LIFTED DIVISION FOR LIFTED HUGIN BELIEF PROPAGATION

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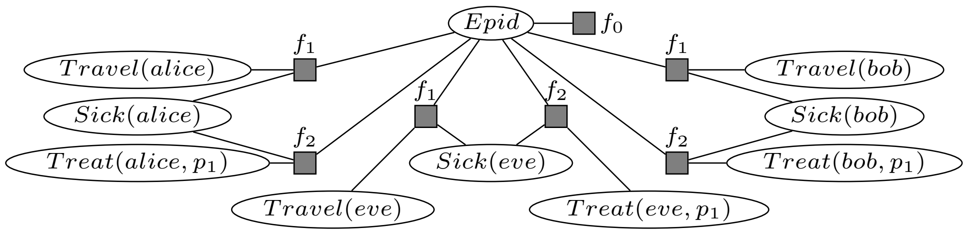
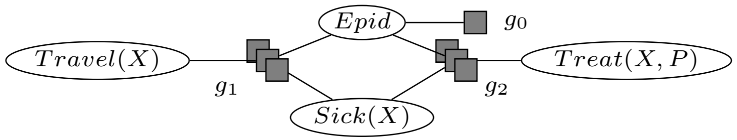
PROPOSITIONAL & LIFTED INFERENCE

- Probabilistic graphical models
  - Propositional, e.g., factor graph
  - Lifted, e.g., parfactor graph
    - Logical variables to encode symmetries in factor graphs
- Semantics: full joint probability distribution
- Inference task: query answering
  - Set of propositional random variables
    - Lifting: grounded random variables
  - Examples: $P(Epid), P\big(Sick(eve)\big)$

- Factor graph



- Parfactor graph



- Equivalent with $X \in \{alice, eve, bob\}, P \in \{p_1\}$

QUERY ANSWERING: (LIFTED) VARIABLE ELIMINATION

- Eliminate all non-query random variables
  - Absorb evidence
  - Sum-out random variables
  - Combine factors (multiply)
  - Lifted operators to handle lifting preconditions
- Division only propositional
  - Remove a factor from a larger factor
  - Part of message passing algorithm called Hugin

Lifted division?
Lifted Hugin?

Operator	Propositional	Lifted
absorb	✓	✓
sum-out	✓	✓
multiply	✓	✓
		Additional operators: • Count-convert • Split • Expand • Count-normalise • Ground
divide	✓	✗

CONTRIBUTIONS

Lifted Division

- Closes another gap in the suite of lifted operators
- In paper
  - Proofs for
    - Correctness
    - Complexity

Operator 1 DIVIDE

Input:

(1) $g_1 = \forall \boldsymbol{x}_1 \in \pi_{\boldsymbol{X}_1}(C_1) : \ \phi_1(\mathcal{A}_1)_{|C_1}, g_1 \in G$

(2) $g_2 = \forall \boldsymbol{x}_2 \in \pi_{\boldsymbol{X}_2}(C_2) : \ \phi_2(\mathcal{A}_2)_{|C_2}, g_2 \notin G$

(3) $\theta = \{\boldsymbol{Z}_1 \rightarrow \boldsymbol{Z}_2\}$, an alignment, $\boldsymbol{Z}_i \subseteq lv(\mathcal{A}_i)$ for $i = 1,2$

Preconditions:

(1) $\mathcal{A}_2 \subseteq \mathcal{A}_1$

(2) $m = \text{NCOUNT}_{lv(\mathcal{A}_1)\setminus \boldsymbol{Z}_1 | \boldsymbol{Z}_1}\left(\pi_{\boldsymbol{X}_1}(C_1)\right)$

(3) $\forall \boldsymbol{a}_2 \in val(\mathcal{A}_2) : \phi_2(\boldsymbol{a}_2) \neq 0$ or $g_1 = g_2 \prod_i g_i$

Output: $\forall \boldsymbol{x} \in \pi_{\boldsymbol{X}_1\theta \cup \boldsymbol{X}_2}(C) : \phi(\mathcal{A})_{|C}$ such that

$\mathcal{A} = \mathcal{A}_1\theta$

$C = C_1\theta \bowtie C_2$

for each $\boldsymbol{a} \in val(\mathcal{A})$,

$\boldsymbol{a}_1 = \pi_{\mathcal{A}_1\theta}(\{\boldsymbol{a}\})$

$\boldsymbol{a}_2 = \pi_{\mathcal{A}_2}(\{\boldsymbol{a}\})$

$$\phi(\boldsymbol{a}) = \begin{cases} \phi_1(\boldsymbol{a}_1)/\phi_2^{1/m}(\boldsymbol{a}_2) & \text{if } \phi_2(\boldsymbol{a}_2) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Postcondition:

$G \equiv G \setminus \{g_1\} \cup \{\text{DIVIDE}(g_1, g_2, \theta)\} \cup \{g_2\}$

Lifted Hugin belief propagation

- Closes another gap between propositional and lifted inference algorithms
- In paper
  - Proof for complexity → independent of degree
  - Empirical study
    - Advantage lifted Hugin vs. propositional Hugin
    - Lifted Hugin vs. lifted Shafer-Shenoy for graphs with high / low degree