

Exercises for the book

Semantic Web Databases

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Chapter 8 – Parallel Databases

1. Let H be the finite set of hash functions with signature $U \rightarrow \{0, 1, \dots, m-1\}$. H is called universal, if for every pair of different keys $x, y \in U$ the number of hash functions $h \in H$ with $h(x) = h(y)$ is exactly $|H|/m$. In other words, the chance of a collision between x and y with $x \neq y$ is exactly $1/m$ for an arbitrarily chosen hash function $h \in H$, i.e., the chance is as big as the chance of a collision when using hash functions, which map all elements from U arbitrarily to $\{0, 1, \dots, m-1\}$. Note that for the last kind of hash functions a table of size $|U|$ would be necessary for the administration of the mapping from U to $\{0, 1, \dots, m-1\}$.

The following class H_{prim} of hash functions is universal:

Let m be a prime. We divide a key x in $r + 1$ subcomponents, such that $x = \langle x_0, x_1, \dots, x_r \rangle$. The only condition is, that the maximal value of a subcomponent must be smaller than m . Let $a = \langle a_0, a_1, \dots, a_r \rangle$ be a sequence of $r + 1$ elements, which are arbitrarily chosen from $\{0, 1, \dots, m-1\}$. Then we define the corresponding hash function $h_a \in H_{prim}$ by $h_a(x) = (\sum_{i=0}^r a_i * x_i) \bmod m$. Note that we can prove the universality of H_{prim} by using number theory.

- a) Calculate the hash values of 3, 20 and 82 using the hash function $h_{\langle 5, 8, 9 \rangle}(x) = (\sum_{i=0}^r a_i * x_i) \bmod 13$. Let the range of a subcomponent x_i be $\{0, \dots, 7\}$. The subcomponent can be calculated using $x_i = (x \div 8^i) \bmod 8$.
 - b) Prove: If we require $a_i > 0$ for each $a_i \in \{a_0, a_1, \dots, a_r\}$, then H_{prim} is not universal any more.
2. Let us assume we want to use 3 threads for parallel operand result computation and the following operatorgraph. Which operand results (A to H) are optimal to be computed in parallel?

