



Lecture

Quantum Computing

(CS5070)

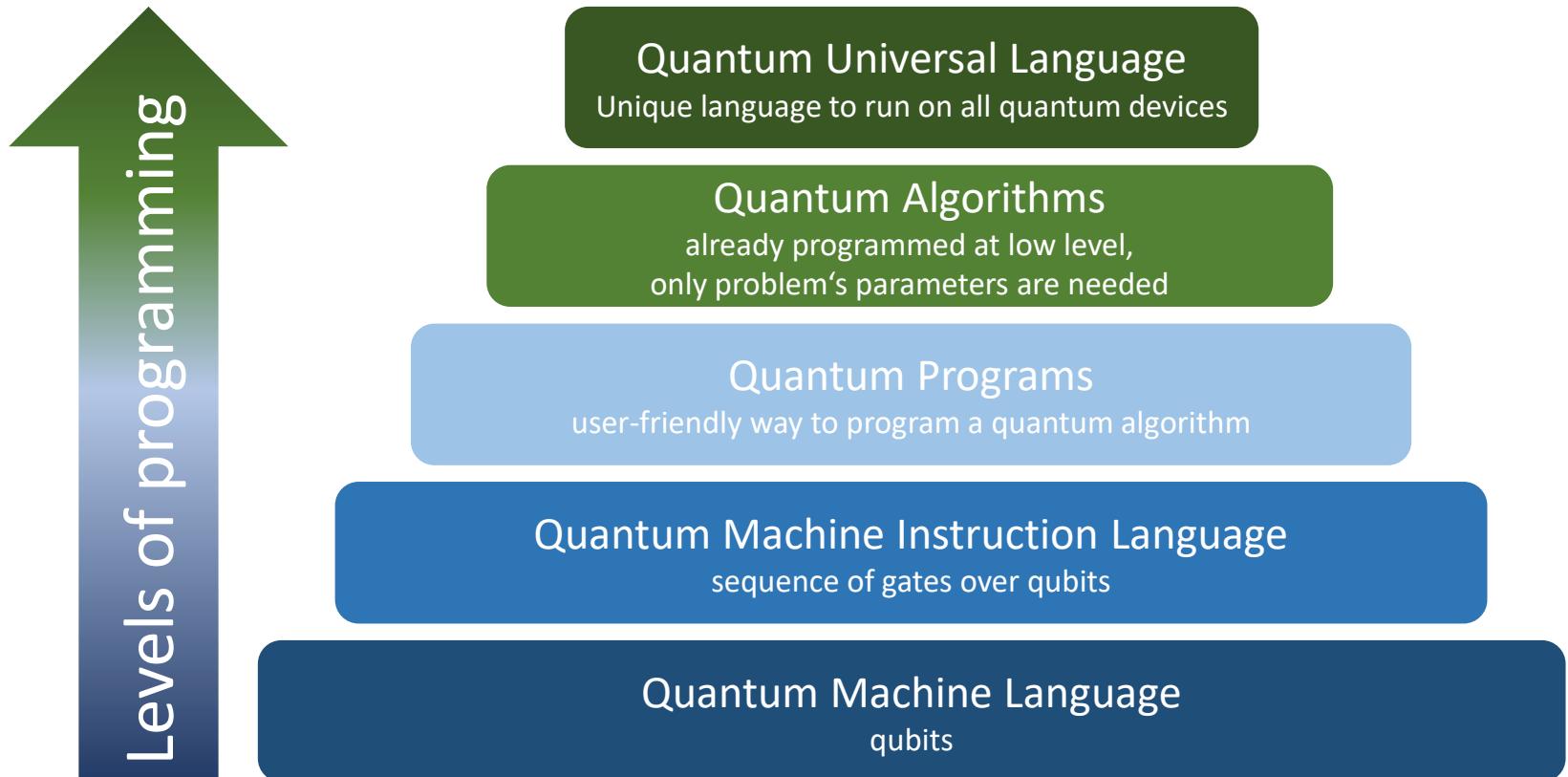
Qiskit and Deutsch-Jozsa

Algorithm

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<https://www.ifis.uni-luebeck.de/~groppe>

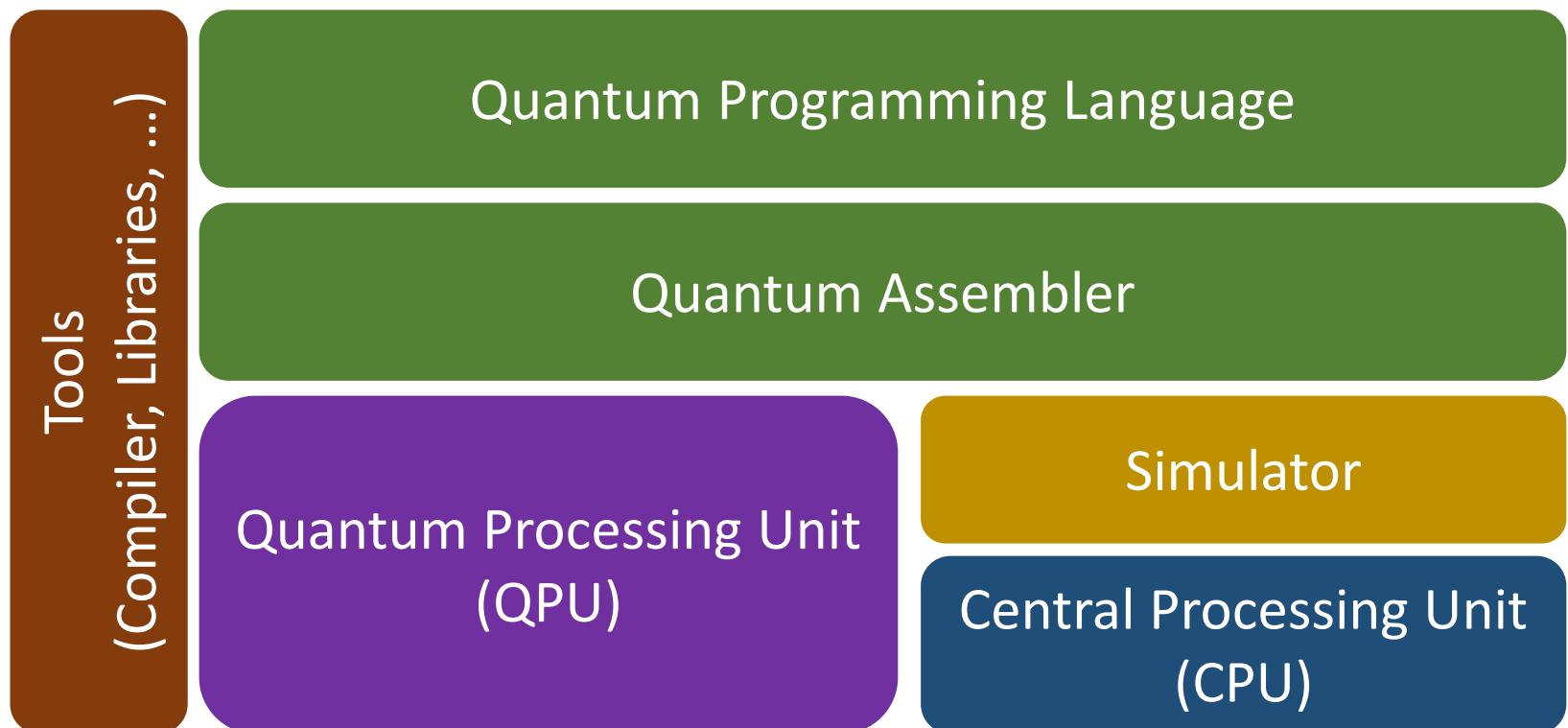


Levels of Quantum Programming

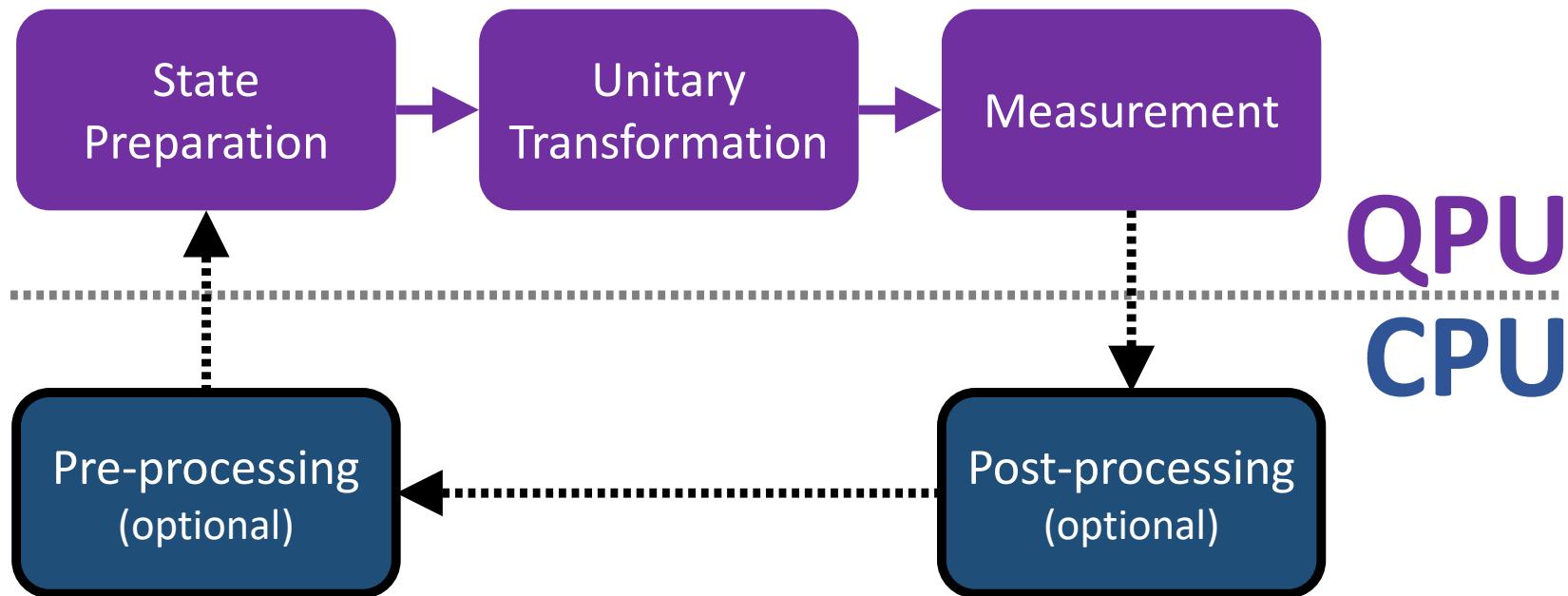




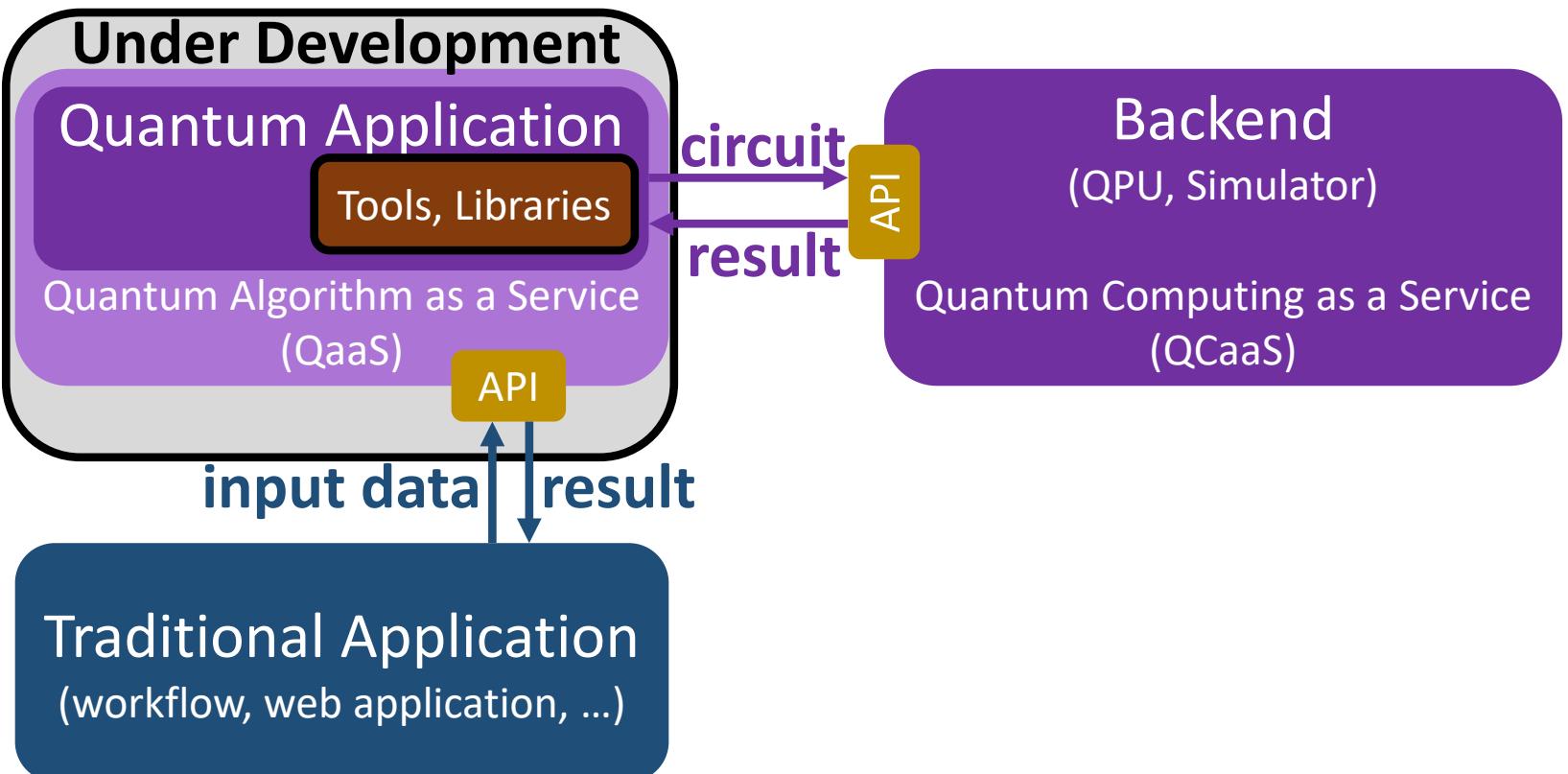
Language and Tools Stack



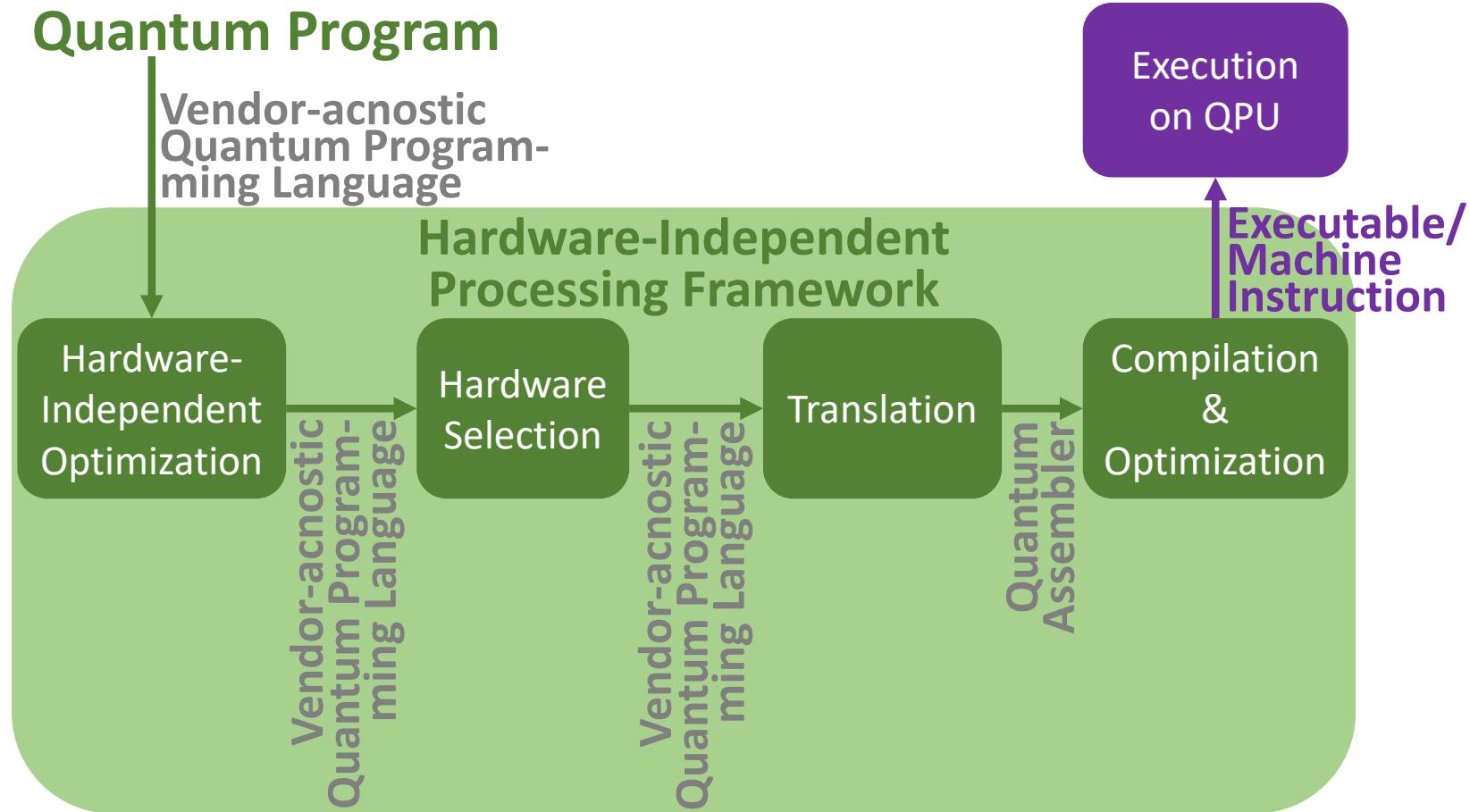
CPU and QPU Processing Steps



Quantum Algorithms as a Service



Hardware Independent Translation



Overview Quantum Programming Languages & their Stacks

Quantum Universal Languages	XACC							
Provider	IBM	Rigetti	DWave	Xanadu	Google	Microsoft	Qilimajaro	ETH Zürich
Full-stack Libraries	Qiskit	Forest			Cirq	Quantum Development Kit		Silq
Quantum algorithms	Qiskit ml, finance chemistry, opt.	Grove	DWave Ocean	Strawberry Fields	OpenFermion -Cirq	Q#		
Quantum circuits	Qiskit Terra	pyquil			Cirq		Qibo	
Assembly language	Open QASM	Quil	QMASM	Blackbird	Open QASM/ Quil	Internal Quantum Simulator	Open QASM	Internal Quantum Simulator
Hardware	Quantum Device							

This list is not complete. "Quantum Language" refers to specialized programming languages and also to libraries to write quantum programs on top of well-known programming languages for classical computation.



Qiskit

- Installation
 - Local installation via Python, Anaconda and Jupyter
 - Cloud
 - [IBM Quantum \(Lab\)](#)
 - Graphically build quantum circuits
 - Jupyter-notebooks for Qiskit without installation
 - [Quantum system details](#) like status, topology, calibration data, and access details of IBM quantum systems
 - Running quantum circuits on real quantum computers
 - Step-by-step tutorials and guides
 - [Strangeworks](#)
 - Hardware-agnostic, software-inclusive, collaborative environment for quantum development
 - [Support of many important quantum frameworks](#) without installation and configuration
 - Running quantum circuits on real quantum computers

IBM Quantum Composer

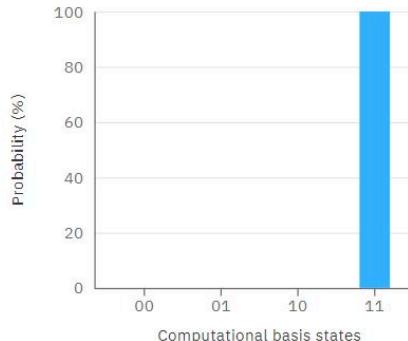
My first circuit Saved A Bell state Visualizations seed 3749 ▾

OpenQASM 2.0 ▾

Open in Quantum Lab

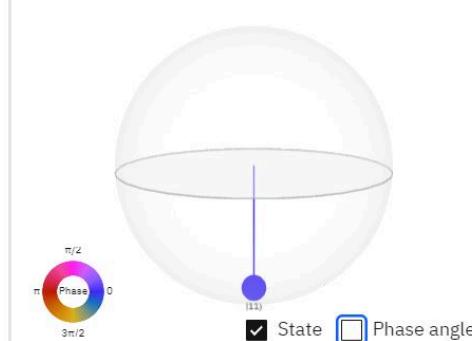
```
OPENQASM 2.0;
include "qelib1.inc";
qreg q[2];
creg c[2];
h q[0];
cx q[0],q[1];
measure q[0] -> c[0];
```

Probabilities



Computational basis states	Probability (%)
00	0
01	0
10	0
11	100

Q-sphere



State Phase angle



Qiskit Library/Python

- Importing important packages, creating and drawing first quantum circuit with one qubit initialized with $|0\rangle$...

Program:

```
from qiskit import QuantumCircuit
import kaleidoscope.qiskit
from kaleidoscope import bloch_sphere
from qiskit.visualization import array_to_latex
from IPython.display import Latex

circuit = QuantumCircuit(1)
circuit.draw('latex', initial_state=True)
```

Output:

$q : |0\rangle$ —

Printing the state vector & visualizing in the bloch sphere...

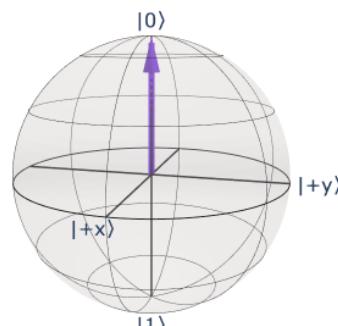
- Determine state vector of circuit for printing and visualizing

Program:

```
display(array_to_latex(circuit.statevector(), prefix="State = "))
bloch_sphere(circuit.statevector())
```

Output:

$$State = \begin{bmatrix} 1 & 0 \end{bmatrix}$$



Extending the circuit...

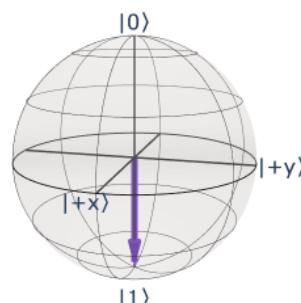
- Add a NOT(X) gate to the circuit resulting in $|1\rangle\dots$

Program:

```
circuit.x(0)
circuit.draw('latex', initial_state=True)
display(array_to_latex(circuit.statevector(), prefix="State = "))
bloch_sphere(circuit.statevector())
```

Output:

$$State = \begin{bmatrix} 0 & 1 \end{bmatrix}$$





Matrix representations of gates

- Printing the matrix representations of gates (here X gate)...

Program:

```
from qiskit.circuit.library import *
xgate = XGate()
print('X gate matrix:')
array_to_latex(xgate.to_matrix())
```

Output:

X gate matrix:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

A circuit with a Hadamard transform...

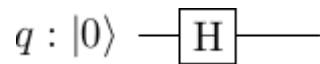
Program:

```
circuit = QuantumCircuit(1)
circuit.h(0)
circuit.draw('latex', initial_state=True)
```

Program:

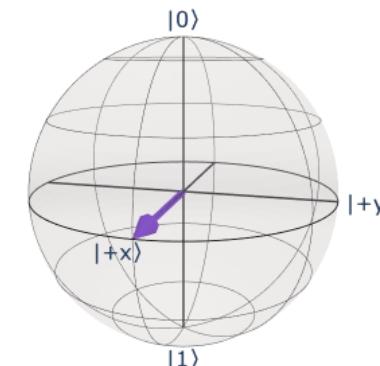
```
display(array_to_latex(circuit.statevector(),
prefix="|+\rangle = "))
bloch_sphere(circuit.statevector())
```

Output:



Output:

$$|+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

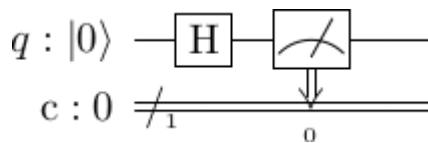


Adding a measurement...

Program:

```
circuit = QuantumCircuit(1,1)
circuit.h(0)
circuit.measure(0,0)
circuit.draw('latex', initial_state=True)
```

Output:



- Open QASM output of the circuit...

Program:

```
circuit.qasm()
```

Output:

```
OPENQASM 2.0;
include "qelib1.inc";
qreg q[1];
creg c[1];
h q[0];
measure q[0] -> c[0];
```

Simulating a circuit

- Initialize simulator, execute a job with 1000 shots and plot the resultant histogram...

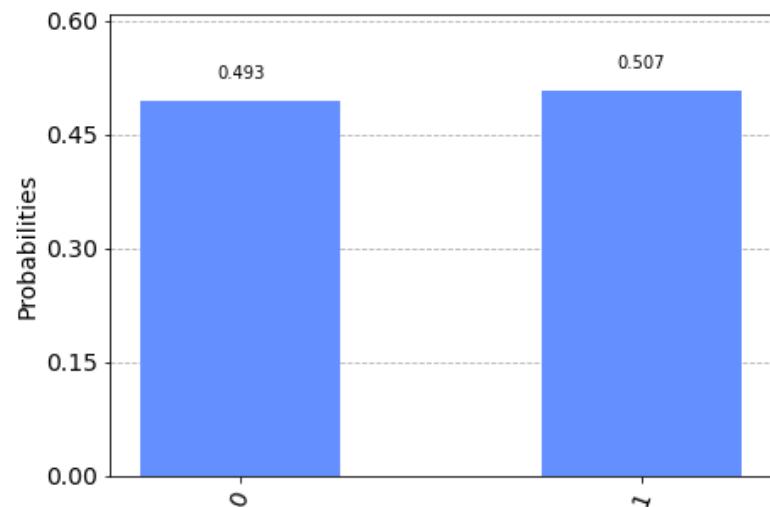
Program:

```
from qiskit import BasicAer,
                  execute
from qiskit.visualization
import plot_histogram

local_simulator =
    BasicAer.get_backend('qasm_simulator')

job = execute(circuit,
              backend=local_simulator,
              shots=1000)
plot_histogram(job.result().get_counts())
```

Output:

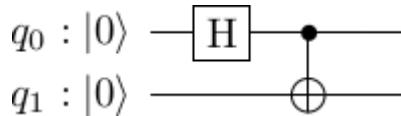


Circuits with 2 Qubits (Correlated Bell State)

Program:

```
circuit = QuantumCircuit(2)
circuit.h(0)
circuit.cx(0, 1)
circuit.draw('latex', initial_state=True)
```

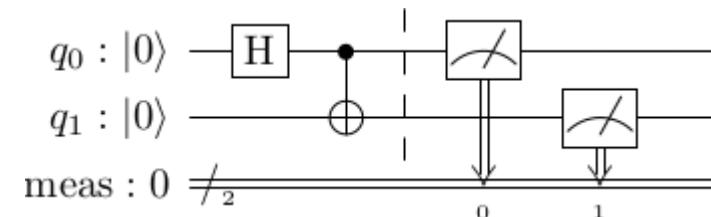
Output:



Program:

```
circuit.measure_all()
circuit.draw('latex', initial_state=True)
```

Output:



Program:

```
array_to_latex(circuit.statevector(),
prefix="\text{CNOT}|0{+}\rangle = ")
```

Output:

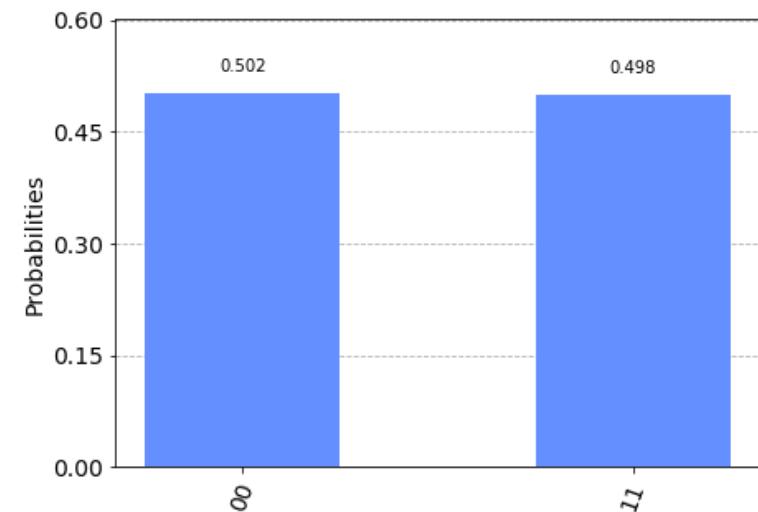
$$\text{CNOT}|0+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Simulating the Correlated Bell State

Program:

```
local_simulator =  
    BasicAer.get_backend('qasm_simulator')  
  
job = execute(circuit, backend=local_simulator,  
             shots=5000)  
plot_histogram(job.result().get_counts())
```

Output:

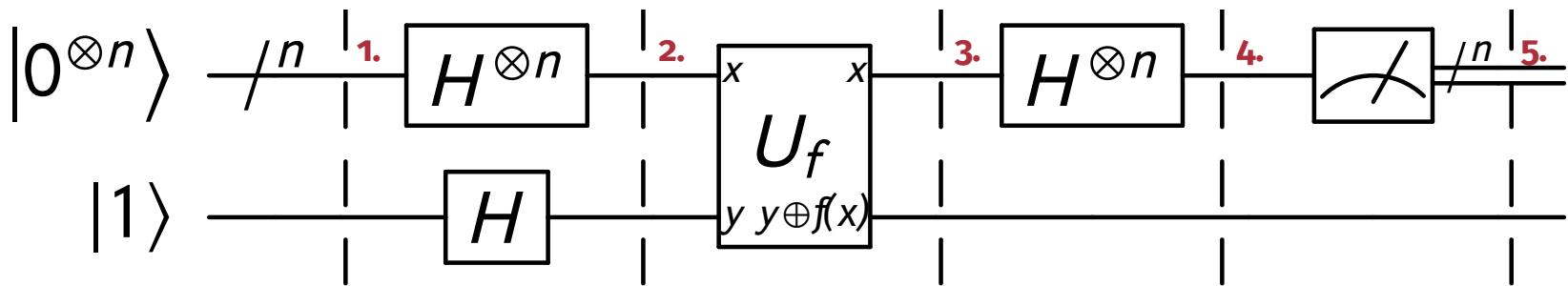


Deutsch-Jozsa Algorithm

- **Purpose:** shows that the power of state superposition in a quantum algorithm can significantly reduce the runtime complexity compared to its classical counterpart **by exponential speedup**
- **Problem** to be solved
 - **Objective:** Determine the nature of a function, $f : \{0, 1\}^n \rightarrow \{0, 1\}$, which takes as input an n -bit integer and returns either 0 or 1
 - We are **guaranteed** that this function
 - is either constant (returning either 0 or 1 on all inputs), **or**
 - perfectly balanced (returning 0 on half of the inputs and 1 on the other half)
 - **Goal:** Determine of which type (constant or balanced) **the function is with minimum number of function executions**

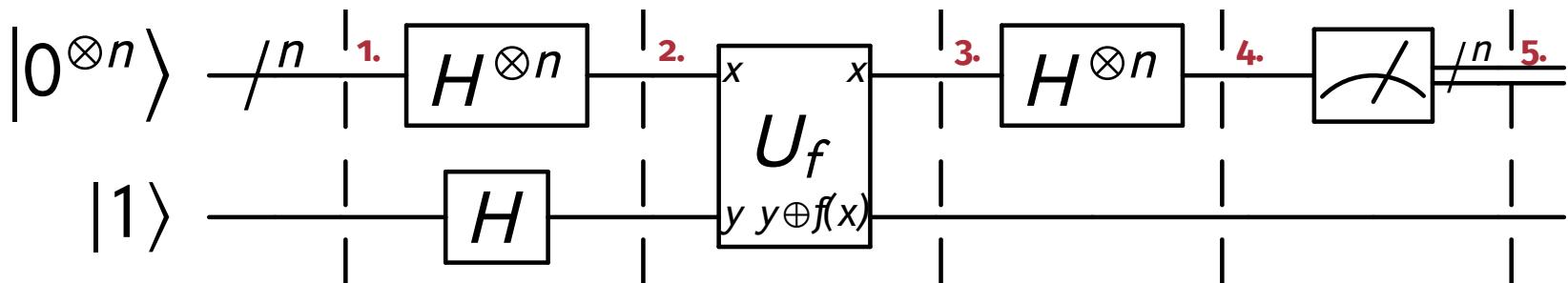
- References:
 - $n = 1$, Deutsch algorithm: D. Deutsch. Quantum theory, the Church-Turing principle and the universal quantum computer. *Proceedings of the Royal Society A*. 400 (1818): 97–117, 1985 ↗
 - $n > 1$, Deutsch-Jozsa algorithm: D. Deutsch, R. Jozsa. Rapid solutions of problems by quantum computation. *Proceedings of the Royal Society of London A*. 439: 553–558, 1992 ↗

Deutsch-Jozsa Algorithm - Idea & Circuit



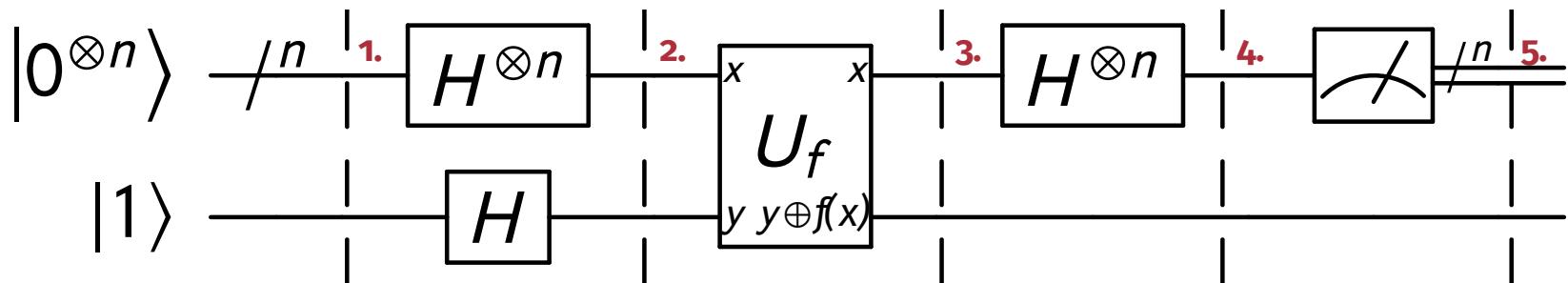
- **Step 1 & 2:** Bring n qubits into superposition with all values $0, 1, \dots, 2^n - 1$ having the same probability

Deutsch-Jozsa Algorithm - Idea & Circuit



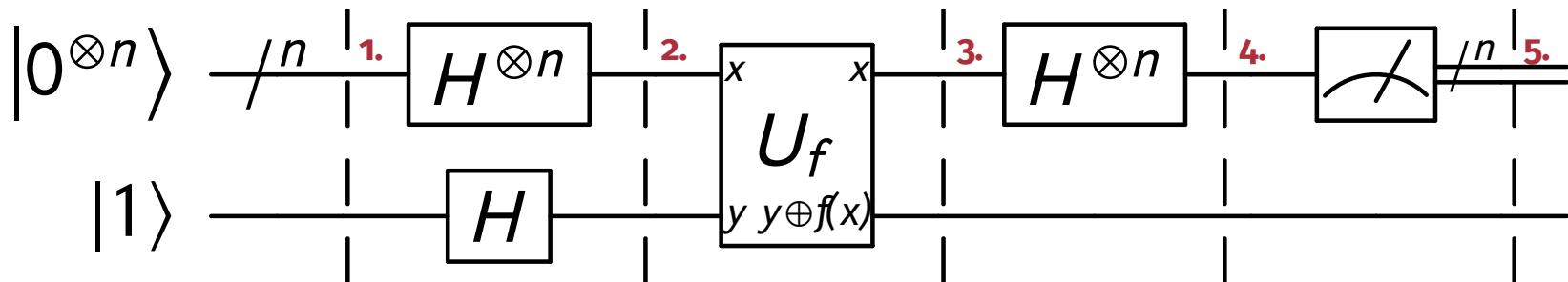
- **Step 1 & 2:** Bring n qubits into superposition with all values $0, 1, \dots, 2^n - 1$ having the same probability
- **Step 3:** Apply the oracle function on all input values simultaneously
 - **Constant** functions do not change the state (up to a global phase)
 - **Balanced** functions cause state changes for half of the input values

Deutsch-Jozsa Algorithm - Idea & Circuit



- **Step 1 & 2:** Bring n qubits into superposition with all values $0, 1, \dots, 2^n - 1$ having the same probability
- **Step 3:** Apply the oracle function on all input values simultaneously
 - **Constant** functions do not change the state (up to a global phase)
 - **Balanced** functions cause state changes for half of the input values
- **Step 4 & 5:** Interfere the result and check output
 - **Constant** functions: No quantum state change & Hadamard is reversible
 \Rightarrow Output of the circuit = Input of the circuit
 - **Balanced** functions: Quantum state changes of previous step
 \Rightarrow Output of the circuit \neq Input of the circuit

Deutsch-Jozsa Algorithm - Details of Step 1



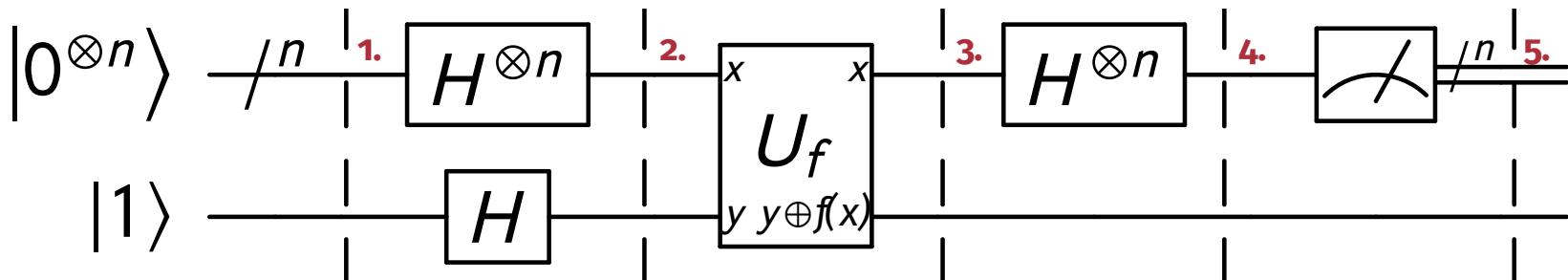
$n = 1$:

$$\begin{aligned} |\psi_1\rangle &= |0\rangle|1\rangle = |0\rangle \otimes |1\rangle \\ &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\ &= |01\rangle \end{aligned}$$

Arbitrary $n \geq 1$:

$$\begin{aligned} |\psi_1\rangle &= |0\rangle \cdots |0\rangle |1\rangle = |0\rangle \otimes \cdots \otimes |0\rangle \otimes |1\rangle \\ &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ &= |0 \cdots 01\rangle \end{aligned}$$

Deutsch-Jozsa Algorithm - Details of Step 2



$n = 1$:

Arbitrary $n \geq 1$:

$$H_2 = H \otimes H = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

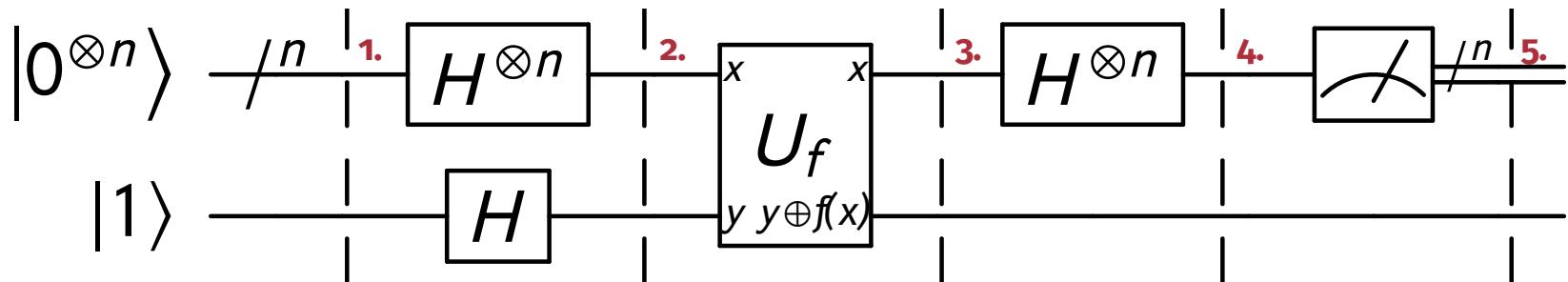
$$H_{n+1} = H \otimes \cdots \otimes H$$

$$|\psi_2\rangle = H_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)}{2}$$

$$|\psi_2\rangle = H_{n+1} |0\cdots 01\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x\rangle (|0\rangle - |1\rangle)$$

Deutsch-Jozsa Algorithm - Details of Step 3



$n = 1$:

Apply the oracle function in gate mapping

$$|x\rangle|b\rangle \mapsto |x\rangle|b \oplus f(x)\rangle$$

$$|x\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \text{ and } |b\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$ b \oplus f(x)\rangle$	$b = 0$	$b = 1$
$f(x) = 0$	$ 0\rangle$	$- 1\rangle$
$f(x) = 1$	$ 1\rangle$	$- 0\rangle$

$$|\psi_3\rangle = |x\rangle|b \oplus f(x)\rangle$$

(where \oplus is addition modulo 2)

Arbitrary $n \geq 1$:

The result of applying the quantum oracle is

$$\frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x\rangle(|f(x)\rangle - |1 \oplus f(x)\rangle).$$

For each x , $f(x)$ is either 0 or 1. Testing these two possibilities, we see the above state is equal to

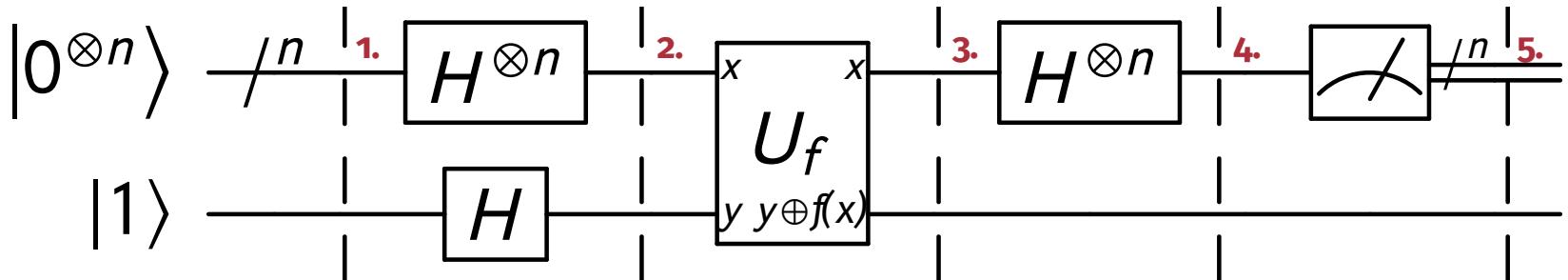
$$|\psi_3\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} |x\rangle(|0\rangle - |1\rangle).$$

At this point the last qubit $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ may be

ignored and therefore below is remained:

$$\frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} |x\rangle.$$

Deutsch-Jozsa Algorithm - Details of Step 3



$n = 1$: Examples of oracles:

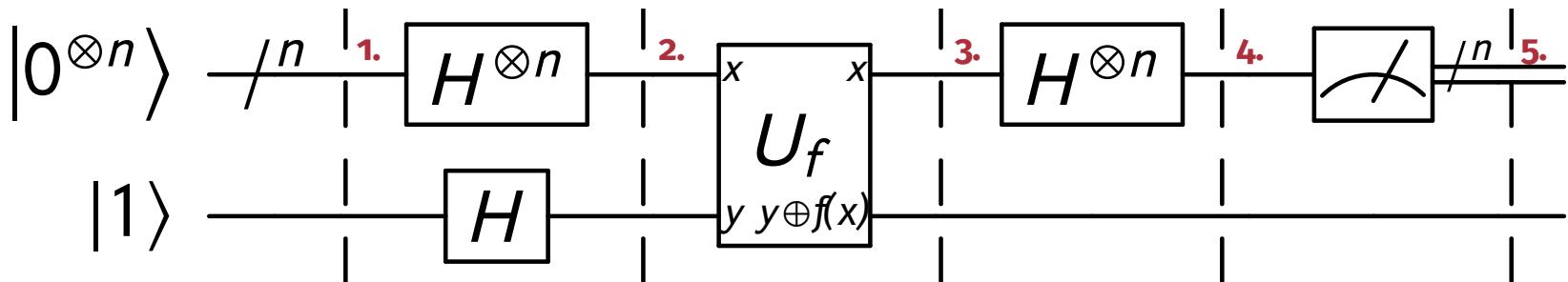
	$ x\rangle b \oplus f(x)\rangle$	$f(0) = 0$	$f(0) = 1$
$f(1) = 0$	$\frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$	$\frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$	
$f(1) = 1$	$\frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$	$\frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$	

Arbitrary $n \geq 1$: Example oracle:

$f(x_1, x_2) = x_1 \oplus x_2$	$x_1 = 0$	$x_1 = 1$
$x_2 = 0$	0	1
$x_2 = 1$	1	0

$$\begin{aligned}
\psi_3 &= \frac{|00\rangle\otimes(|0\oplus 0\rangle - |1\oplus 0\oplus 0\rangle) + |01\rangle\otimes(|0\oplus 1\rangle - |1\oplus 0\oplus 1\rangle)}{2\sqrt{2}} \\
&\quad + \frac{|10\rangle\otimes(|1\oplus 0\rangle - |1\oplus 1\oplus 0\rangle) + |11\rangle\otimes(|1\oplus 1\rangle - |1\oplus 1\oplus 1\rangle)}{2\sqrt{2}} \\
&= \frac{|00\rangle\otimes(|0\rangle - |1\rangle) - |01\rangle\otimes(|0\rangle - |1\rangle) - |10\rangle\otimes(|0\rangle - |1\rangle) + |11\rangle\otimes(|0\rangle - |1\rangle)}{2\sqrt{2}} \\
&= \frac{1}{2} (|00\rangle - |01\rangle - |10\rangle + |11\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \\
&= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)
\end{aligned}$$

Deutsch-Jozsa Algorithm - Details of Step 4



$n = 1$:

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$\begin{aligned} |\psi_4\rangle &= (H \otimes I)|\psi_3\rangle \\ &= (H \otimes I)|x\rangle|b \oplus f(x)\rangle \end{aligned}$$

Arbitrary $n \geq 1$:

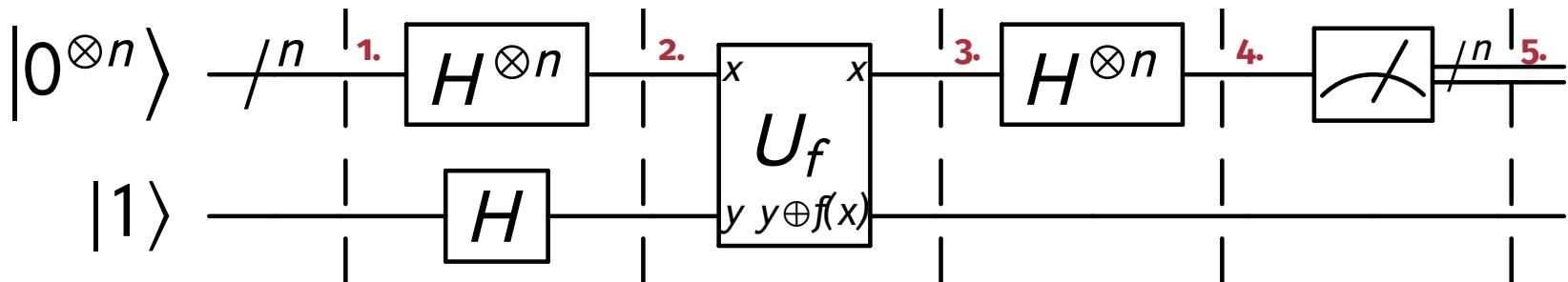
Hadamards transform to:

$$\begin{aligned} |\psi_4\rangle &= \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} \left[\frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} (-1)^{x \cdot y} |y\rangle \right] \\ &= \frac{1}{2^n} \sum_{y=0}^{2^n-1} \left[\sum_{x=0}^{2^n-1} (-1)^{f(x)} (-1)^{x \cdot y} \right] |y\rangle \end{aligned}$$

where

$x \cdot y = x_0 y_0 \oplus x_1 y_1 \oplus \dots \oplus x_{n-1} y_{n-1}$ is the sum of the bitwise product (where \oplus is addition modulo 2).

Deutsch-Jozsa Algorithm - Details of Step 4



$n = 1$: Examples of oracles:

	$f(0) = 0$	$f(0) = 1$
$f(1) = 0$	$\frac{1}{2\sqrt{2}} \begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \end{bmatrix}$	$\frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ -2 \\ 2 \end{bmatrix}$
$f(1) = 1$	$\frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 2 \\ -2 \end{bmatrix}$	$\frac{1}{2\sqrt{2}} \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

Arbitrary $n \geq 1$: Example oracle:

$f(x_1, x_2) = x_1 \oplus x_2$	$x_1 = 0$	$x_1 = 1$
$x_2 = 0$	0	1
$x_2 = 1$	1	0

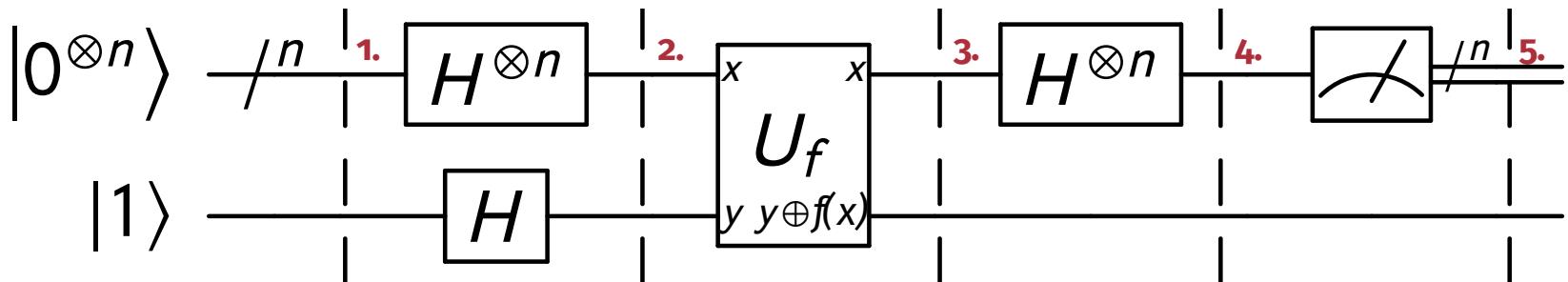
$$\psi_3 = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

The result of applying Hadamards on the first and second qubit is:

$$|\psi_4\rangle = |1\rangle \otimes |1\rangle \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

\Rightarrow It will be $|11\rangle$ measured in the next step!

Deutsch-Jozsa Algorithm - Details of Step 5



$n = 1$: Examples of oracles:

$|0\rangle$ is measured (neglecting noise) as 1st

qubit for $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$, rest is analogous

00	01
01	10
10	11

	$f(0) = 0$	$f(0) = 1$
$f(1) = 0$	$ 0\rangle \Rightarrow$ constant	$ 1\rangle \Rightarrow$ balanced
$f(1) = 1$	$ 1\rangle \Rightarrow$ balanced	$ 0\rangle \Rightarrow$ constant

Arbitrary $n \geq 1$:

The probability of measuring $|0\rangle^{\otimes n}$ is:

$$\left| \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)} \right|^2$$

which evaluates to

- 1 (i.e., $|0\rangle^{\otimes n}$ is measured)
if $f(x)$ is constant (constructive interference) and
- 0 (i.e., $|0\rangle^{\otimes n}$ is not measured)
if $f(x)$ is balanced (destructive interference).



Constant Oracles in Qiskit

Oracle 0 : Constant 0

Program:

```
from qiskit import QuantumRegister
input = QuantumRegister(1, name='input')
output = QuantumRegister(1, name='output')
constant0 = QuantumCircuit(input, output,
                           name='oracle')
oracle0 = constant0.to_instruction()
constant0.draw('latex', initial_state=True)
```

Output:

input : $|0\rangle$ —

output : $|0\rangle$ —

Oracle 1 : Constant 1

Program:

```
input = QuantumRegister(1, name='input')
output = QuantumRegister(1, name='output')
constant1 = QuantumCircuit(input, output,
                           name='oracle')
constant1.x(output)
oracle1 = constant1.to_instruction()
constant1.draw('latex', initial_state=True)
```

Output:

input : $|0\rangle$ —————

output : $|0\rangle$ — [X] —————

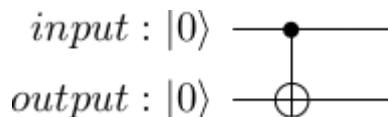
Balanced Oracles in Qiskit

Oracle I: Identity

Program:

```
input = QuantumRegister(1, name='input')
output = QuantumRegister(1, name='output')
identity = QuantumCircuit(input, output,
                           name='oracle')
identity.cx(input, output)
oracleI = identity.to_instruction()
identity.draw('latex', initial_state=True)
```

Output:

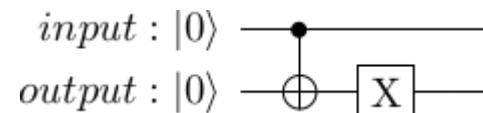


Oracle X: Invert/Not/X

Program:

```
input = QuantumRegister(1, name='input')
output = QuantumRegister(1, name='output')
invert = QuantumCircuit(input, output,
                        name='oracle')
invert.cx(input, output)
invert.x(output)
oracleX = invert.to_instruction()
invert.draw('latex', initial_state=True)
```

Output:



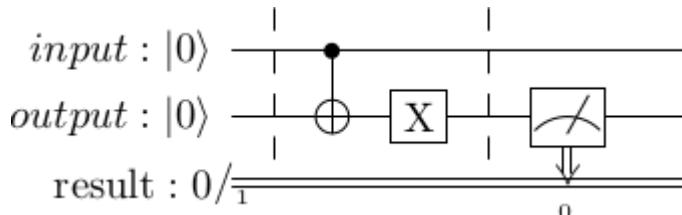
Run an Oracle

Oracle X: Invert

Program:

```
from qiskit import ClassicalRegister
result = ClassicalRegister(1, name='result')
circuit = QuantumCircuit(input, output, result)
circuit.barrier()
circuit.compose(invert, inplace=True)
circuit.barrier()
circuit.measure(output, result)
circuit.draw('latex', initial_state=True)
```

Output:



Program:

```
execute(circuit, backend=local_simulator,
shots=1000).result().get_counts()
```

Output:

```
{'1': 1000}
```

Simulating Deutsch's Algorithm

Program:

```
qr = QuantumRegister(2, name='qubits')
cr = ClassicalRegister(1, name='result')
circuit = QuantumCircuit(qr, cr)
circuit.x(qr[1])
circuit.h(qr)
# oracle0, oracle1, oracleI or oracleX:
circuit.append(oracle0, [qr[0], qr[1]])
circuit.h(qr[0])
circuit.measure(qr[0], cr[0])
circuit.draw('latex', initial_state=True,
            justify="right")
```

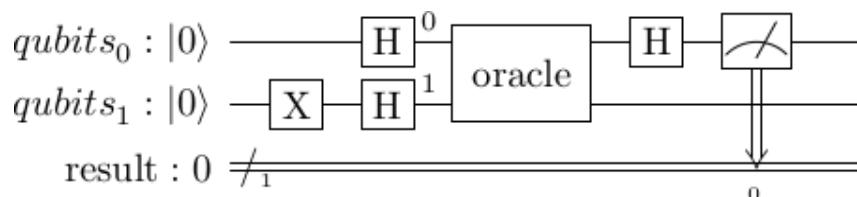
Program:

```
counts = execute(circuit,
                 backend=local_simulator,
                 shots=1).result().get_counts()
counts['BALANCED'] = counts.pop('1', None)
counts['CONSTANT'] = counts.pop('0', None)
counts
```

Output:

```
{ 'BALANCED': None,
  'CONSTANT': 1}
```

Output:





Running Deutsch's Algorithm on a real Quantum Computer

Program:

```
import qiskit.tools.jupyter
from qiskit import IBMQ
from qiskit.providers.ibmq import least_busy
provider = IBMQ.load_account()
least_busy_device = least_busy(provider.backends(simulator=False,
    filters=lambda b: b.configuration().n_qubits >= 2))
least_busy_device
```

Output:

```
IBMQBackend('ibmq_lima') from IBMQ(hub='ibm-q', group='open', project='main')
```

Some information about ibmq_lima:

n_qubits	5
quantum_volume*	8
pending_jobs	2
basis_gates	['id', 'rz', 'sx', 'x', 'cx', 'reset']
max_shots	20000
max_experiments	100

Job Status and Result

Program:

```
job.status()
```

Output:

JobStatus.RUNNING: 'job is actively running'

Program:

```
job.wait_for_final_state()  
job.status()
```

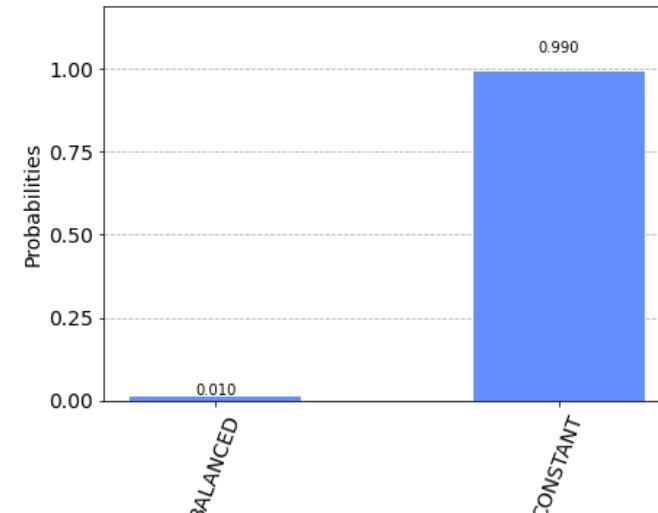
Output:

JobStatus.DONE: 'job has successfully run'

Program:

```
counts = job.result().get_counts()  
counts['BALANCED'] = counts.pop('1', None)  
# wrong answer:  
counts['CONSTANT'] = counts.pop('0', None)  
plot_histogram(counts)
```

Output:

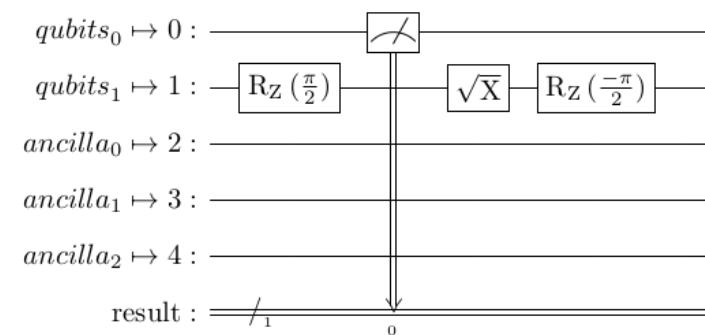
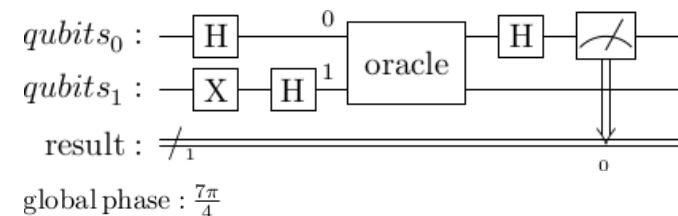
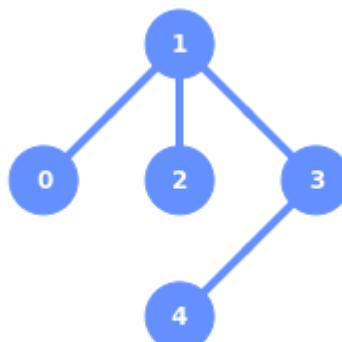


Physical Mapping of Circuit

Program:

```
from qiskit.visualization import plot_gate_map
from qiskit import transpile
display(plot_gate_map(least_busy_device))
job = execute(circuit, backend=least_busy_device, shots=1000, initial_layout=[0, 4])
display(circuit.draw('latex'))
transpile(circuit, least_busy_device).draw('latex')
```

Output:





Summary and Conclusions

- Qiskit
 - introduction
 - gates, circuits
 - output, visualizations (e.g., bloch sphere)
- Deutsch-Jozsa algorithm
 - problem to solve
 - why it works...
 - realization in Qiskit
 - running on a real quantum computer