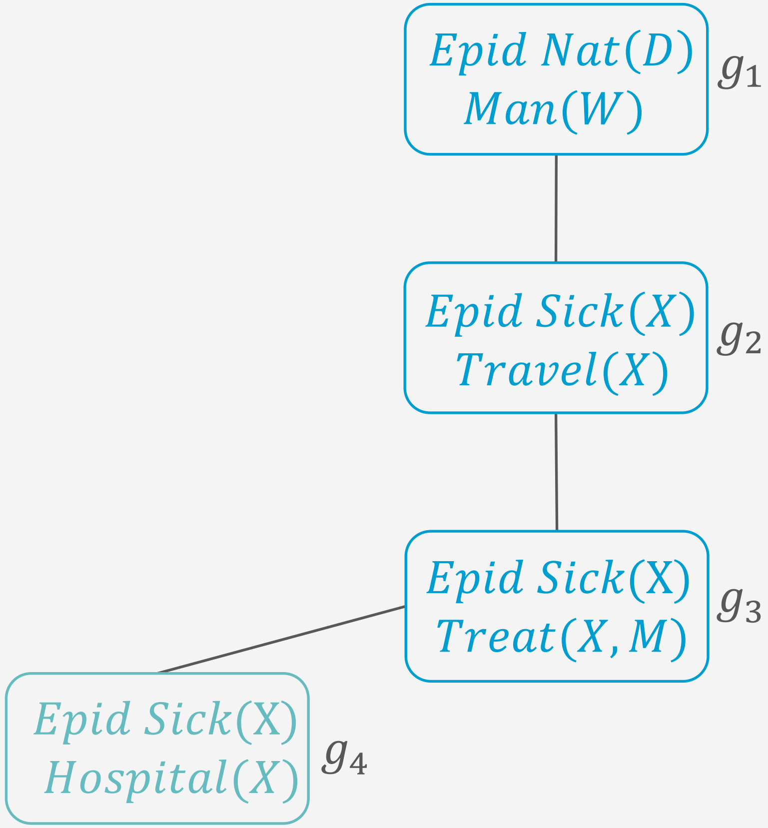
Adaptive Inference

- After changes in queries, evidence, model: Avoid starting from scratch to fast reach the point of answering queries again
  → adaptive inference
- Small, local model changes may preserve much of tree
  - If only local changes, up to half of messages still valid
    - Only resend messages
      if local model or incoming information changed



Adaptive Inference: Changes

- Queries: no change
- Evidence: changes local models
  - New observations incoming
- Model
  - Potentials: changes local models
  - Domain sizes: changes local models
    - Nice property of relational models: No effect on model structure!
      - E.g., more people in $\mathrm{dom}(X)$
      - Propositional models: number of variables changes, which changes the tree structure
  - Parfactors (addition, deletion): changes tree structure

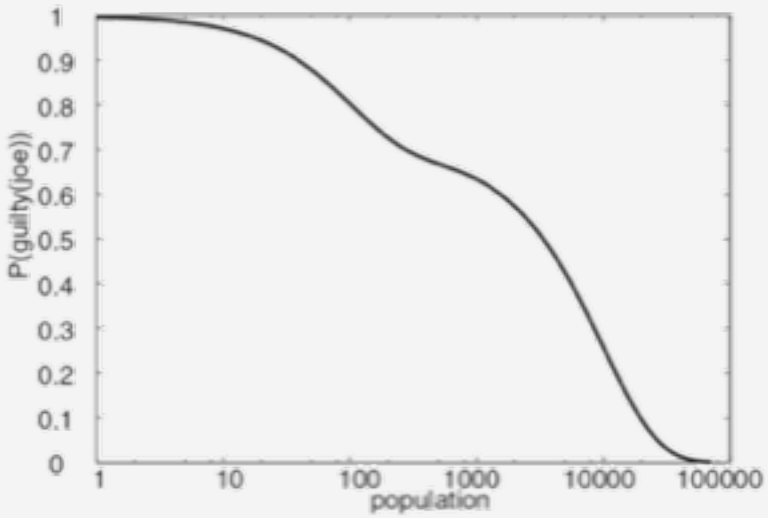


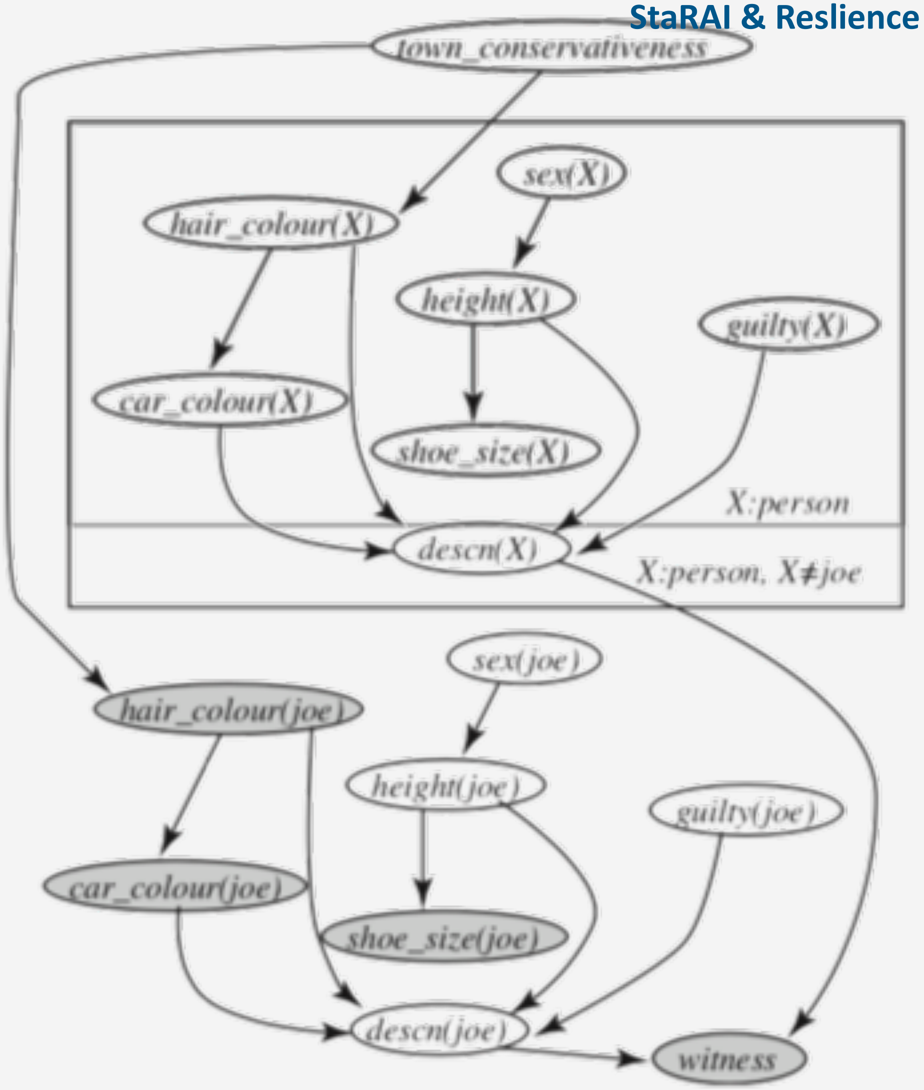
Changing Domains

Algorithms for More Resilient Inference

Changing Domains

- Keep semantics as before
  - Assume that parfactors accurately describe world
- Posterior probabilities change depending on domain sizes
  - Example by Poole (2003)

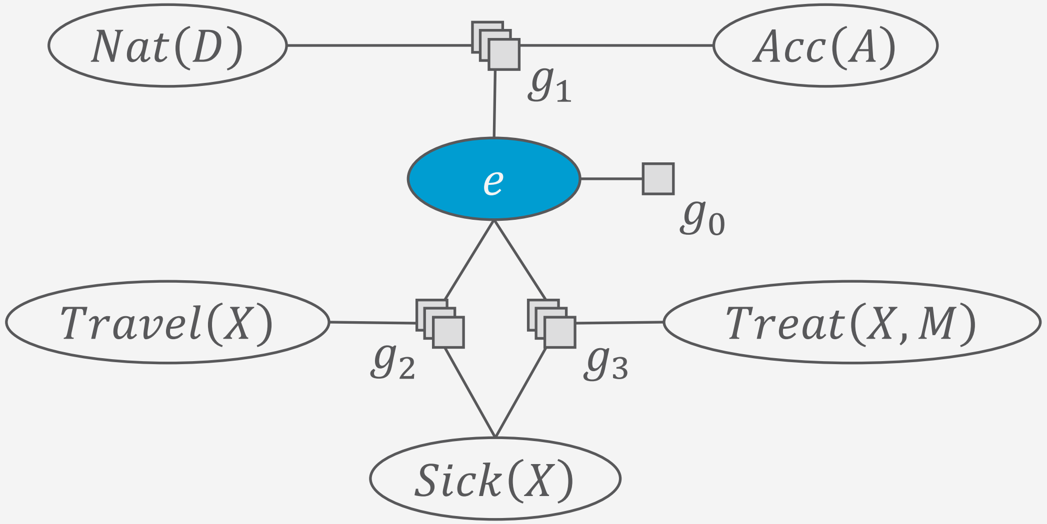




… Without Effects

- (Conditional) Independence
  PRVs, containing logical variables $\boldsymbol{X}$, that are (conditionally) independent from query terms
  → domains of $\boldsymbol{X}$ have no influence on query results
- E.g., given $Epid = e$,
  - $\mathcal{D}(D)$ and $\mathcal{D}(W)$ do not matter for queries regarding $Travel$, $Sick$, and $Treat$
  - $\mathcal{D}(X)$ and $\mathcal{D}(M)$ do not matter for queries regarding Nat and Man

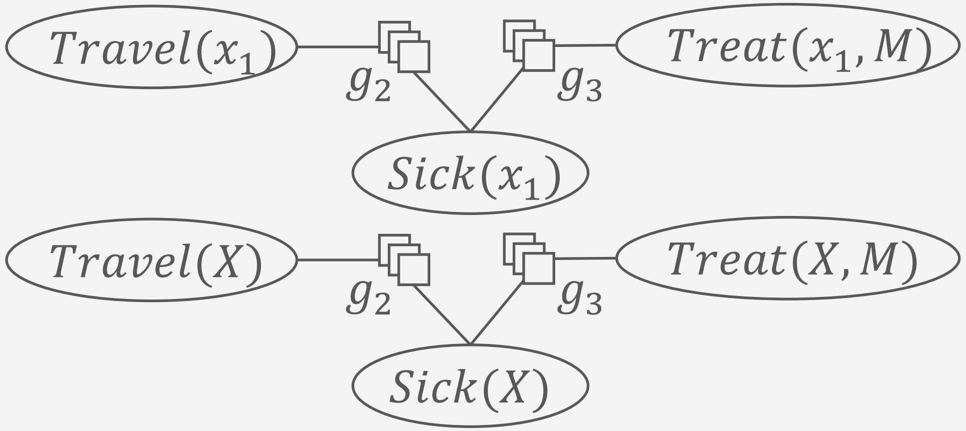
→ Partly invariant under increasing domain sizes



… Without Effects

- A simple case of so-called projectivity
  After shattering, query terms are independent of model parts containing logical variables $\boldsymbol{X}$ → domains of $\boldsymbol{X}$ have no influence on query results
  - Depends on model structure
  - More by Jaeger and Schulte (2018)
- E.g., $P(Sick(x_1))$
  - $\mathcal{D}(X) = \{x_1, \dots, x_n\}$
  - After shattering:
    - $\mathcal{D}(X) = \{x_2, \dots, x_n\}$
    - Upper part independent from lower part; $\mathcal{D}(X)$ irrelevant

→ Partly invariant under increasing domain sizes



Growing Domain Sizes

- Let domain size n grow
  - With grounding semantics, posteriors change
    - Can lead to extreme behaviour in the posteriors
- Example: $Epid$ gets more and more neighbours with n rising

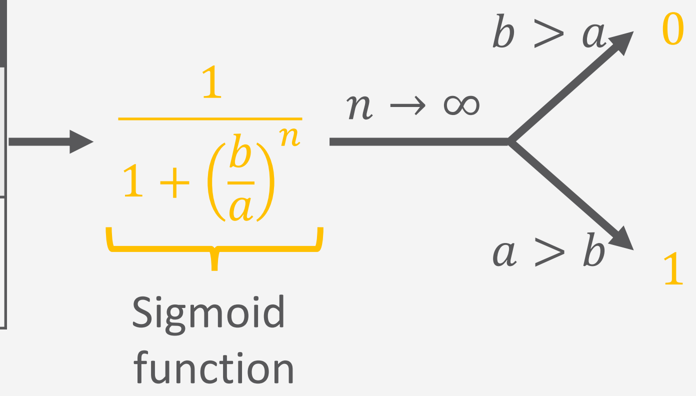
$$P(Epid) \propto \left(\sum_{s \in r(Sick(X))} g(Epid, Sick(x) = s) \right)^n$$

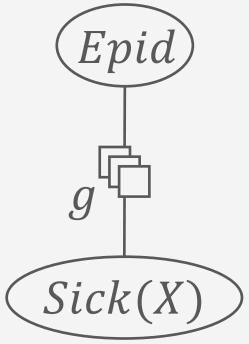
$$= \left(g'(Epid)\right)^n = g''(Epid) = g^\alpha(Epid)$$

$Epid$	g'
$false$	a
$true$	b

$Epid$	g''
$false$	a^n
$true$	b^n

$Epid$	g^α
$false$	$\frac{a^n}{a^n + b^n}$
$true$	$\frac{b^n}{a^n + b^n}$





Growing Domain Sizes

- How to avoid extreme behaviour?

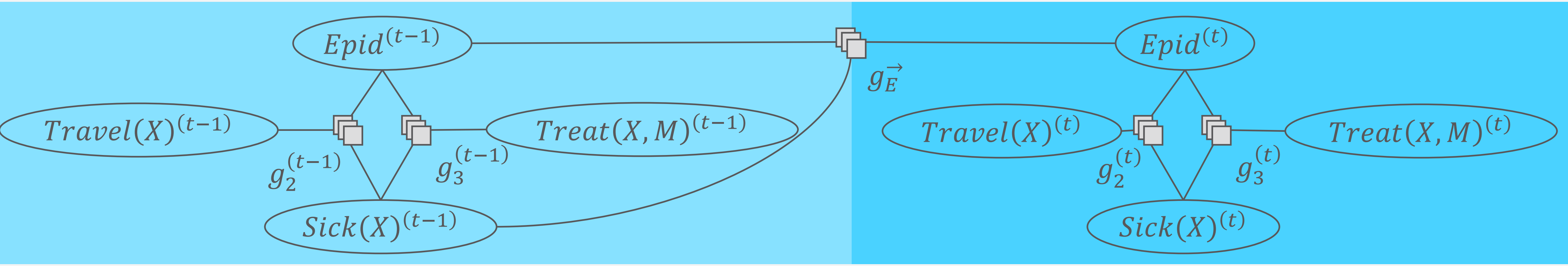
→ Adapt values in model dependent on domain size

- Approach for MLNs: Domain-size aware MLNs
  - Assume predicates $P_1, \dots, P_m$ occur in a first-order formula F
    - Count number of connections c_j for each predicate P_j given *new* domains
    - Build a connection vector $[c_1, \dots, c_m]$
    - Choose $\max_{c_i}[c_1, \dots, c_m]$ as scaling-down factor
    - Instead of $\max$, other functions possible
    - Works best if the values in $[c_1, \dots, c_m]$ do not vary that much
  - Given an MLN with a set of formulas F_i with weights w_i
    - Rescale each w_i with scaling-down factor s_i computed for F_i as $\frac{w_i}{s_i}$
- Analogous approach possible for parfactors

Keeping Inference Going over Time

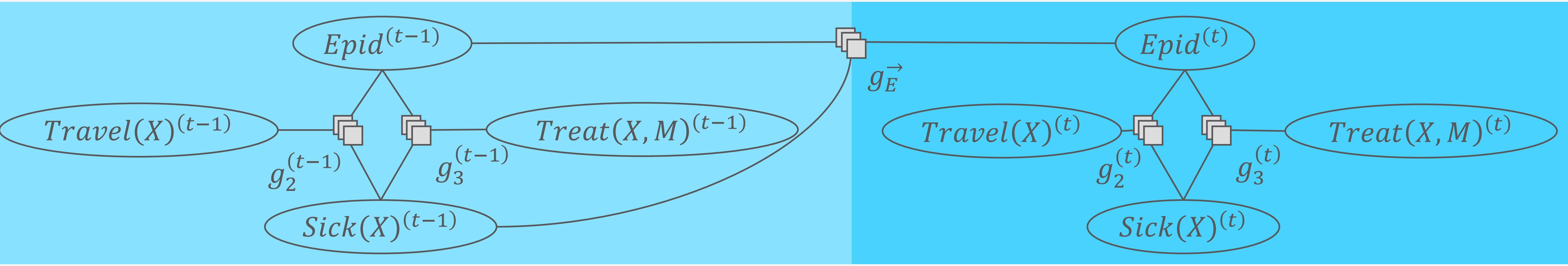
Algorithms for More Resilient Inference

Dynamic Probabilistic Relational Models & Temporal Queries



- Marginal distribution queries: $P\left(A_\pi^i \middle| E_{0:t}\right)$
  - Hindsight: $\pi < t$ (Was there an epidemic $t - \pi$ days ago?)
  - Filtering: $\pi = t$ (Is there currently an epidemic?)
  - Prediction: $\pi > t$ (Will there be an epidemic in $\pi - t$ days?)
- Assignment queries on temporal sequence

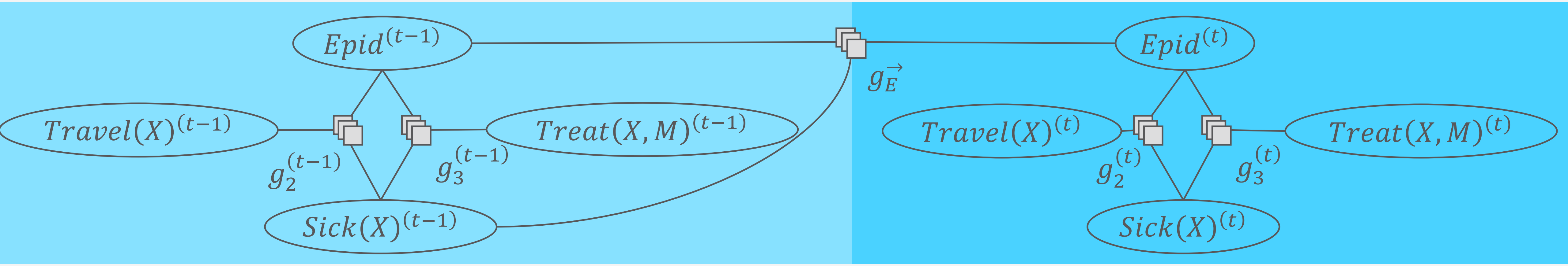
Reasoning over Time: Interfaces



- Main idea: Use temporal conditional independences for temporal QA on smaller model
  - Normally only a subset of random variables influence next time step → interface variables
  - State description of interface variables from time slice $t-1$ suffice to perform inference on time slice t

→ Makes present independent from past / future

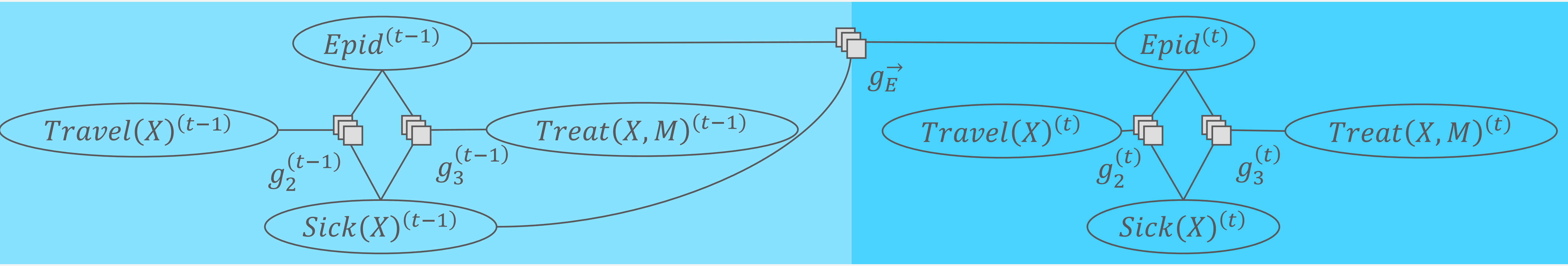
Reasoning over Time: Interfaces



- Procedure
  - Build a helper structure of clusters ((first-order) junction tree)
  - Proceed forward one time step at a time (forward message), using the same structure (vanilla junction tree)
  - Answer queries on the structure in each time step

Algorithms:
- Propositional: Interface Algorithm (Murphy, 2002)
- Lifted: Lifted Dynamic Junction Tree Algorithm (Gehrke et al, 2018)

Taming Reasoning

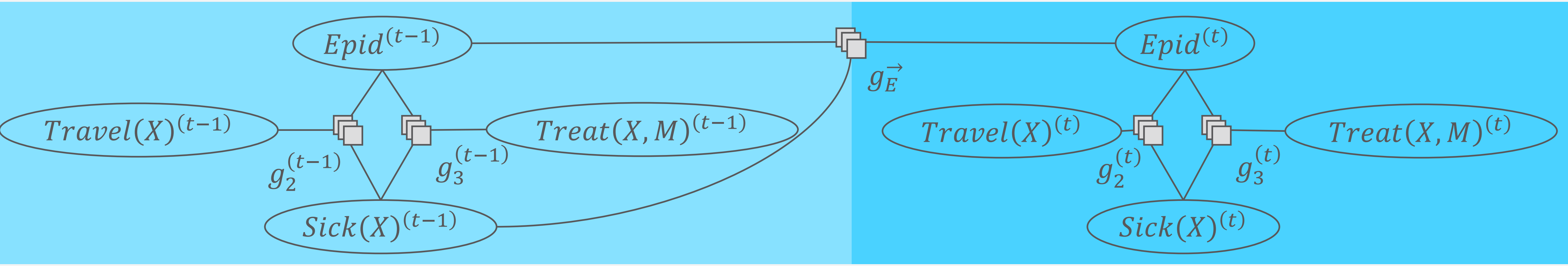


- Evidence can ground a model over time
- Non-symmetric evidence
  - Observe evidence for some instances in one time step
  - Observe evidence for a subset of these instances in another time step
  - Split the logical variable slowly over time

Forward message carries over splits, leading to slowly grounding a model over time

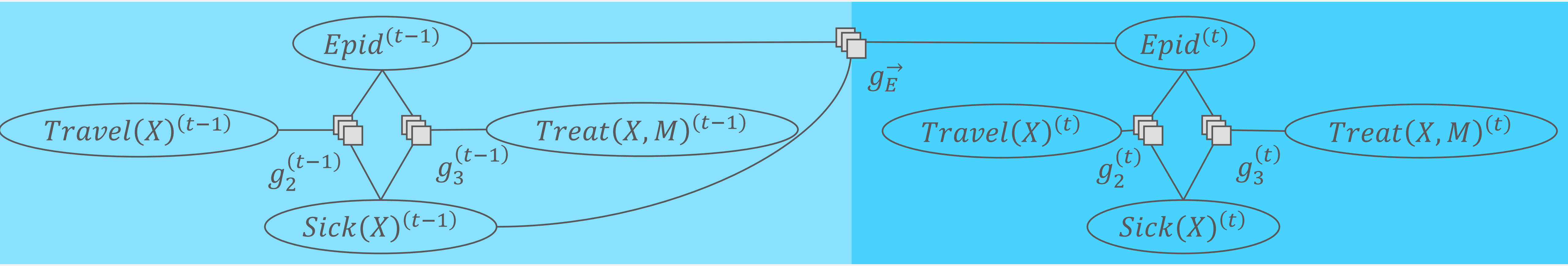
Gehrke et al. (2020)

Undoing Splits



- Need to undo splits to
  keep reasoning polynomial w.r.t. domain sizes
- Where can splits be undone efficiently?
  - When moving from one time step to the next, i.e., in the forward message
- How to undo splits?
  - Find approximate symmetries
  - Merge based on groundings
- Is it reasonable to undo splits?
  - Effect of slight differences in evidence?
  - Impact of evidence vs. temporal model

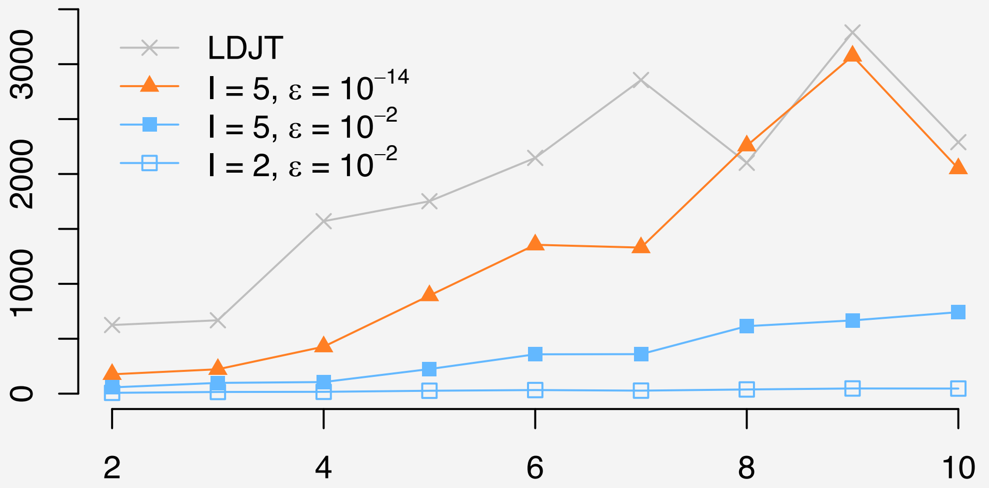
Is It Reasonable to Undo Splits?



- Approximate forward message
- For each time step the temporal behaviour is multiplied on the forward message
- Indefinitely bounded error due to temporal behaviour

Results

- DBSCAN for Clustering
- ANOVA for checking fitness of clusters
- Right: runtimes
- Below: approximation error



π	Max	Min	Average
0	0.0001537746121	0.0000000001720	0.0000191206488
2	0.0000000851654	0.0000000000001	0.0000000111949
4	0.0000000000478	0	0.0000000000068

Gehrke et al. (2020)

Agenda

- Probabilistic Relational Models (PRMs)
  - Application example
  - Semantics
  - Query answering / basic inference
- Algorithms for More Resilient Inference
  - Cluster trees for efficient multi-query inference
  - Adaptive inference in cluster trees
  - Changing domains
  - Keeping inference going over time
- Summary

The Finish Line

- PRMs as a compact encoding of a full joint
  - Exploit symmetries
- Lifted inference
  - Use information about regular structures to speed up inference
- More resilient inference
  - Multi-query answering using junction tree as helper structure
  - Adaptive inference to get to the point of answering queries again fast
  - Changing domains with minimal effect
  - Keeping inference going over time

What else is there to do? – Oh, so much…
- Approximating symmetries
- Generalising lifting operators
- More robust learning algorithms
- Privacy
- Ethical behaviour
- Explainability
- …

Thank you!

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Ordered topic-wise and then alphabetically

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