Exact Lifted Inference on Relational Models

Statistical Relational AI

Tutorial at BTW 2019

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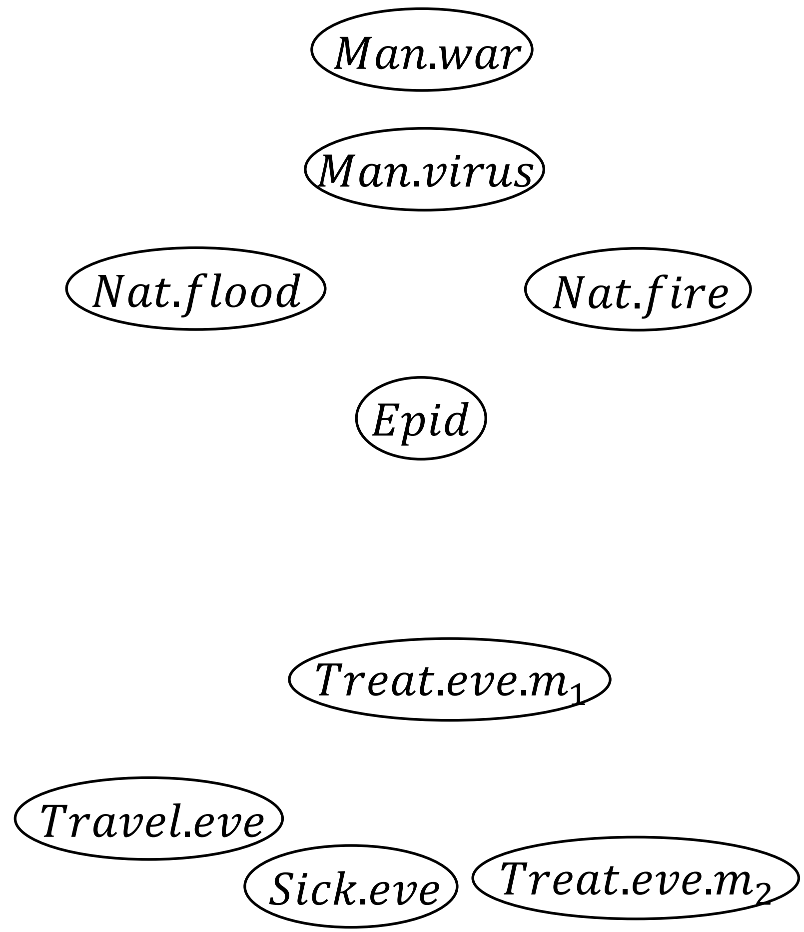
Propositional Models: Worlds

- Characterise world by random variables
  - E.g., $Epid$
  - Possible values (range): $r(Epid) = \{true, false\}$
- 1 world = specific values for variables
- Probability per world
- Joint probability distribution P_F over all possible worlds



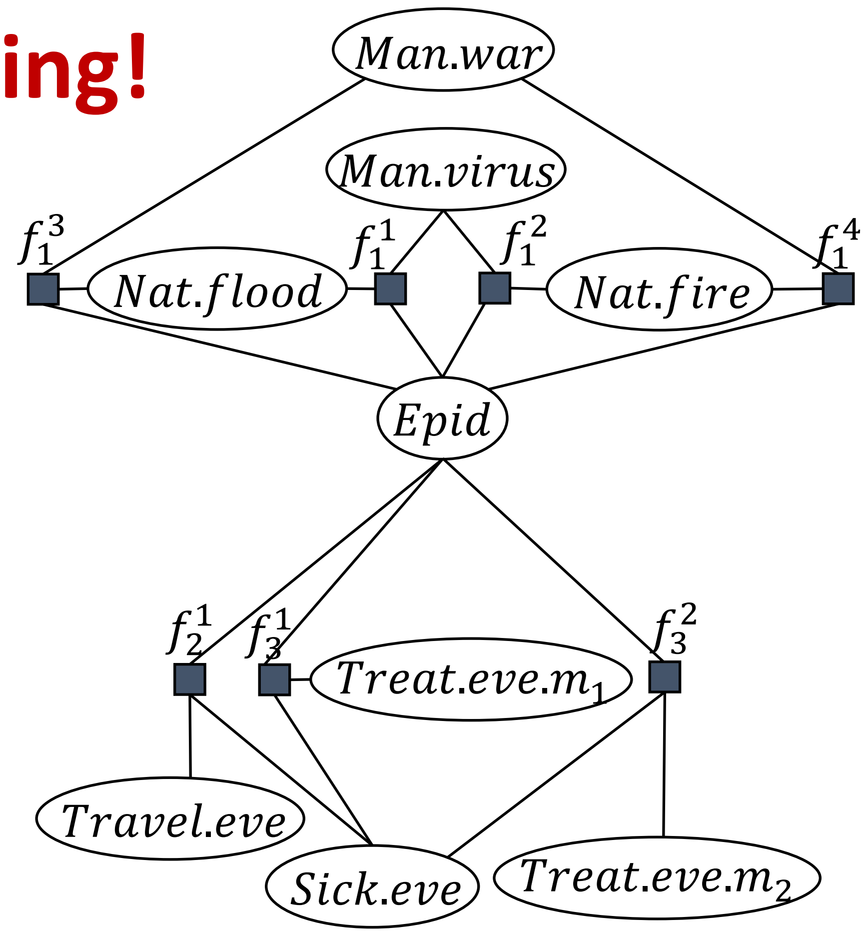
$\mathbf{2^9 = 512}$ **possible worlds**

Propositional Models: Factors

- Full joint P_F as product of factors **Sparse encoding!**
  - Model F
  - $rv(F), rv(f)$ random variables in F, f

$$P_F = \frac{1}{Z} \prod_{i=1}^{n} f_i$$

$$Z = \sum_{v \in r(rv(F))} \prod_{i=1}^{n} f_i(\pi_{rv(f_i)}(v))$$



$7 \cdot 2^3 = 56$ entries

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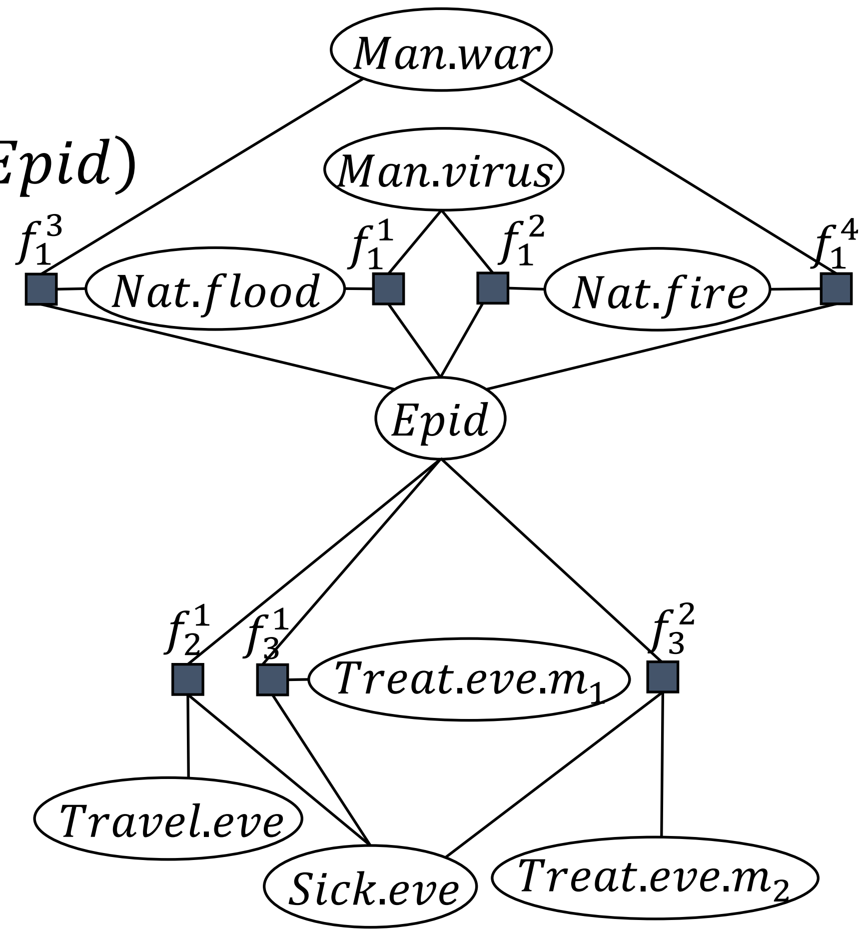
Propositional Models: Factors

- Factors
  - Potentials
  - $f_2^1(Travel.eve, Sick.eve, Epid)$

$Travel.eve$	$Epid$	$Sick.eve$	f_2^1
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

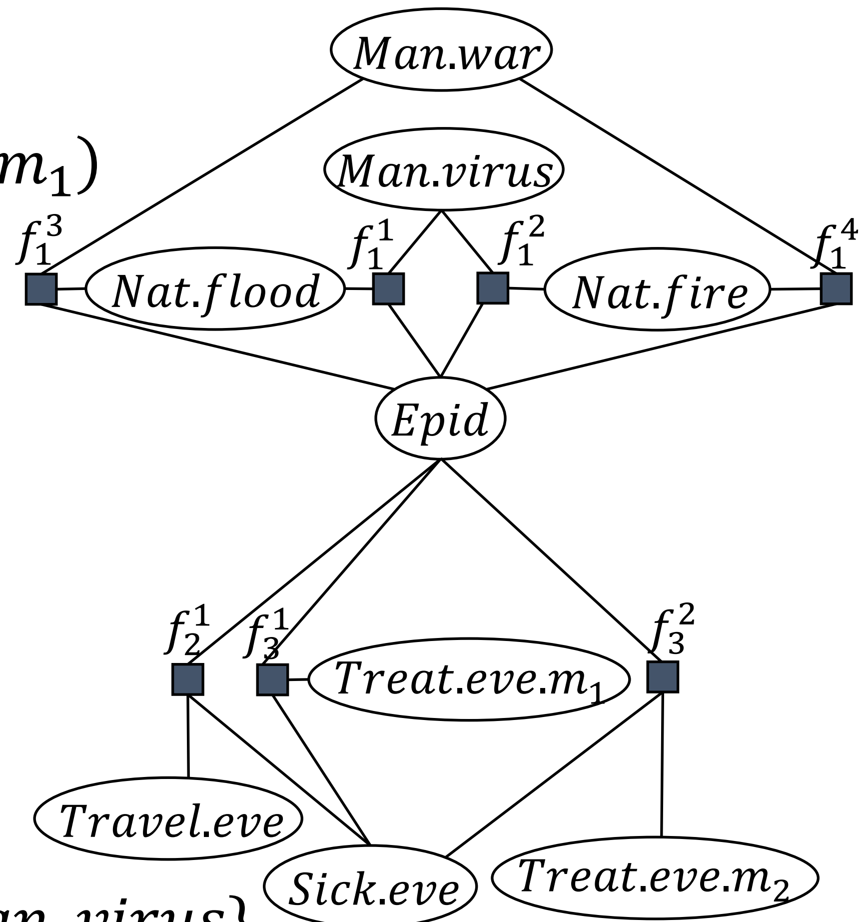


Query Answering (QA): Queries

- Marginal distribution
  - $P(Sick.eve)$
  - $P(Travel.eve, Treat.eve.m_1)$
- Conditional distribution
  - $P(Sick.eve|Epid)$
  - $P(Epid|Sick.eve = true)$
- Most probable assignment

$$\underset{v \in r(R)}{\operatorname{argmax}} \sum_{v \in r(R)} P_F(v), R \subseteq rv(F)$$

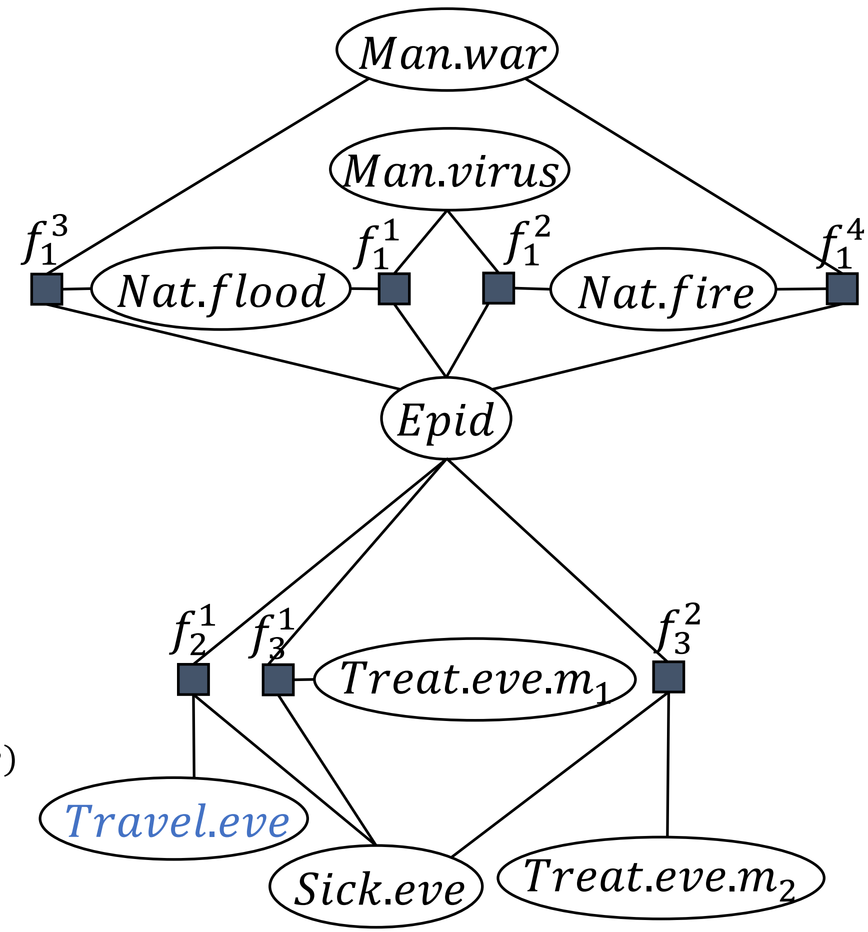
  - MPE: $R = rv(F)$
  - MAP: $R = \{Travel.eve, Man.virus\}$



QA: Variable Elimination (VE)

Pearl (1988), Zhang and Poole (1994), Dechter (1999)

- Eliminate all variables not appearing in query
  - Through summing out
- E.g., marginal
  - $P(Travel.eve)$



$$P(Travel.eve) \propto \sum_{e \in r(Epid)} \sum_{s \in r(Sick.eve)} \sum_{m_1 \in r(Treat.eve.m_1)} \sum_{m_2 \in r(Treat.eve.m_2)} \sum_{o \in r(Nat.flood)} \sum_{i \in r(Nat.fire)} \sum_{w \in r(Man.war)} \sum_{v \in r(Man.virus)} P_F$$

QA: Variable Elimination (VE)

Zhang and Poole (1994)

$$P(Travel.eve) \propto \sum_{e \in r(Epid)} \sum_{s \in r(Sick.eve)} f_2^1(Travel.eve, e, s) \sum_{m_1 \in r(Treat.eve.m_1)} f_3^1(e, s, m_1)$$

$$\sum_{m_2 \in r(Treat.eve.m_2)} f_3^2(e, s, m_2)$$

$$f_1'(e) \sum_{o \in r(Nat.flood)} \sum_{i \in r(Nat.fire)} \sum_{w \in r(Man.war)} f_1^1(o, w, e) f_1^2(i, w, e)$$

$$f_1'(o, i, e) \sum_{v \in r(Man.virus)} f_1^3(o, v, e) f_1^4(i, v, e)$$

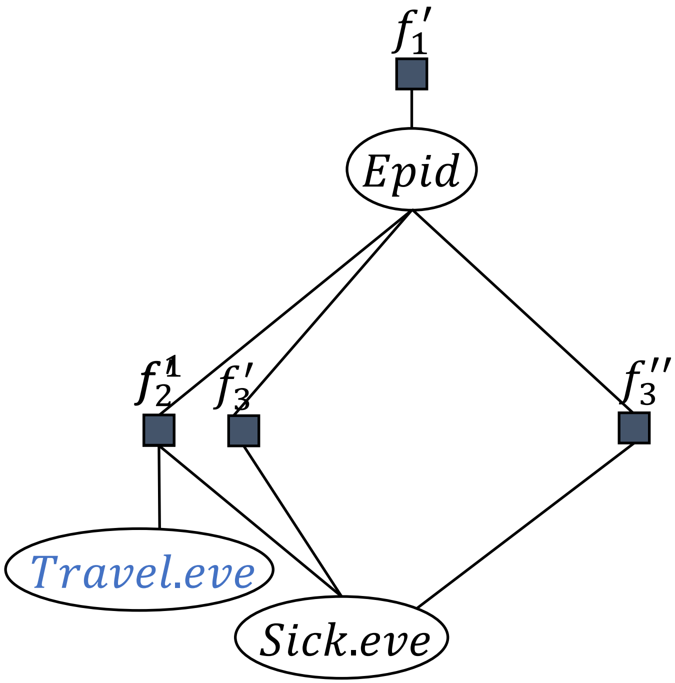
QA: Variable Elimination (VE)

Zhang and Poole (1994)

$$P(Travel.eve) \propto \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f_2^1(Travel.eve, e, s) f_3'(e, s) f_3''(e, s)$$

$$= \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f'(Travel.eve, e, s)$$

$Travel.eve$	$Epid$	$Sick.eve$	f'
false	*false*	*false*	50
false	*false*	*true*	10
false	*true*	*false*	34
false	*true*	*true*	16
true	*false*	*false*	54
true	*false*	*true*	36
true	*true*	*false*	12
true	*true*	*true*	69



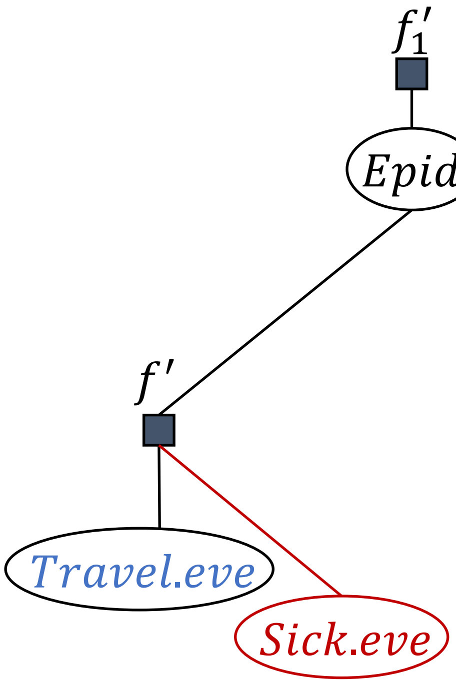
QA: Variable Elimination (VE)

Zhang and Poole (1994)

$$P(\textcolor{blue}{Travel.eve}) \propto \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f_2^1(\textcolor{blue}{Travel.eve}, e, s) f_3'(e,s) f_3''(e,s)$$

$$= \sum_{e \in r(Epid)} f_1'(e) \textcolor{red}{\sum_{s \in r(Sick.eve)}} f'(\textcolor{blue}{Travel.eve}, e, \textcolor{red}{s})$$

$Travel.eve$	$Epid$	~~$Sick.eve$~~	f'	Σ
false	*false*	~~*false*~~	50	**60**
false	*false*	~~*true*~~	10	
false	*true*	~~*false*~~	34	**50**
false	*true*	~~*true*~~	16	
true	*false*	~~*false*~~	54	**90**
true	*false*	~~*true*~~	36	
true	*true*	~~*false*~~	12	**81**
true	*true*	~~*true*~~	69	



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QA: Variable Elimination (VE)

Zhang and Poole (1994)

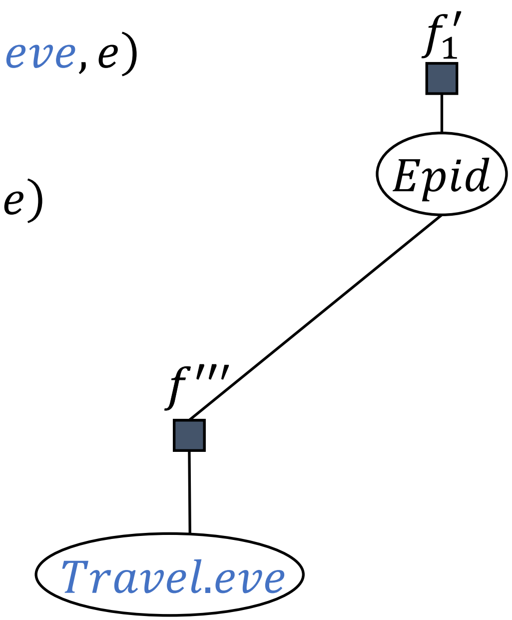
$$P(Travel.eve) \propto \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f_2^1(Travel.eve, e, s) f_3'(e, s) f_3''(e, s)$$

$$= \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f'(Travel.eve, e, s)$$

$$= \sum_{e \in r(Epid)} f_1'(e) f''(Travel.eve, e)$$

$$= \sum_{e \in r(Epid)} f'''(Travel.eve, e)$$

$Travel.eve$	$Epid$	f'''
false	*false*	90
false	*true*	100
true	*false*	135
true	*true*	162



QA: Variable Elimination (VE)

Zhang and Poole (1994)

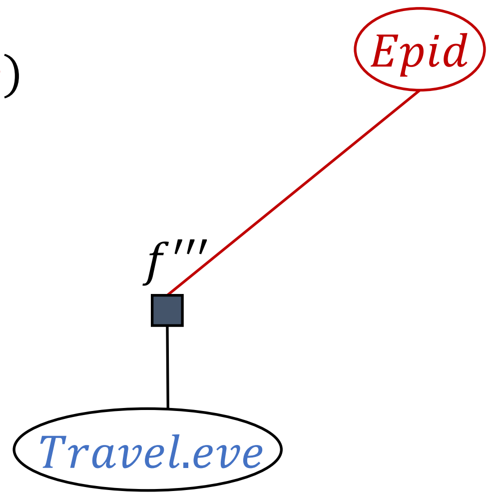
$$P(\color{blue}{Travel.eve}) \propto \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f_2^1(\color{blue}{Travel.eve}, e, s) f_3'(e,s) f_3''(e,s)$$

$$= \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f'(\color{blue}{Travel.eve}, e, s)$$

$$= \sum_{e \in r(Epid)} f_1'(e) f''(\color{blue}{Travel.eve}, e)$$

$$= \color{red}{\sum_{e \in r(Epid)}} f'''(\color{blue}{Travel.eve}, \color{red}{e})$$

$Travel.eve$	$\cancel{Epid}$	f'''	Σ
$false$	$\cancel{false}$	90	190
$false$	$\cancel{true}$	100	
$true$	$\cancel{false}$	135	297
$true$	$\cancel{true}$	162	



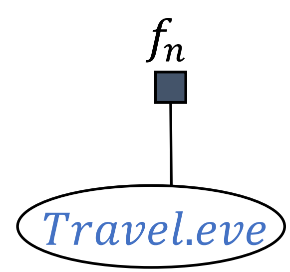
QA: Variable Elimination (VE)

Zhang and Poole (1994)

$$
\begin{aligned}
P(\color{blue}{Travel.eve}) &\propto \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f_2^1(\color{blue}{Travel.eve}, e, s) f_3'(e,s) f_3''(e,s) \\
&= \sum_{e \in r(Epid)} f_1'(e) \sum_{s \in r(Sick.eve)} f'(\color{blue}{Travel.eve}, e, s) \\
&= \sum_{e \in r(Epid)} f_1'(e) f''(\color{blue}{Travel.eve}, e) \\
&= \sum_{e \in r(Epid)} f'''(\color{blue}{Travel.eve}, e) = f(\color{blue}{Travel.eve}) = f_n(\color{blue}{Travel.eve})
\end{aligned}
$$

$Travel.eve$	f
false	190
true	297

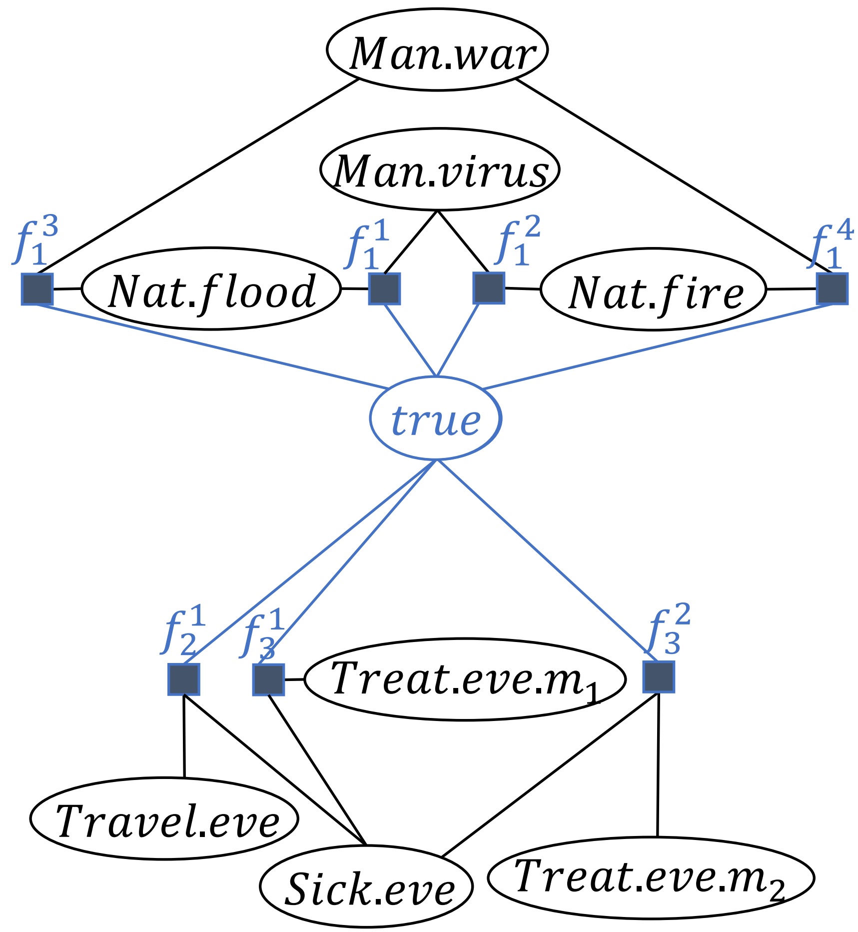
$Travel.eve$	f_n
false	0.39
true	0.61



QA: Variable Elimination (VE)

- Eliminate all variables not appearing in query
  - Through summing out
- E.g., conditional
  - $P(Travel.eve | Epid = true)$
  - Absorb $Epid = true$ in all factors
  - Sum out remaining variables

Zhang and Poole (1994)

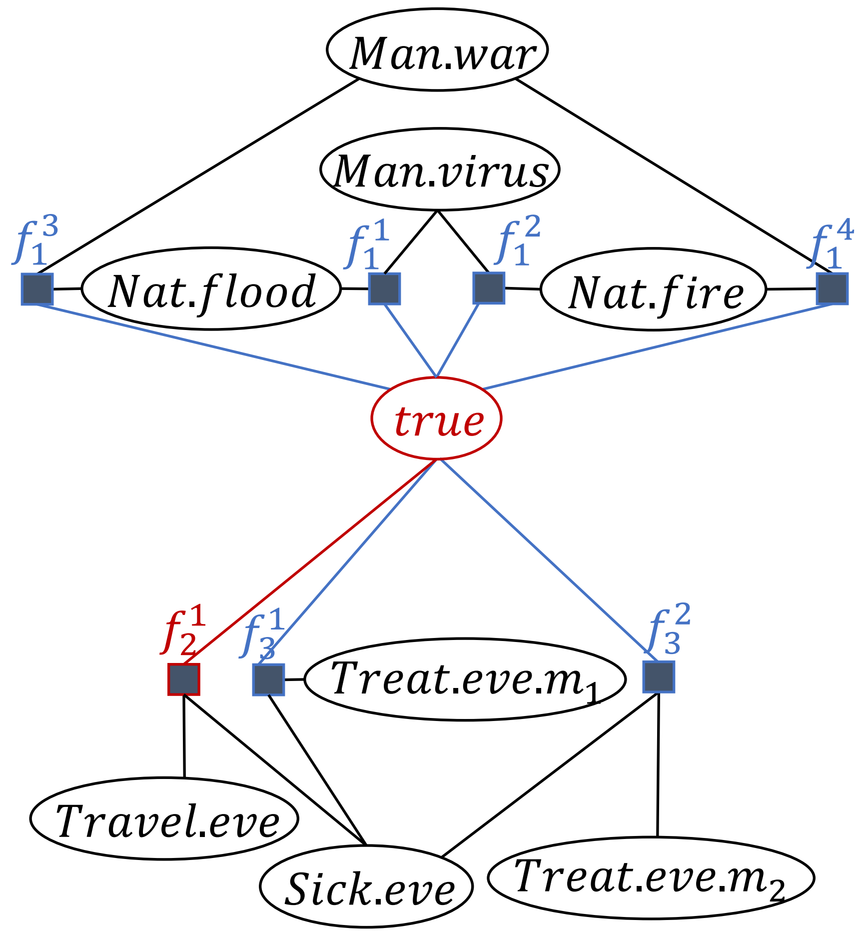


QA: Evidence Absorption

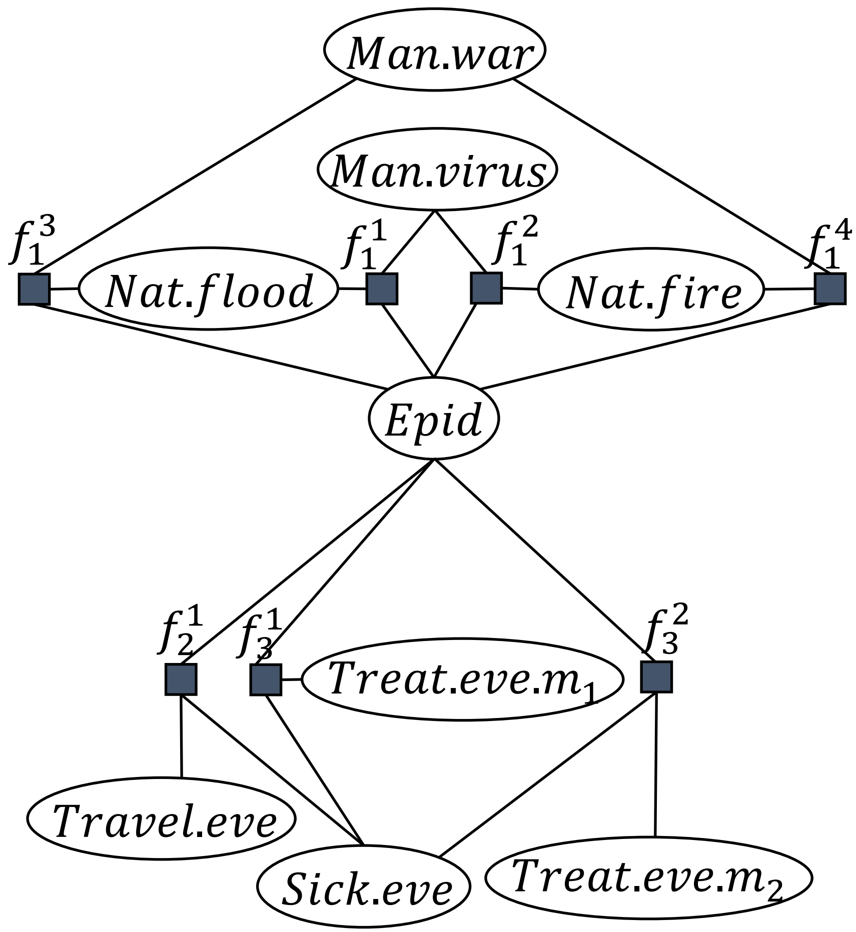
Zhang and Poole (1994)

- Absorb $Epid = true$ in all factors, e.g., f_2^1
- Possiby eliminate variable
-

$Travel.eve$	$Sick.eve$	f_2^1 e	f_2^1
$false$	$false$	4	4
$false$	$true$	6	6
$true$	$false$	2	2
$true$	$true$	9	9
$true$	$false$	$false$	~~4~~ 0
$true$	$false$	$true$	~~6~~ 0
$true$	$true$	$false$	2
$true$	$true$	$true$	9

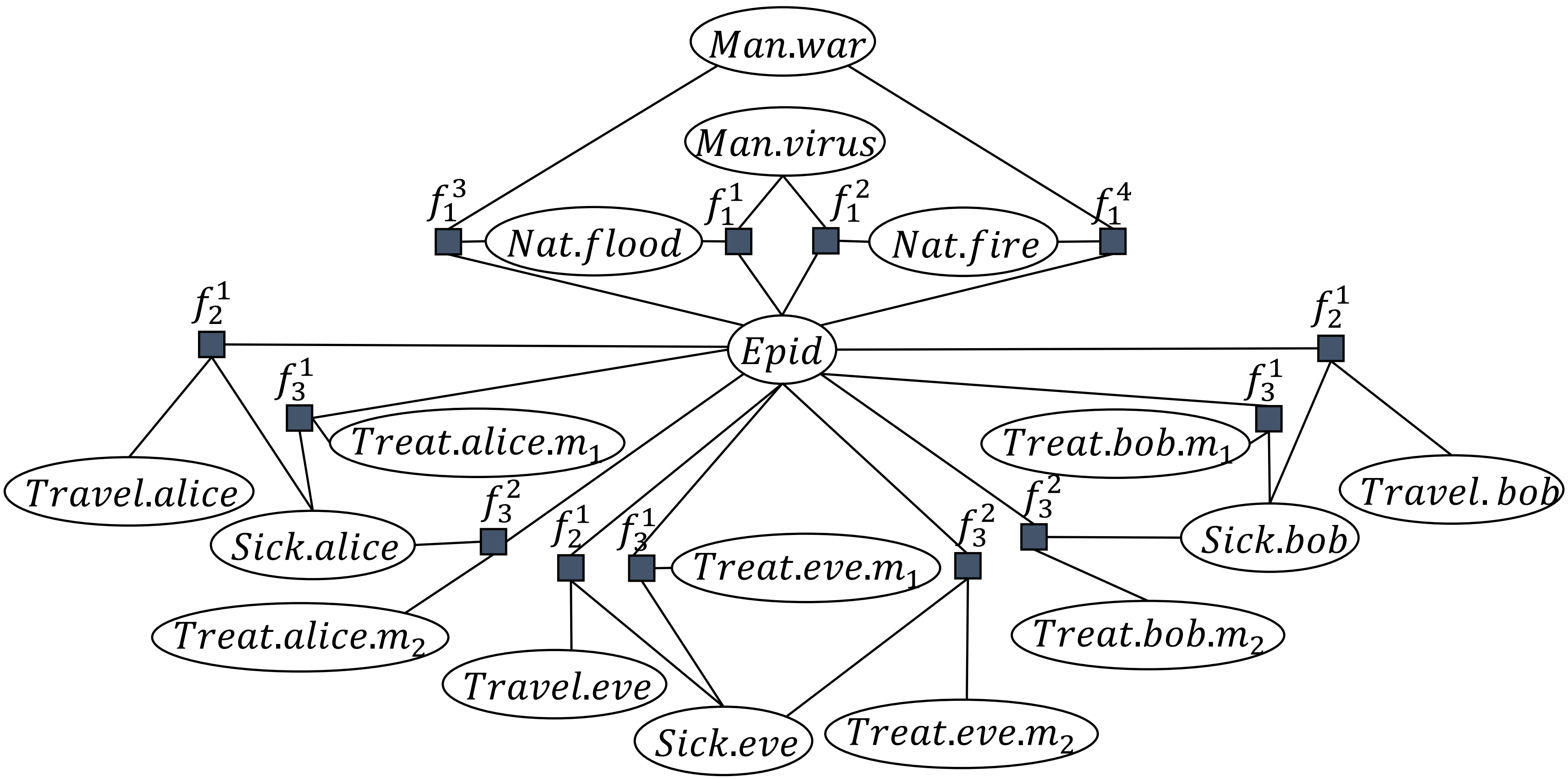


Problem: Models Explode



$7 \cdot 2^3 = 56$ entries in 7 factors, 9 variables

Problem: Models Explode

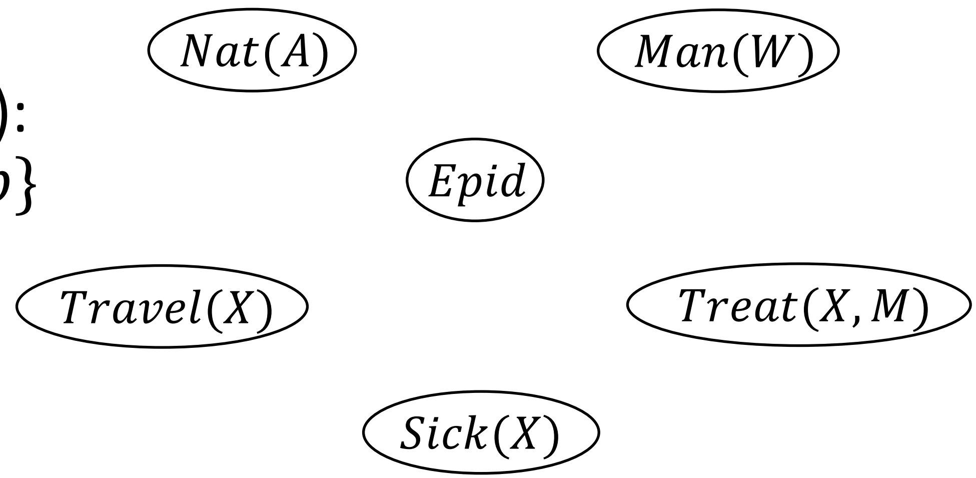


$13 \cdot 2^3 = 104$ entries in 13 factors, 17 variables

Solution: Lifting

Poole (2003)

- Parameterised random variables = PRV
  - With logical variables
  - E.g., X
  - Possible values (domain): $\mathcal{D}(X) = \{alice, eve, bob\}$
- Inputs to factors!



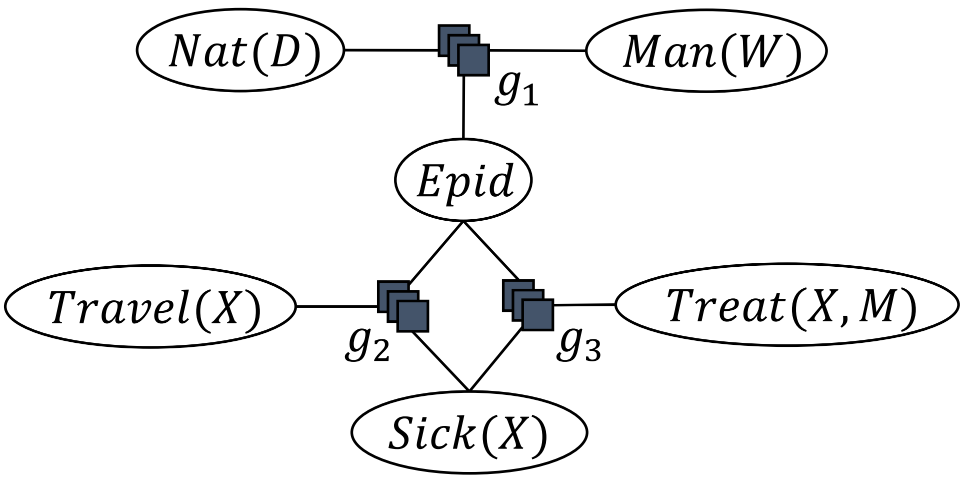
Solution: Lifting

Poole (2003)

- Factors with PRVs = parfactors
  - (Graphical) Model G
  - E.g., g_2

Sparse encoding!

$Travel(X)$	$Epid$	$Sick(X)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9



$3 \cdot 2^3 = 24$ entries in 3 parfactors, 6 PRVs

Solution: Lifting

Poole (2003)

- Grounding
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

$Travel(eve)$	$Epid$	$Sick(eve)$	f_2^1
false	false	false	5
false	false	true	0
false	true	false	4
false	true	true	6
true	false	false	4
true	false	true	6
true	true	false	2
true	true	true	9

$Travel(X)$ $Epid$

$Travel(alice)$	Epi…		
$false$	$fals$…		
$false$	$fals$…		
$false$	tru…		
$false$	tru…		
$true$	$fals$…		
$true$	$fals$…		
$true$	$true$	$false$	2
$true$	$true$	$true$	9

…(bob)	$Epid$	$Sick(bob)$	f_2^3
…e	$false$	$false$	5
…e	$false$	$true$	0
…e	$true$	$false$	4
…e	$true$	$true$	6
…e	$false$	$false$	4
…e	$false$	$true$	6
…e	$true$	$false$	2
$true$	$true$	$true$	9

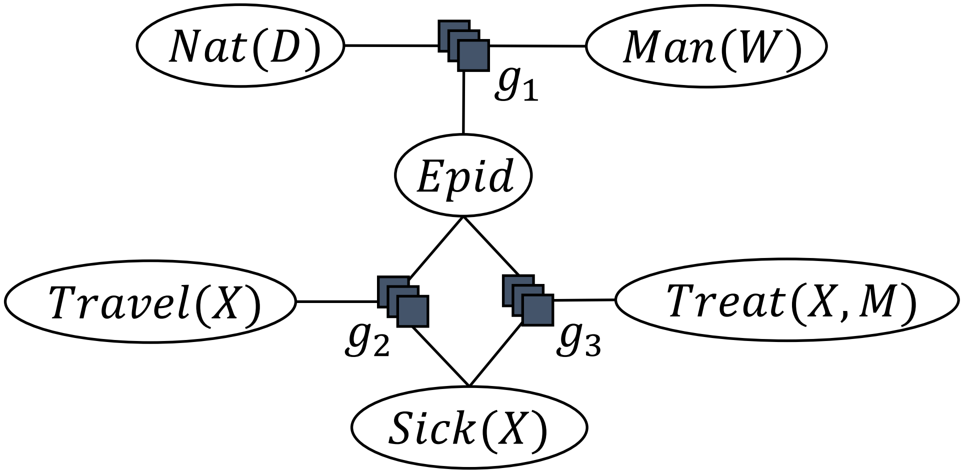
Solution: Lifting

Poole (2003)

- Joint probability distribution P_G by grounding

$$P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$$

$$Z = \sum_{v \in r(rv(gr(G)))} \prod_{f \in gr(G)} f_i(\pi_{rv(f_i)}(v))$$



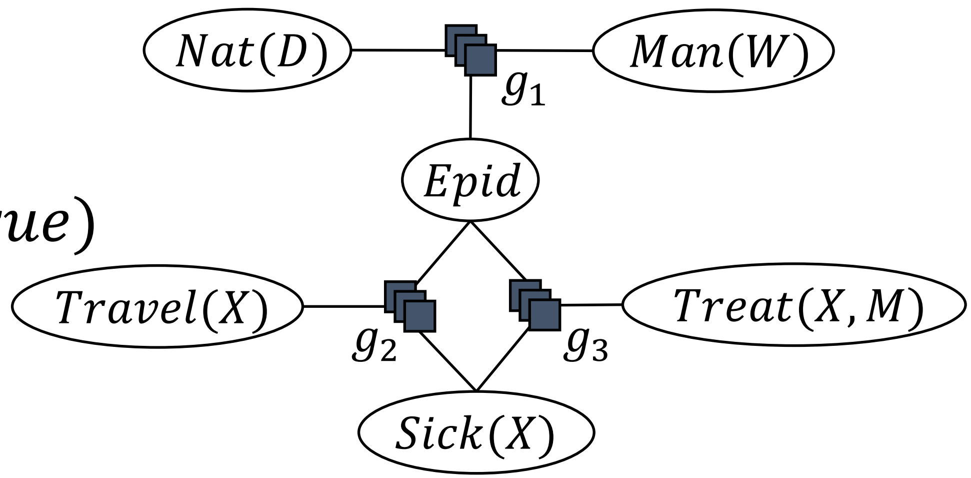
QA: Queries

- Marginal distribution
  - $P(Sick(eve))$
  - $P(Travel(eve,)\ Treat(eve, m_1))$
- Conditional distribution
  - $P(Sick(eve)|Epid)$
  - $P(Epid|Sick(eve) = true)$
- Most probable assignment

$$\underset{v \in r(R)}{\operatorname{argmax}} \sum_{v \in r(R)} P_F(v)\, , R \subseteq rv(gr(G))$$

  - MPE: $R = rv(gr(G))$
  - MAP: $R = \{Travel(eve), Man(virus)\}$

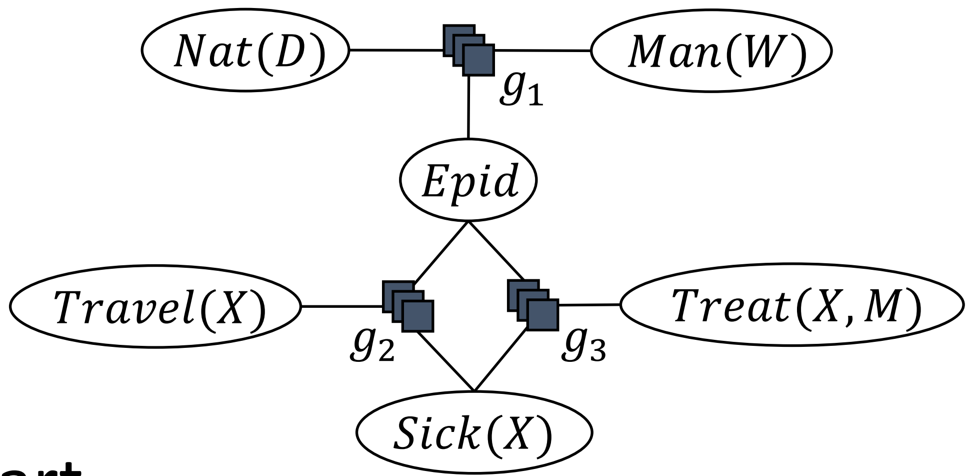
Avoid groundings!



QA: Lifted VE (LVE)

Poole (2003), de Salvo Braz et al. (2005, 2006), Milch et al. (2008), Taghipour et al. (2012, 2013)

- Eliminate all variables not appearing in query
- Lifted summing out
  - Sum out representative instance
  - Exponentiate result for isomorphic instances
- Preconditions exist!
- Count conversion (not part of this tutorial)

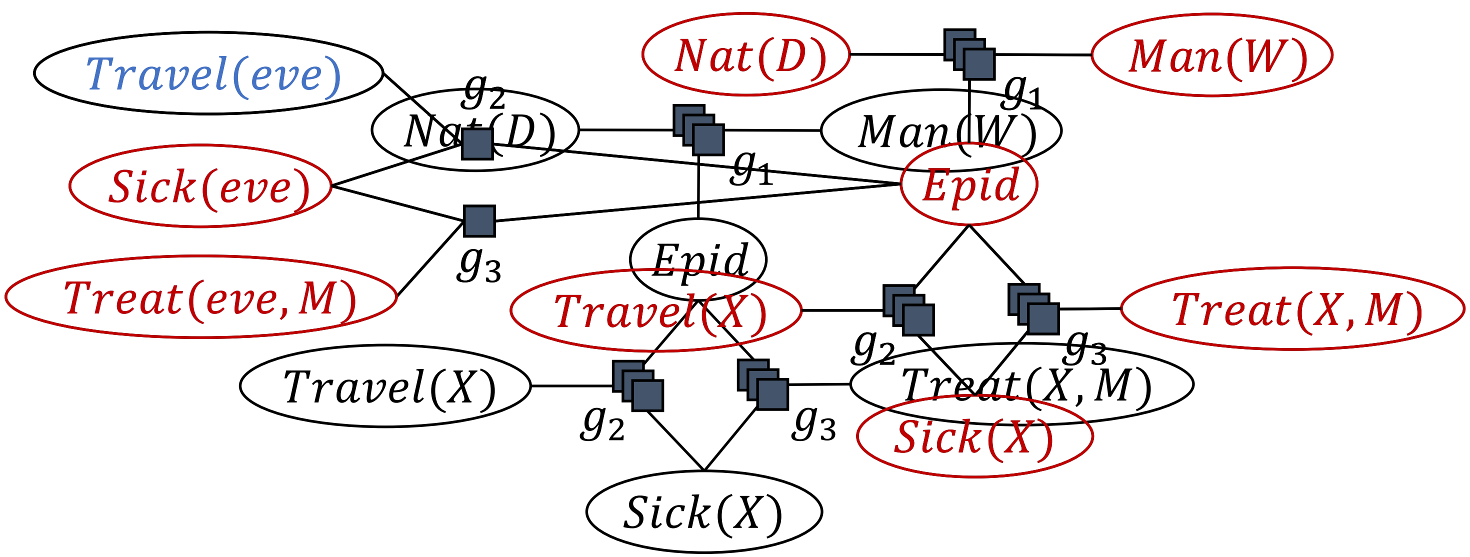


Avoid groundings!

QA: LVE in Detail

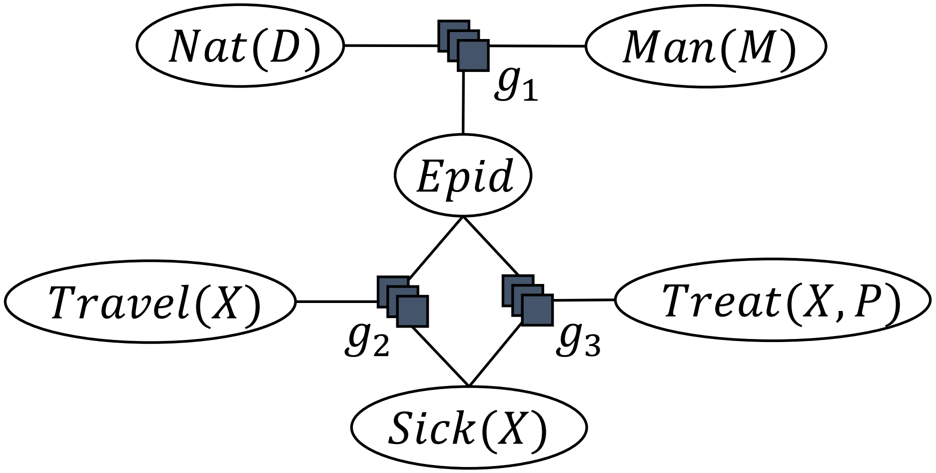
Poole (2003), de Salvo Braz et al. (2005, 2006), Milch et al. (2008), Taghipour et al. (2013, 2013a)

- E.g., marginal
  - $P(Travel(eve))$
  - Split w.r.t. $Travel(eve)$
  - Eliminate all non-query variables
  - Normalise



Problem: Many Queries

- Set of queries
  - $P(Travel(eve))$
  - $P(Sick(bob))$
  - $P(Treat(eve, m_1))$
  - $P(Epid)$
  - $P(Nat(flood))$
  - $P(Man(virus))$
  - Combinations of variables
- Under evidence
  - $Sick(X') = true$
  - $X' \in \{alice, eve\}$

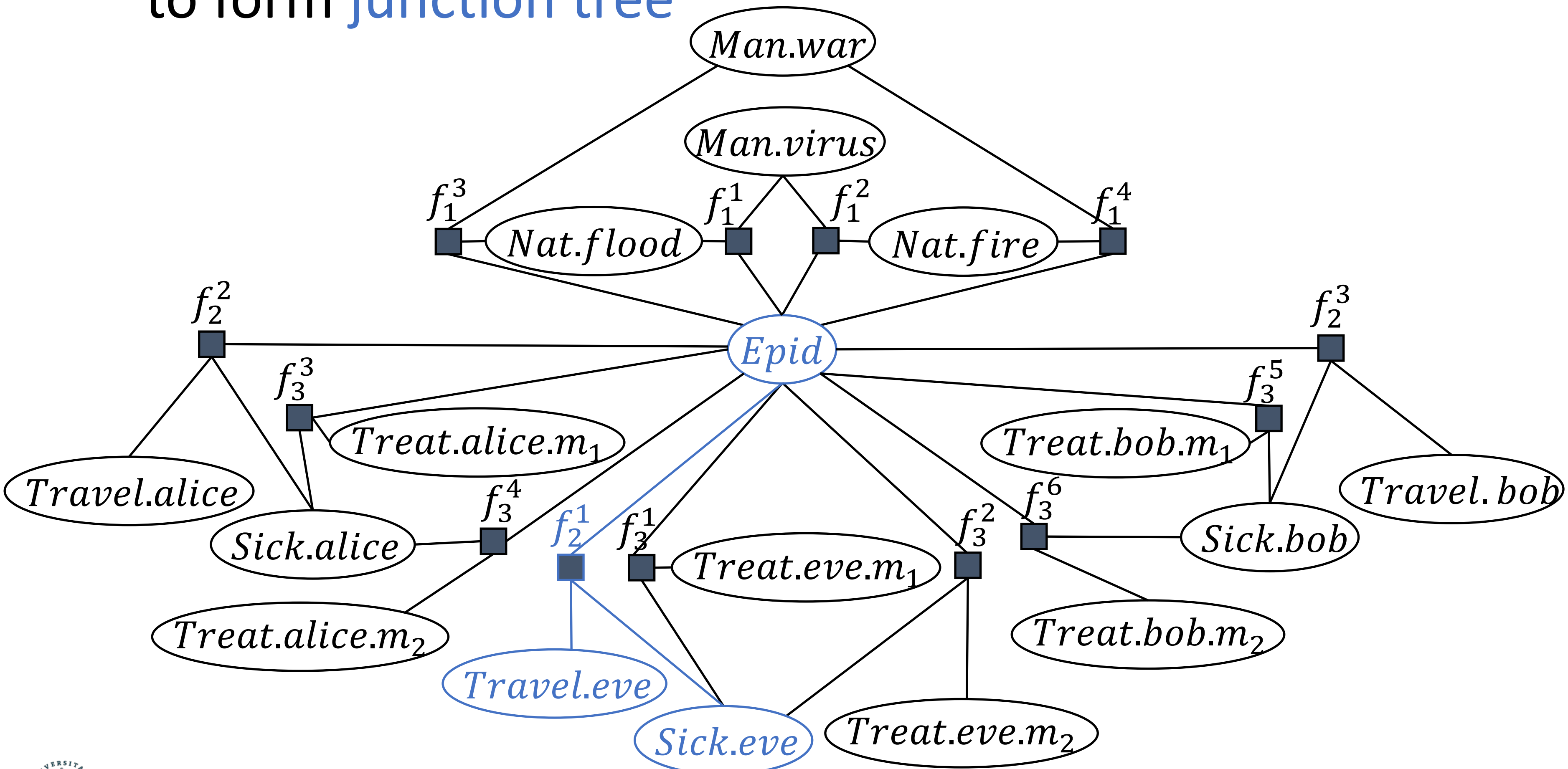


- (L)VE starts with complete model for QA

Solution: Junction Tree Algorithm

Lauritzen and Spiegelhalter (1988), Shafer and Shenoy (1989), Jensen et al. (1990)

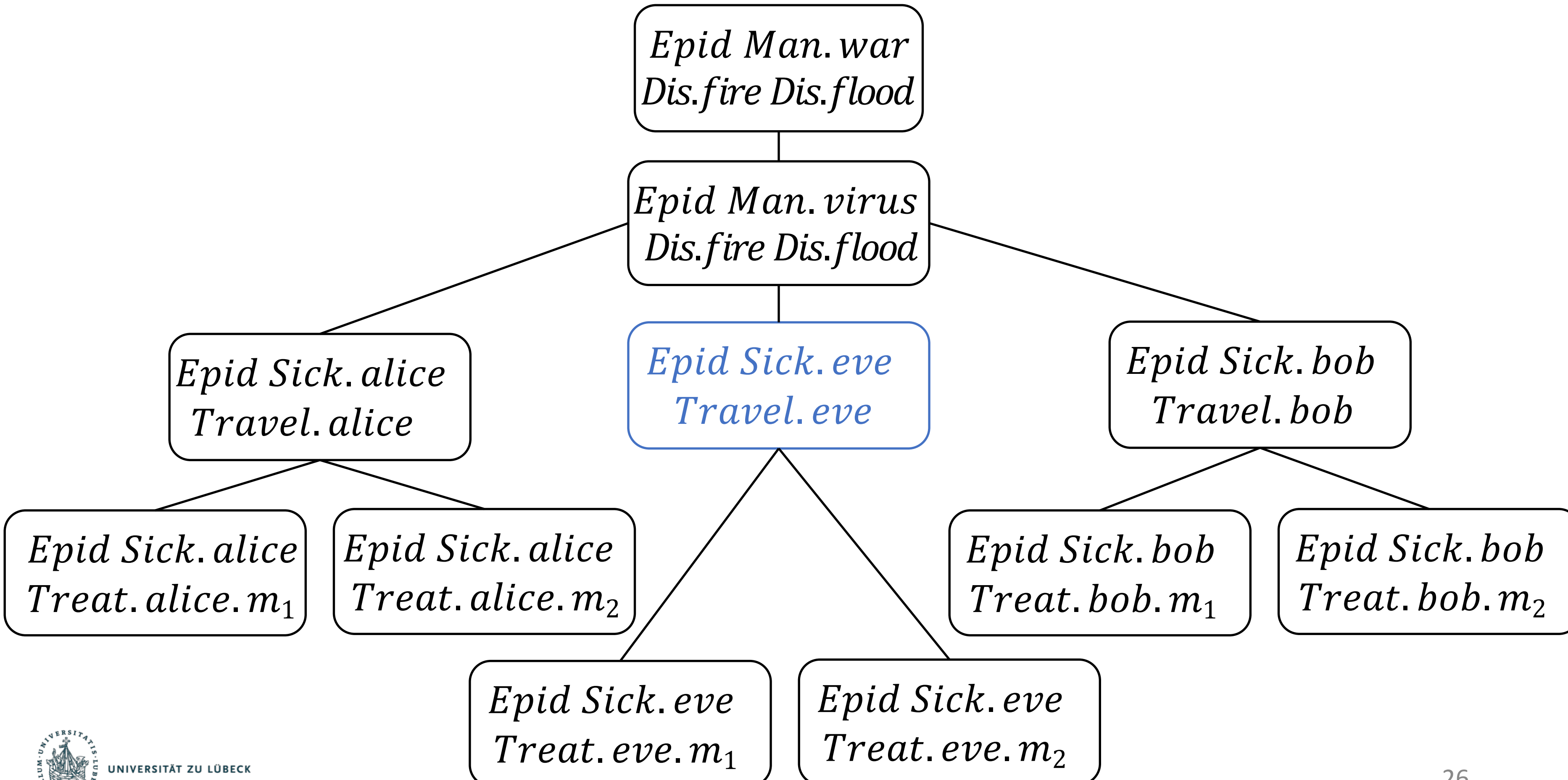
- Identify clusters in model to form junction tree



Junction Tree: Data Structure

Lauritzen and Spiegelhalter (1988), Shafer and Shenoy (1989), Jensen et al. (1990)

- Clusters: sets of variables from underlying model



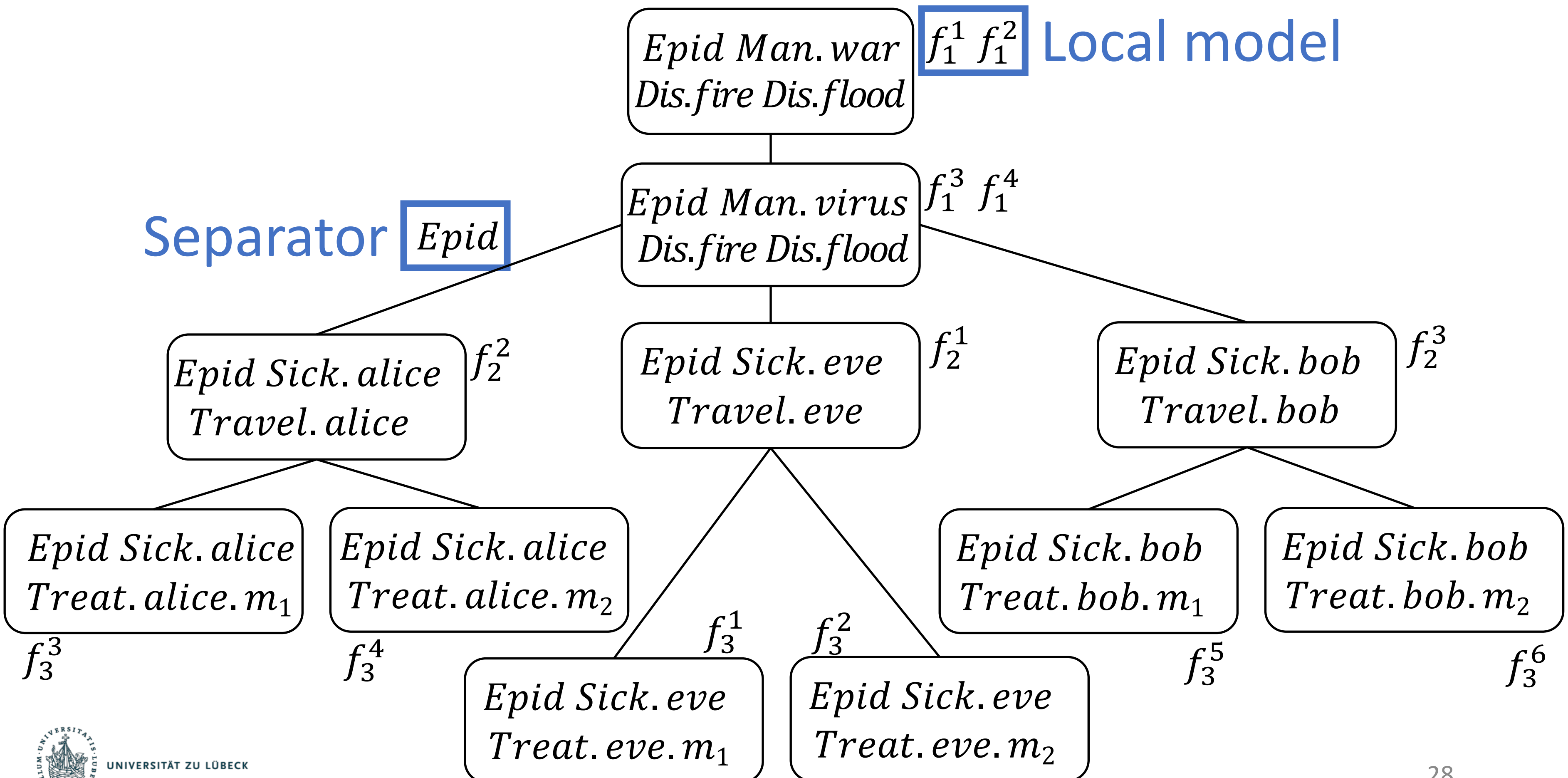
Junction Tree: Definition

Lauritzen and Spiegelhalter (1988)

- Junction tree $J = (V, E)$ for model F
  - V set of nodes, a node = cluster C
  - E set of edges
  1. A cluster C_i is a set of variables from F
  2. For every factor f in F, its arguments appear in some cluster C_i
  3. If a variable from F appears in clusters C_i and C_j, it must appear in every cluster C_k on the path between nodes i and j.
- Each cluster has a local model F_i : set of factors
  - factor arguments subset of cluster
  - $\bigcup_{i \in V} F_i = F$ (F_i partition F)
- Separator: shared variables of edges $(i, j) \in E$

Junction Tree: Components

- Clusters sufficient for QA after some preprocessing



Junction Tree: Message Passing

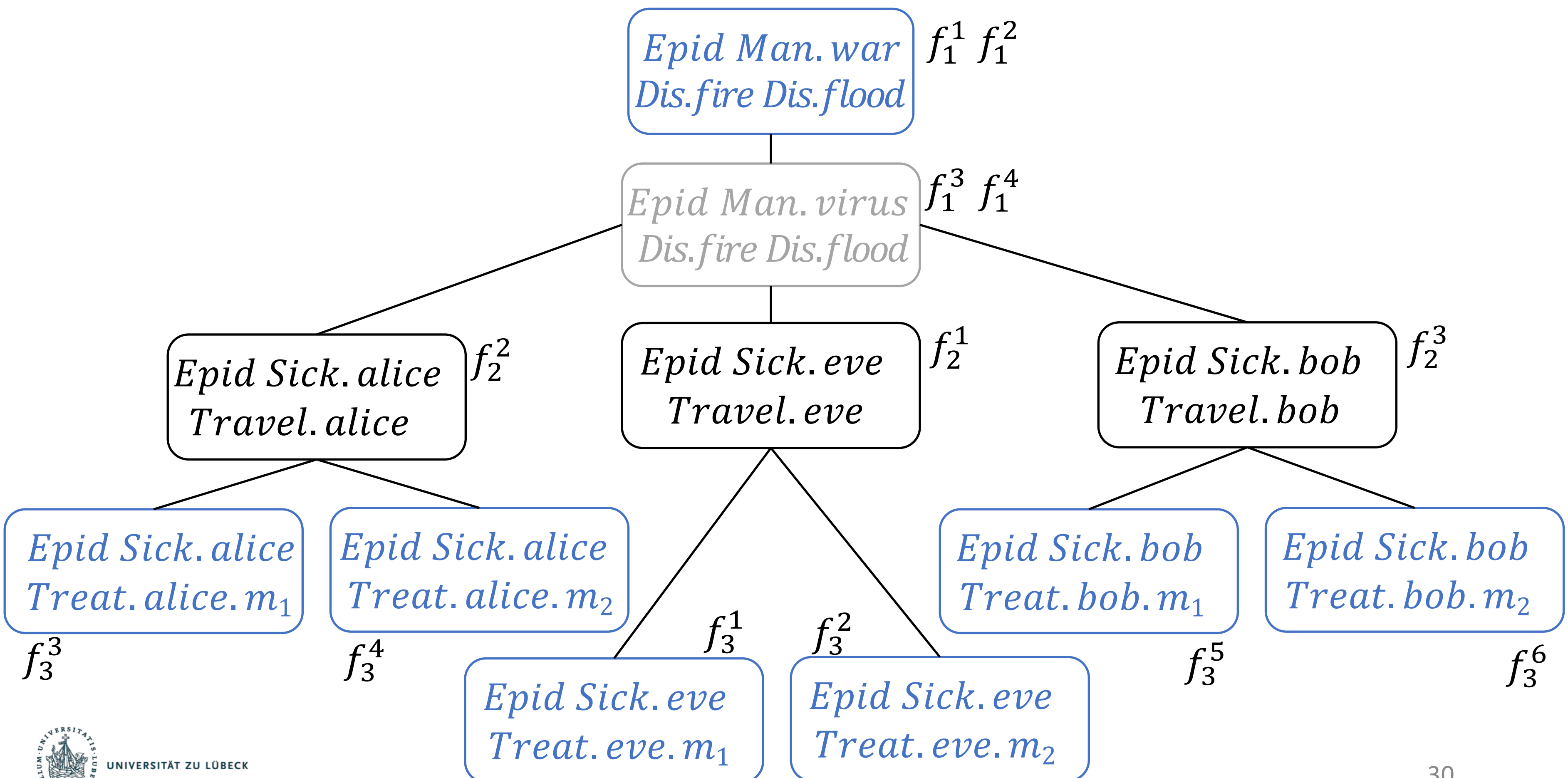
Lauritzen and Spiegelhalter (1988), Shenoy and Shafer (1990)

- Distribute local information by messages
  1. If a cluster received messages from all neighbours but one, it sends message to remaining neighbour
  2. If a cluster received all messages, it send messages to all neighbours that have not received a message yet
  - Message: VE on local model and other messages with query over separator
- Local computations correctness:
  - Valid junction tree (w.r.t. properties)
  - Combination & marginalisation (in the form of multiplication & summing out)
- QA on local models and messages
  - Use cluster that contains query variable

Junction Tree: Messages

- From periphery to centre and back

Shafer and Shenoy (1989)



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Junction Tree: Messages

Shafer and Shenoy (1989)

- Message: eliminate all non-separator variables
  - E.g., cluster with f_3^3 in local model

$Sick.alice$	$Epid$	$\sum$	$\cancel{eat.alice.m_1}$	f_3^3	$\sum$
$false$	$false$	11	$\cancel{false}$	9	11
$false$	$true$	7	$\cancel{true}$	2	
$true$	$false$	8	$\cancel{false}$	3	7
$true$	$true$	15	$\cancel{true}$	4	
$true$	$false$		$\cancel{false}$	1	8
$true$	$false$		$\cancel{true}$	7	
$true$	$true$		$\cancel{false}$	6	15
$true$	$true$		$\cancel{true}$	9	

$Epid$
$Sick.alice$

m_{m_1}

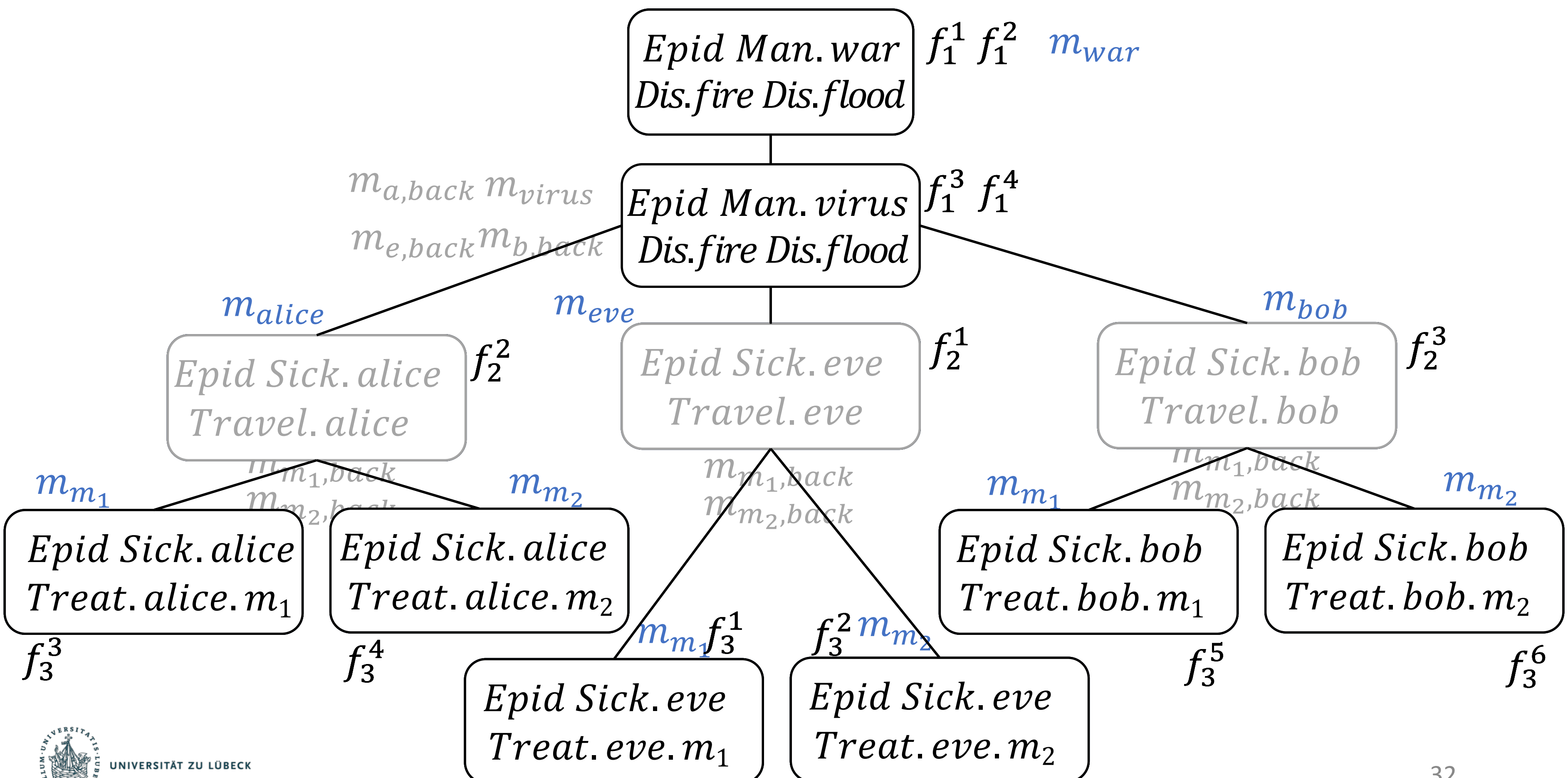
$Epid\ Sick.alice$
$Treat.alice.m_1$

f_3^3

Junction Tree: Messages

Shafer and Shenoy (1989)

- From periphery to centre and back



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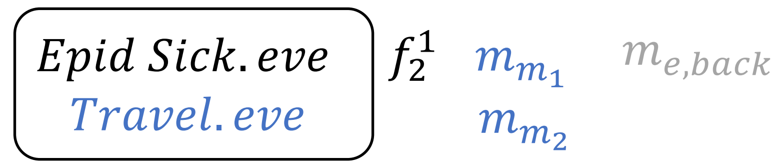
QA: Query Answering with JT

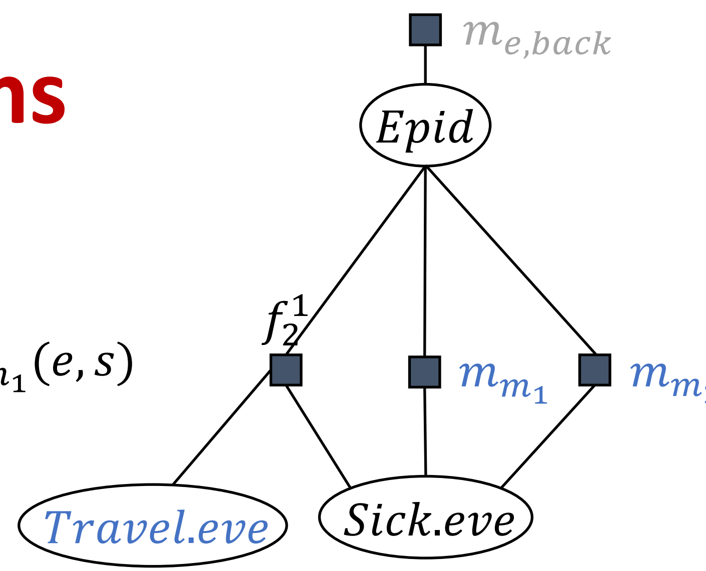
Shafer and Shenoy (1989)

- Eliminate all variables not appearing in query
  - Through summing out
- E.g., marginal
  - $P(Travel.eve)$



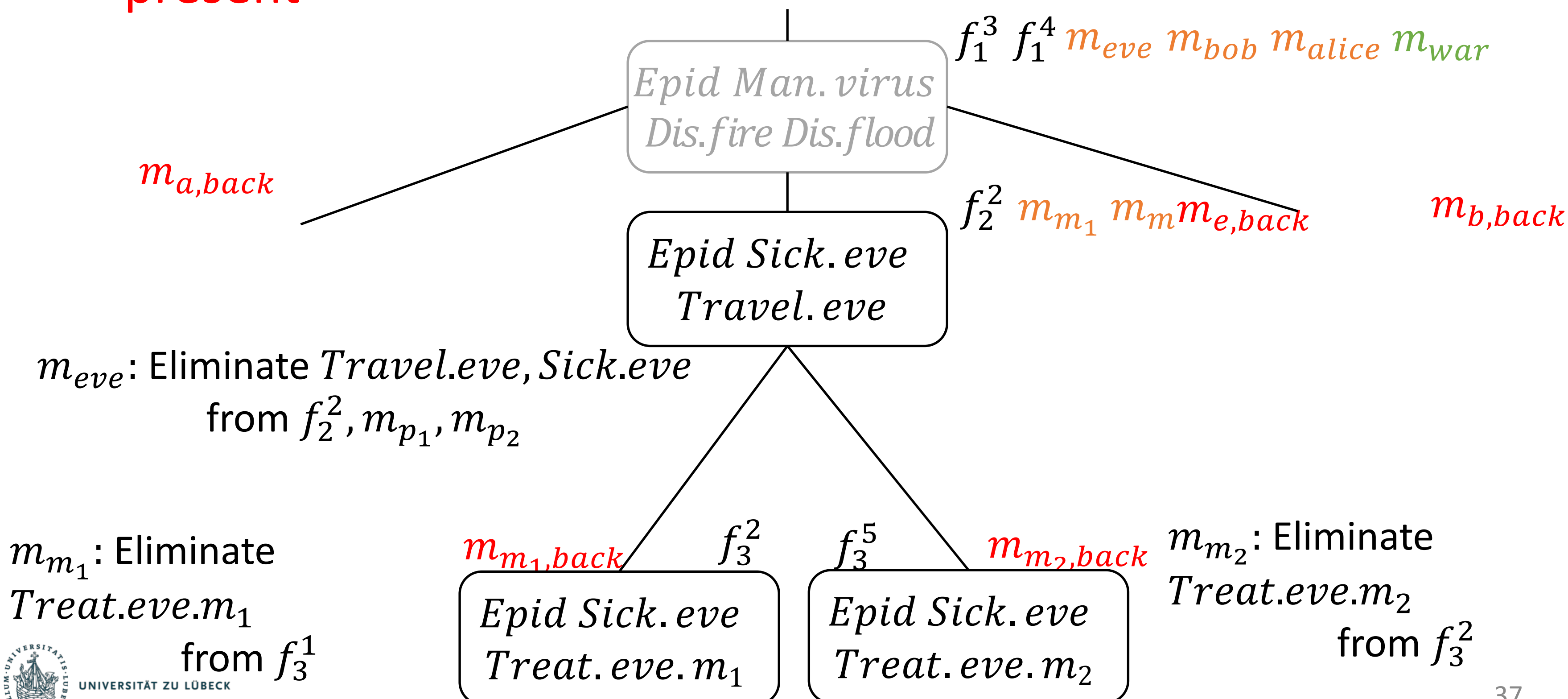
2 sums

$$P(Travel.eve) \propto \sum_{e \in r(Epid)} m_{e,back}(e) \sum_{s \in r(Sick.eve)} f_2^1(Travel.eve, e, s) m_{m_1}(e,s) m_{m_1}(e,s)$$



Junction Tree: Symmetry→ Inefficiency

- Identical messages incoming
- Information already present
- Calculating identical messages + sending information partially present



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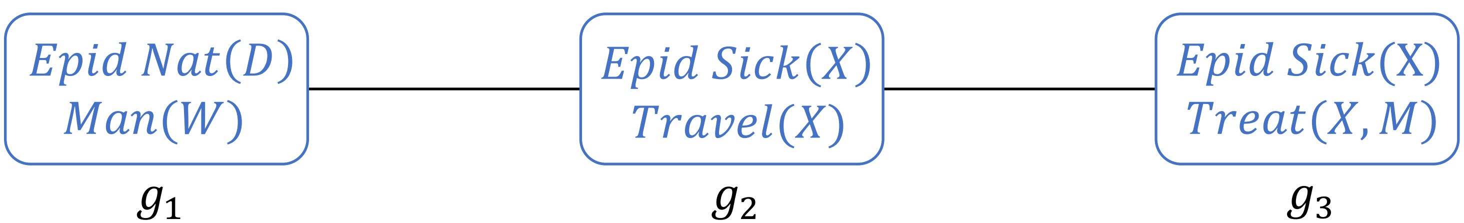
First-order Junction Tree: FO jtree

Braun and Möller (2016)

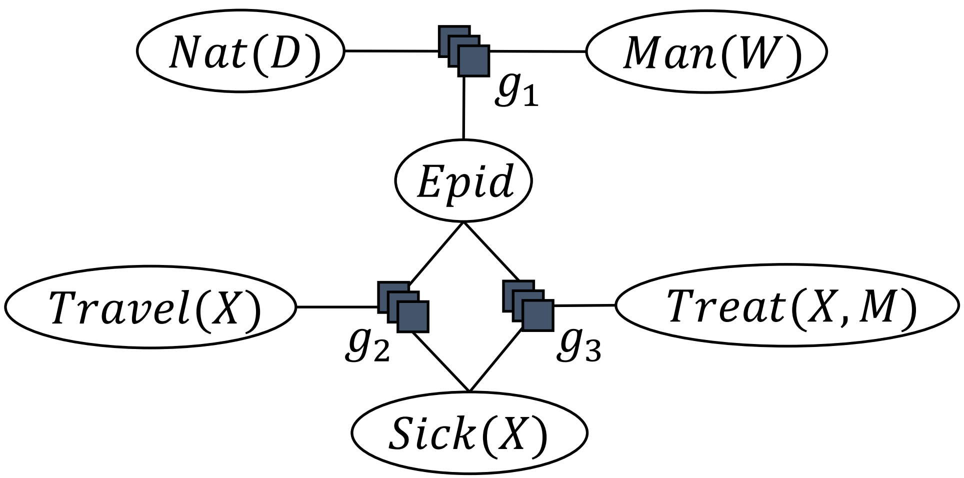
- Parameterised clusters = parclusters
  - Nodes of FO jtree
  - Shared PRVs between neighbours = separators
  - Parclusters have local models of parfactors
- FO jtree for parfactor model G:
  - Analogous definition, junction tree properties apply
  - Message passing, QA analogous



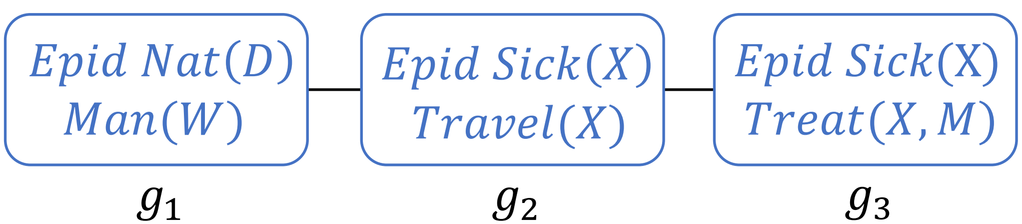
Sparse encoding!

Lifted Junction Tree Algorithm: LJT

Braun and Möller (2017)



Queries on grounded PRVs, e.g.,
$Travel(eve), Treat(eve, p_1), Epid$



- Input
  - Model G
  - Evidence $\boldsymbol{E}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J
  3. Pass messages in J
     - Inbound
     - Outbound
  4. Answer queries $\boldsymbol{Q}$

LJT: Example Input

Braun and Möller (2017)

- Model $G = \{g_i\}_{i=1}^{3}$
  - $g_1\big(Epid, Nat(D), Man(W)\big)$
  - $g_2(Travel(X), Epid, Sick(X)\ )$
  - $g_3\big(Epid, Sick(X), Treat(X, M)\big)$

  → Including function specification
- Evidence $\boldsymbol{E} = \{Sick(alice) = true, Sick(eve) = true\}$
- Queries $\boldsymbol{Q} = \{Travel(eve), Epid\}$
- Algorithm
  1. Build FO jtree J for G

FO Jtree Construction

- Propositional junction tree construction
  - Triangulation, compute maximum spanning tree, …
  - Hypergraph partitioning
  - Decomposition tree (dtree), clusters, …
- First-order: logical variables

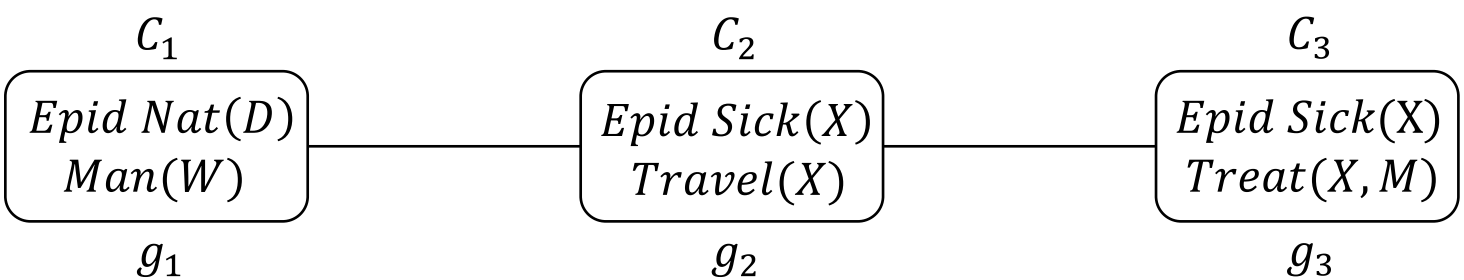
Taghipour et al. (2013b)

  - First-order decomposition trees (FO dtrees)
  - FO dtrees have node properties (cutset, context, cluster)
  - (FO) dtree + clusters = (FO) jtree
  - Heuristic to build an FO dtree
    (logical variables guide the construction)

Lifted Junction Tree Algorithm: LJT

Braun and Möller (2017)

- Input
  - Model G
  - Evidence $\boldsymbol{E}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G



  2. Enter evidence $\boldsymbol{E}$ into J

LJT: Enter Evidence

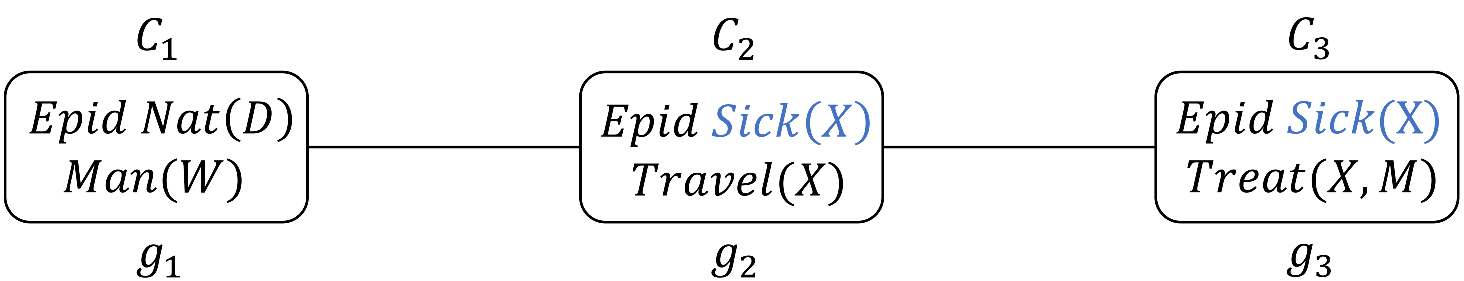
- Evidence as a set of events
  - $\boldsymbol{E} = \{Sick(eve) = true, Sick(alice) = true\}$
- Evidence as a parfactor
  - $g_E(Sick(X'))$
  - $X' \in \{eve, alice\}$
  - Function specification

$Sick(X')$	g_E
$false$	0
$true$	1

- At every parcluster that contains evidence variables, enter evidence

LJT: Enter Evidence

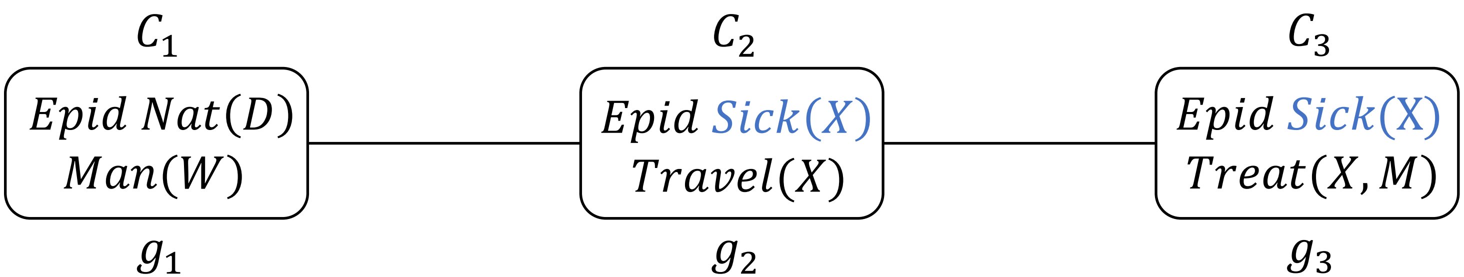
- At every parcluster that contains evidence variables
  - $g_E\big(Sick(X')\big), X' \in \{eve, alice\}$
  - Parclusters
    - $Sick(X') \nsubseteq C_1$
    - $Sick(X') \subset C_2$
    - $Sick(X') \subset C_3$



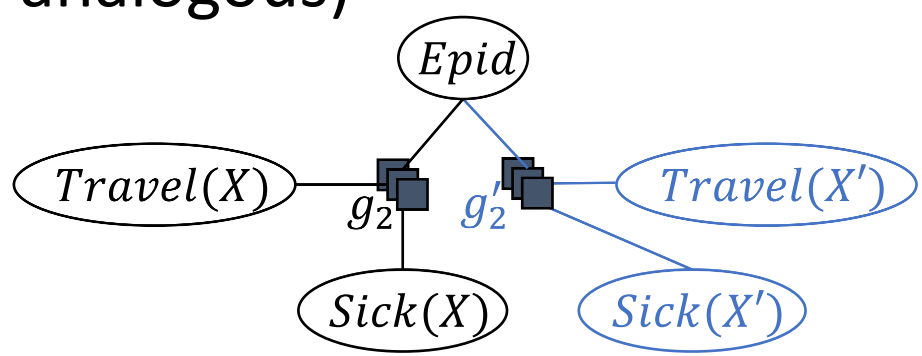
- Enter evidence at C_2 and C_3

LJT: Enter Evidence

- At every parcluster that contains evidence variables
  - $g_E\big(Sick(X')\big), X' \in \{eve, alice\}$
  - Parclusters C_2 and C_3



- Enter evidence at C_2 (C_3 analogous)
  - Split local model
    - $X \in \{bob, \ldots\}$
  - Absorb evidence in g_2'



Lifted Junction Tree Algorithm: LJT

Braun and Möller (2017)

- Input
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  - Evidence $\boldsymbol{E}$
  - Queries $\boldsymbol{Q}$
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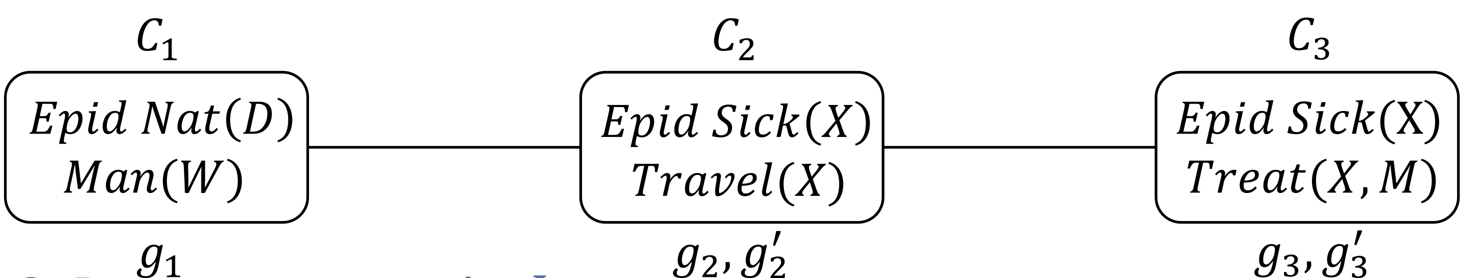
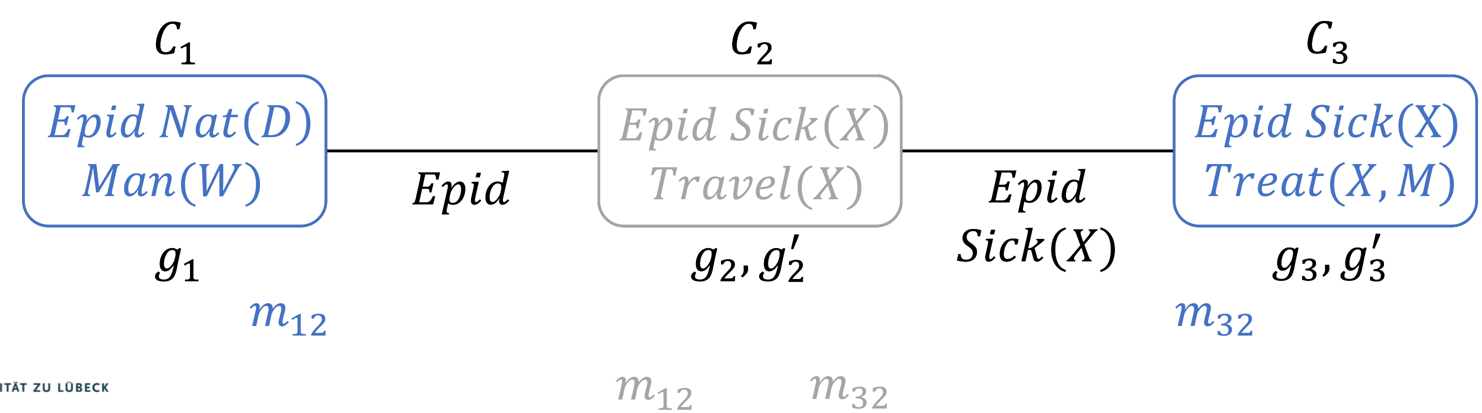


  3. Pass messages in J

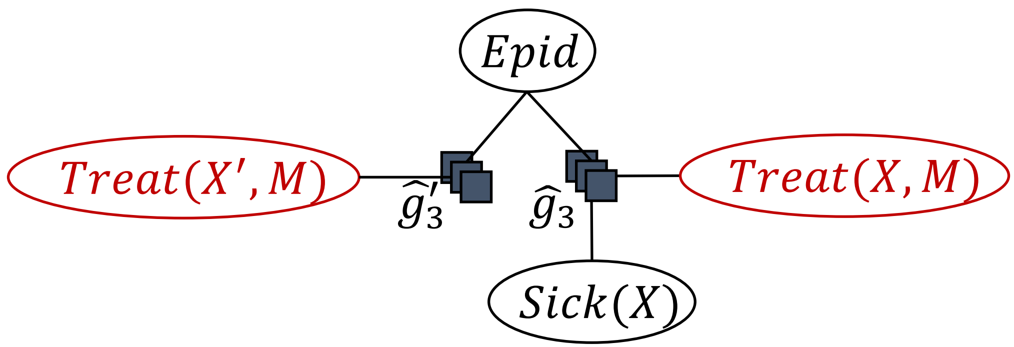
LJT: Pass Messages

- Separators
- Messages
  - Inbound
    - m_{12} from C_1 to C_2 over $Epid$
    - m_{32} from C_3 to C_2 over $Epid, Sick(X)$
  - Outbound
    - m_{21} from C_1 to C_2 over $Epid$
    - m_{23} from C_3 to C_2 over $Epid, Sick(X)$

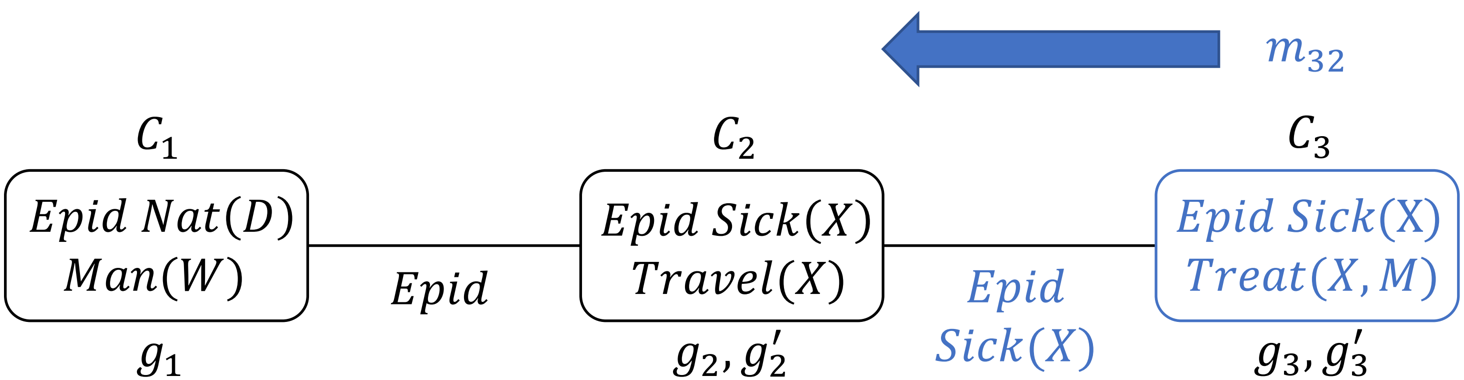


LJT: Example Message Inbound

- m_{32} from C_3 to C_2
  - Eliminate $Treat(X,P), Treat(X',P)$



$$m_{32} = \{\hat{g}_3, \hat{g}_3'\}$$

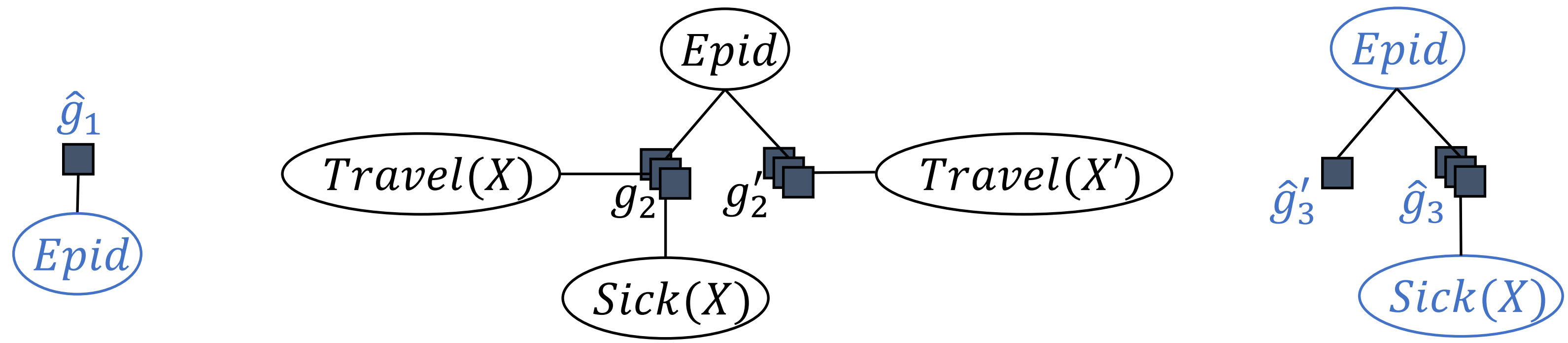


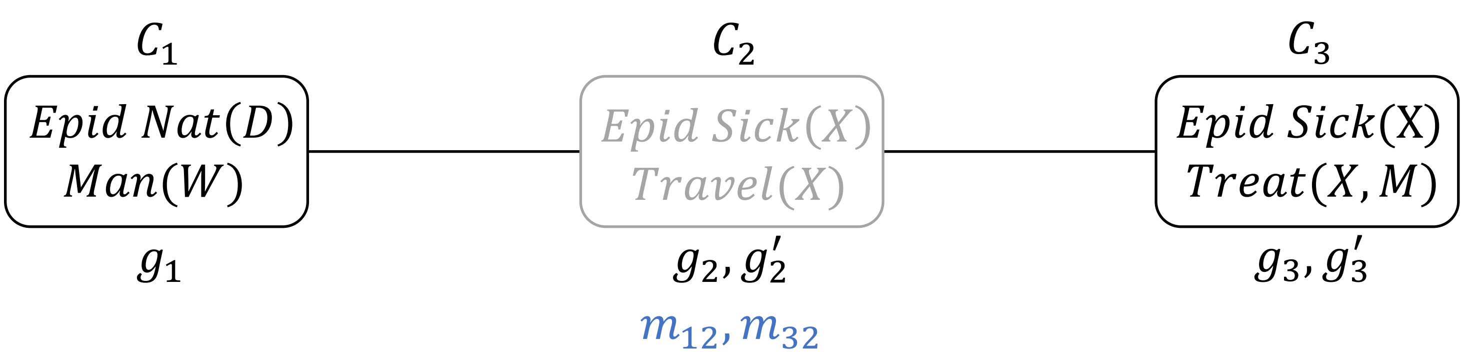
LJT: Messages at C_2

- After m_{12} and m_{32} arrived

$m_{12} = \{\hat{g}_1\}$

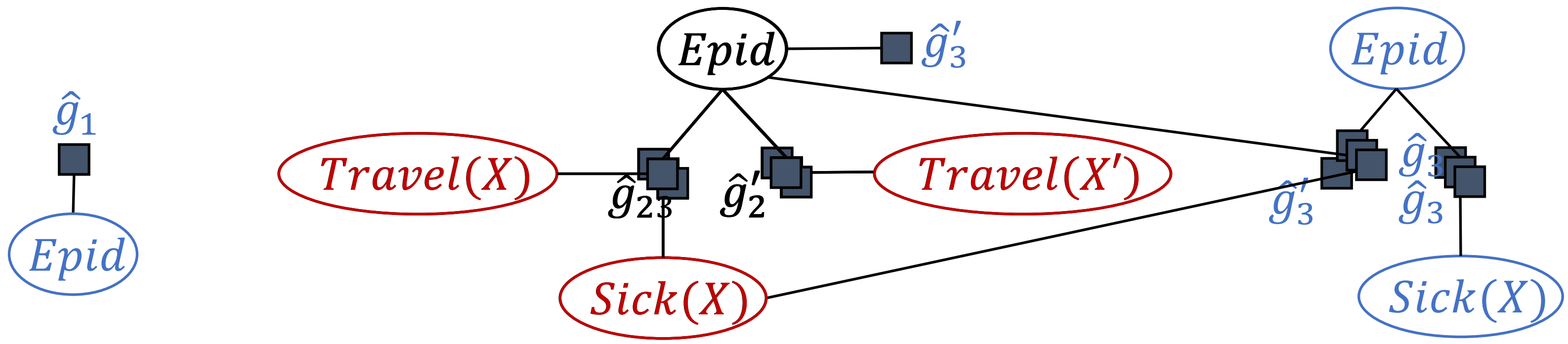
$m_{32} = \{\hat{g}_3, \hat{g}_3'\}$



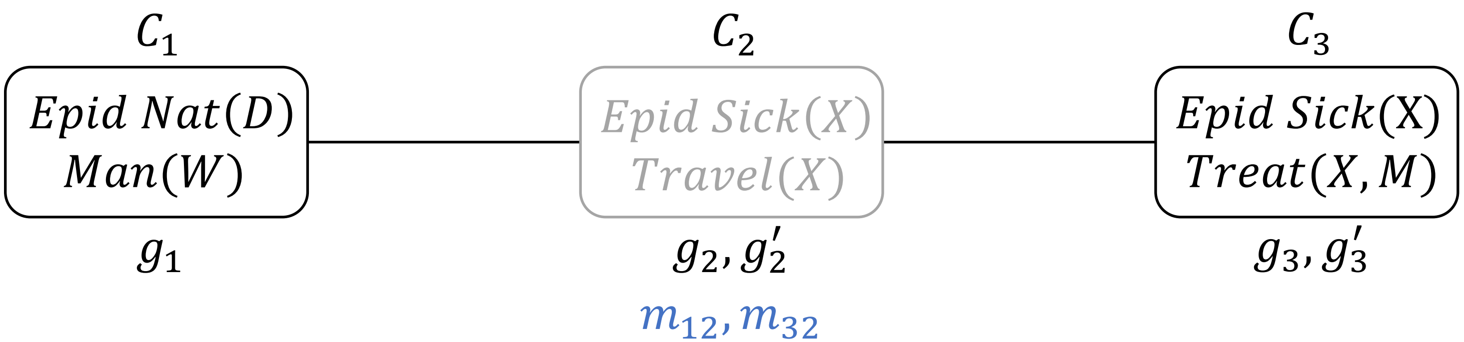


LJT: Example Message Outbound

- m_{21} from C_2 to C_1
  - Eliminate $Sick(X), Travel(X), Travel(X')$ from g_2, g_2', m_{32}

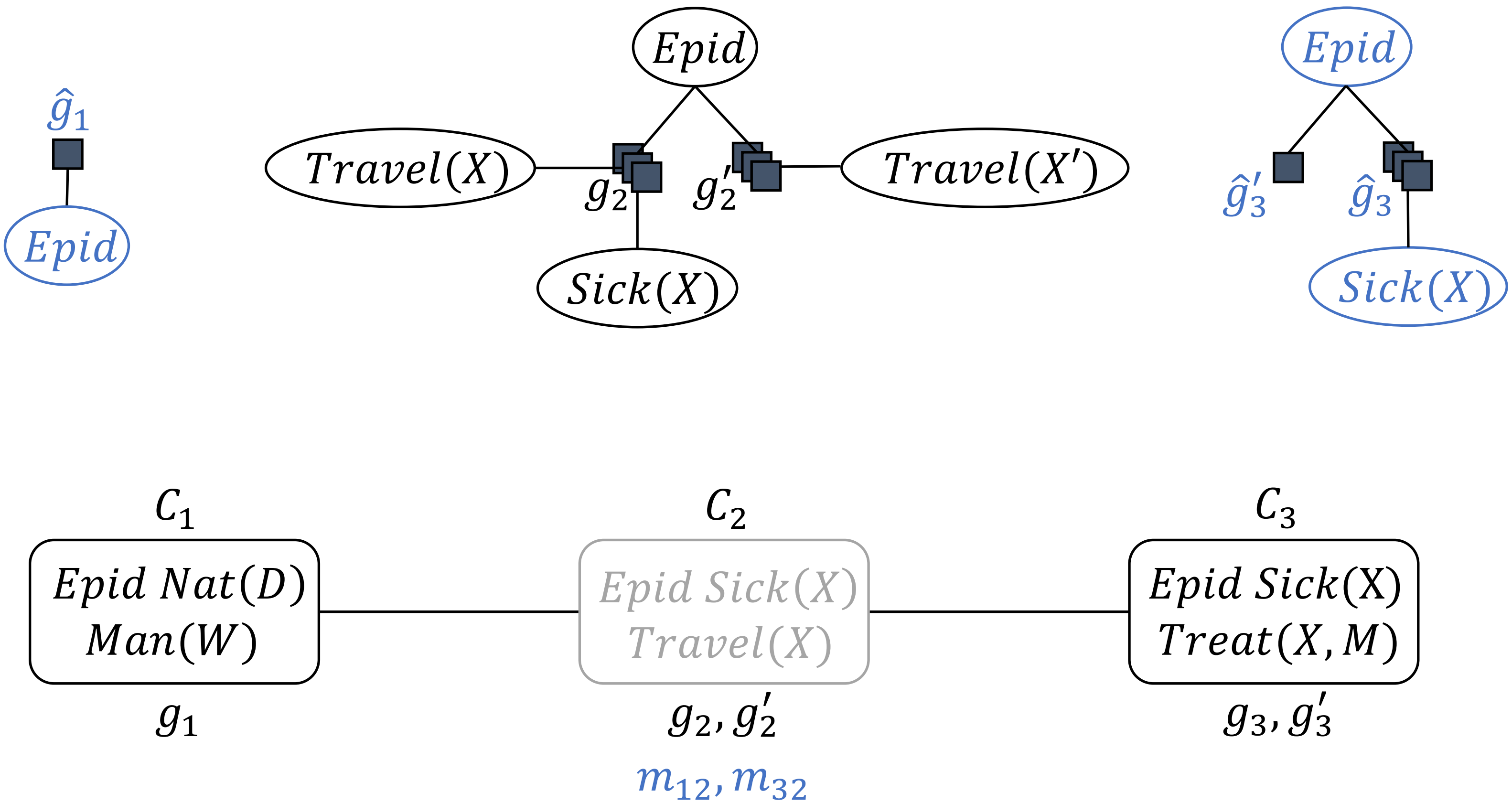


$$m_{21} = \{\hat{g}_{23}, \hat{g}_2', \hat{g}_3'\}$$



LJT: Example Message Outbound

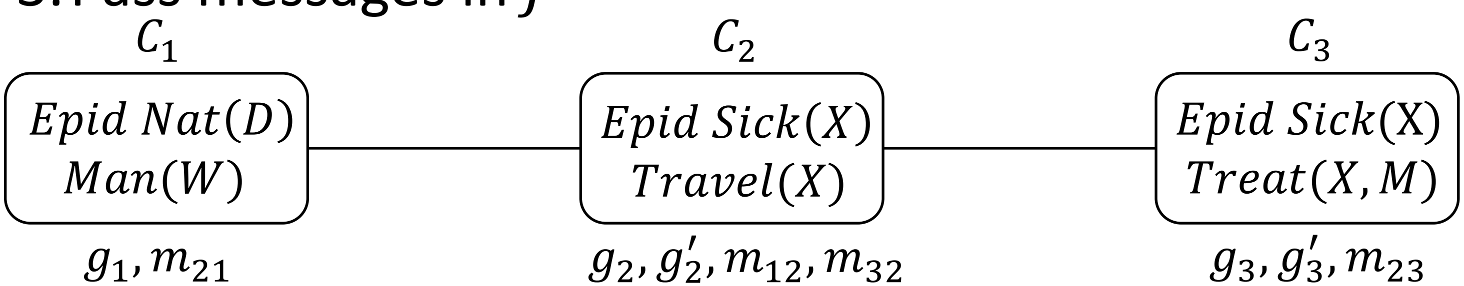
- m_{23} from C_2 to C_3
  - Eliminate $Travel(X), Travel(X')$ from g_2, g_2', m_{12}



Lifted Junction Tree Algorithm: LJT

Braun and Möller (2017)

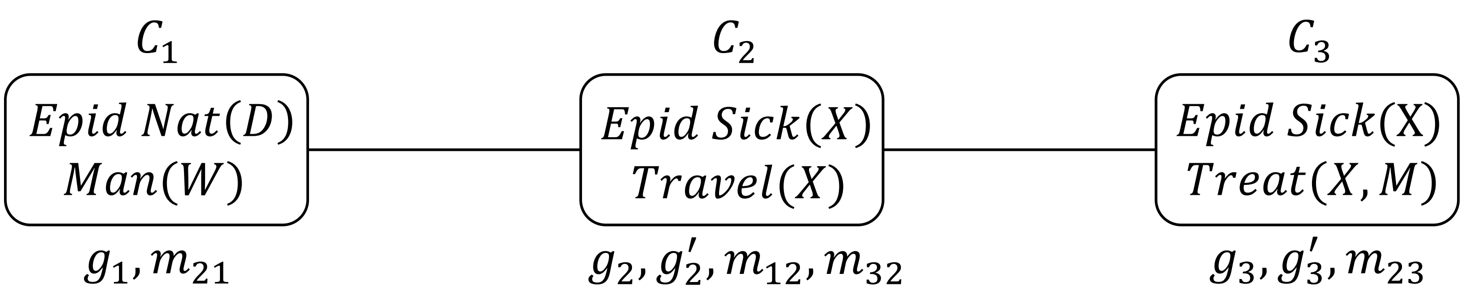
- Input
  - Model G
  - Evidence $\boldsymbol{E}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J
  3. Pass messages in J



  4. Answer queries $\boldsymbol{Q}$

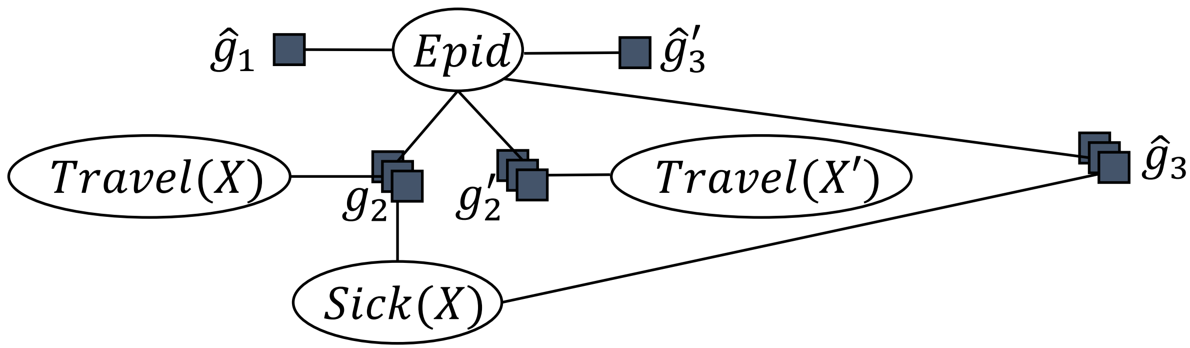
LJT: Answer Queries

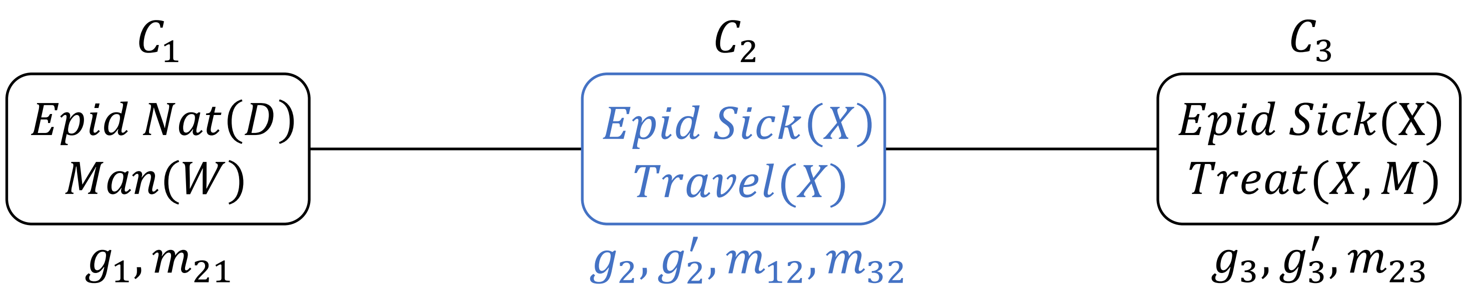
- Queries $\boldsymbol{Q} = \{Travel(eve), Epid\}$
- For each query Q
  - Find parcluster that contains Q
  - Extract submodel of local model and messages
  - Use LVE to answer Q



LJT: Answer Queries

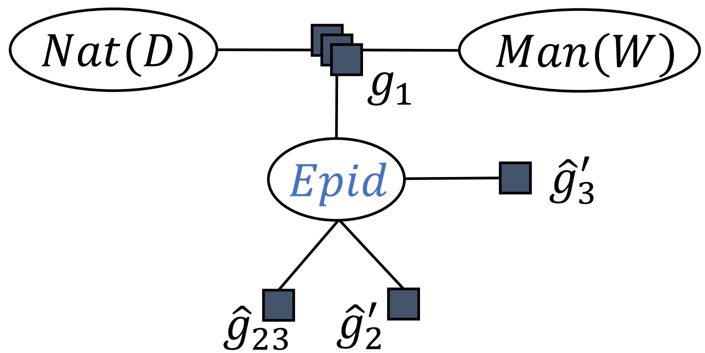
- $Q_1 = Travel(eve)$
  - Find parcluster: C_2
  - Extract submodel: $G' = \{g_2, g_2', m_{12}, m_{32}\}$
  - Answer $Travel(eve)$ with LVE

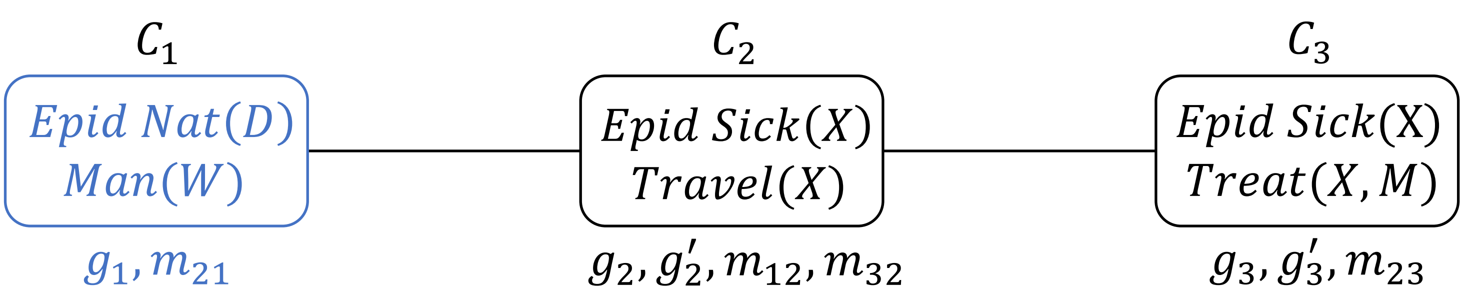




LJT: Answer Queries

- $Q_2 = Epid$
  - Find parcluster: C_1 (any of the three parclusters)
  - Extract submodel: $G' = \{g_1, m_{21}\}$
  - Answer $Epid$ with LVE





LJT: Analysis

- Static overhead
  - Construction
  - Evidence entering
  - Message passing
- Payoff during QA
  - Multiple queries
  - Complexity of LVE for one query = Complexity of message pass in LJT

Queries all under the same evidence

$$\boldsymbol{E} = \{Sick(eve) = true,$$
$$Sick(alice) = true\}$$

Extending LVE and LJT

Probability queries, based on LVE

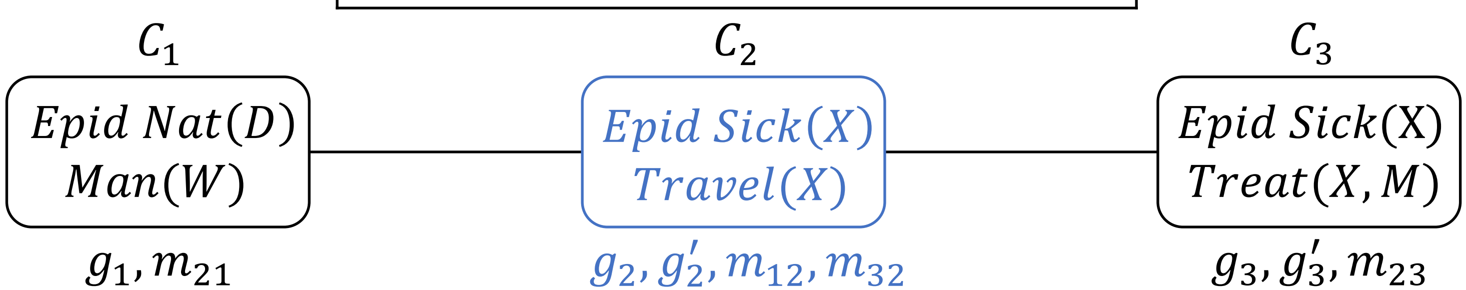
- Liftable models (fusion)
- Conjunctive queries
  - $P\big(Epid, Travel(eve)\big)$
- Isomorphic query terms (parameterised queries)
  - $P\big(Sick(eve), Sick(alice), Sick(bob)\big) \triangleq P\big(Sick(X)\big)$

Isomorphic Conjunctive Queries

Braun and Möller (2018a)

- $Q = Sick(alice) \wedge Sick(eve) \wedge Sick(bob)$
- Same parclu
- But
  - Split mode … query
  - Eliminate r … tical sums, large inter
  - Normalise

$Sick(alice)$	$Sick(eve)$	$Sick(bob)$	g
false	*false*	*false*	1
false	*false*	*true*	2
false	*true*	*false*	2
false	*true*	*true*	3
true	*false*	*false*	2
true	*false*	*true*	3
true	*true*	*false*	3
true	*true*	*true*	4



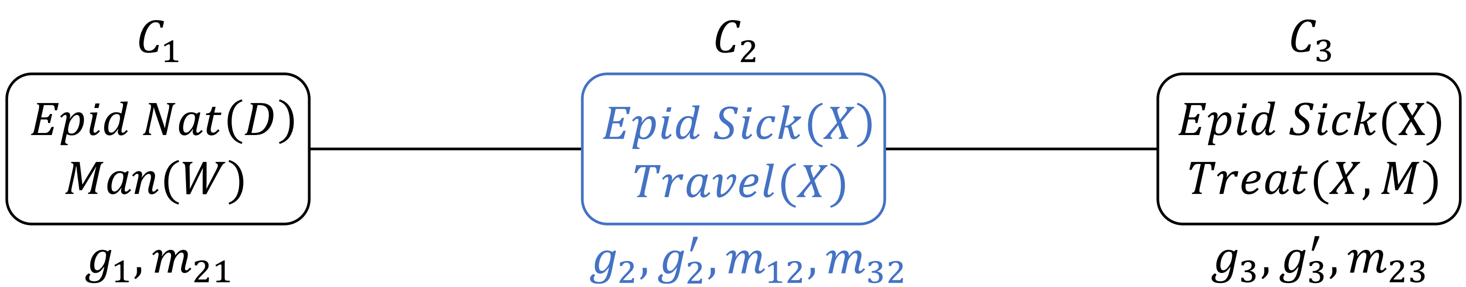
Parameterised Queries

Braun and Möller (2018a)

- Compact query representation
- Lifted computations during LVE
- Compact result representation

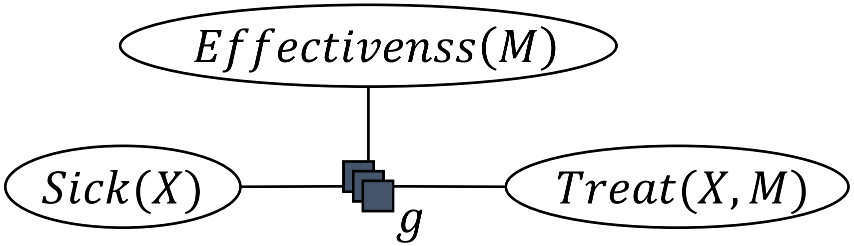
- $Q = Sick(X)$
- Result: $\#_X[Sick(X)]$

$\#_X[Sick(X)]$	g
[0,3]	1
[1,2]	2
[2,1]	3
[3,0]	4



Grounding Parameterised Queries

- $g\big(Sick(X), Treat(X,P), Effectivenss(P)\big)$



- $Q = Treat(X,M)$
  - LVE grounds one logical variable
- $Q = Sick(X), Treat(X,M), Effectivenss(M)$
  - LVE grounds one logical variable
  - Compare forbidden queries in PDBs (Dalvi & Suciu 12)

Extending LVE and LJT

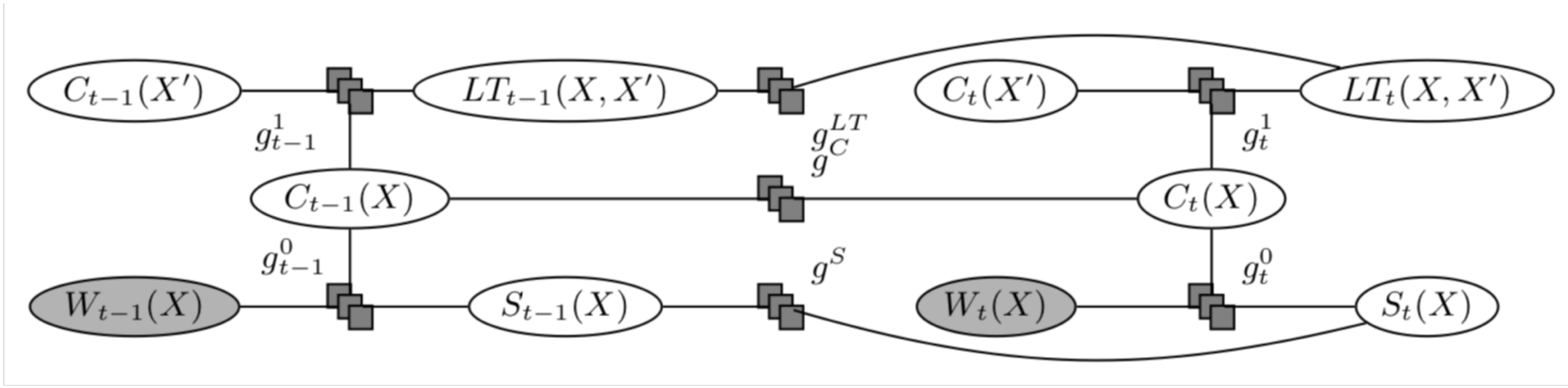
Additional inference targets

- Most probable assignment (MPE, MAP)
  - New argmax operators
- Dynamic inference (filtering, prediction, hindsight)
  - Lifted dynamic junction tree algorithm
- Adaptive inference (incremental changes)
  - Evidence, model structure, parfactors
    $\longrightarrow$ Adaptive steps of LJT

What about time?

Murphy (2002), Gehrke et al. (2018, 2019)

- Based on interface algorithm, dynamic Bayes nets
- Dynamic parfactor graphs
  - Another application area: Health care



- Lifted Dynamic Junction Algorithm
  - Answer filtering, prediction, hindsight queries

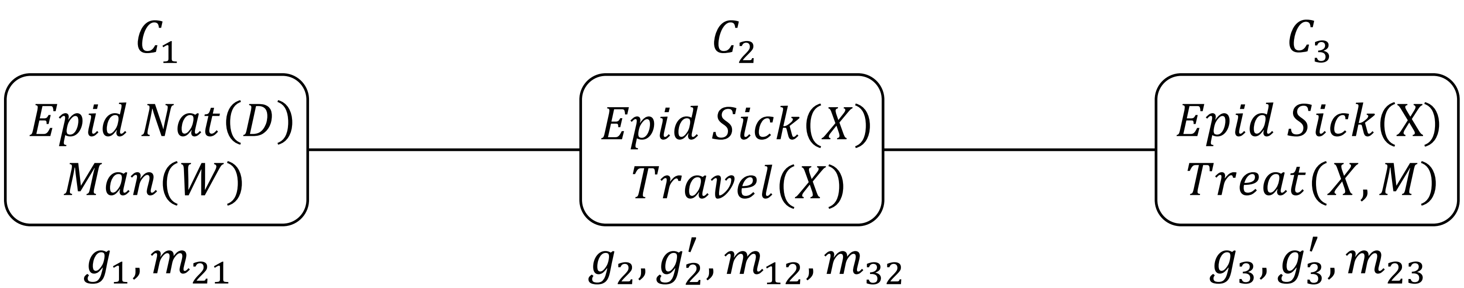
Does it have to be LVE in LJT?

Braun and Möller (2018c)

LJT with LVE &

First-order Knowledge Compilation (FOKC)
to solve a WFOMC problem

- LVE for evidence entering and message passing
- FOKC for query answering



Does it work in practice?

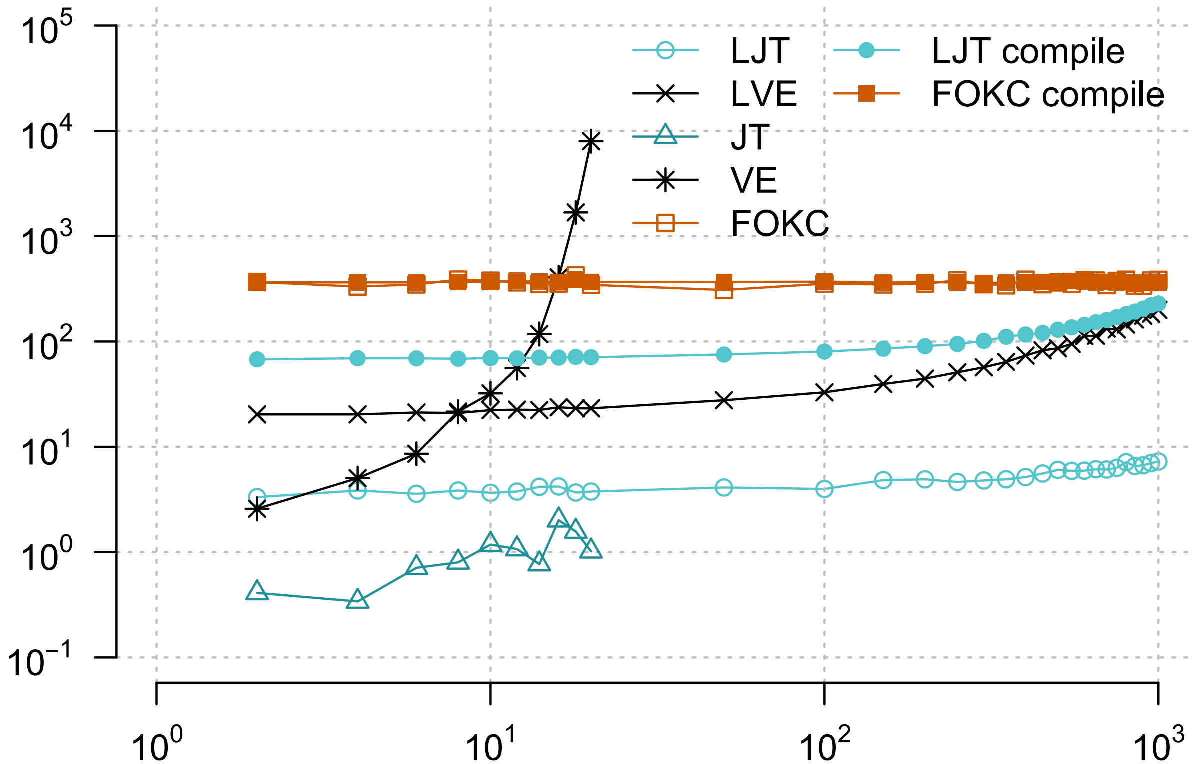
- Prototype implementation
  - LVE by Taghipour
    https://dtai.cs.kuleuven.be/software/gcfove
  - LJT
    (in preparation)

MISSING

Scalable implementation with DB backing

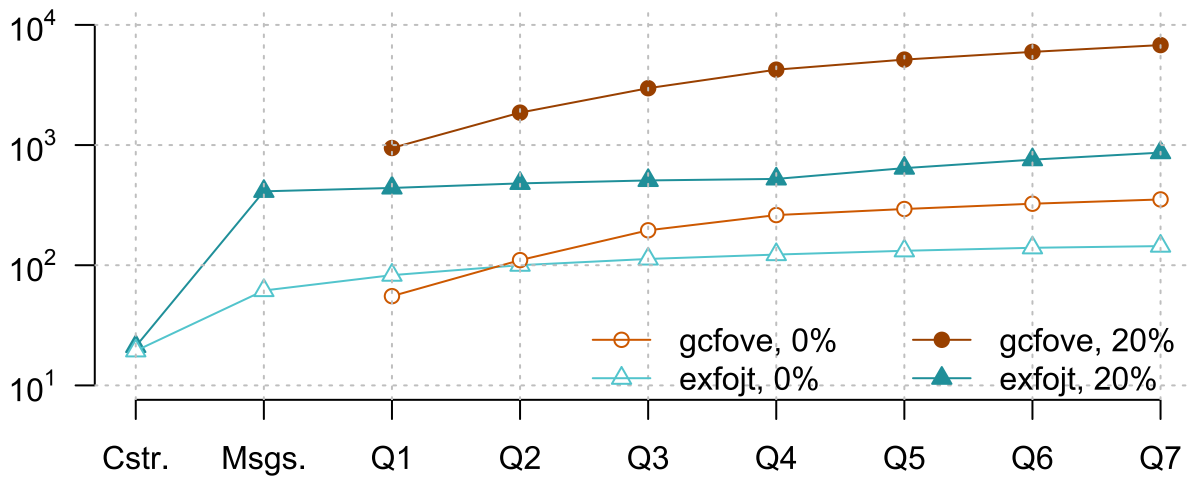
Message passing = Propagation in advance
as in TensorLog (Cohen 2016, Cohen et al. 2017)

Grounded vs. Lifted Inference



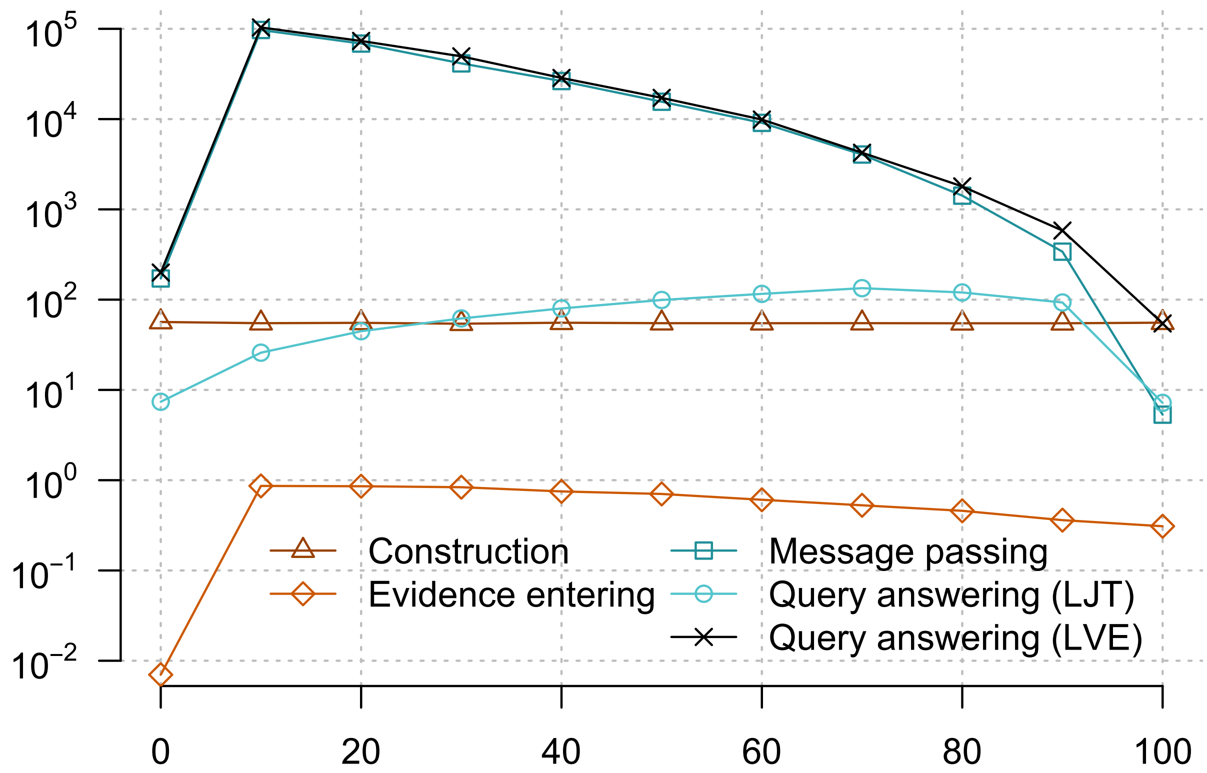
Runtimes in milliseconds, one query, domain sizes between 2 and 1000, evidence at 0%

LJT: Payoff versus LVE



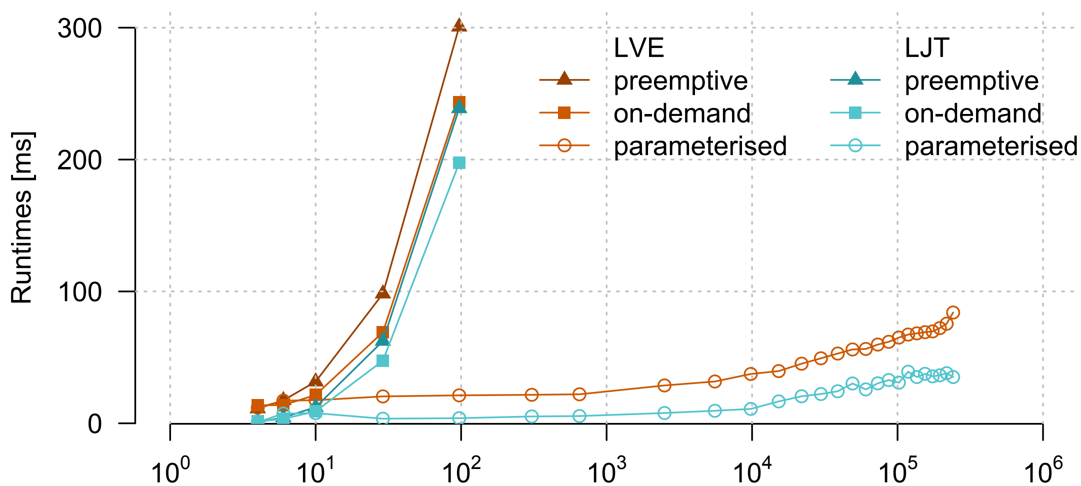
Accumulated runtimes in milliseconds , seven queries, grounded model size: 100,000
Evidence at 0% and 20% on PRVs with one parameter

LJT: Evidence



Runtimes in milliseconds for each LJT step
Evidence from 0% to 100% in 10% steps on PRVs with one parameter

LJT: Parameterised Queries



Runtimes in milliseconds, one query, grounded model size between 10 and 100,000

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Outlook

- Continue optimising
  - Parallelisation
  - Caching
- From discrete over interval to continuous ranges
- Learning?
  - Structure
  - Potentials
  - Symmetries
  - *Transfer learning*
- Open world?
  - Unknown domains
  - Unknown behaviour

Wrap-up Exact Lifted Inference

- Parfactor models for sparse encoding
  - Factorisation of full joint distribution
  - Logical variables to model objects
- Algorithms for exact query answering
  - LVE for single inference
  - LJT for repeated inference
  - Extensions possible
    - Parameterised, conjunctive queries
    - Temporal models
    - Incremental changes
    - Assignment queries

Next: Approximate Lifted Inference

Mission and Schedule of the Tutorial*

Providing an overview and a synthesis of StaR AI

- Introduction — 10 min ✓
  - StaR AI
- Overview: Probabilistic relational modeling — 40 min ✓
  - Semantics (grounded-distributional, maximum entropy)
  - Inference problems and their applications
  - Algorithms and systems
  - Scalability (limited expressivity, knowledge compilation, approximation)
- Scalability by lifting
  - Exact lifted inference — 40+50 min ✓
  - Approximate lifted inference — 30 min
- Summary — 10 min

*Thank you to the SRL/StaRAI crowd for all their exciting contributions! The tutorial is necessarily incomplete. Apologies to anyone whose work is not cited

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Work @ IFIS

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Work @ IFIS

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  Marcel Gehrke, Tanya Braun, and Ralf Möller. Lifted Dynamic Junction Tree Algorithm. In *Proceedings of the International Conference on Conceptual Structures*, 2018.
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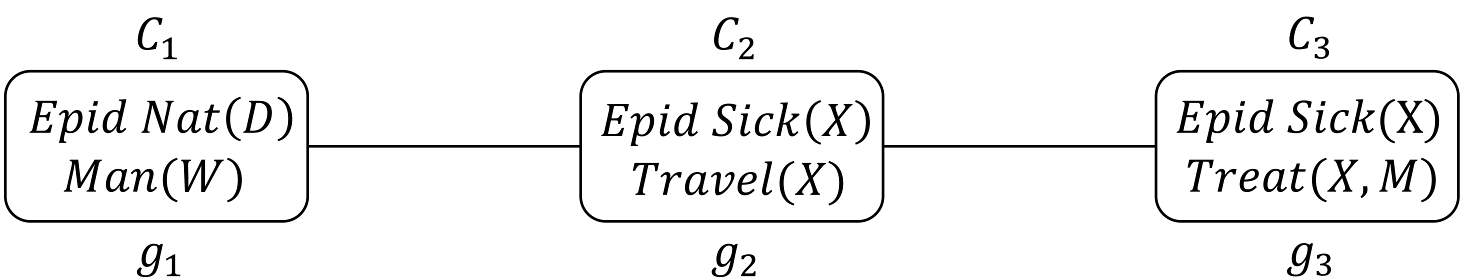
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Relational Forward Backward Algorithm for Multiple Queries. In *FLAIRS-32 Proceedings of the 32nd International Florida Artificial Intelligence Research Society Conference*, 2019.

Appendix

Extensions to LVE and LJT

New Evidence? Back to Step 2

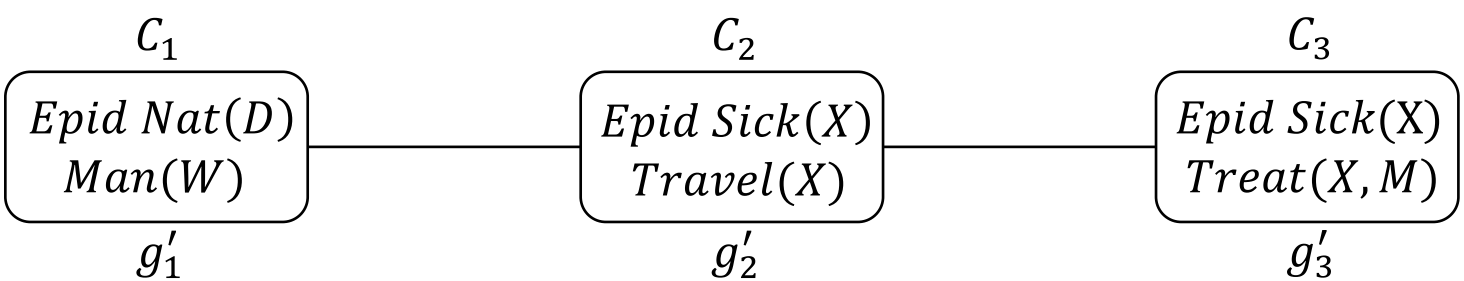
- Input
  - Model G
  - Evidence $\boldsymbol{E} = \{Epid = true\}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G



  2. Enter evidence $\boldsymbol{E}$ into J

And continue…

- Input
  - Model G
  - Evidence $\boldsymbol{E} = \{Epid = true\}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J



  3. Pass messages

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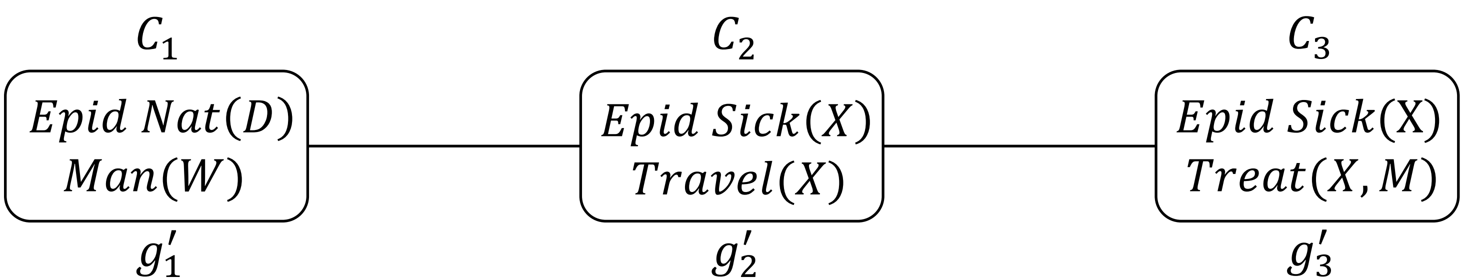
What about incremental changes?

Braun and Möller (2018)

- Input
  - Model G
  - Evidence $\boldsymbol{E} = \{Epid = true\} \cup \{Travel(eve) = true\}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G

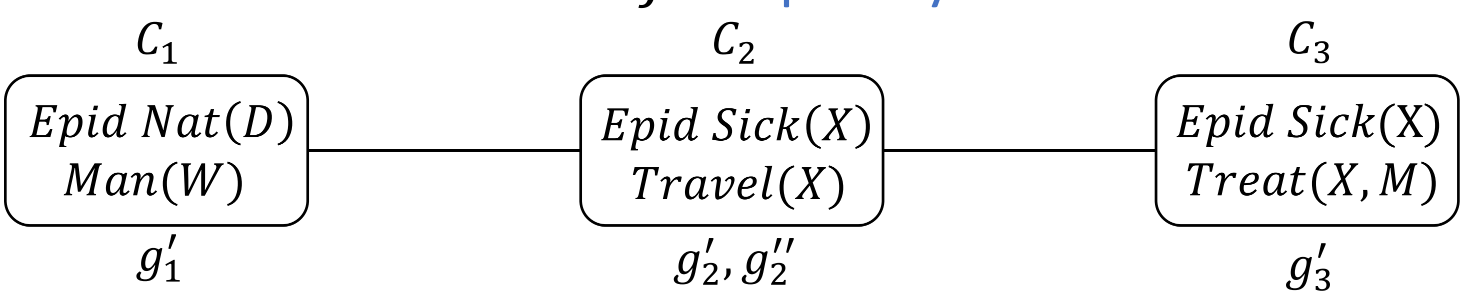


  2. Enter evidence $\boldsymbol{E}$ into J incrementally

What about incremental changes?

Braun and Möller (2018)

- Input
  - Model G
  - Evidence $\boldsymbol{E} = \{Epid = true\} \cup \{Travel(eve) = true\}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J adaptively



  3. Pass messages adaptivly

Adaptive Messages

Braun and Möller (2018)

- Calculate new message only if
  - Local model changed
  - Incoming message changed
  - Existing messages invalid
- Else
  - Send an empty message
  - Existing messages still valid

C_3 calculates new messages m_{21}, m_{23}

C_1, C_2 send empty messages

C_1: $Epid\ Nat(D)$, $Man(W)$ — g_1', m_{21}

C_2: $Epid\ Sick(X)$, $Travel(X)$ — $g_2', g_2'', m_{12}, m_{32}$

C_3: $Epid\ Sick(X)$, $Treat(X, M)$ — g_3', m_{23}

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Changes: Adaptive Inference

Braun and Möller (2018)

- Changes
  - Domains
  - Parfactors
    - Changing the structure: new/removed argument
    - Without changing the model structure: new range
  - Evidence (additional, retracted, new)
- Adaptive LJT
  - Adapt FO jtree J to changes in G
  - Adaptively enter evidence
    - Changes in evidence $\boldsymbol{E}$
    - Changes in J
  - Adaptively pass messages
  - Answer queries

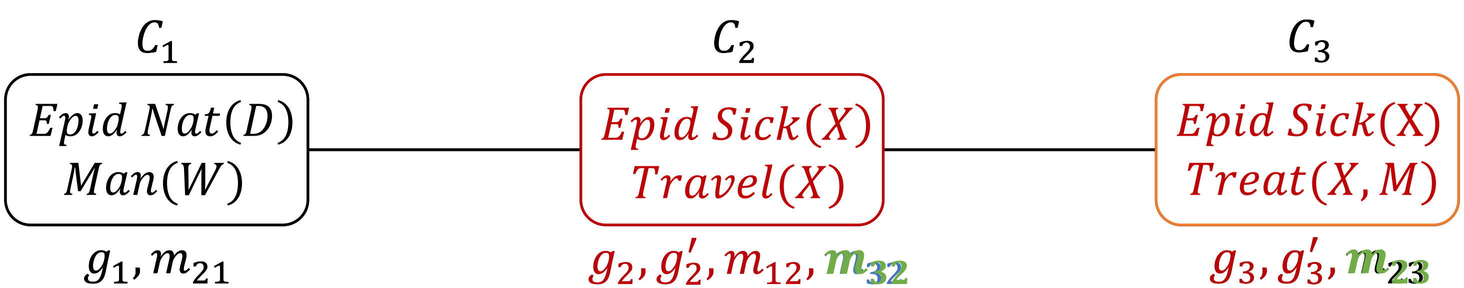
J may require reconstruction after a while

LJT: Conjunctive queries?

Koller and Friedman (2009), Braun and Möller (2017a)

- Conjunctive query: $Epid \wedge Travel(eve)$
  - QA as before: Eliminate all non-query variables

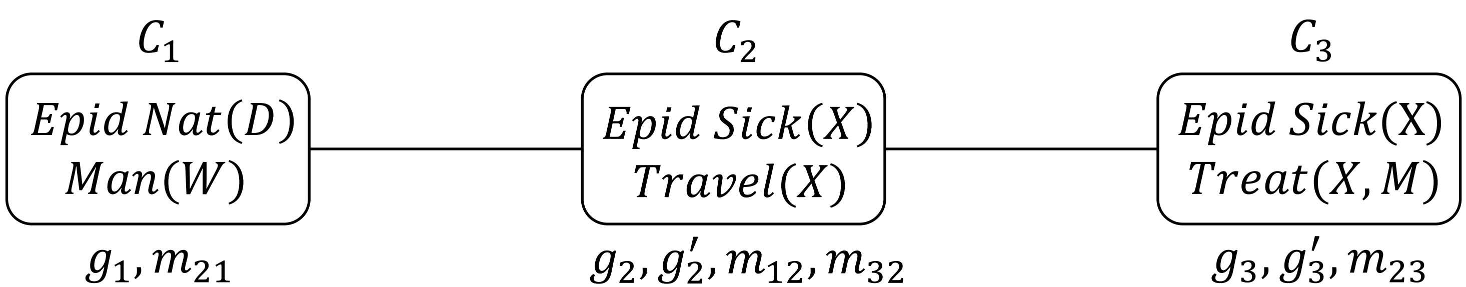
- Conjunctive query: $Travel(eve) \wedge Treat(eve, m_1)$
  - **Query variables not in one parcluster!**
  - **Duplicate information in messages!**



LJT: Answer Conjunctive Queries

Koller and Friedman (2009), Braun and Möller (2017a)

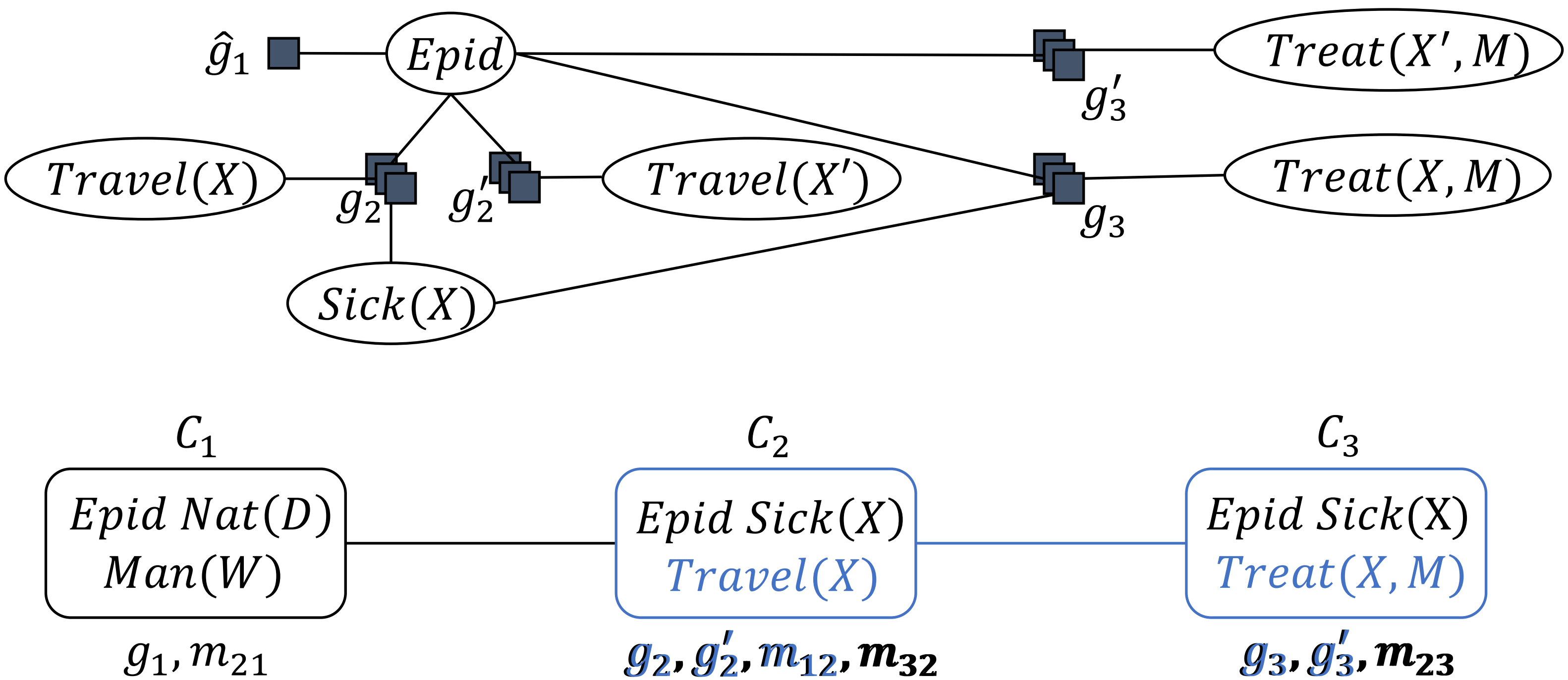
- Queries $\boldsymbol{Q} = \{Q_1, Q_2, Q_3, Q_4\}$
  - $Q_1 = Travel(eve) \land Treat(eve, m_1)$
  - $Q_2 = Epid \land Travel(eve)$
  - $Q_3 = Travel(eve)$
  - $Q_4 = Epid$
- For each query Q
  - Find subtree that covers the query variables
  - Extract submodel without duplicate information
  - Use LVE to answer query



LJT: Answer Conjunctive Queries

Braun and Möller (2017a)

- $Q_1 = Travel(eve) \wedge Treat(eve, m_1)$
  - Find subtree that covers the query variables
  - Extract submodel without duplicate information
  - Use LVE to answer query

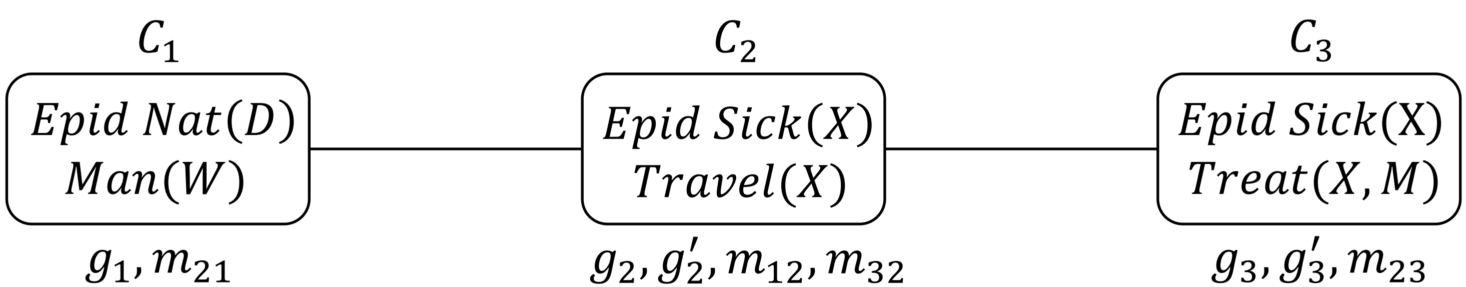


LJT: Answer Conjunctive Queries

Braun and Möller (2017a)

- Queries $\boldsymbol{Q} = \{Q_1, Q_2, Q_3, Q_4\}$
  - $Q_1 = Travel(eve) \wedge Treat(eve, m_1)$
  - $Q_2 = Epid \wedge Travel(eve)$
  - $Q_3 = Travel(eve)$
  - $Q_4 = Epid$

  Subtree = Single parcluster
- For each query Q
  - Find subtree that covers the query variables
  - Extract submodel without duplicate information
  - Use LVE to answer query



QA: Most probable assignment

Dawid (1992), Dechter (1999), de Salvo Braz et al. (2006), Apsel and Brafman (2012), Braun and Möller (2018b)

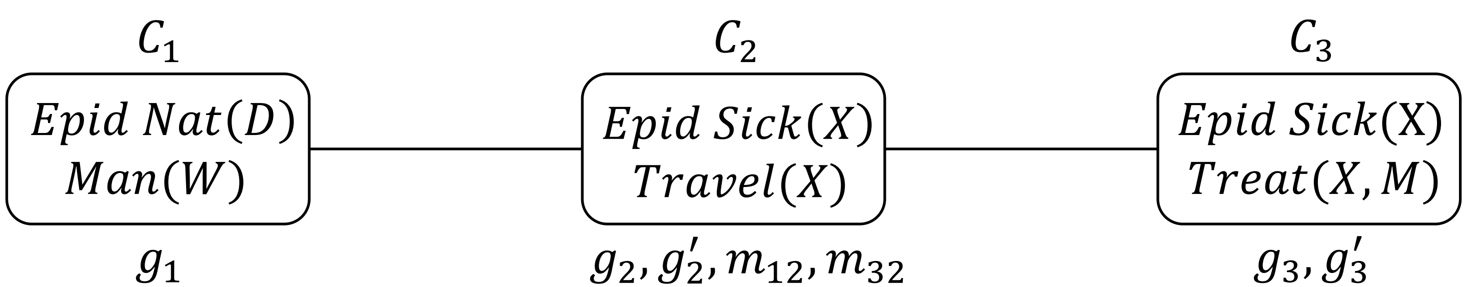
- To all variables: most probable explanation (MPE)

$$\underset{rv(G)}{\operatorname{argmax}} P_G$$

  - (L)VE: maxing out (instead of summing out)
  - (L)JT: message calculation by maxing out
  - Ismorphic instances: identical most probable assignment
- As before: construction, evidence entering
- Message passing
  - Only one message pass (from periphery to centre)
  - Max out remaining variables at centre

LJT: MPE

- $Epid = false$
- $\forall D, W : Nat(D) = false, Man(W) = false$
- $\forall X' \in \{alice, eve\}, P : Travel(X') = true, Treat(X', P) = true$
- $\forall X, M : Travel(X) = true, Sick(X) = true, Treat(X, M) = true$



QA: Most probable assignment

Dechter (1999), Sharma et al. (2018)
Braun and Möller (2018b)

- To subset of variables: maximum a posteriori (MAP)

$$\underset{Travel.eve,Epid}{\operatorname{argmax}} \sum_{\substack{r(Nat(D),Man(W),Sick(X),Treat(X,P)),\\ r(Travel(X)),X\neq eve}} P_G$$

  - *MAP a more general case of MPE*
  - **$\sum$ and $\max$ not commutative!**
- As before: construction, evidence entering, message passing (with summing out)
- Answer MAP query: get submodel (as before)
  - Sum out non-query variables, max out query variables
- Assignment to complete parclusters safe

Does LJT lift what is liftable?

Braun and Möller (2017)

- Original LJT had avoidable groundings

C_1: $Epid\ Nat(D)$ $Man(W)$ — g_1, m_{21}

C_2: $Epid\ Sick(X)$ $Treat(X,M)\ Eff(M)$ — $g_2, g_2', m_{12}, \boldsymbol{m_{32}}$

$Treat(X,M)$

C_3: $Precond(X)$ $Treat(X,M)$ — $g_3, g_3', \boldsymbol{m_{23}}$

- After fusion, answer is YES!

C_1: $Epid\ Nat(D)$ $Man(W)$ — g_1, m_{21}

C_2: $Epid\ Sick(X)\ Precond(X)$ $Treat(X,M)\ Eff(M)$ — $g_2, g_2', g_3, g_3', m_{12}$

Appendix

FO jtree construction

FO Jtree Construction

- Propositional junction tree construction
  - Triangulation, compute maximum spanning tree, …
  - Hypergraph partitioning
  - Decomposition tree (dtree), clusters, …
- First-order: logical variables

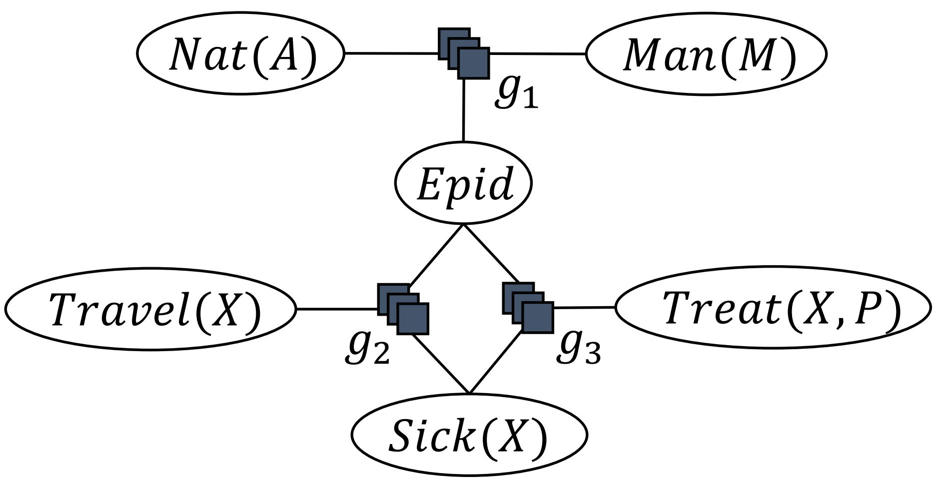
Taghipour et al. (2013b)

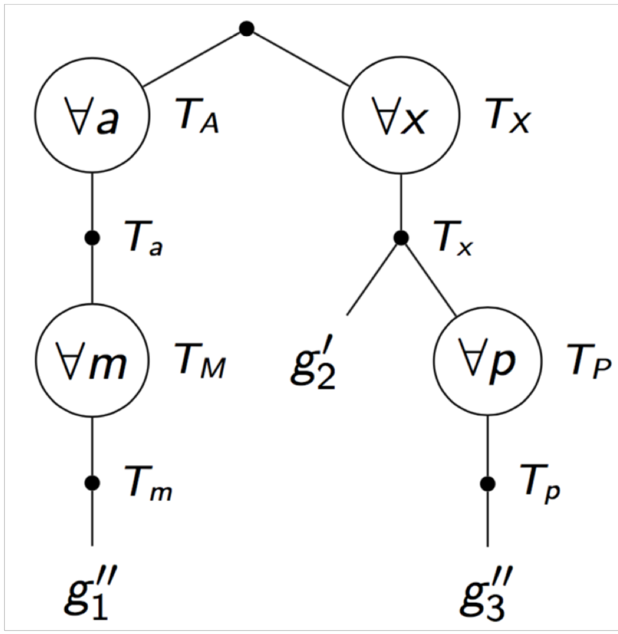
  - First-order decomposition trees (FO dtrees)
  - FO dtrees have node properties (cutset, context, cluster)
  - (FO) dtree + clusters = (FO) jtree
  - Heuristic to build an FO dtree
    (logical variables guide the construction)

FO Dtree: Data Structure

Taghipour et al. (2013b)

- (L)VE decomposes model into subproblems
- Represent as tree





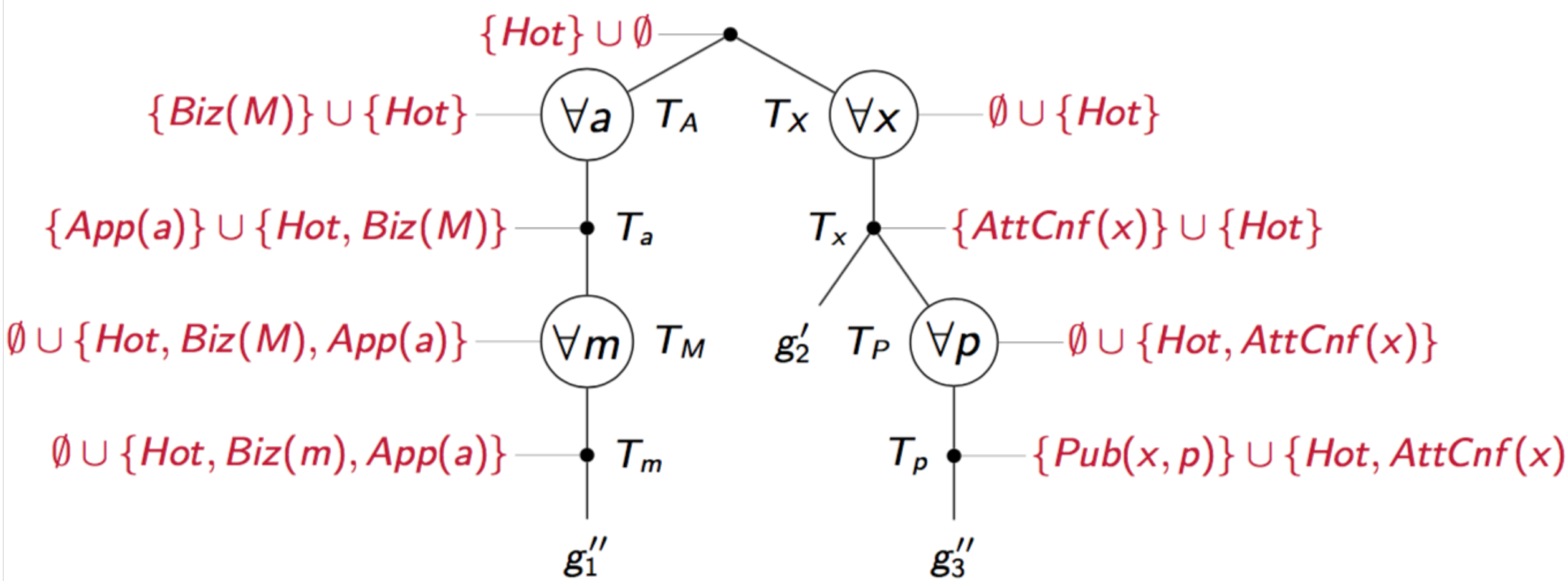
FO Dtree: Cutset, Context, Cluster

Taghipour et al. (2013b)

$$cutset(T) = \bigcup_{T_i, T_j \in child(T)} RV(T_i) \cap RV(T_j) \setminus acutset(T),\ acutset(T) = \bigcup_{T' \in ancestor(T)} cutset(T')$$

$$context(T) = RV(T) \cap acutset(T),\ RV(T) = \bigcup_{T' \in child(T)} RV(T'),\ RV(L) = RV(\phi_L),\ L \text{ leaf}$$

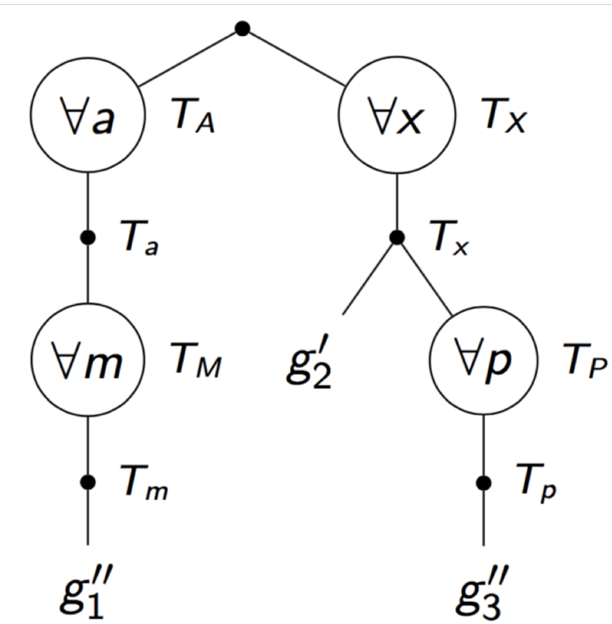
$$cluster(T) = cutset(T) \cup context(T),\ cluster(L) = RV(\phi_L),\ L \text{ leaf}$$

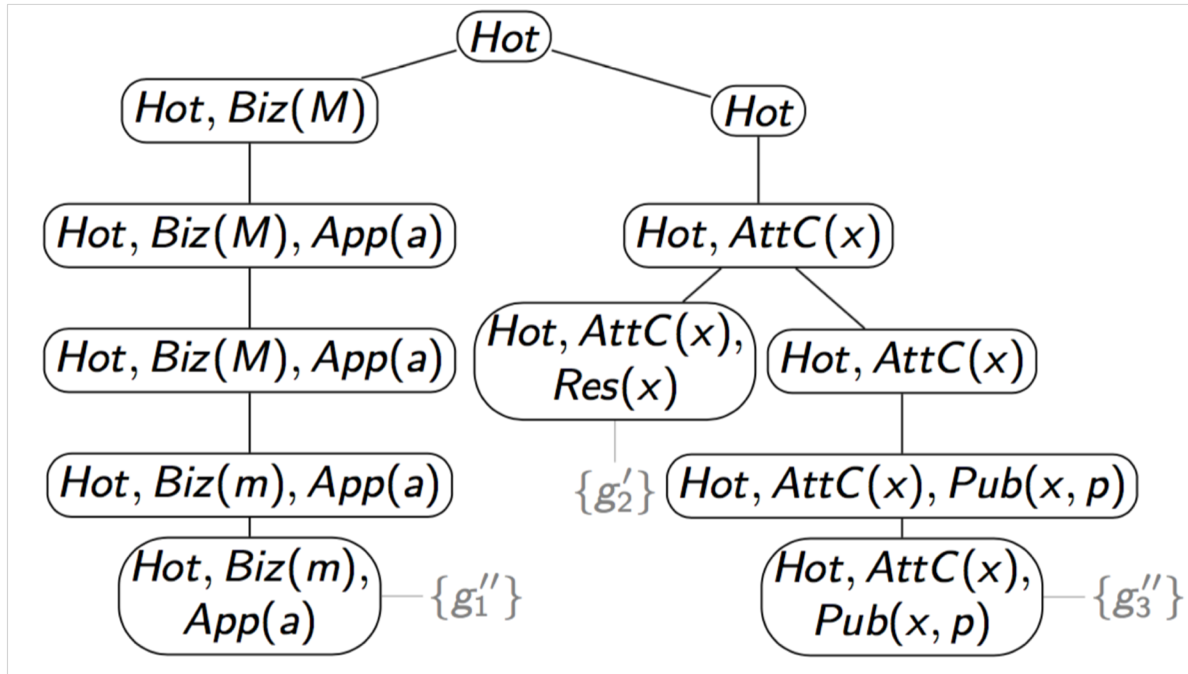


FO Dtree to FO Jtree

Braun and Möller (2016)

- Compute clusters per node and minimise





Minimising an FO Jtree

Braun and Möller (2016)

- FO jtree is minimal
  - If by removing a variable from any parcluster, the FO jtree stops being an FO jtree
- Merge parclusters
  - If neighbouring parclusters are subsets of each other