# Restricting the Maximum Number of Actions for Decision Support under Uncertainty

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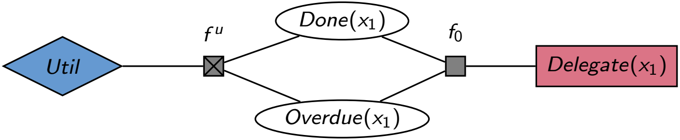
September 19, 2020

# Decision Making in Probabilistic Graphical Models

Decision Factor Graph $F = \{f_i\}_{i=1}^n$ with Utilities and Actions



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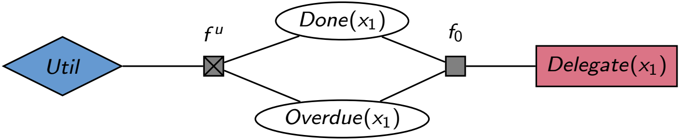


Maximum expected utility (MEU) query
given evidence $\mathbf{e}$ over all possible action assignments $\mathbf{a}$

$$MEU(F, \mathbf{e}) = (\mathbf{a}^*, EU(F, \mathbf{e}, \mathbf{a}^*)) \qquad a^* = \arg\max_{\mathbf{a}} EU(F, \mathbf{e}, \mathbf{a})$$

$EU$ = expected utility of $\mathbf{a}$ in $F$ given $\mathbf{e}$

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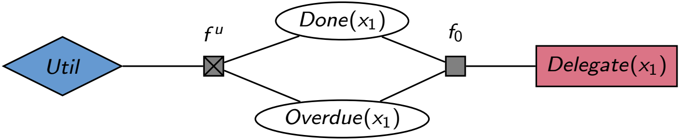


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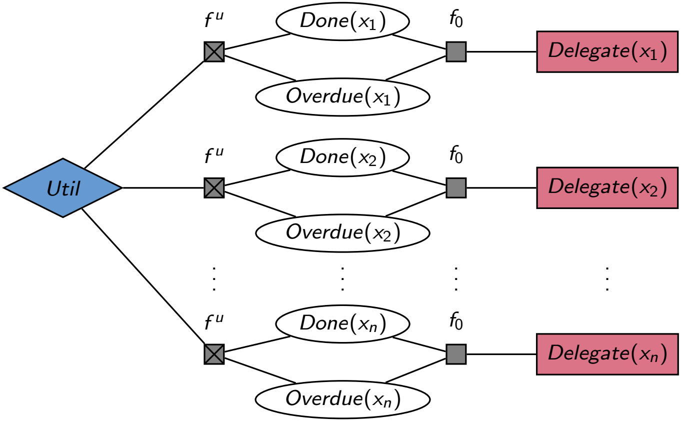
**Exponential in the number of actions!**

# Relational Domains

## Indistinguishable Constants




With $n = 100$: $2^{100}$ possible assignments

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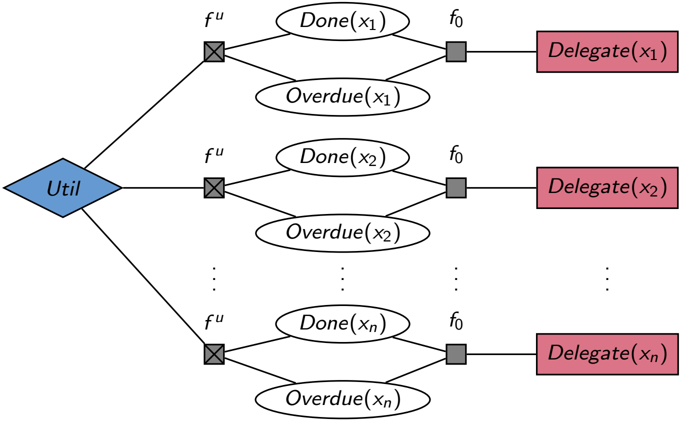
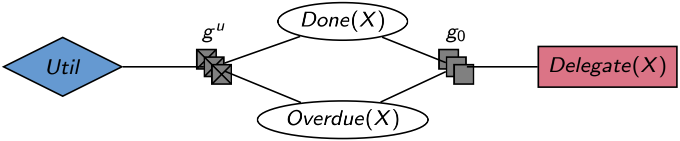



With $n = 100$: $2^{100}$ possible assignments

**Treat identically until evidence makes them distinguishable**

# Groups in Probabilistic Relational Models

Decision Parfactor Graph $G = \{g_i\}_{i=1}^n$ with *Parameterised* Utilities and Actions




Evidence **e** makes constants distinguishable, e.g., with $|dom(X)| = 100$

$$Overdue(X_1) = true, dom(X_1) = \{x_1, x_2, \dots, x_{10}\}$$

$$Overdue(X_2) = false, dom(X_2) = \{x_{11}, x_{12}, \dots, x_{20}\}$$

$$Overdue(X_3) \text{ N/A}, dom(X_3) = \{x_{21}, x_{12}, \dots, x_{100}\}$$

**Constants still indistinguishable within group**

# Decision Making in Probabilistic Relational Models

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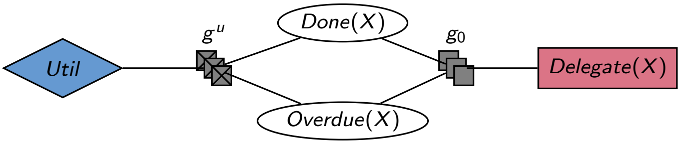


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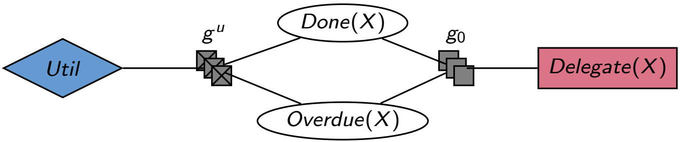


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**Exponential in the number of groups in domains!**

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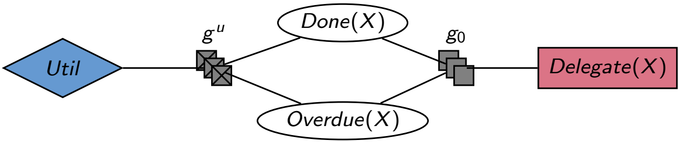


**Instead of $2^{100}$ possible assignments**

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$$Overdue(X_3)\ \text{N/A}, dom(X_3) = \{x_{21}, x_{12}, \dots, x_{100}\}$$

**$\rightarrow 3$ groups: $2^3$ possible assignments**

# Conference Contribution

## Problem

- Constraints on resources may render action assignments invalid.
- Error-prone to check each assignment with complex restrictions.

**Iterating over too many assignments or not enough.**

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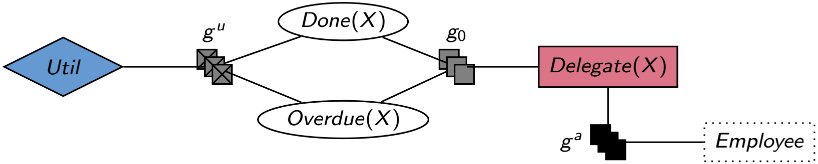
## Approach: ReLiA

- Build a graph out of the resource restrictions
- Solve a max-flow problem in the graph
- Return all max-flows with maxed out capacities

**Iterate only over the necessary assignments.**

# ReLiA: Restricting Lifted Assignments

## Resources and Action Parfactors




Action parfactors indicate

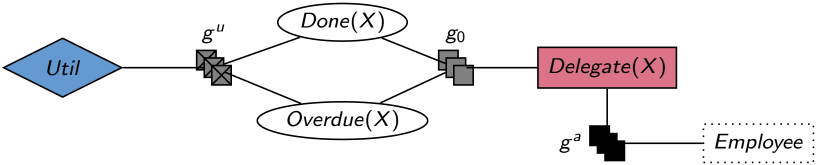
- how many resource units are needed per individual action and
- how many times an action can be assigned.

E.g., 15 employees that carry out tasks

Employees needed for one task: 1    Times task executable: 20

# ReLiA: Restricting Lifted Assignments

Resource Graph and Max-flow Problem

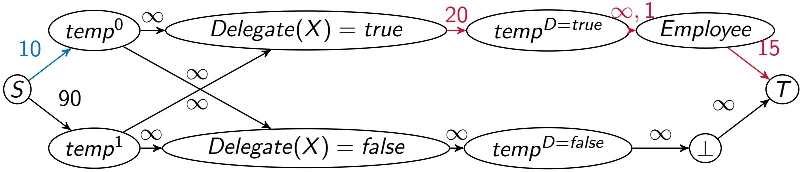



15 employees

Employees needed for one task: 1

Times task executable: 20

$Overdue(X_1) = true, dom(X_1) = \{x_1, x_2, \ldots, x_{10}\}$, rest N/A



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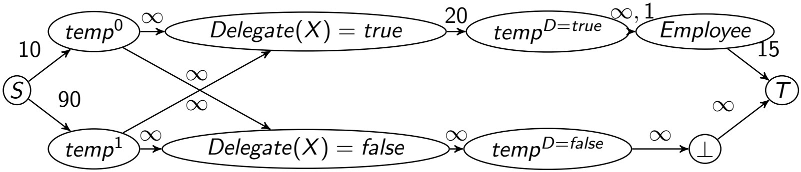


Max-flow problem provides how many actions are executable, need all flows with max-flow when sending max capacities, e.g.:

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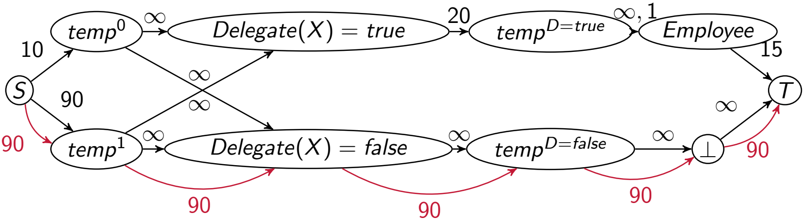


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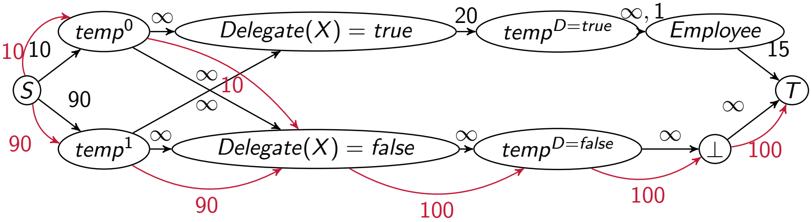


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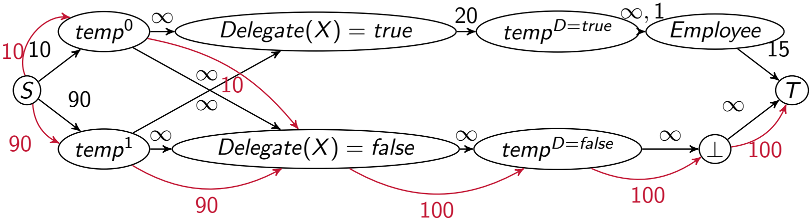


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$\forall x \in dom(X_1, X_2) : Delegate(x) = false$

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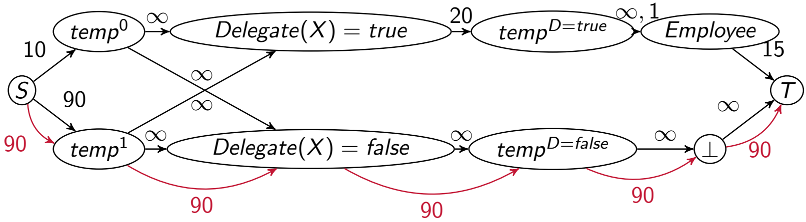


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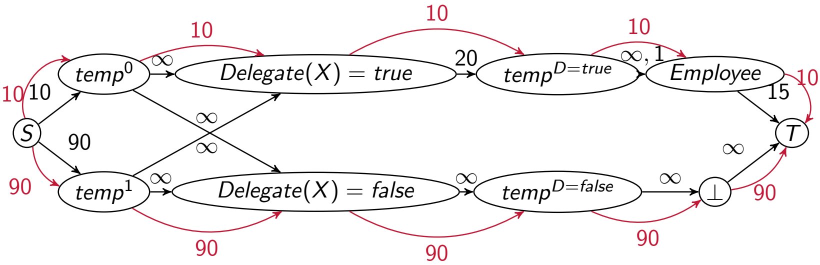


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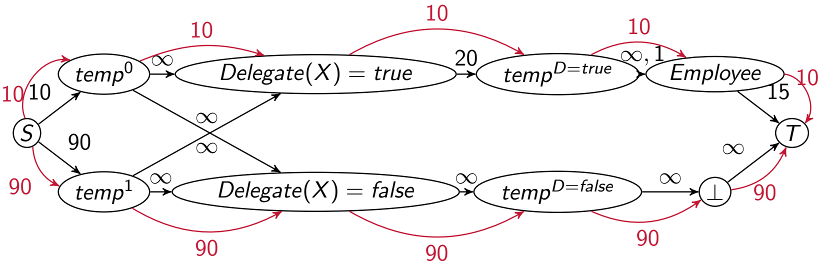


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$$\forall x \in dom(X_1) : D(x) = true, \forall x \in dom(X_2) : D(x) = false$$

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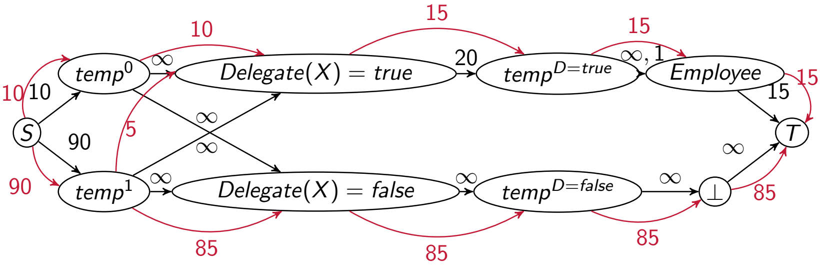


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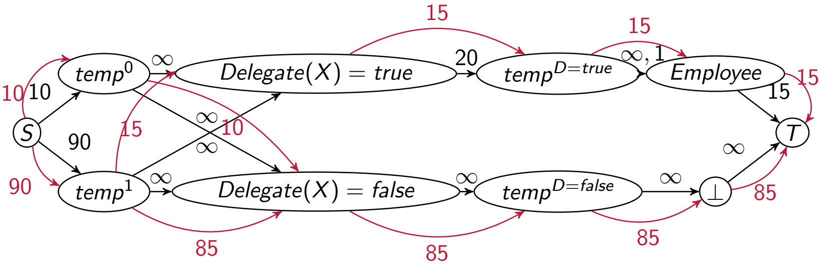


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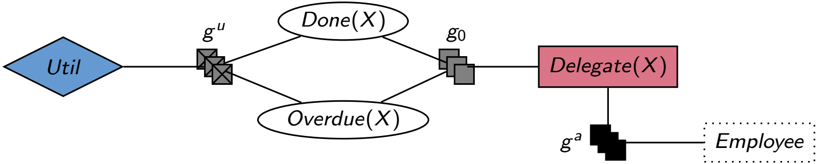


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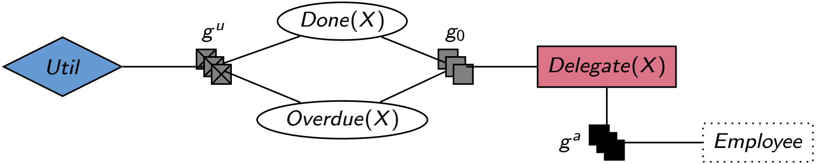


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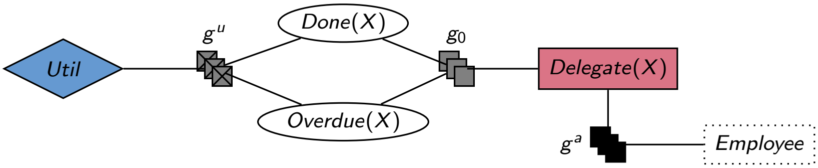


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