StarAI

Semantics & Symmetries in Exact Lifted Inference

Tutorial ECAI 2020

Tanya Braun, Marcel Gehrke, Ralf Möller
Universität zu Lübeck

UNIVERSITÄT ZU LÜBECK

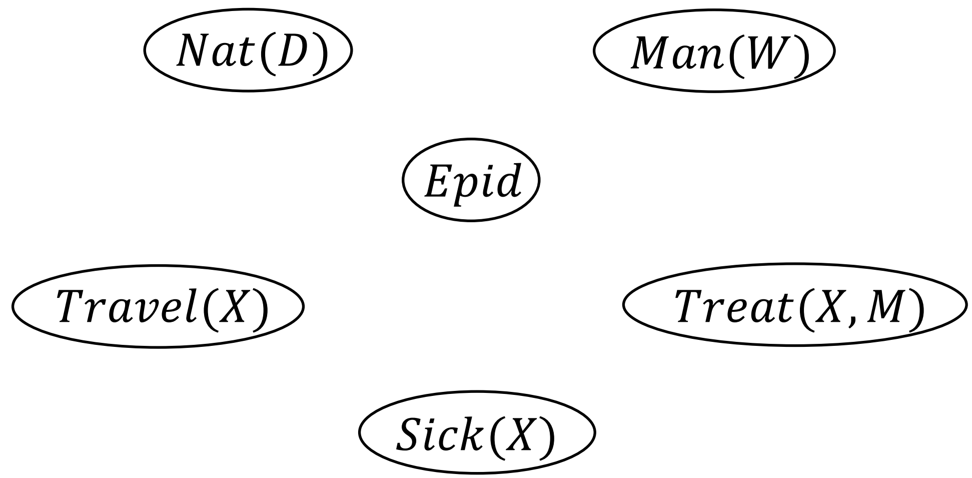
Agenda

- Probabilistic relational models (PRMs) [Ralf]
  - Application example
  - Semantics, static vs. dynamic behavior
  - Query answering / basic inference
  - Variants: raw PRMs, MLNs, …
- Exact symmetries and changing domains in static PRMs [Tanya]
- Stable inference over time in dynamic PRMs [Marcel]
- Summary [Tanya]

Goal: Overview of central ideas

Application: Epidemics

- Atoms: Parameterised random variables = PRVs
  - With logical variables
    - E.g., X, M
    - Possible values (domain):
    $\mathcal{D}(X) = \{alice, eve, bob\}$
    $\mathcal{D}(M) = \{injection, tablet\}$
  - With range
    - E.g., Boolean
    - $range(Travel(X))$
    $r(Travel(X))$



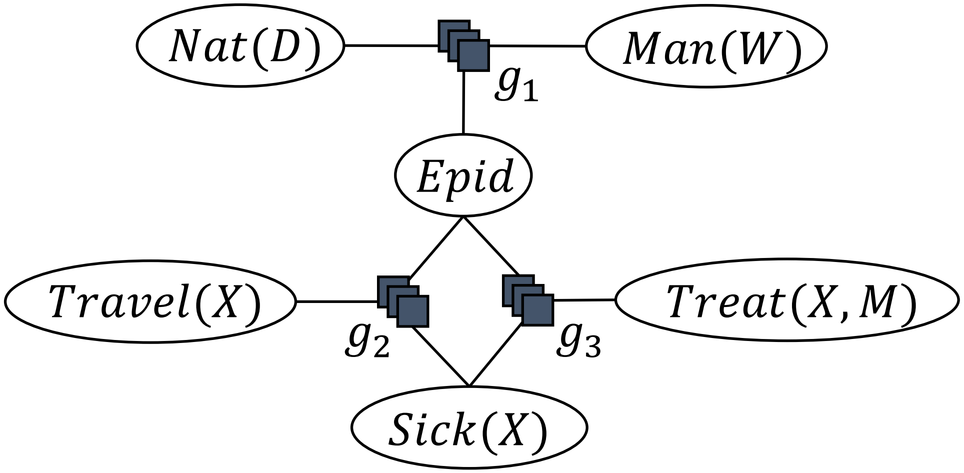
$Nat(D)$ = *natural disaster (D)*
$Man(W)$ = *man-made disaster (W)*

Encoding the Joint Distribution

- Factors with PRVs = parfactors
  - (Graphical) Model G
  - E.g., g_2

$Travel(X)$	$Epid$	$Sick(X)$	g_2
false	*false*	*false*	5
false	*false*	*true*	0
false	*true*	*false*	4
false	*true*	*true*	6
true	*false*	*false*	4
true	*false*	*true*	6
true	*true*	*false*	2
true	*true*	*true*	9

Sparse encoding of joint distribution



Nat(D) = natural disaster (D)
Man(W) = man-made disaster (W)

$3 \cdot 2^3 = 24$ entries in 3 parfactors, 6 PRVs

UNIVERSITÄT ZU LÜBECK

Factors

- Grounding
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

$Travel(X)$	$Epid$

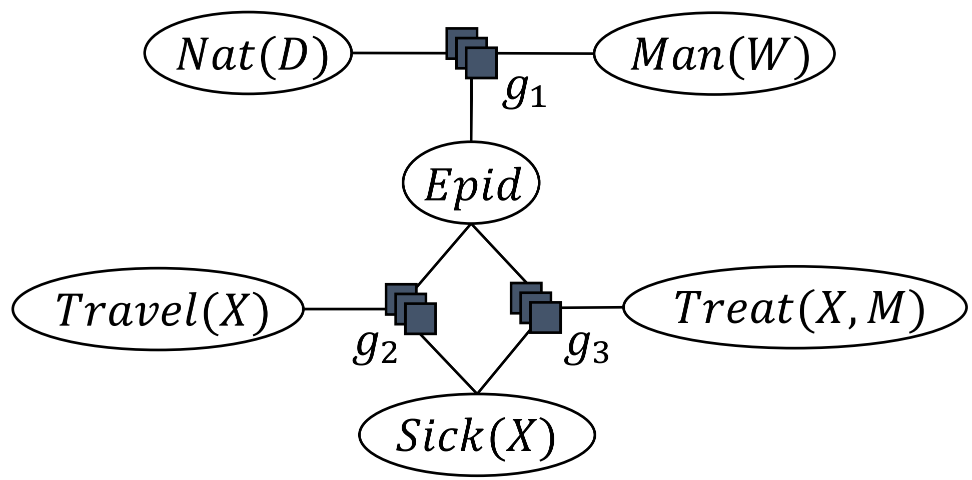
$Travel(alice)$	Epi		
$false$	$fals$		
$false$	$fals$		
$false$	tru		
$false$	tru		
$true$	$fals$		
$true$	$fals$		
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$Travel(eve)$	$Epid$	$Sick(eve)$	f_2^1
false	false	false	5
false	false	true	0
false	true	false	4
false	true	true	6
true	false	false	4
true	false	true	6
true	true	false	2
true	true	true	9

(bob)	$Epid$	$Sick(bob)$	f_2^3
se	$false$	$false$	5
se	$false$	$true$	0
se	$true$	$false$	4
se	$true$	$true$	6
e	$false$	$false$	4
e	$false$	$true$	6
e	$true$	$false$	2
$true$	$true$	$true$	9

Semantics of a PRM

- Joint probability distribution P_G by grounding



$$P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$$

$$Z = \sum_{v \in r(rv(gr(G)))} \prod_{f \in gr(G)} f_i(\pi_{rv(f_i)}(v))$$

$\pi_{variables}(v)$ = projection of v onto *variables*

Multiplication

Multiplication: „join“ over shared randvars and a product of potentials

$$f_2^2(Travel(eve), Epid, Sick(eve)) \cdot f_3^2(Epid, Sick(eve), Treat(eve, m_1)) = f_{23}(Travel(eve), Epid, Sick(eve), Treat(eve))$$

$Travel(eve)$	$Epid$	$Sick(eve)$	f_2^2
false	*false*	*false*	5
false	*false*	*true*	0
false	*true*	*false*	4
false	*true*	*true*	6
true	*false*	*false*	4
true	*false*	*true*	6
true	*true*	*false*	2
true	*true*	*true*	9

•

$Epid$	$Sick(eve)$	$Treat(eve, m_1)$	f_3^2
false	*false*	*false*	9
false	*false*	*true*	1
false	*true*	*false*	4
false	*true*	*true*	2
true	*false*	*false*	2
true	*false*	*true*	6
true	*true*	*false*	6
true	*true*	*true*	9

Multiplication: Join and Product

$Travel(eve)$	$Epid$	$Sick(eve)$	f_2^2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$\bullet$

$Epid$	$Sick(eve)$	$Treat(eve, m_1)$	f_3^2
$false$	$false$	$false$	9
$false$	$false$	$true$	1
$false$	$true$	$false$	4
$false$	$true$	$true$	2
$true$	$false$	$false$	2
$true$	$false$	$true$	6
$true$	$true$	$false$	6
$true$	$true$	$true$	9

$Travel(eve)$	$Epid$	$Sick(eve)$	$Treat(eve, m_1)$	f_{23}
$false$	$false$	$false$	$false$	$5 \cdot 9$

Multiplication: Join and Product

$Travel(eve)$	$Epid$	$Sick(eve)$	f_2^2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

•

$Epid$	$Sick(eve)$	$Treat(eve, m_1)$	f_3^2
$false$	$false$	$false$	9
$false$	$false$	$true$	1
$false$	$true$	$false$	4
$false$	$true$	$true$	2
$true$	$false$	$false$	2
$true$	$false$	$true$	6
$true$	$true$	$false$	6
$true$	$true$	$true$	9

$Travel(eve)$	$Epid$	$Sick(eve)$	$Treat(eve, m_1)$	f_{23}
$false$	$false$	$false$	$false$	$5 \cdot 9$
$false$	$false$	$false$	$true$	$5 \cdot 1$

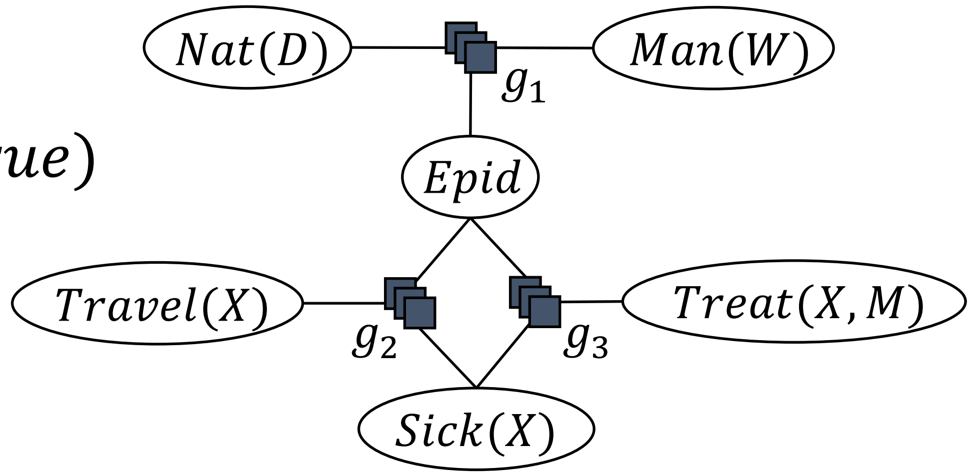
Result

$Travel(eve)$	$Epid$	$Sick(eve)$	$Treat(eve,m_1)$	f_{23}
false	*false*	*false*	*false*	45
false	*false*	*false*	*true*	5
false	*false*	*true*	*false*	0
false	*false*	*true*	*true*	0
false	*true*	*false*	*false*	8
false	*true*	*false*	*true*	24
false	*true*	*true*	*false*	36
false	*true*	*true*	*true*	54
true	*false*	*false*	*false*	36
true	*false*	*false*	*true*	4
true	*false*	*true*	*false*	24
true	*false*	*true*	*true*	12
true	*true*	*false*	*false*	4
true	*true*	*false*	*true*	12
true	*true*	*true*	*false*	54
true	*true*	*true*	*true*	81

Queries

- Marginal distribution
  - $P(Sick(eve))$
  - $P(Travel(eve,)\ Treat(eve,m_1))$
- Conditional distribution
  - $P(Sick(eve)|Epid)$
  - $P(Epid|Sick(eve) = true)$
- Assignment queries
  - MPE
  - MAP

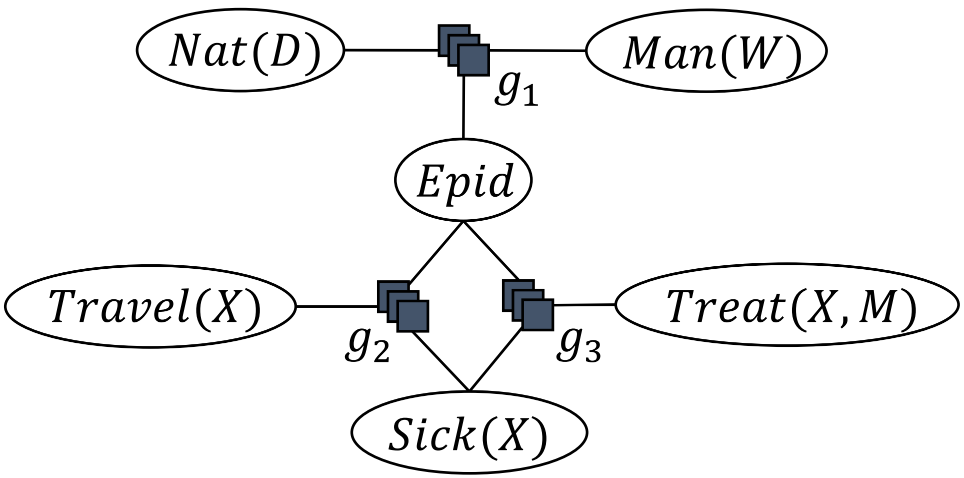
Avoid groundings!



QA: Lifted Variable Elimination (LVE)

[Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a]

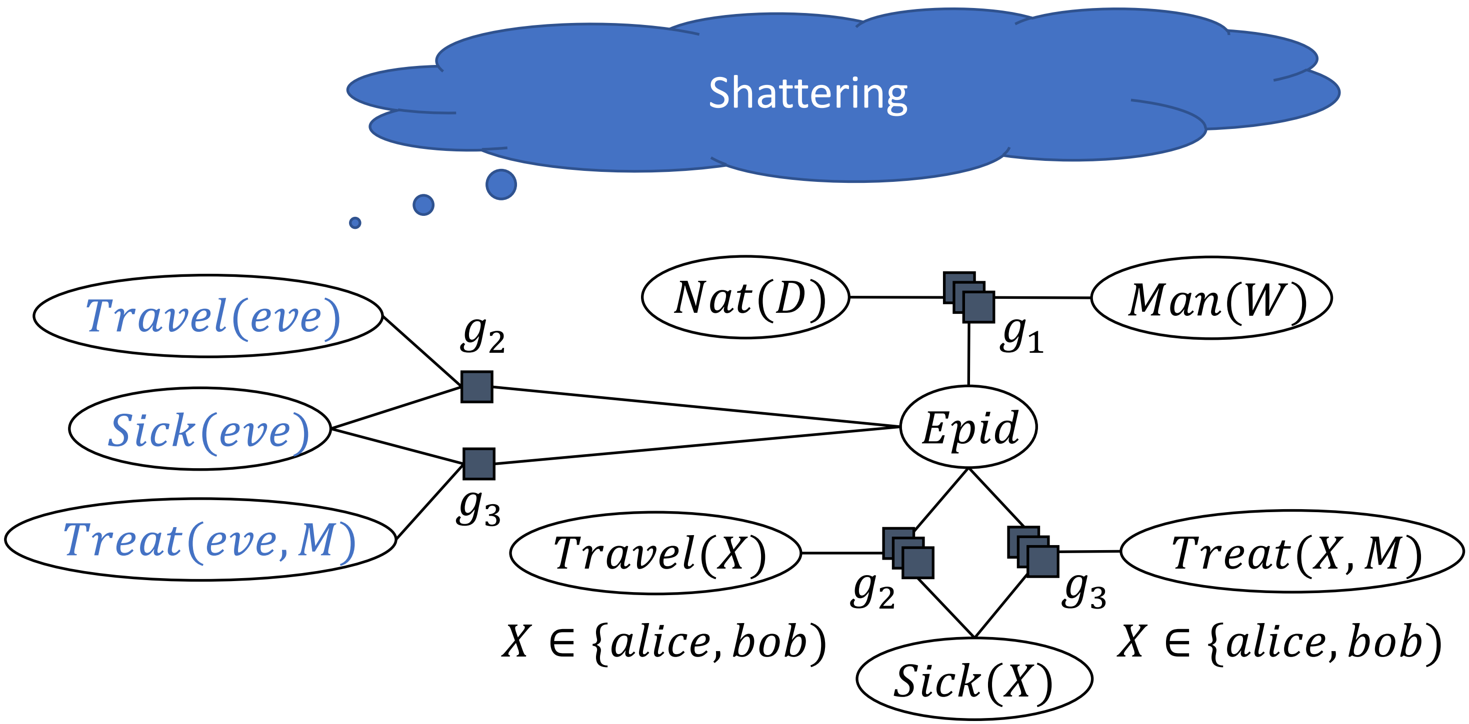
- Eliminate all variables not appearing in query
- Lifted summing out
    1. Sum out representative instance as in propositional variable elimination
    2. Exponentiate result for isomorphic instances



Avoid groundings!

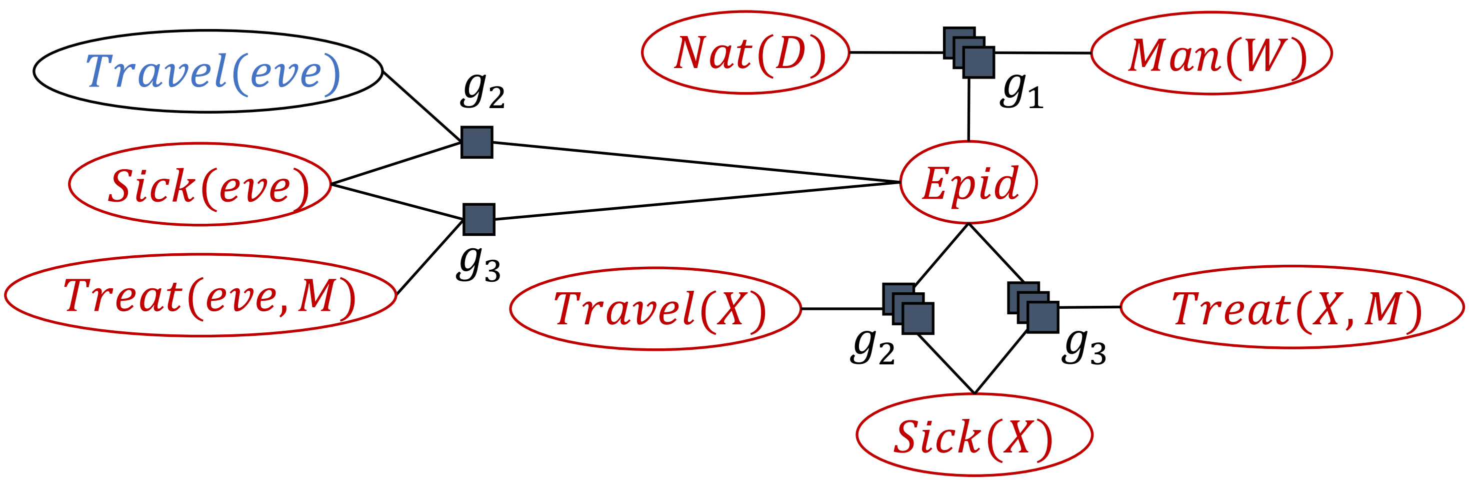
QA: LVE in Detail

- E.g., marginal
  - $P(Travel(eve))$
  - Split atoms $P(\dots, X, \dots)$ w.r.t. eve if eve in $\mathcal{D}(X)$



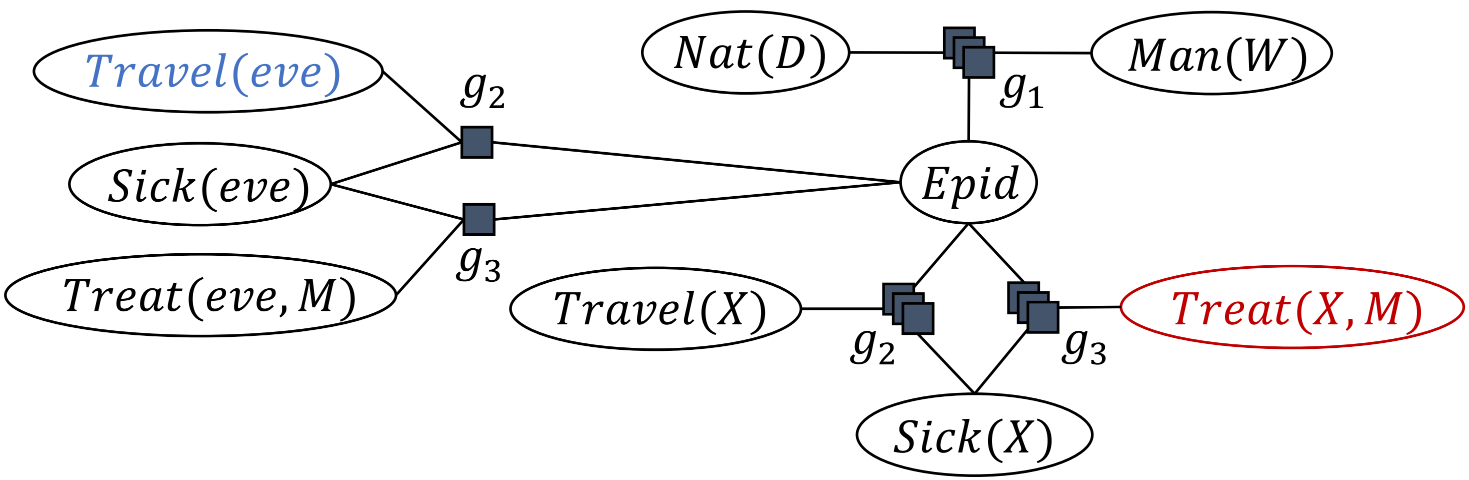
QA: LVE in Detail

- E.g., marginal
  - $P(Travel(eve))$
  - Split atoms $P(\dots, X, \dots)$ w.r.t. eve if eve in $\mathcal{D}(X)$
  - Eliminate all non-query variables



QA: LVE in Detail

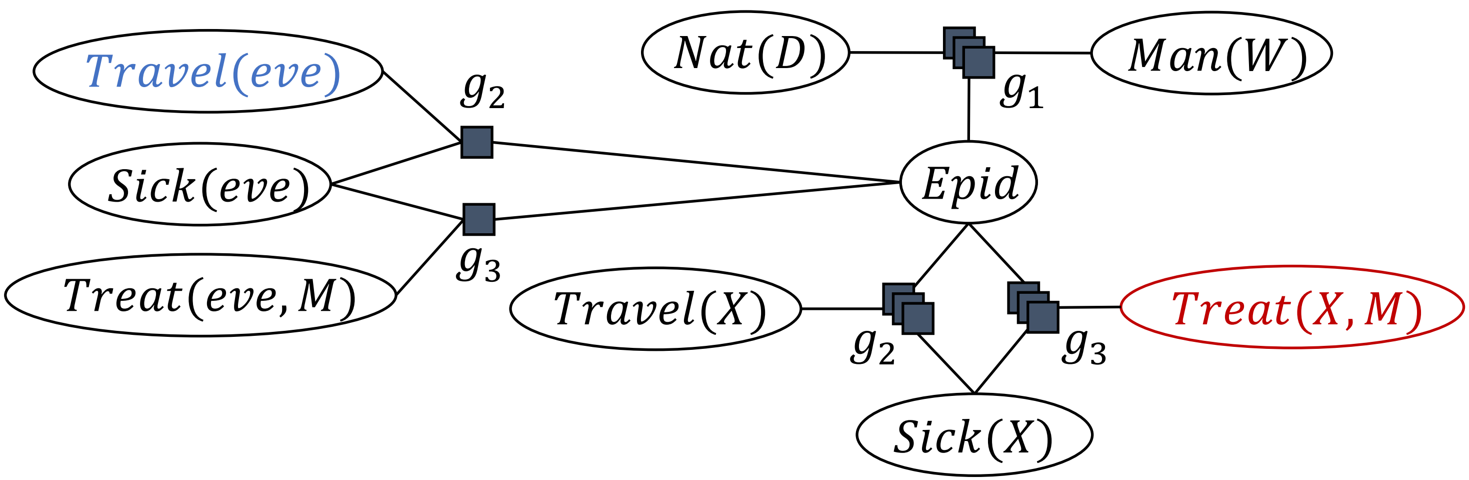
- Eliminate $Treat(X, M)$
  - Appears in only one g: g_3
  - Contains all logical variables of g_3: X, M
  - For each X constant: the same number of M constants
  - ✓ Preconditions of lifted summing out fulfilled, lifted summing out possible



LVE in Detail: Lifted Summing Out

- Eliminate $Treat(X, M)$ by lifted summing out
  1. Sum out representative
  2. Exponentiate for indistinguishable objects

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t) \right)^{|M|}$$



LVE in Detail: Lifted Summing Out

- Eliminate $Treat(X, M)$
  1. Sum out representative
  2. Exponentiate for indistinguishable objects

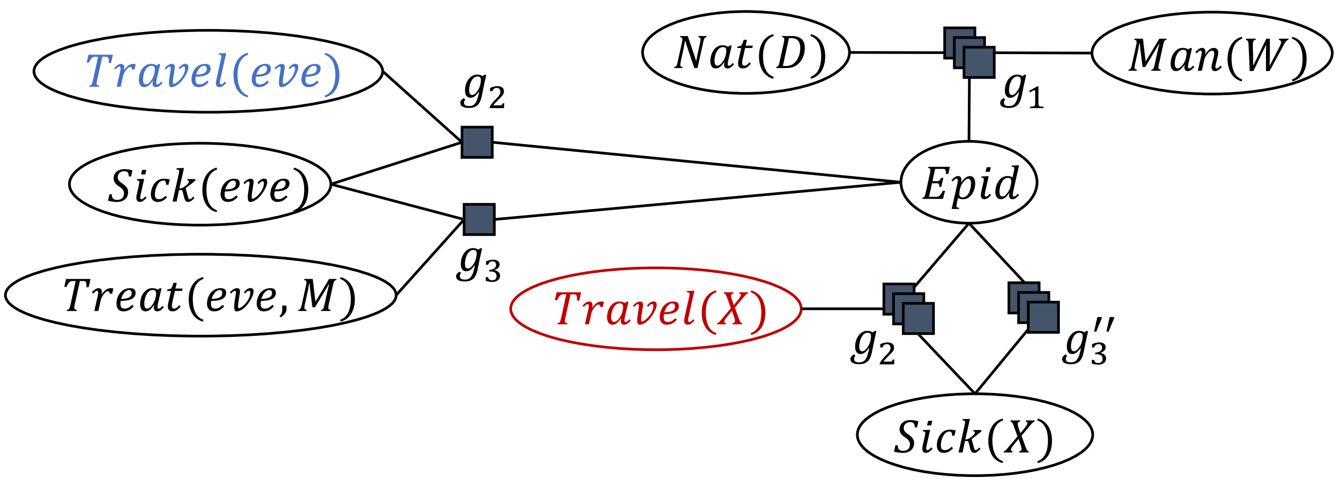
$$\left(\sum_{t \in r(Treat(X,M))} g_3(e, s, t) \right)^{|M|}$$

$Epid$	$Sick(X)$	$Treat(X, M)$	g_3
$false$	$false$	$false$	5
$false$	$false$	$true$	1
$false$	$true$	$false$	3
$false$	$true$	$true$	2
$true$	$false$	$false$	5
$true$	$false$	$true$	4
$true$	$true$	$false$	1
$true$	$true$	$true$	7

$Epid$	$Sick(X)$	g_3'	g_3''
$false$	$false$	6	$6^2 = 36$
$false$	$true$	5	$5^2 = 25$
$true$	$false$	9	$9^2 = 81$
$true$	$true$	8	$8^2 = 64$

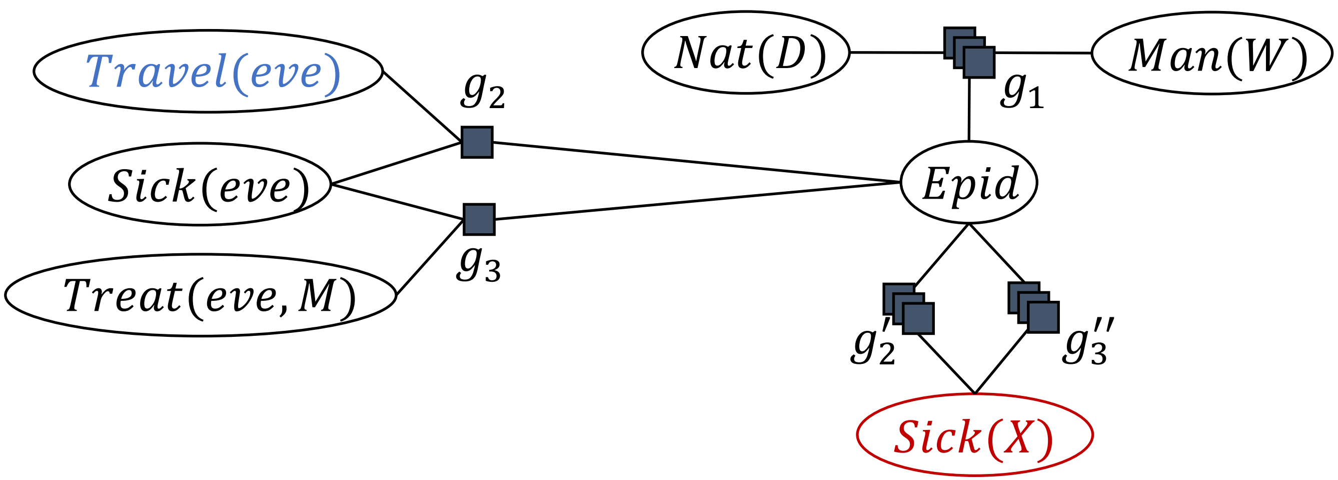
QA: LVE in Detail

- After eliminating $Treat(X, M)$
  - Eliminate $Travel(X)$
    - Does not eliminate logical variable (unlike M)
    - Yields $g_2'\big(Epid, Sick(X)\big)$



QA: LVE in Detail

- After eliminating $Treat(X, M), Travel(X)$
  - Eliminate $Sick(X)$
    - Requires multiplication of g_2' and g_3''
    - Eliminates X
    - Yields g_{23}''



Lifted Multiplication

- Multiplication: „join“ over shared randvars and a product of potentials

$$g'_2(Epid, Sick(X)) \cdot g''_3(Epid, Sick(X)) = g''_{23}(Epid, Sick(X))$$

$Epid$	$Sick(X)$	g'_2
$false$	$false$	5
$false$	$true$	0
$true$	$false$	4
$true$	$true$	6

$\cdot$

$Epid$	$Sick(X)$	g''_3
$false$	$false$	9
$false$	$false$	1
$false$	$true$	4
$false$	$true$	2

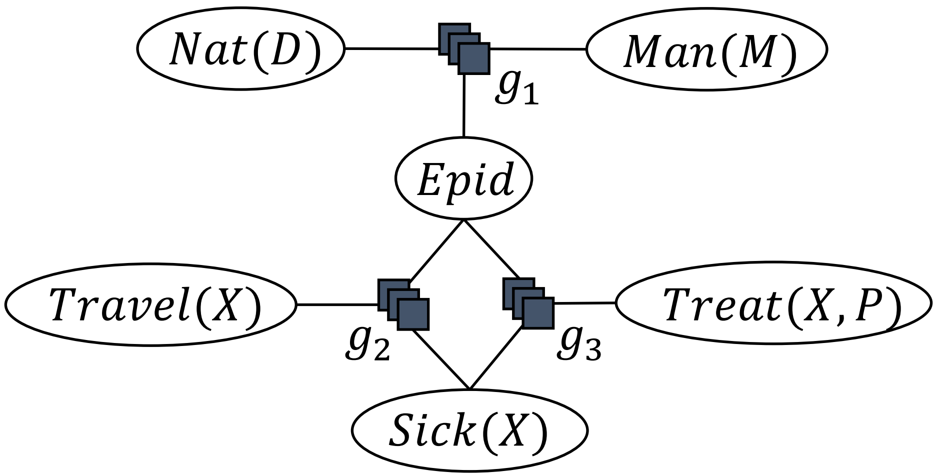
$=$

$Epid$	$Sick(X)$	g''_{23}
$false$	$false$	$5 \cdot 9$
$false$	$false$	$0 \cdot 1$
$false$	$true$	$4 \cdot 4$
$false$	$true$	$6 \cdot 2$

UNIVERSITÄT ZU LÜBECK

Many Queries: LJT

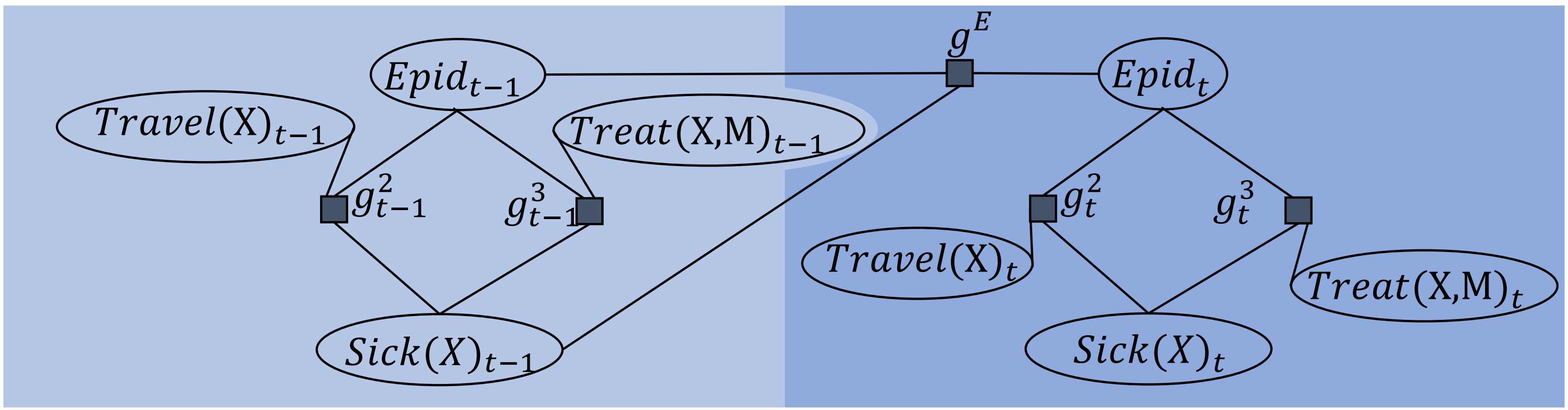
- Set of queries
  - $P(Travel(eve))$
  - $P(Sick(bob))$
  - $P(Treat(eve, m_1))$
  - $P(Epid)$
  - $P(Nat(flood))$
  - $P(Man(virus))$
  - Combinations of variables
- Under evidence
  - $Sick(X') = true$
  - $X' \in \{alice, eve\}$



- **Challenges:**
  - **Do not start from scratch for every query**
  - **Support QA on subset of atoms**
  - **Avoid groundings**
- **Challenges (among others) mastered as part of Tanya‘s dissertation**

Dynamic PRMs

- Marginal distribution query: $P\left(A_{\pi}^{i} \middle| E_{0:t}\right)$ w.r.t. the model:
  - Hindsight: $\pi < t$ (Was there an epidemic $t - \pi$ days ago?)
  - Filtering: $\pi = t$ (Is there currently an epidemic?)
  - Prediction: $\pi > t$ (Will there be an epidemic in $\pi - t$ days?)
- MPE, MAP on temporal sequence



- **Define the interface for relational case (avoid groundings)**
- **Taming reasoning w.r.t. evidence over time (avoid creeping groundings)**

PRMs and Variants

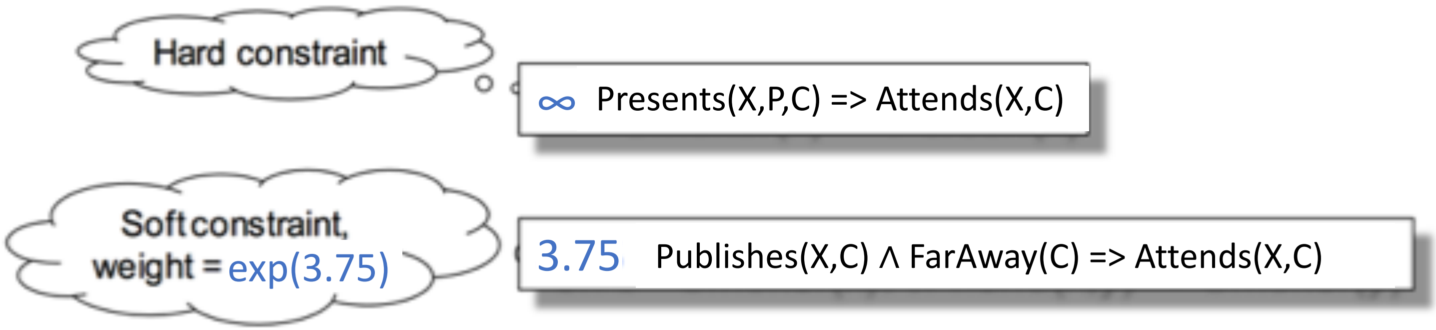
- Probabilistic Relational Models (raw PRMs)
  [Poole 03, Taghipour et al. 13, Braun & M 16-19, Gehrke, Braun & M 18-19]
- Markov Logic Networks (MLNs) [Richardson & Domingos 06]
  - Use logical formulas to specify potential functions
- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Use density functions to specify potential functions

PRMs and Variants

- Probabilistic Relational Models (raw PRMs)
  [Poole 03, Taghipour et al. 13, Braun & M 16-19, Gehrke, Braun & M 18-19]
- Markov Logic Networks (MLNs) [Richardson & Domingos 06]
  - Use logical formulas to specify potential functions
- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Use density functions to specify potential functions

Markov Logic Networks (MLNs)

- Weighted formulas for modelling constraints [Richardson & Domingos 06]



- An MLN is a set of constraints $(w, \Gamma(\boldsymbol{x}))$
  - w = weight
  - $\Gamma(\boldsymbol{x})$ = FO formula
- $weight$ of a world = product of $\exp(w)$ terms
  - for all MLN rules $(w, \Gamma(\boldsymbol{x}))$ and groundings $\Gamma(\boldsymbol{a})$ that hold in that world
- Probability of a world $= \frac{weight}{Z}$
  - Z = sum of weights of all worlds (no longer a simple expression!)

Why exp?

Factorization

- Log-linear models
- Let D be a set of constants and $\omega \in \{0,1\}^m$ a world with m atoms w.r.t. D

$$weight(\omega) = \prod_{\{(w,\Gamma(\boldsymbol{x}))\in MLN \mid \exists \boldsymbol{a}\in D^n : \omega\models\Gamma(\mathbf{a})\}} \exp(w)$$

$$\ln\big(weight(\omega)\big) = \sum_{\{(w,\Gamma(\boldsymbol{x}))\in MLN \mid \exists \boldsymbol{a}\in D^n : \omega \models \Gamma(\boldsymbol{\pi}_\Gamma(\boldsymbol{a}))\}} w$$

  - Sum allows for component-wise optimization during weight learning
- $Z = \sum_{\omega\in\{0,1\}^m} \ln\big(weight(\omega)\big)$
- $P(\omega) = \dfrac{\ln\big(weight(\omega)\big)}{Z}$

Log-linear representation

MLN Reasoning

- Answering queries for *m* discrete atoms

$$P(Attends(alice)) = \sum_{\omega \in \{FALSE, TRUE\}^m \wedge \omega \vDash Attends(alice)} P(\omega)$$

- Approximated lifted reasoning for MLNs
  Lifted Weighted Model Counting [Gogate & Domingos 11]
  Lifted MAP [Sarkhel et al. 14]
- Reasoning: Google AI approach [Cohen et al. 17]
  - Translation to Tensorflow: TensorLog
  - QA based on belief propagation
    (fast but only approximate)

Transforming MLNs into PRMs …

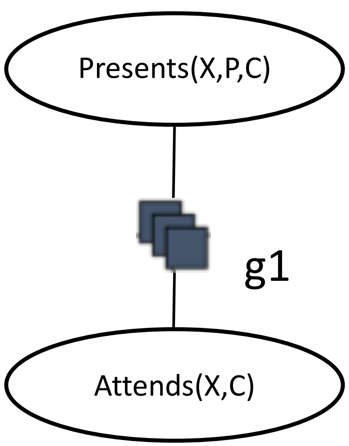
[van den Broeck 13]

Hard constraint

∞ Presents(X,P,C) => Attends(X,C)

Soft constraint, weight = exp(3.75)

3.75 Publishes(X,C) ∧ FarAway(C) => Attends(X,C)



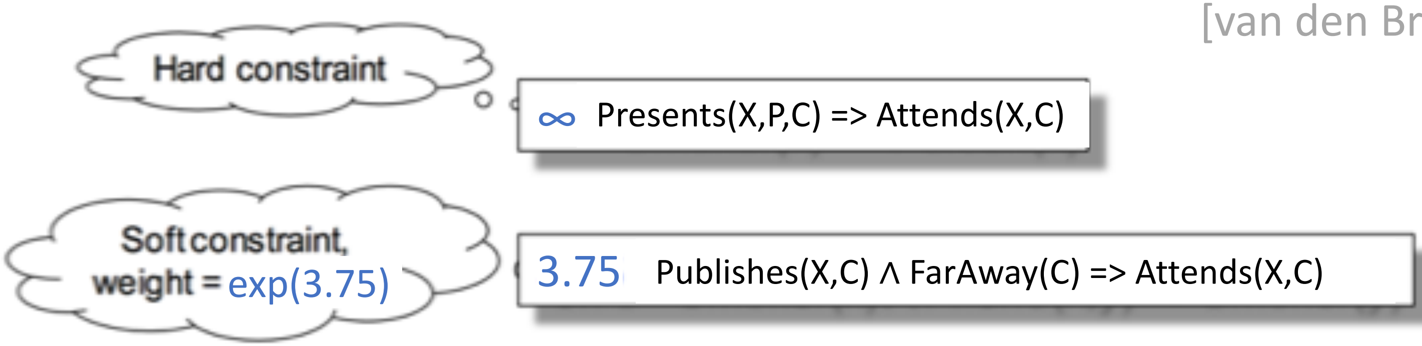
Presents(X,P,C)	Attends(X,C)	g1
TRUE	TRUE	1000
TRUE	FALSE	1
FALSE	TRUE	1000
FALSE	FALSE	1000

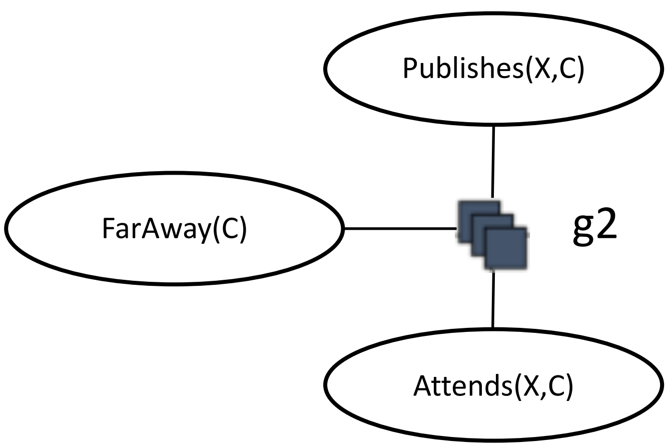
$(X, P, C) \in Dom(X) \times Dom(P) \times Dom(C)$

Possibly shattering required

Transforming MLNs into PRMs …

[van den Broeck 13]

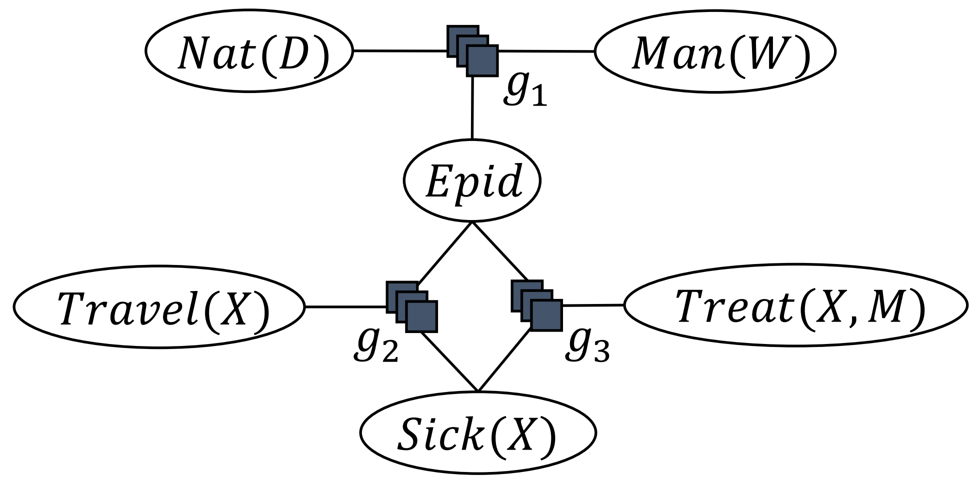




Publishes(X,C)	FarAway(C)	Attends(X,C)	g2
TRUE	TRUE	TRUE	exp(3.75)
TRUE	FALSE	FALSE	exp(3.75)
FALSE	TRUE	FALSE	exp(3.75)
FALSE	FALSE	TRUE	exp(3.75)
TRUE	TRUE	FALSE	1
TRUE	FALSE	TRUE	exp(3.75)
FALSE	TRUE	TRUE	exp(3.75)
FALSE	FALSE	FALSE	exp(3.75)

Encoding a Joint Distribution

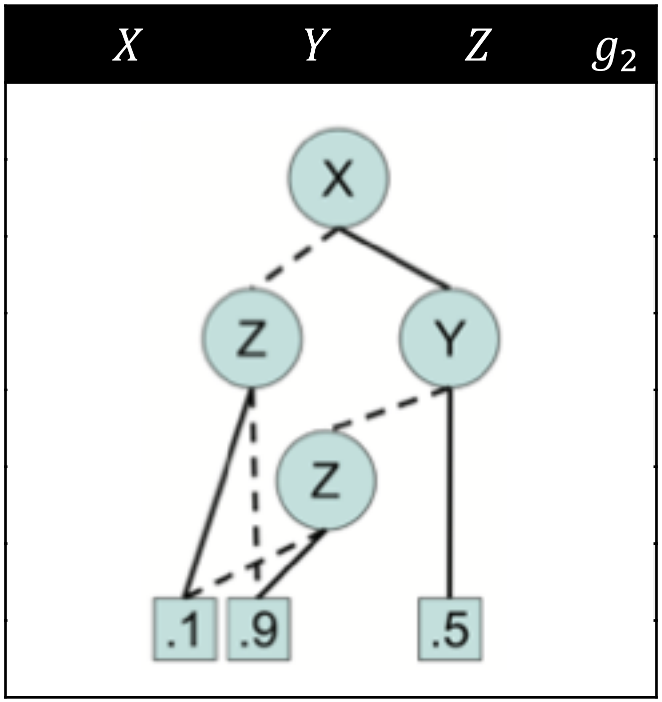
$Travel(X)$	$Epid$	$Sick(X)$	g_2
false	*false*	*false*	5
~~*false*~~	~~*false*~~	~~*true*~~	~~0~~
false	*true*	*false*	4
false	*true*	*true*	6
true	*false*	*false*	4
true	*false*	*true*	6
true	*true*	*false*	2
true	*true*	*true*	9



Nat(D) = natural disaster (D)
Man(W) = man-made disaster (W)

Prefer 0 potentials

Encoding a Joint Distribution

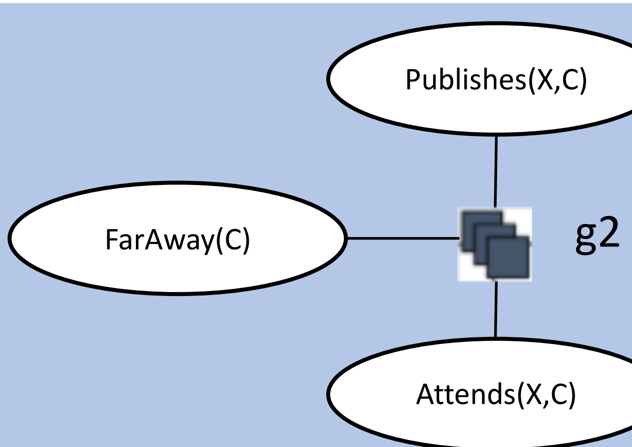


- Alternative: Represent potential functions using algebraic decision diagrams (ADDs)
  [Chavira & Darwiche 07]
  - Requires "multiplications" used for elimination
    to be defined over ADDs

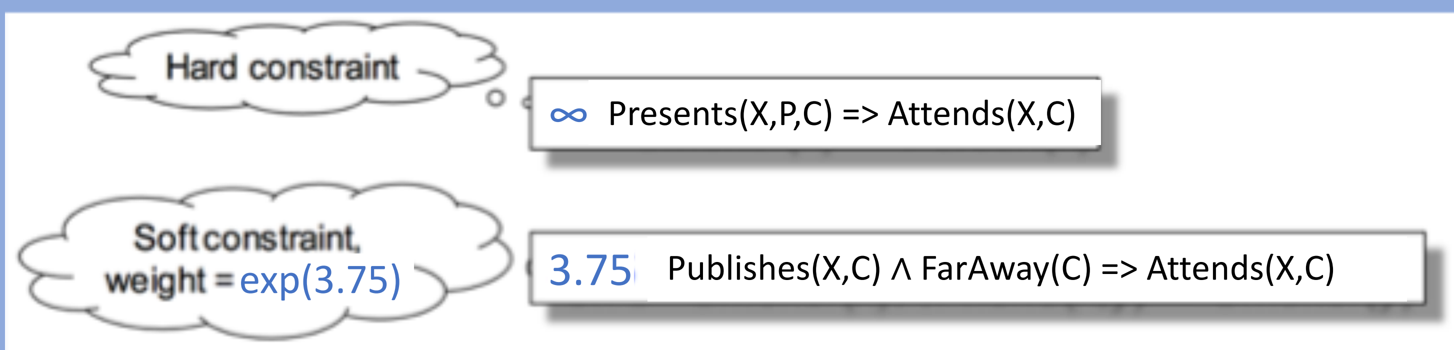
X	Y	Z	$f(.)$
x_1	y_1	z_1	0.9
x_1	y_1	z_2	0.1
x_1	y_2	z_1	0.9
x_1	y_2	z_2	0.1
x_2	y_1	z_1	0.1
x_2	y_1	z_2	0.9
x_2	y_2	z_1	0.5
x_2	y_2	z_2	0.5

MLNs to PRMs and back?

- Represent/approximate potential functions using formulas with parfactor input atoms, e.g., for (approximate) explanation purposes

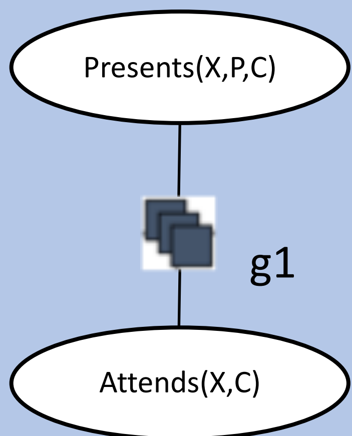


Publishes(X,C)	FarAway(C)	Attends(X,C)	g2
TRUE	TRUE	TRUE	exp(3.75)
TRUE	FALSE	FALSE	exp(3.75)
FALSE	TRUE	FALSE	exp(3.75)
FALSE	FALSE	TRUE	exp(3.75)
TRUE	TRUE	FALSE	1
TRUE	FALSE	TRUE	exp(3.75)
FALSE	TRUE	TRUE	exp(3.75)
FALSE	FALSE	FALSE	exp(3.75)



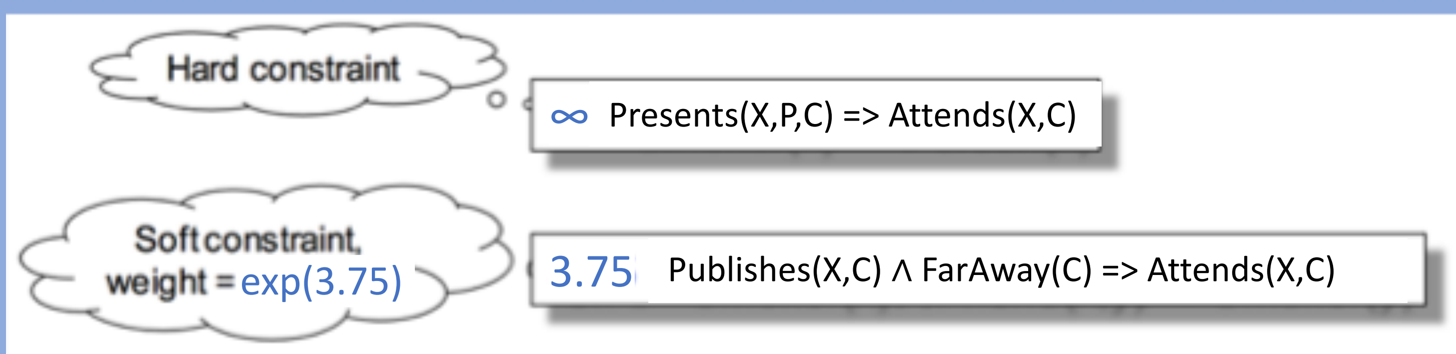
MLNs to PRMs and back?

- Represent/approximate potential functions using formulas with parfactor input atoms, e.g., for (approximate) explanation purposes



Presents(X,P,C)	Attends(X,C)	g1
TRUE	TRUE	1000
TRUE	FALSE	1000
FALSE	TRUE	1
FALSE	FALSE	1000

$(X, P, C) \in \mathrm{Dom}(X) \times \mathrm{Dom}(P) \times \mathrm{Dom}(C)$



Agenda

- Probabilistic relational models (PRMs) [Ralf]
  - Application example
  - Semantics, static vs. dynamic behavior
  - Query answering / basic inference
  - Variants: raw PRMs, MLNs, …
- Exact Symmetries and Changing Domains in static PRMs [Tanya]
- Stable inference over time in dynamic PRMs [Marcel]
- Summary [Tanya]

Bibliography

Alphabetically sorted

Bibliography

- [Ahmadi et al. 13]
Babak Ahmadi, Kristian Kersting, Martin Mladenov, and Sriraam Natarajan. Exploiting Symmetries for Scaling Loopy Belief Propagation and Relational Training. In Machine Learning. 92(1):91-132, 2013
- [Bach, Broecheler, Huang, Getoor 17]
Stephen H. Bach, Matthias Broecheler, Bert Huang, Lise Getoor. Hinge-Loss Markov Random Fields and Probabilistic Soft Logic. In: J. Mach. Learn. Res. 18:. 109:1-109:67, 2017
- [Chavira and Darwiche 07]
Mark Chavira, Adnan Darwiche. Compiling Bayesian Networks Using Variable Elimination. In: Proc. IJCAI 2007: 2443-2449
- [Cohen et al. 17] Cohen, William W., Fan Yang and Kathryn Mazaitis. “TensorLog: Deep Learning Meets Probabilistic DBs.” ArXiv abs/1707.05390 (2017)
- [De Salvo Braz et al. 05]
Rodrigo de Salvo Braz, Eyal Amir, and Dan Roth. Lifted First-order Probabilistic Inference. IJCAI-05 Proceedings of the 19th International Joint Conference on Artificial Intelligence, 2005
- [De Salvo Braz et al. 06]
Rodrigo de Salvo Braz, Eyal Amir, and Dan Roth. MPE and Partial Inversion in Lifted Probabilistic Variable Elimination. AAAI-06 Proceedings of the 21st Conference on Artificial Intelligence, 2006
- [Gogate and Domingos 11]
Vibhav Gogate and Pedro Domingos. Probabilistic Theorem Proving. In: Proc. UAI 2011: 256–265
- [Kersting et al. 09]
Kristian Kersting, Babak Ahmadi, and Sriraam Natarajan. Counting Belief Propagation. In UAI-09 Proceedings of the 25th Conference on Uncertainty in Artificial Intelligence, 2009
- [Milch et al. 08]
Brian Milch, Luke S. Zettelmoyer, Kristian Kersting, Michael Haimes, and Leslie Pack Kaelbling. Lifted Probabilistic Inference with Counting Formulas. In AAAI-08 Proceedings of the 23rd AAAI Conference on Artificial Intelligence, 2008
- [Poole 03]
David Poole. First-order probabilistic inference. IJCAI 2003: 985-991
- [Poole & Mackworth 03]
David Poole, Alan Mackworth. Artificial Intelligence: Foundations of Computational Agents, 2nd Edition, Cambridge University Press, 2017

Bibliography

- [Richardson & Domingos 06]
  Matthew Richardson, Pedro Domingos. Markov logic networks. In: J. Machine Learning. Band 62. Nr. 1-2. 2006. 107–136
- [Russell & Norvig 16] Stuart Russell, Peter Norvig, Artificial Intelligence: A Modern Approach, Pearson, 2016
- [Sarkhel, Venugopal et al. 14]
  Somdeb Sarkhel, Deepak Venugopal, Parag Singla, Vibhav Gogate:. Lifted MAP Inference for Markov Logic Networks. AISTATS 2014: 859-867
- [Singla and Domingos 08]
  Parag Singla and Pedro Domingos. Lifted First-order Belief Propagation. In AAAI-08 Proceedings of the 23rd AAAI Conference on Artificial Intelligence, 2008
- [Tagipour et al. 13]
  Nima Taghipour, Daan Fierens, Jesse Davis, and Hendrik Blockeel. Lifted Variable Elimination: Decoupling the Operators from the Constraint Language. Journal of Artificial Intelligence Research, 47(1):393–439, 2013
- [Taghipour et al. 13a]
  Nima Taghipour, Daan Fierens, Jesse Davis, and Hendrik Blockeel. Lifted Variable Elimination: Decoupling the Operators from the Constraint Language. Journal of Artificial Intelligence Research, 47(1):393–439, 2013
- [van den Broeck 13]
  Guy Van den Broeck. Lifted Inference and Learning in Statistical Relational Models, PhD thesis, KU Leuven, 2013

Bibliography

- [Braun & Möller 16]
  Tanya Braun and Ralf Möller. Lifted Junction Tree Algorithm. In Proceedings of KI 2016: Advances in Artificial Intelligence, pages 30–42, 2016
- [Braun & Möller 17]
  Tanya Braun and Ralf Möller. Preventing Groundings and Handling Evidence in the Lifted Junction Tree Algorithm. In Proceedings of KI 2017: Advances in Artificial Intelligence, pages 85–98, 2017
- [Braun & Möller 17a]
  Tanya Braun and Ralf Möller. Counting and Conjunctive Queries in the Lifted Junction Tree Algorithm. In Postproceedings of the 5th International Workshop on Graph Structures for Knowledge Representation and Reasoning, 2017
- [Braun & Möller 18]
  Tanya Braun and Ralf Möller. Adaptive Inference on Probabilistic Relational Models. In Proceedings of the 31st Australasian Joint Conference on Artificial Intelligence, 2018
- [Braun & Möller 18a]
  Tanya Braun and Ralf Möller. Parameterised Queries and Lifted Query Answering. In IJCAI-18 Proceedings of the 27th International Joint Conference on Artificial Intelligence, 2018
- [Braun & Möller 18b]
  Tanya Braun and Ralf Möller. Lifted Most Probable Explanation. In Proceedings of the International Conference on Conceptual Structures, 2018
- [Braun & Möller 18c]
  Tanya Braun and Ralf Möller. Fusing First-order Knowledge Compilation and the Lifted Junction Tree Algorithm. In Proceedings of KI 2018: Advances in Artificial Intelligence, 2018
- [Braun & Möller 19]
  Tanya Braun, Ralf Möller: Exploring Unknown Universes in Probabilistic Relational Models, in: Proceedings of AI 2019: Advances in Artificial Intelligence, 2019

Bibliography

- [Gehrke et al. 18]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Lifted Dynamic Junction Tree Algorithm. In Proceedings of the International Conference on Conceptual Structures, 2018

- [Gehrke et al. 18b]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Towards Preventing Unnecessary Groundings in the Lifted Dynamic Junction Tree Algorithm. In Proceedings of KI 2018: Advances in Artificial Intelligence, 2018

- [Gehrke et al. 18c]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Preventing Unnecessary Groundings in the Lifted Dynamic Junction Tree Algorithm. In Proceedings of the AI 2018: Advances in Artificial Intelligence, 2018

- [Gehrke et al. 19]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Relational Forward Backward Algorithm for Multiple Queries. In FLAIRS-32 Proceedings of the 32nd International Florida Artificial Intelligence Research Society Conference, 2019

- [Gehrke et al. 19b]
  Marcel Gehrke, Tanya Braun, Ralf Möller, Alexander Waschkau, Christoph Strumann, and Jost Steinhäuser. Lifted Maximum Expected Utility. In Artificial Intelligence in Health, 2019

- [Gehrke et al. 19c]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Lifted Temporal Maximum Expected Utility. In Proceedings of the 32nd Canadian Conference on Artificial Intelligence, Canadian AI 2019, 2019

- [Gehrke et al. 19d]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Lifted Temporal Most Probable Explanation In Proceedings of the International Conference on Conceptual Structures, 2019

- [Gehrke et al. 19e]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Lifted Taming Reasoning in Temporal Probabilistic Relational Models Technical report

- [Gehrke et al. 19f]
  Marcel Gehrke, Tanya Braun, and Ralf Möller. Uncertain Evidence in Probabilistic Relational Models. In *Proceedings of the 32nd Canadian Conference on Artificial Intelligence, Canadian AI 2019*, 2019

The End *

*PRMs are a true backbone of AI, and this tutorial emphasized only some central topics. We definitely did not cite all publications relevant to the whole field of PRMs here. We would like to thank all our colleagues for making their slides available (see some of the references to papers for respective credits). Slides or parts of it are almost always modified.