Lifted Junction Tree Algorithm

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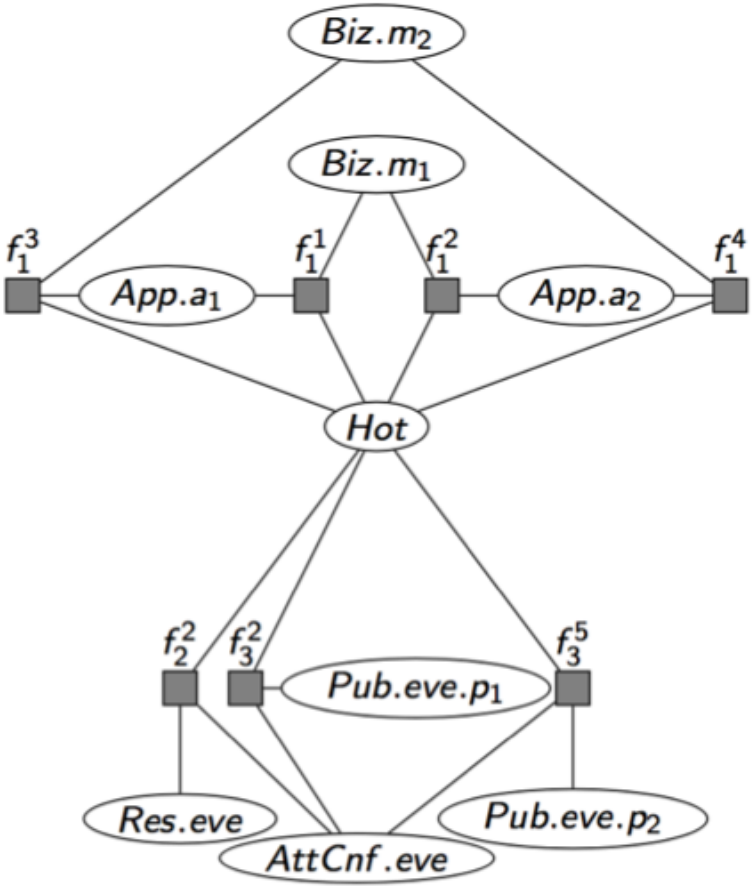
Last Time: Problems with MLN QA

- Grounding

Leads to research about lifted inference:

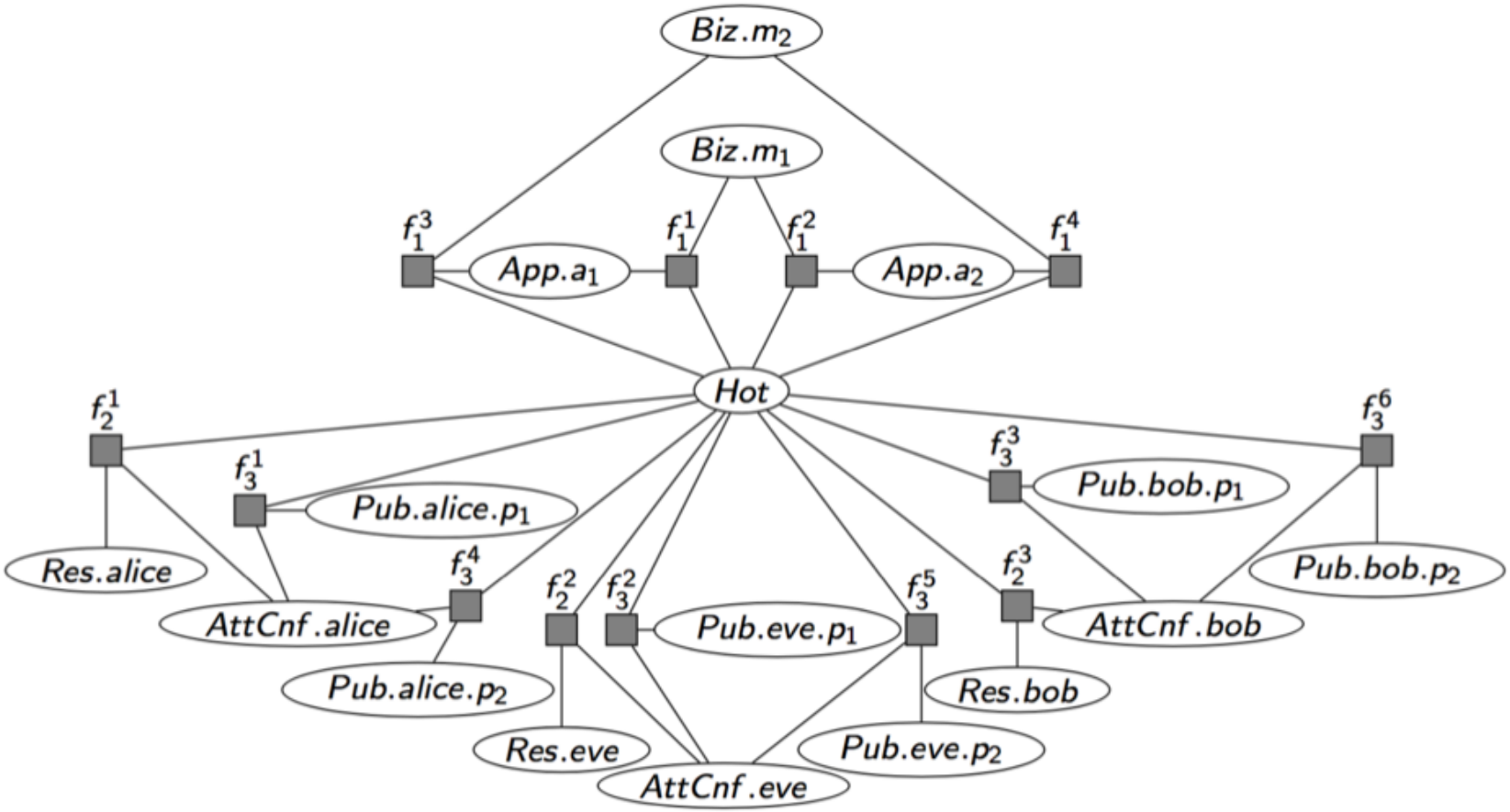
- Probabilistic relational models (PRMs)
- Dynamic probabilistic relational models (DPRMs)

Problem: Models Explode



$7 \cdot 2^3 = 56$ entries in 7 factors, 9 variables

Problem: Models Explode

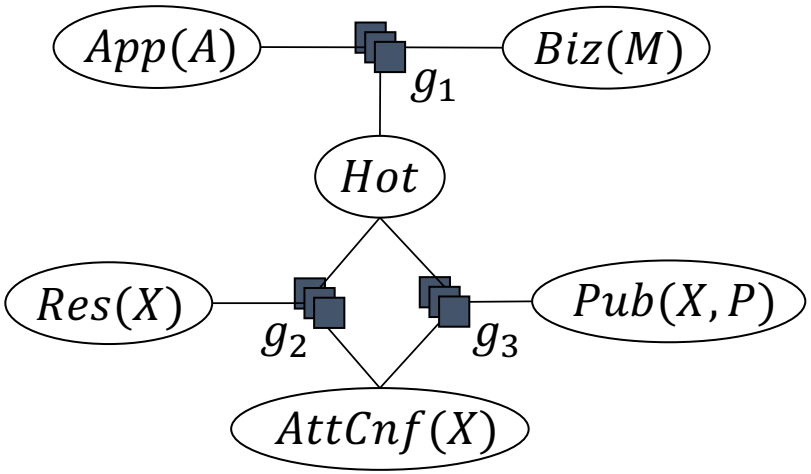


$13 \cdot 2^3 = 104$ entries in 13 factors, 17 variables

Solution: Lifting

Poole and Zhang (2003)

- Parameterised random variables = PRV
  - With logical variables
  - E.g., X
  - Possible values (domain): $\mathcal{D}(X) = \{alice, eve, bob\}$

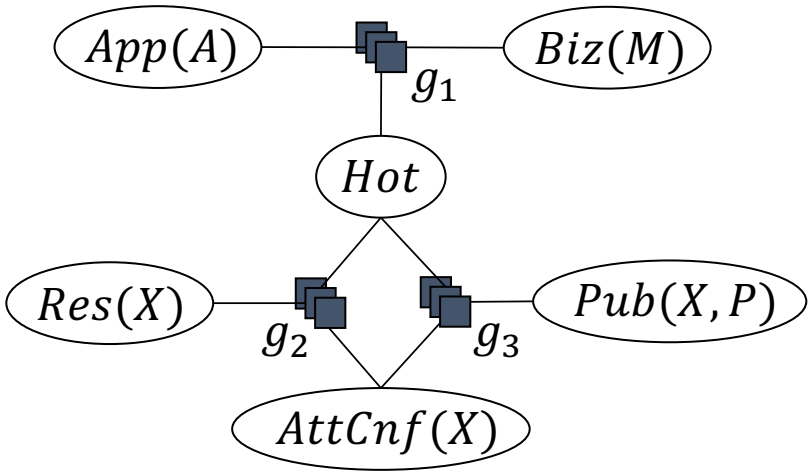


Solution: Lifting

Poole and Zhang (2003)

- Factors with PRVs = parfactors
  - E.g., g_2

$Res(X)$	Hot	$AttCnf(X)$	g_2
0	0	0	5
0	0	1	0
0	1	0	4
0	1	1	6
1	0	0	4
1	0	1	6
1	1	0	2
1	1	1	9



$3 \cdot 2^3 = 24$ entries in 3 factors, 6 PRVs

FOIL Clauses to Parfactors

$$Father(x,z) \wedge Parent(z,y) \Rightarrow Grandfather(x,y)$$

- $g(F(X,Z), P(X,Y), GF(X,Y))$
  - Possible to incorporate weights

$Father(X,Z)$	$Parent(Z,Y)$	$Grandfather(X,Y)$	g
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

PDB to Parfactors

- PDB facts

```
child(liam,eve),0.99
child(dave,eve),0.99
child(liam,bob),0.75
```

- $g\big(Child(X, eve)\big)$

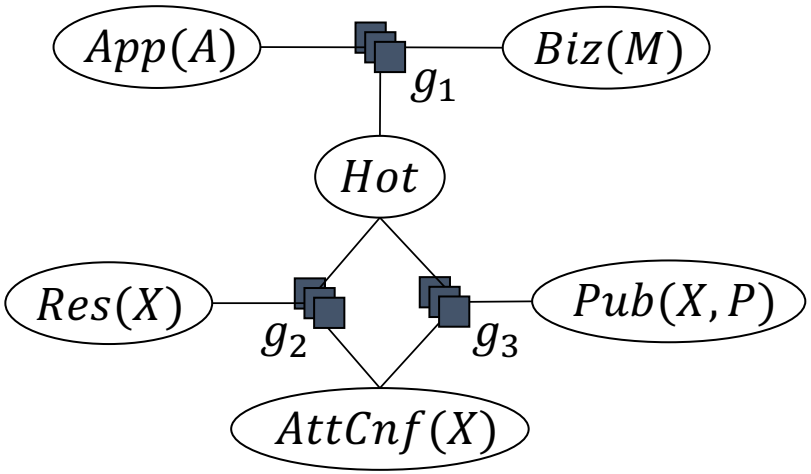
$Child(X, eve)$	g
0	0.01
1	0.99

- $g(Child(liam, eve))$

$Child(liam, eve)$	g
0	0.25
1	0.75

Query Answering (QA): Queries

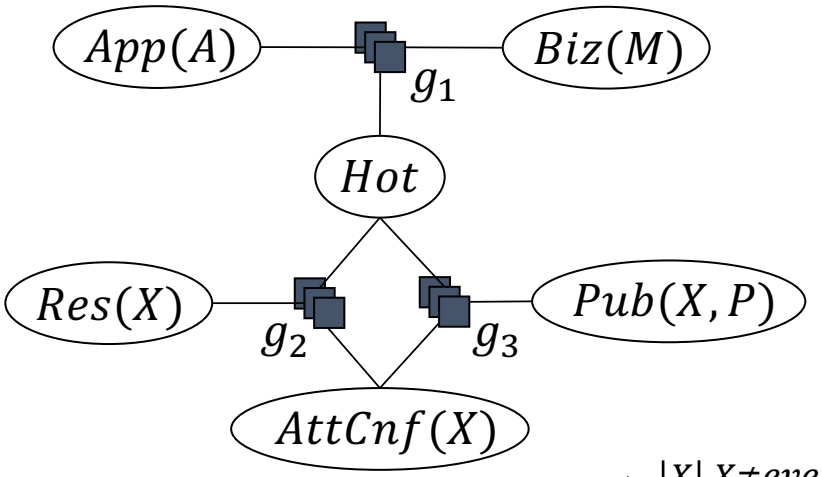
- Likelihood
  - $P(Res(eve) = 1)$
- Marginal distribution
  - $P(Res(eve))$
  - $P(Res(eve), Pub(eve, p_1))$
- Conditional distribution
  - $P(Res(eve)|Hot = 1)$
- Most probable assignment
  - $\underset{rv(G)}{\text{argmax}}\, P_G$
  - $\underset{Res(eve),Biz(m_1)}{\text{argmax}}\, P_G$



QA: Lifted VE (LVE)

Taghipour et al. (2013)

- Eliminate all variables not appearing in query
  - Through lifted summing out
  - Exponentiate result for isomorphic instances
- E.g., marginal
  - $P(Res(eve))$
  - Sum out all non-query variables



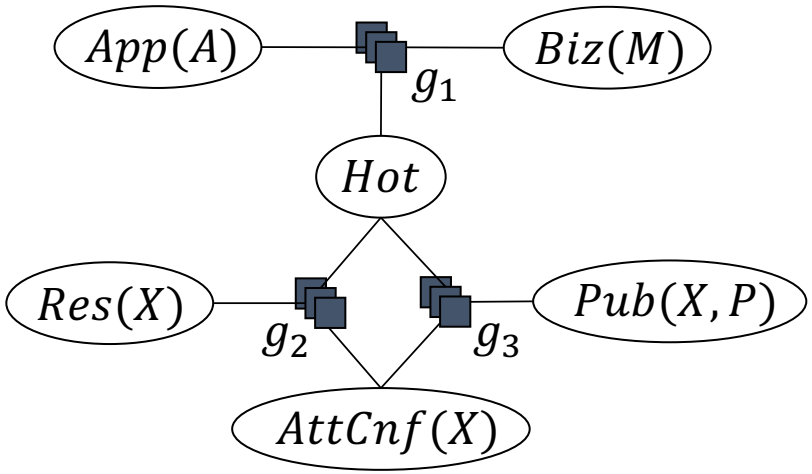
$$\sum_{h \in r(Hot)} \left(\sum_{c \in r(Res(X))} \sum_{c \in r(AttCnf(X))} g_2\,(Res.eve, H, c) \left(\sum_{p \in r(Pub(X,P))} g_3(h, c, p) \right)^{|P|} \right)^{|X|, X \neq eve}$$

$$\left(\sum_{a \in r(App(A))} \sum_{m \in r(Biz(M))} g_1(a, m, h) \right)^{*}$$

* Counting magic

Problem: Many Queries

- Set of queries
  - $P(Res(eve))$
  - $P(AttCnf(eve))$
  - $P(Pub(eve, p_1))$
  - $P(Hot)$
  - $P(App(a_1))$
  - $P(Biz(m_1))$
  - Combinations of variables
- Under evidence
  - $AttCnf(X') = 1$
  - $X' \in \{alice, eve\}$

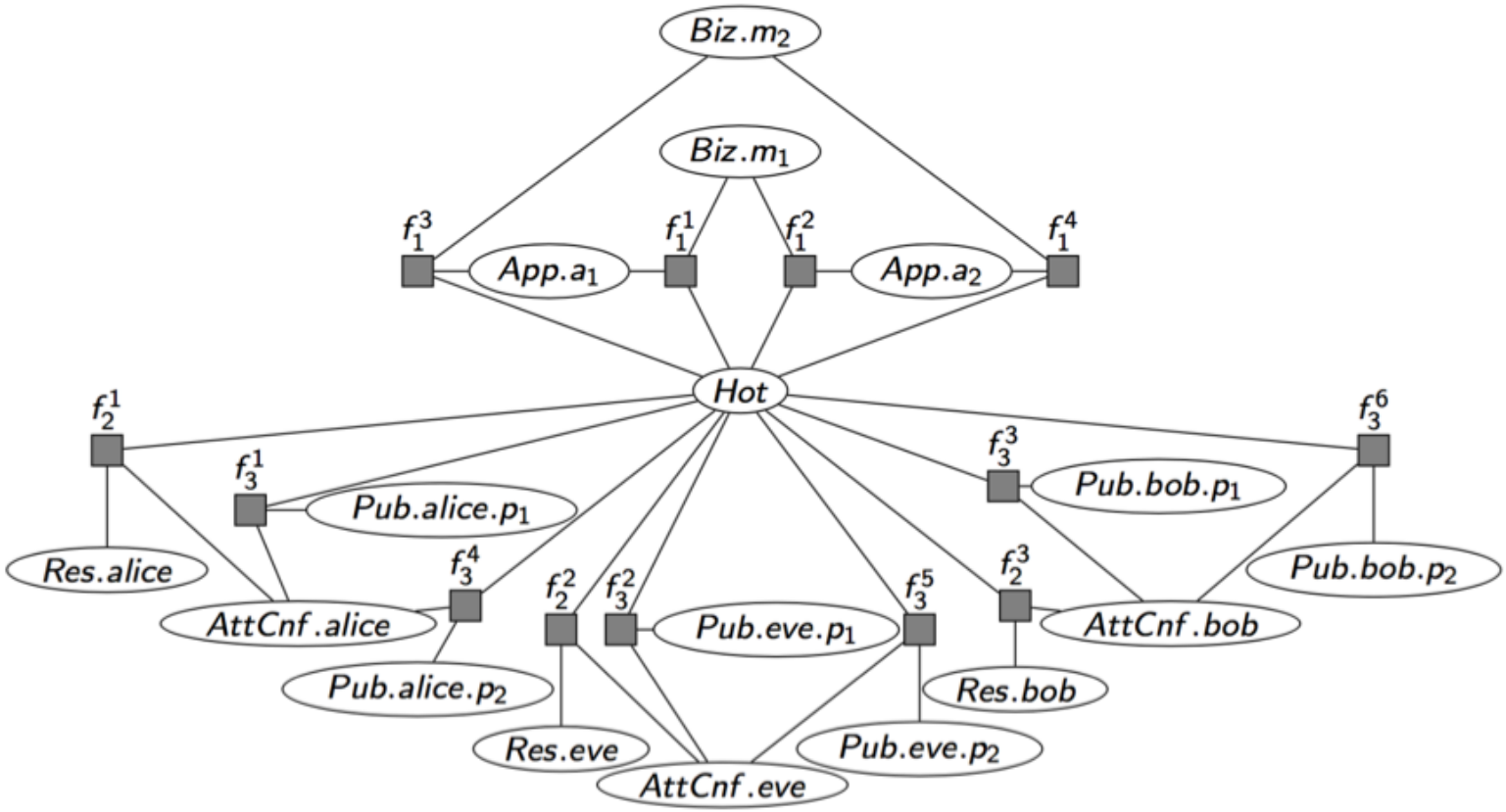


- (L)VE starts with complete model for QA

Solution: Junction Tree Algorithm

Lauritzen and Spiegelhalter (1988)

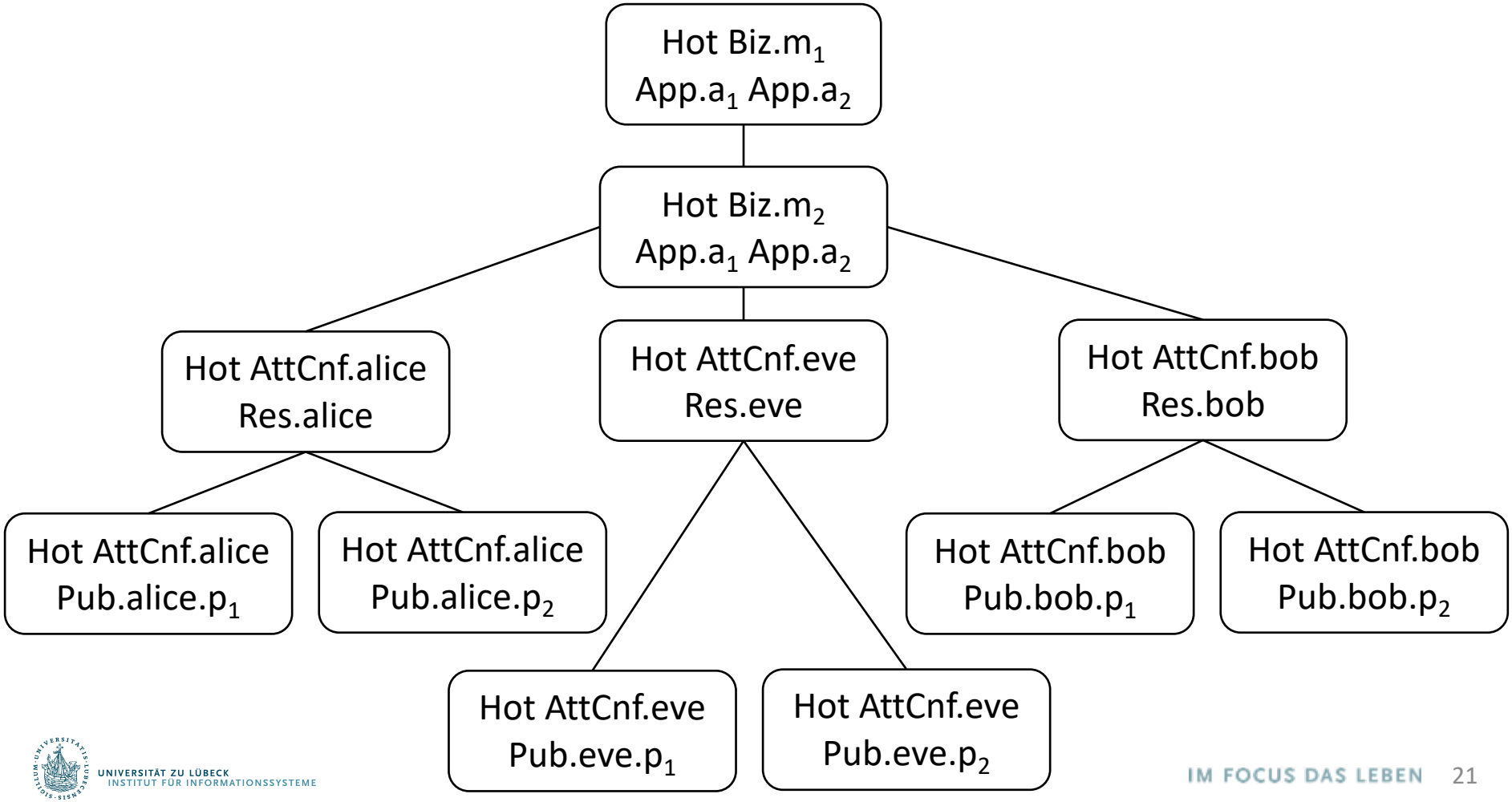
- Identify clusters in model to form junction tree



Junction Tree: Data Structure

Lauritzen and Spiegelhalter (1988)

- Clusters: sets of variables from underlying model



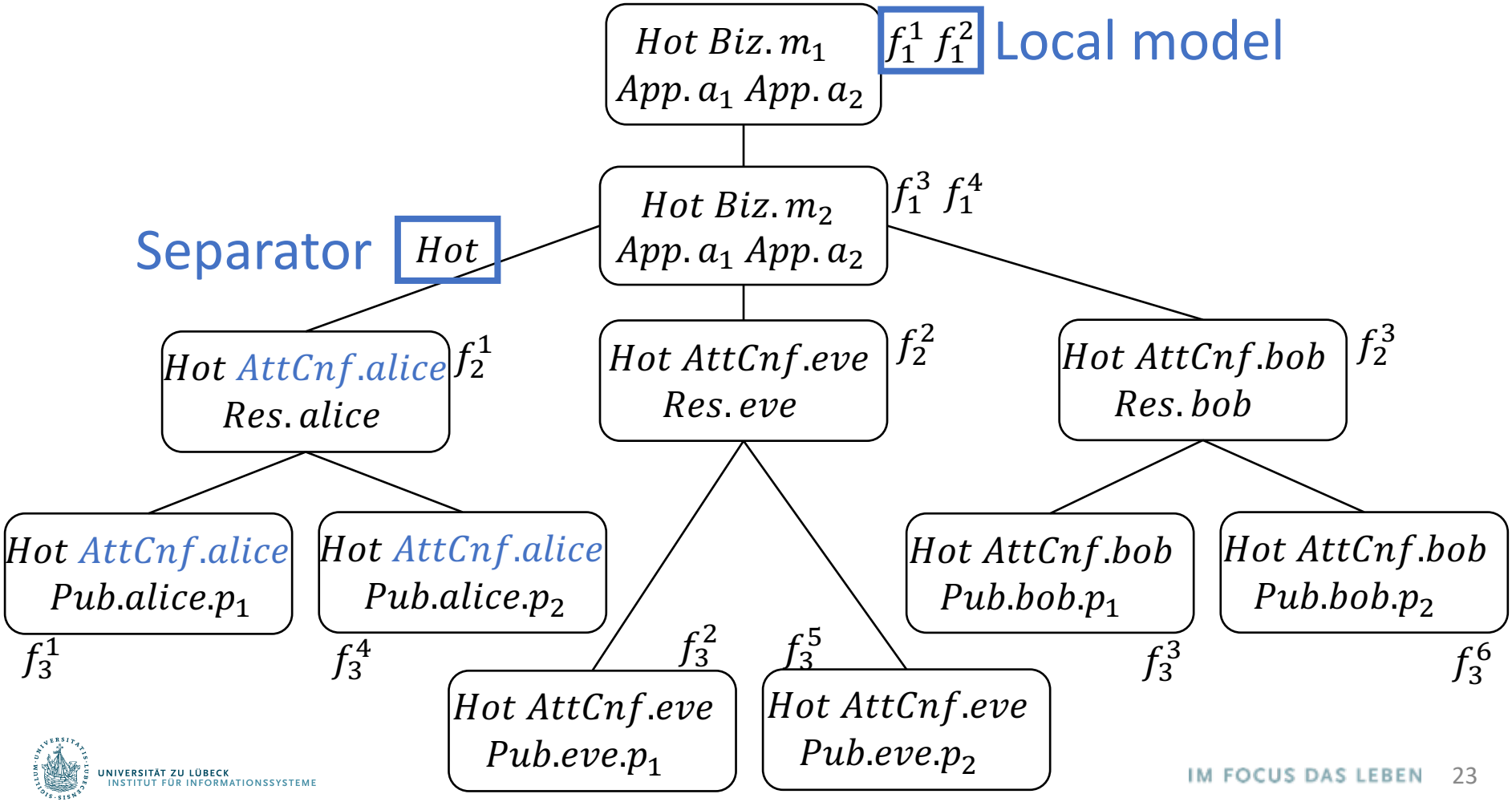
Junction Tree: Definition

Lauritzen and Spiegelhalter (1988)

- Junction tree $J = (V, E)$ for model F
  - V set of nodes, a node = cluster C
  - E set of edges
  1. A cluster C_i is a set of variables from F
  2. For every factor f in F, its arguments appear in some cluster C_i
  3. If a variable from F appears in clusters C_i and C_j, it must appear in every cluster C_k on the path between nodes i and j.
- Each cluster has a local model: set of factors
  - factor arguments subset of cluster
- Separator: shared variables of edges $(i, j) \in E$

Junction Tree: Components

- Clusters sufficient for QA after some preprocessing



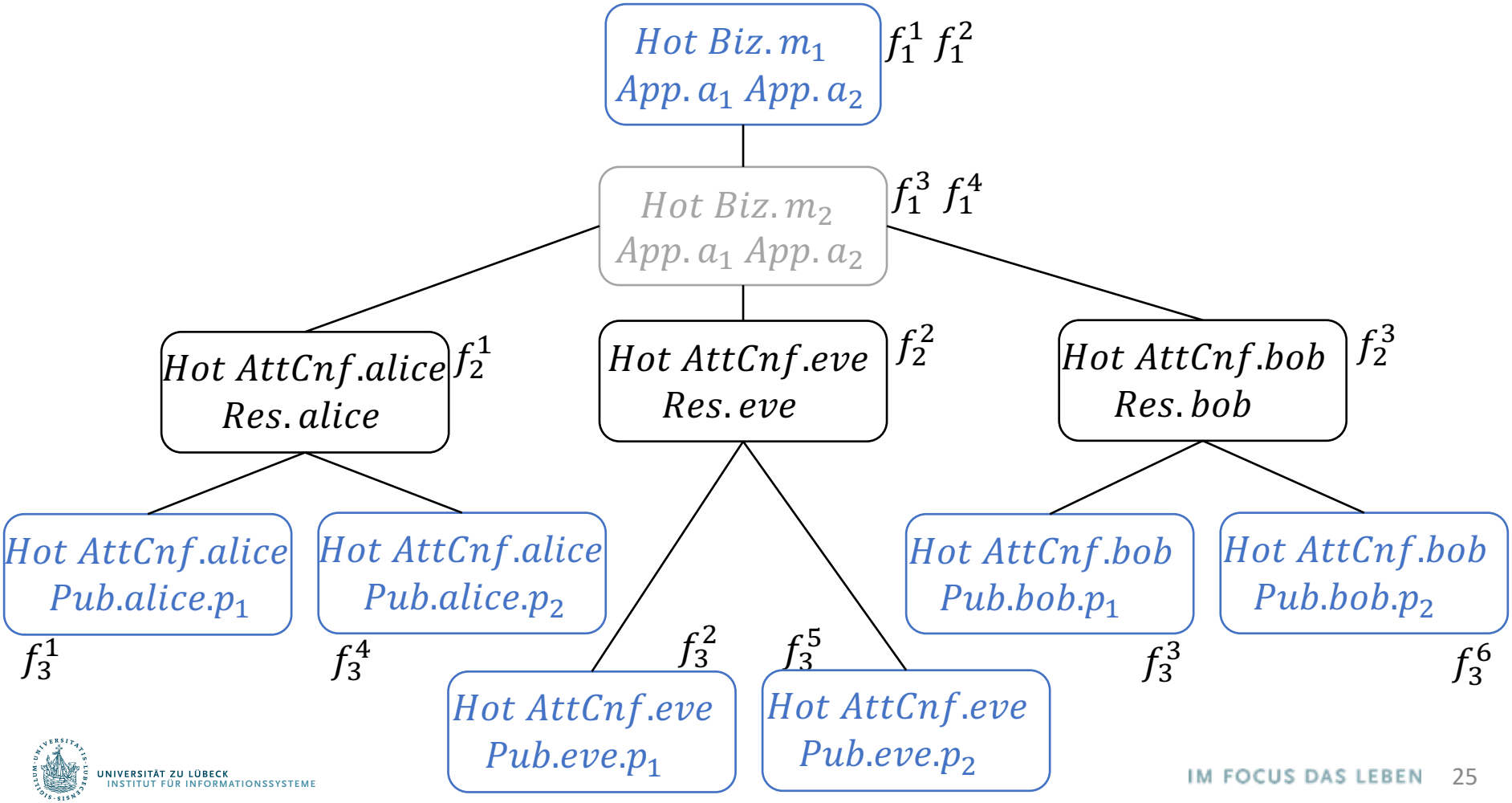
Junction Tree: Message Passing

Lauritzen and Spiegelhalter (1988), Shenoy and Shafer (1990)

- Distribute local information by messages
- Messages from periphery to centre and back
  - VE on local model and other messages
  - I.e., query over separator variables
- Local computations correctness:
  - Valid junction tree
  - combination & marginalisation
    (in the form of multiplication & summing out)
- QA on local models (including messages)
  - Use cluster that contains query variables

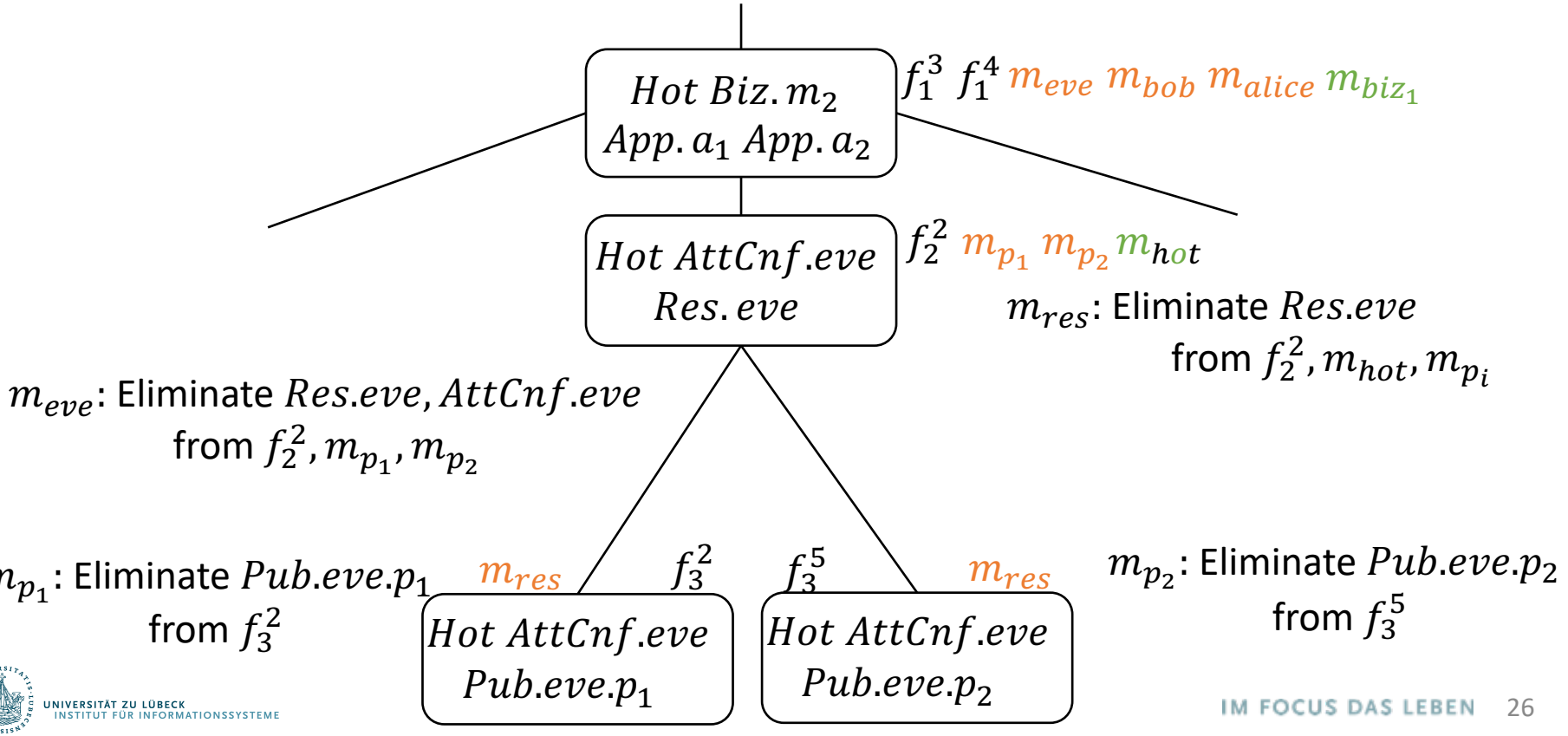
Junction Tree: Messages

- From periphery to centre and back



Junction Tree: Symmetries

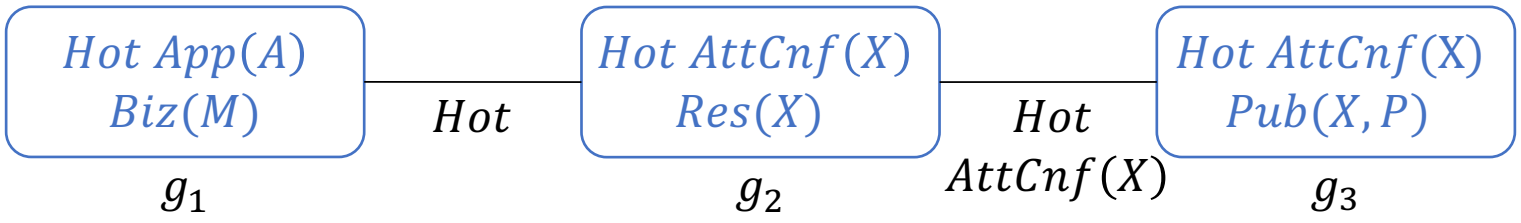
- Identical messages
- Information already present



First-order Junction Tree: FO jtree

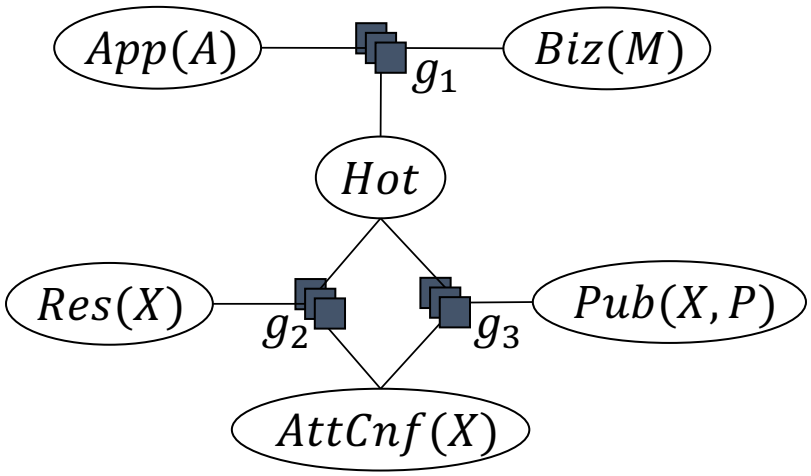
Braun and Möller (2016)

- Parameterised clusters = parclusters
  - Nodes of FO jtree
  - Shared PRVs between neighbours = separators
  - Parclusters have local models of parfactors
- FO jtree for parfactor model G:
  - Analogous definition, junction tree properties apply
  - Message passing, QA analogous

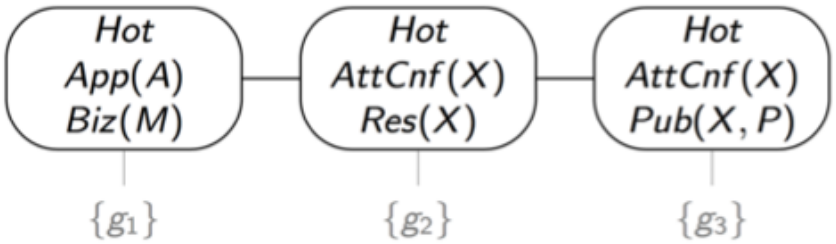


Lifted Junction Tree Algorithm: LJT

Braun and Möller (2017, 2017a)



Queries on grounded PRVs, e.g.,
$Res(eve), Pub(eve, p_1), Hot$



- Input
  - Model G
  - Evidence $\boldsymbol{E}$
  - Queries $\boldsymbol{Q}$
- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J
  3. Pass messages in J
     - Inbound
     - Outbound
  4. Answer queries $\boldsymbol{Q}$

LJT: Example Input

- Model $G = \{g_i\}_{i=1}^{3}$
  - $g_1\big(Hot, App(A), Biz(M)\big)$
  - $g_2(Res(X), Hot, AttCnf(X))$
  - $g_3\big(Hot, AttCnf(X), Pub(X, P)\big)$

  $\rightarrow$ Including function specification
- Evidence $\boldsymbol{E} = \{AttCnf(alice) = 1, AttCnf(eve) = 1\}$
- Queries $\boldsymbol{Q} = \{Res(eve), Res(eve) \wedge Pub(eve, p_1)\}$

- Algorithm
  1. Build FO jtree J for G

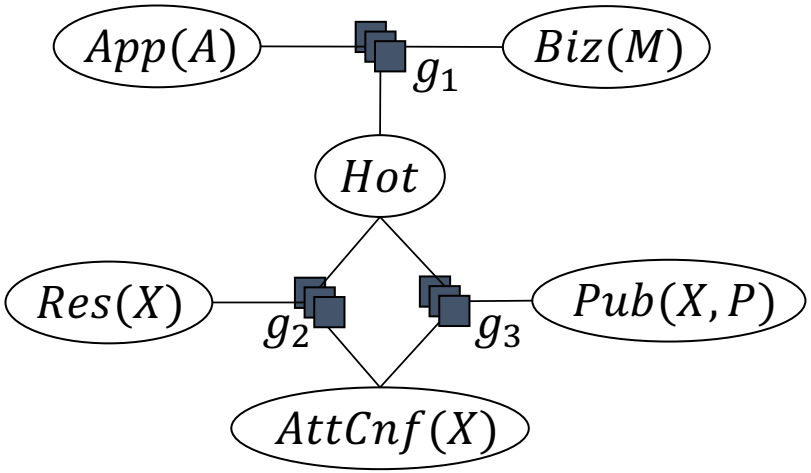
FO Jtree Construction

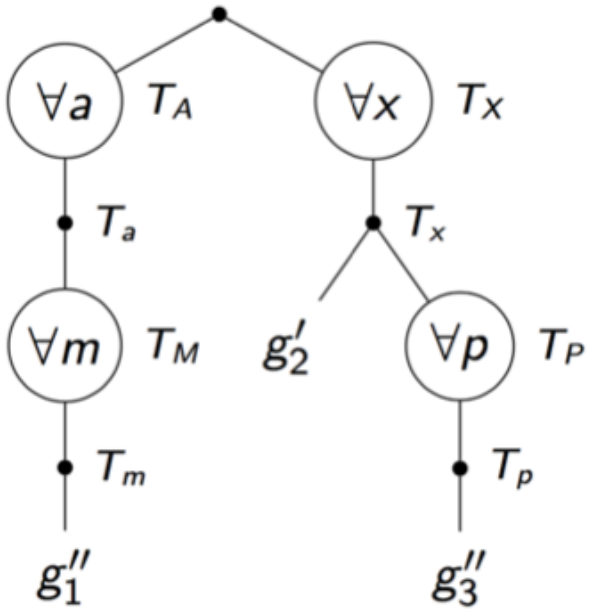
- Propositional jtree construction
  - Triangulation, compute maximum spanning tree, …
  - Hypergraph partitioning
- First-order: logical variables
  - First-order decomposition trees (FO dtrees)
  - FO dtrees have node properties (cutset, context, cluster)
  - (FO) dtree + clusters = (FO) jtree
    (works in propositional case, too)
  - Heuristic to build an FO dtree
    (logical variables guide the construction)

FO Dtree: Data Structure

Taghipour et al. (2013)

- (L)VE decomposes model into subproblems
- Represent as tree





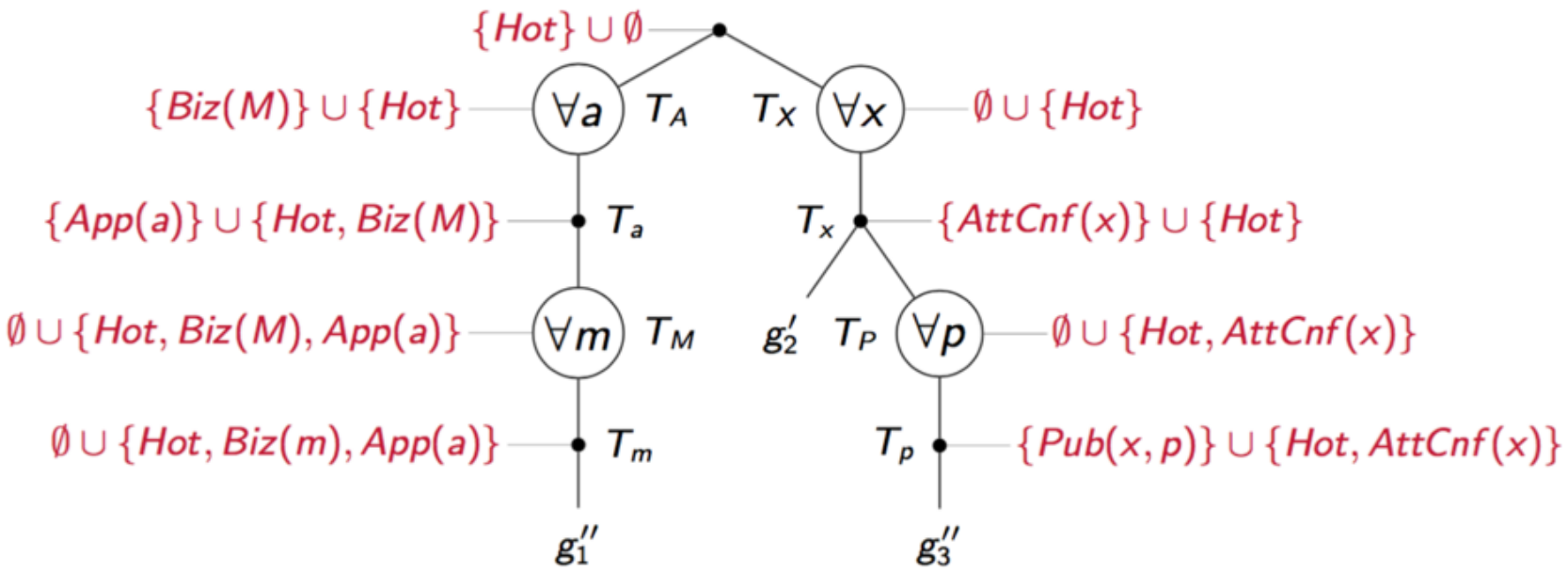
FO Dtree: Cutset, Context, Cluster

Taghipour et al. (2013a)

$$cutset(T) = \bigcup_{T_i, T_j \in child(T)} RV(T_i) \cap RV(T_j) \backslash acutset(T), acutset(T) = \bigcup_{T' \in ancestor(T)} cutset(T')$$

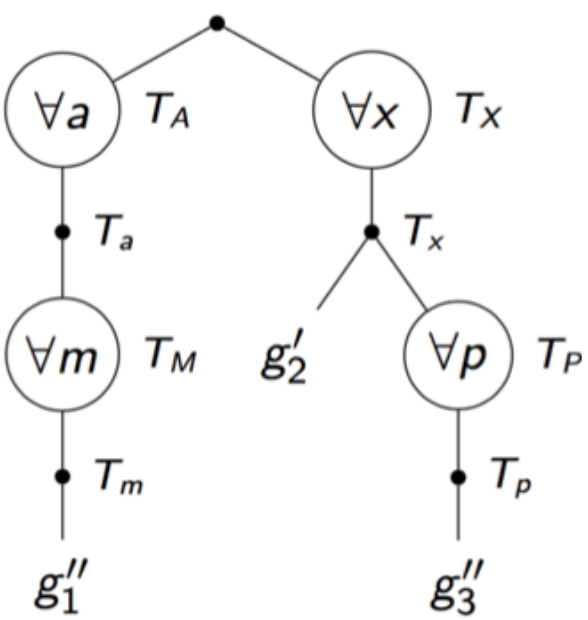
$$context(T) = RV(T) \cap acutset(T), RV(T) = \bigcup_{T' \in child(T)} RV(T'), RV(L) = RV(\phi_L), L \text{ leaf}$$

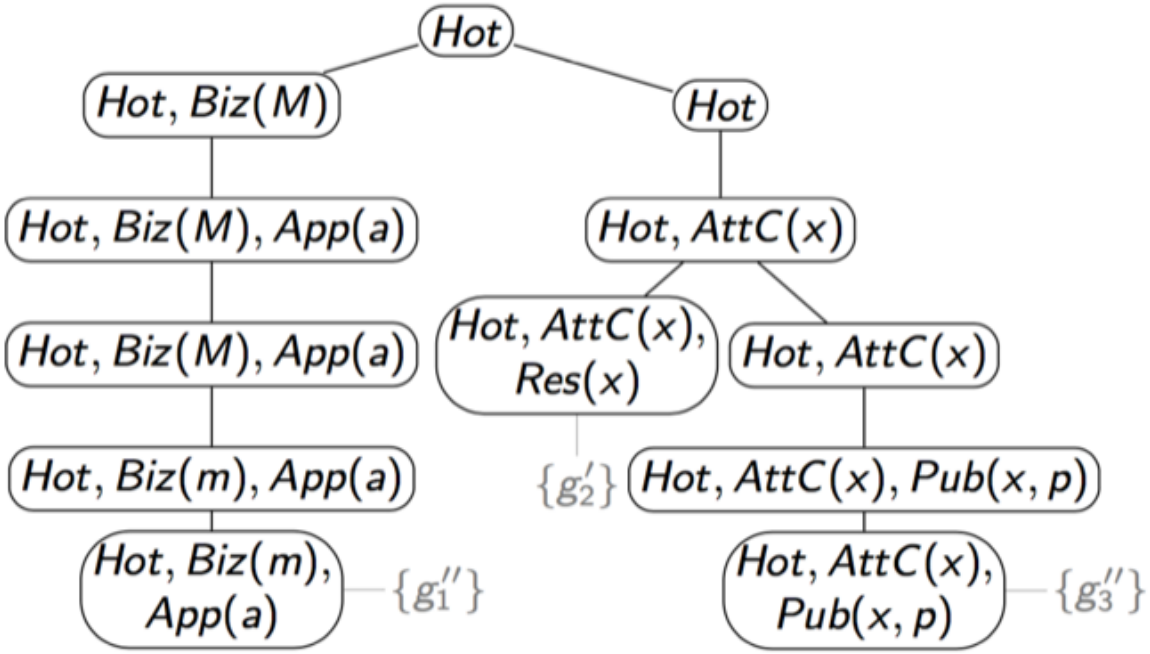
$$cluster(T) = cutset(T) \cup context(T), cluster(L) = RV(\phi_L), L \text{ leaf}$$



FO Dtree to FO Jtree

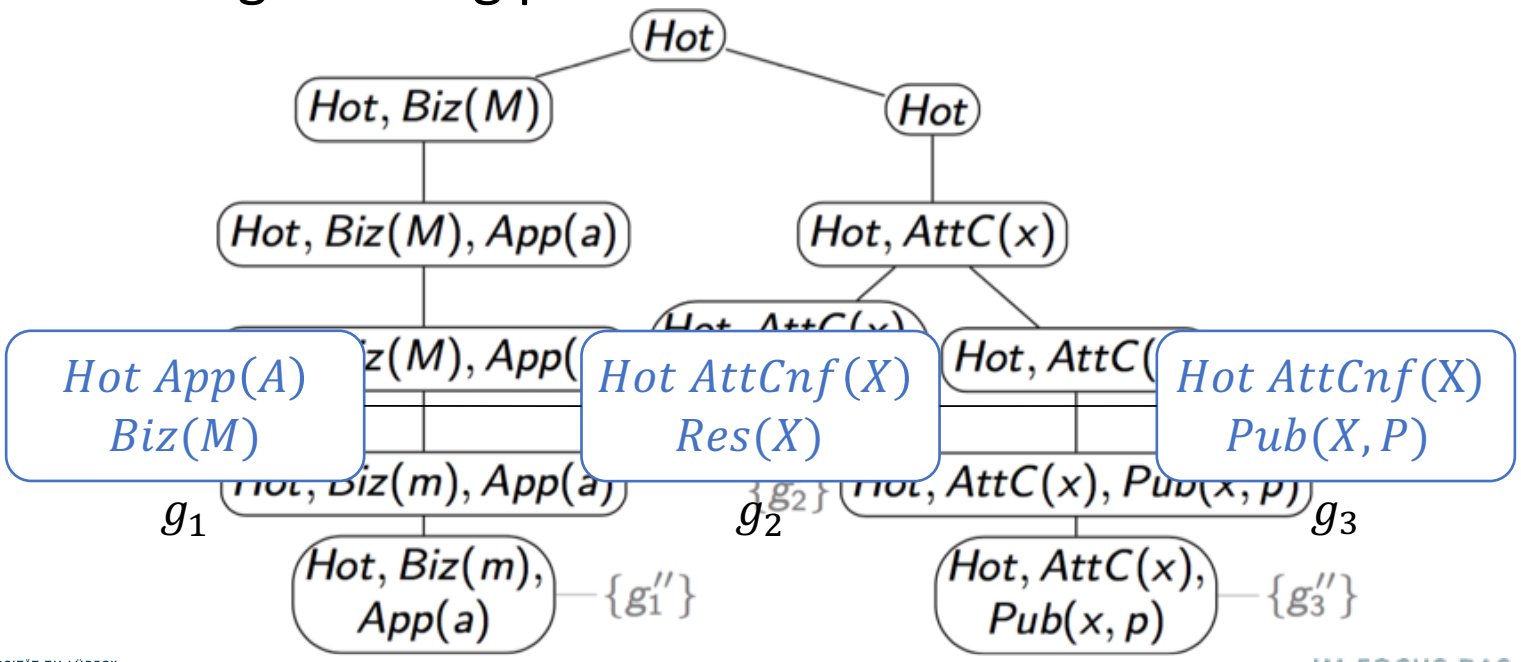
- Compute clusters per node and minimise





Minimising an FO Jtree

- FO jtree is minimal
  - If by removing a variable from any parcluster, the FO jtree stops being an FO jtree
- Merge parclusters
  - If neighbouring parclusters are subsets of each other



Lifted Junction Tree Algorithm: LJT

- Input
  - Model G
  - Evidence $\boldsymbol{E} = \{AttCnf(alice) = 1, AttCnf(eve) = 1\}$
  - Queries $\boldsymbol{Q} = \{Res(eve), Res(eve) \wedge Pub(eve, p_1)\}$
- Algorithm
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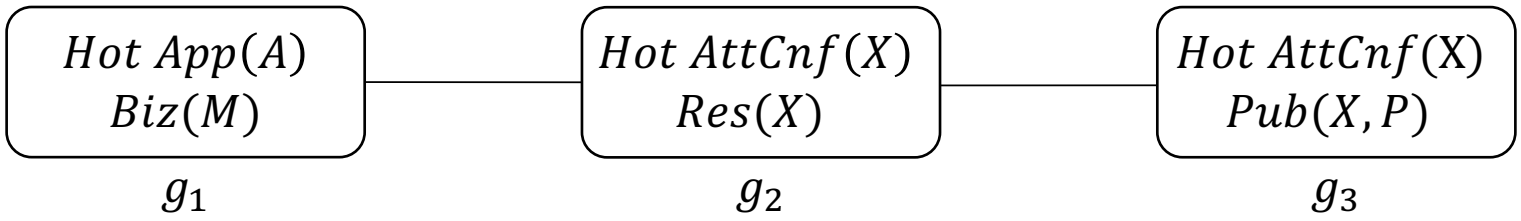
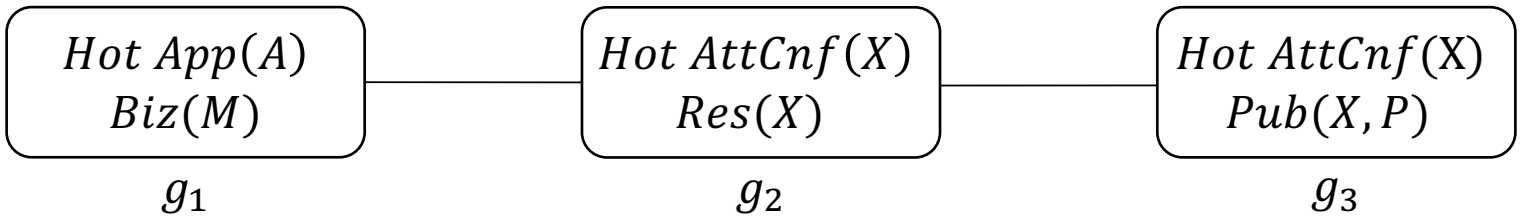


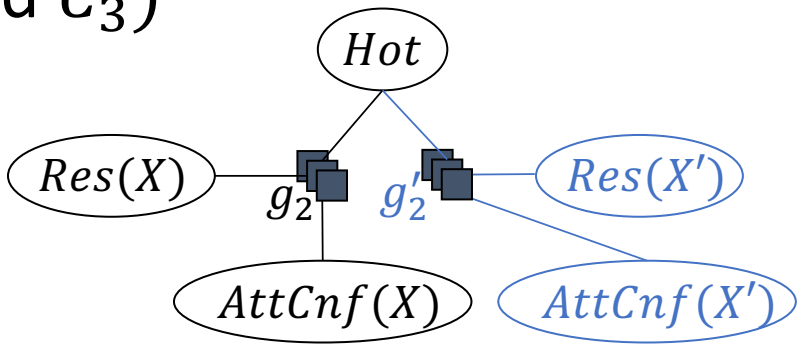
  2. Enter evidence $\boldsymbol{E}$ into J

LJT: Enter Evidence

- At every parcluster that contains evidence variables
  - Evidence $\boldsymbol{E} = \{AttCnf(eve) = 1, AttCnf(alice) = 1\}$
  - Parclusters C_2 and C_3



- Enter evidence, at C_2 (and C_3)
  - Split local model
  - Absorb evidence, in g_2' (possibly exponentiate)



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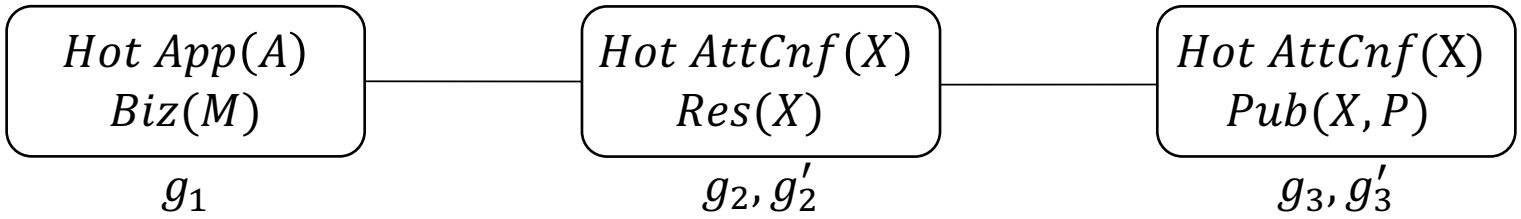
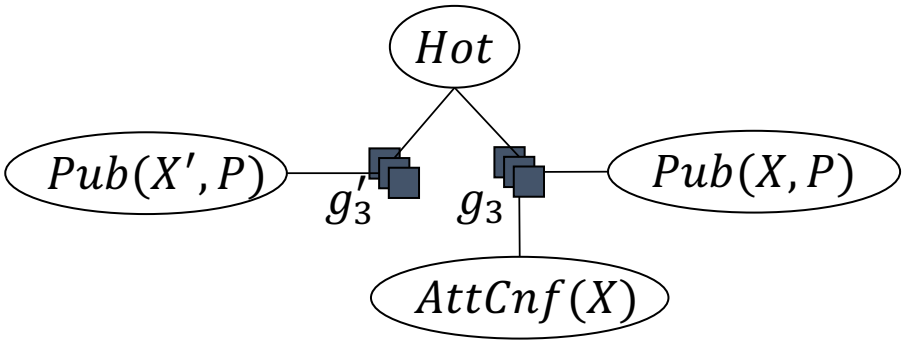
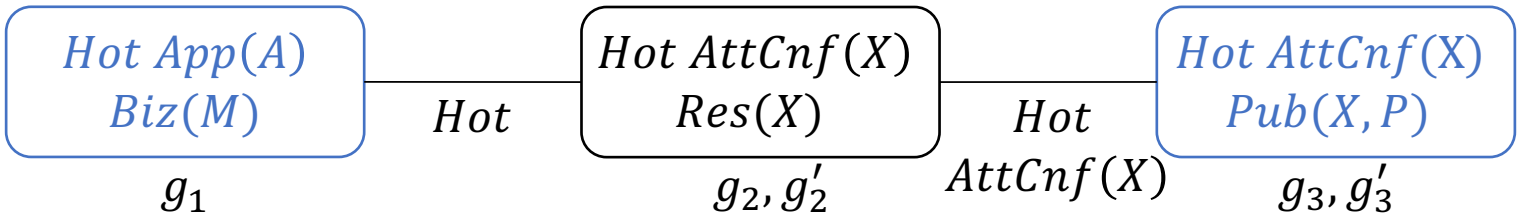


3. Pass messages in J

LJT: Pass Messages

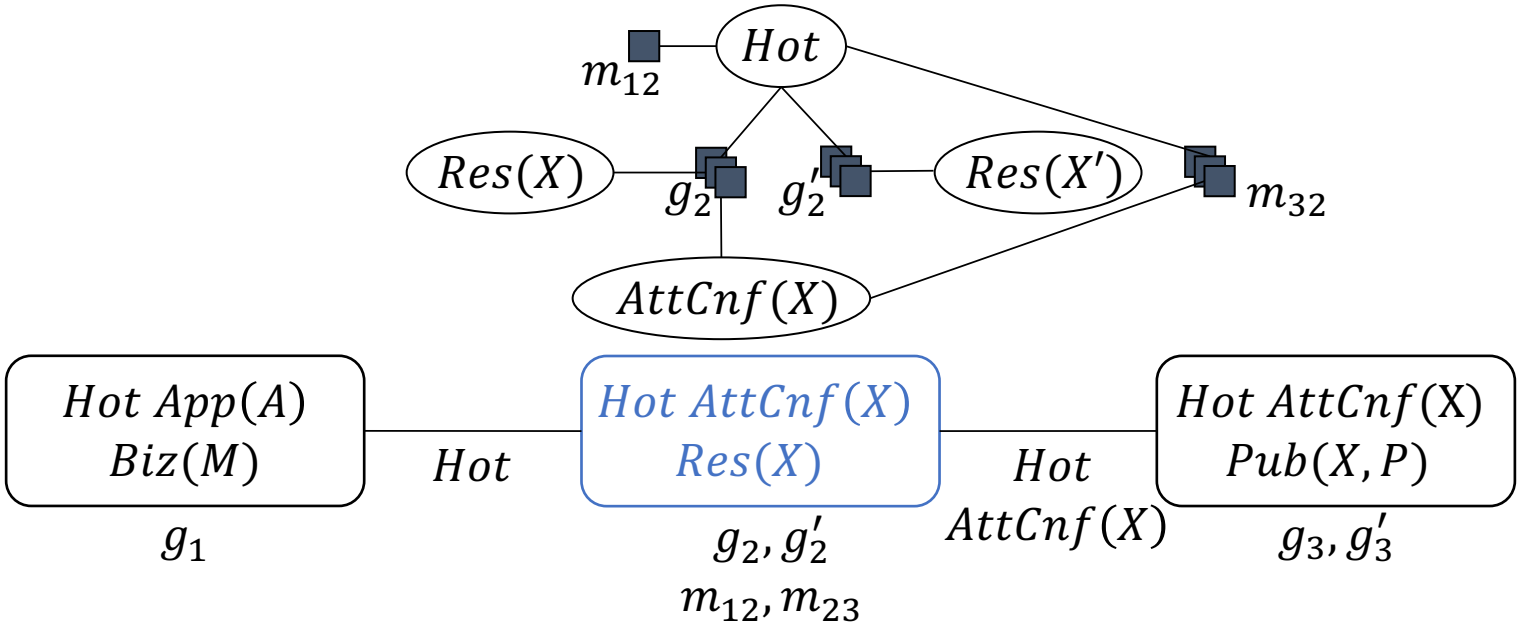
- Inbound
  - m_{12} from C_1 to C_2 over Hot: eliminate $App(A), Biz(M)$
  - m_{23} from C_3 to C_2 over $Hot, AttCnf(X)$: eliminate $Pub(X, P), Pub(X', P)$





LJT: Pass Messages

- Outbound
  - m_{21} from C_2 to C_1 over Hot: eliminate $AttCnf(X), Res(X)$ from g_2, g_2', m_{32}
  - m_{32} from C_2 to C_3 over $Hot, AttCnf(X)$: eliminate $Res(X), Res(X')$ from g_2, g_2', m_{12}



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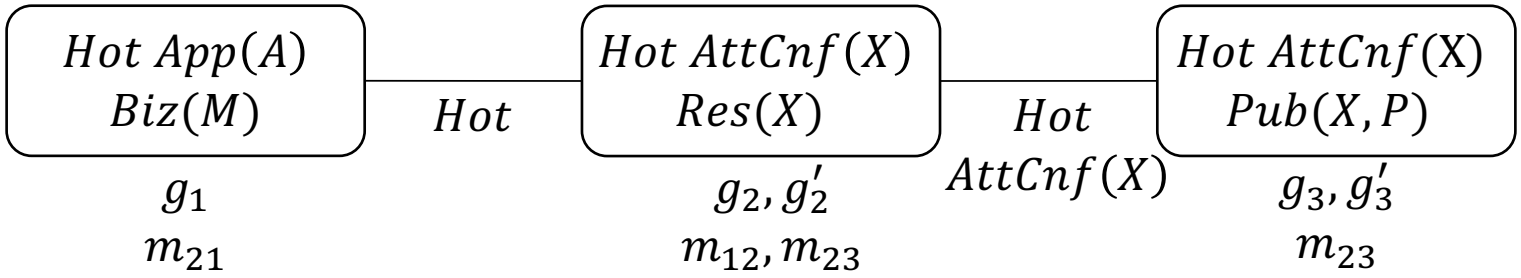
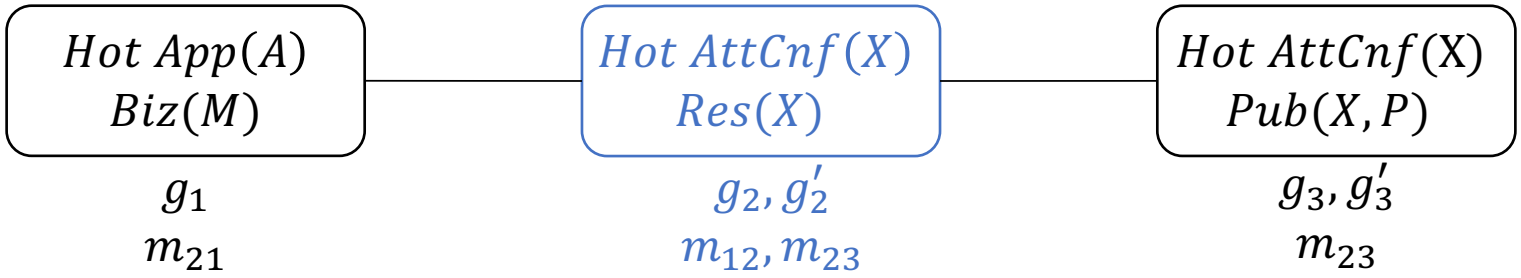


4. Answer queries $\boldsymbol{Q}$

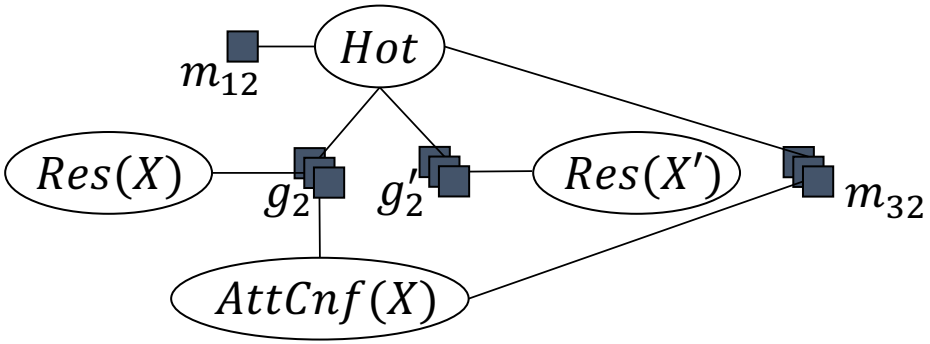
LJT: Answer Queries

- Find subtree that covers the query variables
- Extract submodel without duplicate information
- Use LVE to answer query
- Queries $\boldsymbol{Q} = \{Res(eve), Res(eve) \wedge Pub(eve, p_1)\}$
  - $Q_1 = Res(eve)$



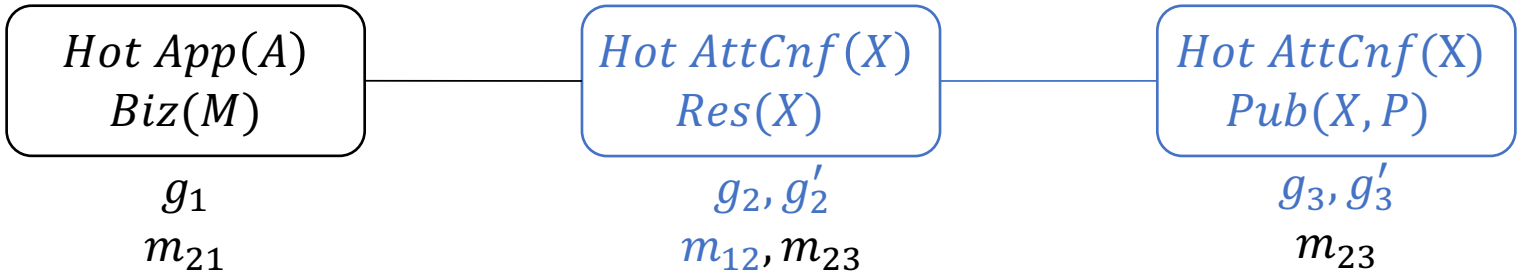
LJT: Answer Queries

- Find subtree that covers the query variables
- Extract submodel without duplicate information
- Use LVE to answer query
- Queries $\boldsymbol{Q} = \{Res(eve), Res(eve) \wedge Pub(eve, p_1)\}$
  - $Q_1 = Res(eve)$
    - Split model
    - Eliminate non-query variables
    - Normalise



LJT: Answer Queries

- Find subtree that covers the query variables
- Extract submodel without duplicate information
- Use LVE to answer query
- Queries $\boldsymbol{Q} = \{Res(eve), Res(eve) \wedge Pub(eve, p_1)\}$
  - $Q_2 = Res(eve) \wedge Pub(eve, p_1)$



LJT: Answer Queries

- Find subtree that covers the query variables
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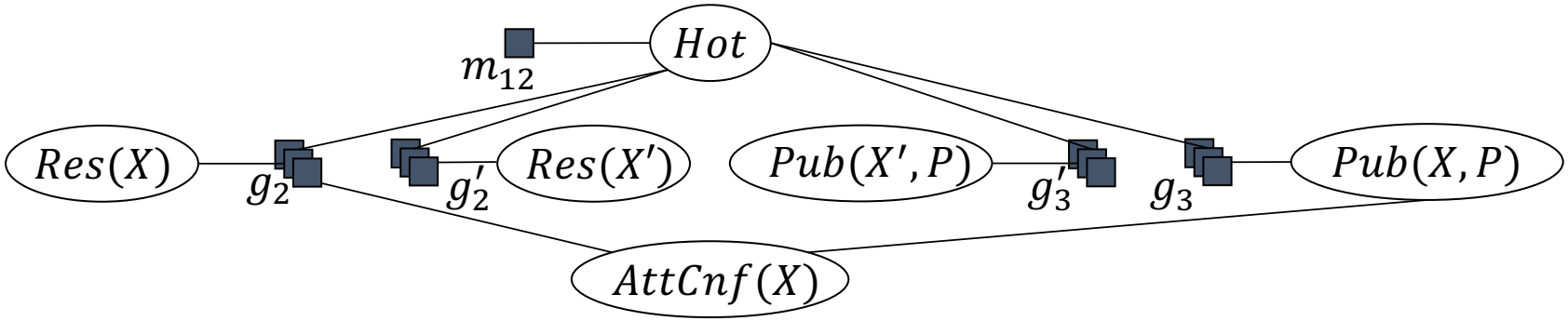
- Queries $\boldsymbol{Q} = \{Res(eve), Res(eve) \wedge Pub(eve, p_1)\}$
  - $Q_2 = Res(eve) \wedge Pub(eve, p_1)$
    - Split model
    - Eliminate non-query variables
    - Normalise

All under evidence
$\boldsymbol{E = \{AttCnf(eve) = 1, AttCnf(alice) = 1\}}$

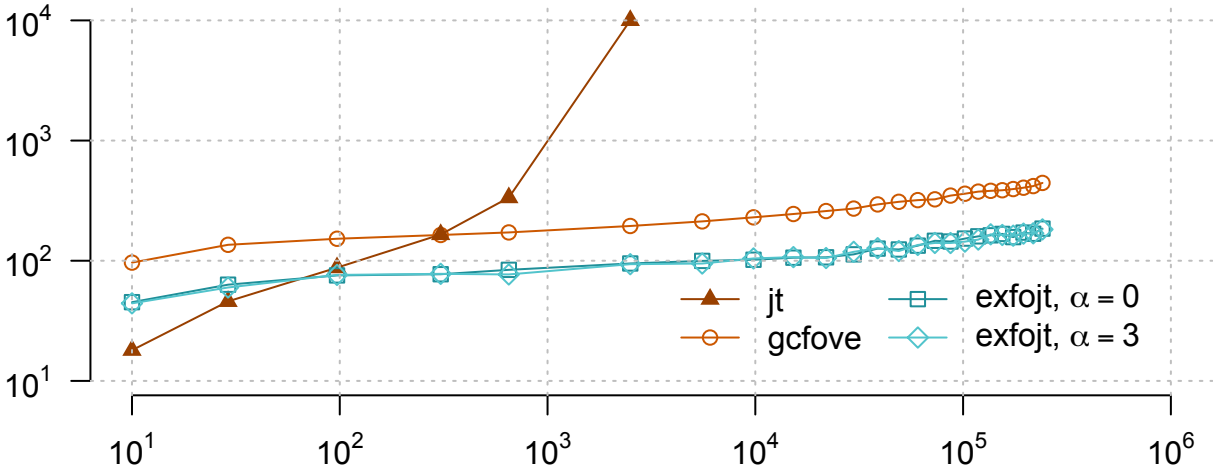


LJT: Performance

- Static overhead
  - Construction
  - Evidence entering
  - Message passing
- Payoff during QA
  - More space required

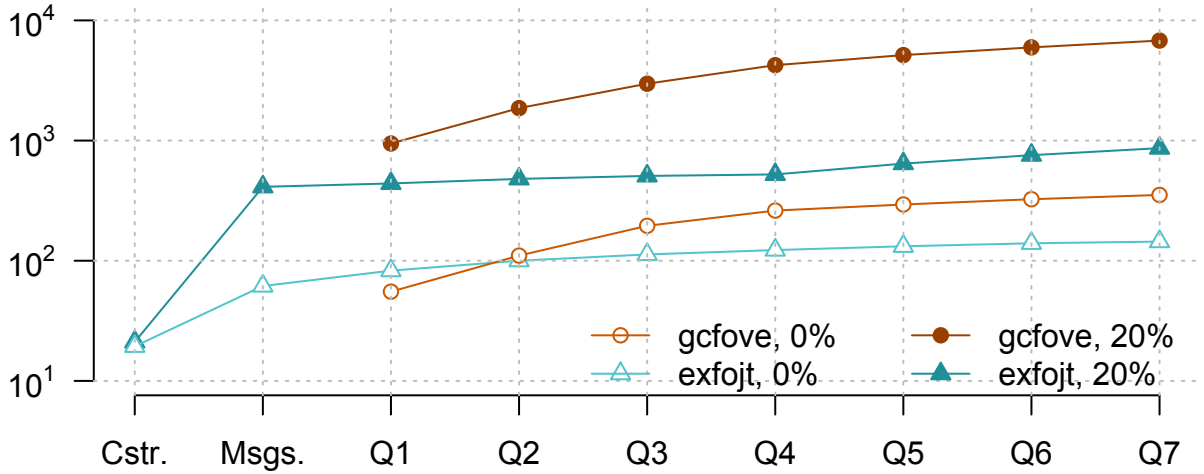
- Prototype implemenation
  - LVE by Taghipour https://dtai.cs.kuleuven.be/software/gcfove
  - LJT

LJT: Grounded versus Lifted



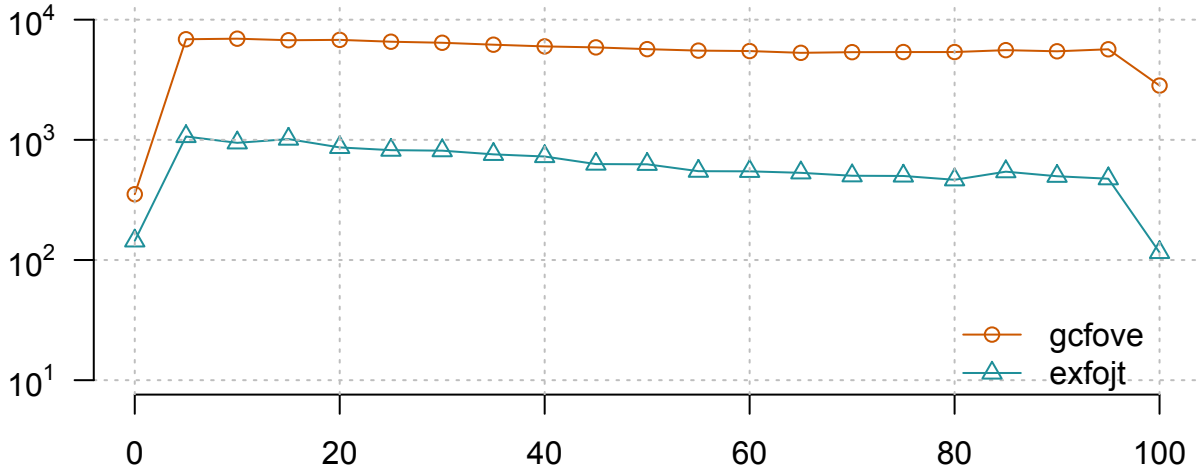
Runtimes in milliseconds , seven queries, grounded model size between 10 and 100,000
Evidence at 0%, α refers to a fusion step not presented here

LJT: Payoff versus LVE



Accumulated runtimes in milliseconds , seven queries, grounded model size: 100,000
Evidence at 0% and 20% on PRVs with one parameter

LJT: Evidence

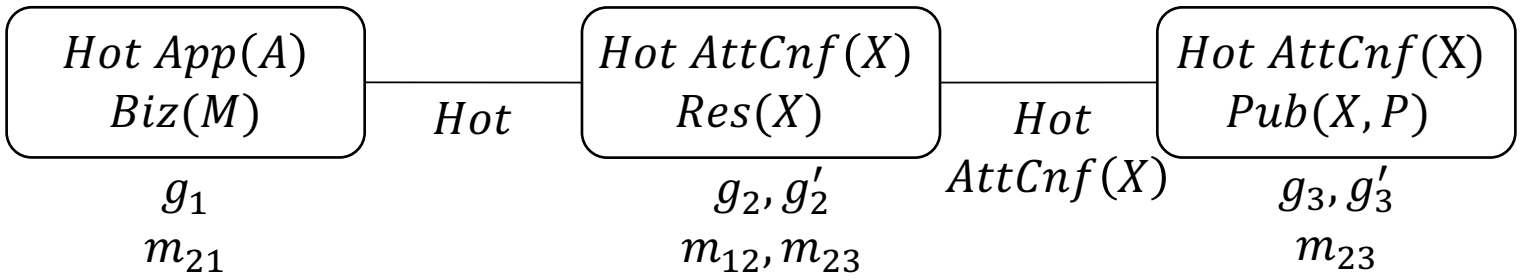


Runtimes in milliseconds , seven queries, grounded model size: 100,000
Evidence from 0% to 100% in 5% steps on PRVs with one parameter

LJT: New Model/Evidence/Queries

- Changing inputs
  - Queries $\boldsymbol{Q}$
    - Continue at Step 4

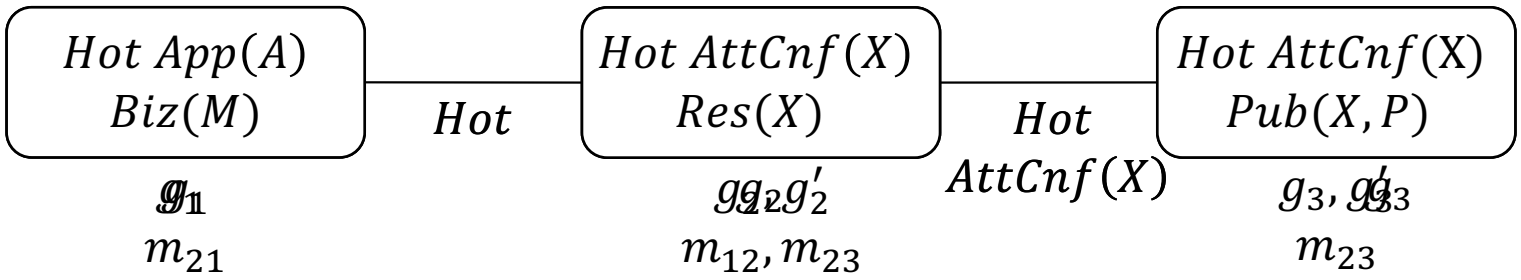
- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J
  3. Pass messages in J
     - Inbound
     - Outbound
  4. Answer queries $\boldsymbol{Q}$



LJT: New Model/Evidence/Queries

- Changing inputs
  - Queries $\boldsymbol{Q}$
    - Continue at Step 4
  - Evidence $\boldsymbol{E}$
    - Restart at Step 2
  - Model G
    - Restart at Step 1
- Incremental changes: at least restart at Step 3

- Algorithm
  1. Build FO jtree J for G
  2. Enter evidence $\boldsymbol{E}$ into J
  3. Pass messages in J
     - Inbound
     - Outbound
  4. Answer queries $\boldsymbol{Q}$



QA: Most probable assignment

Dawid (1992), Dechter (1999)

- to all variables: most probable explanation (MPE)

$$\underset{rv(G)}{\operatorname{argmax}} P_G$$

  - (L)VE: maxing out (instead of summing out)
  - (L)JT: message calculation by maxing out

- to subset of variables: maximum a posteriori (MAP)

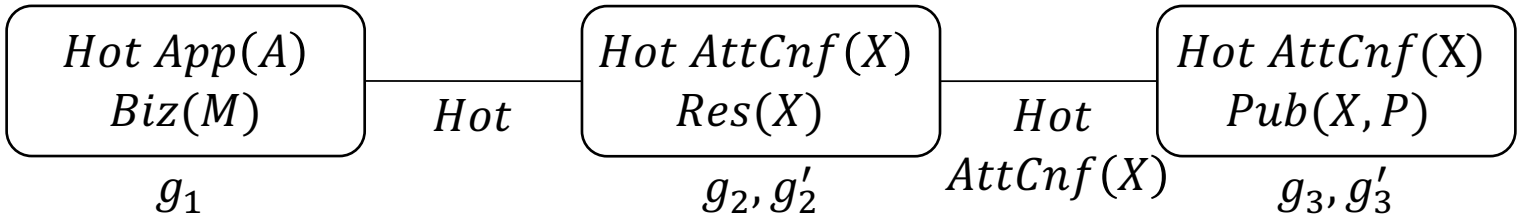
$$\underset{Res.eve,Biz.m_1}{\operatorname{argmax}} \sum_{\substack{r(App(A)),r(Biz(M)),M\neq m_1,\\ r(AttCnf(X)),r(Pub(X,P)),\\ r(Res(X)),X\neq eve,r(Hot)}} P_G$$

  - *MAP a more general case of MPE*
  - $\sum$ **and** $\max$ **not commutative!**

LJT: MPE

Braun and Möller (under review)

- As before: construction, evidence entering
- Message passing
  - Only one message pass (from periphery to centre)
  - Max out remaining variables at centre



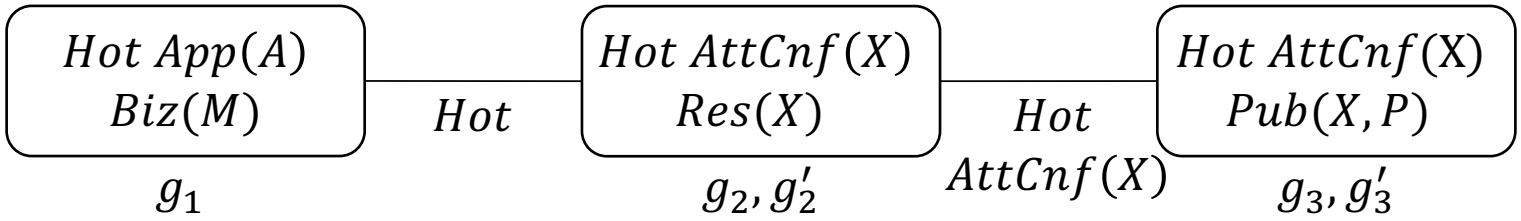
LJT: MPE

- Answering an MPE
  - m_{12} through maxing out $App(A), Biz(M)$
  - m_{32} through maxing out $Pub(X, P)$
  - At C_2, max out $Res(X)$, AttCnf(X)

$$Hot = 0 \quad \forall A, M : App(A) = 0, Biz(M) = 0$$

$$\forall X' \in \{alice, eve\}, P : Res(X') = 1, AttCnf(X') = 1, Pub(X', P) = 1$$

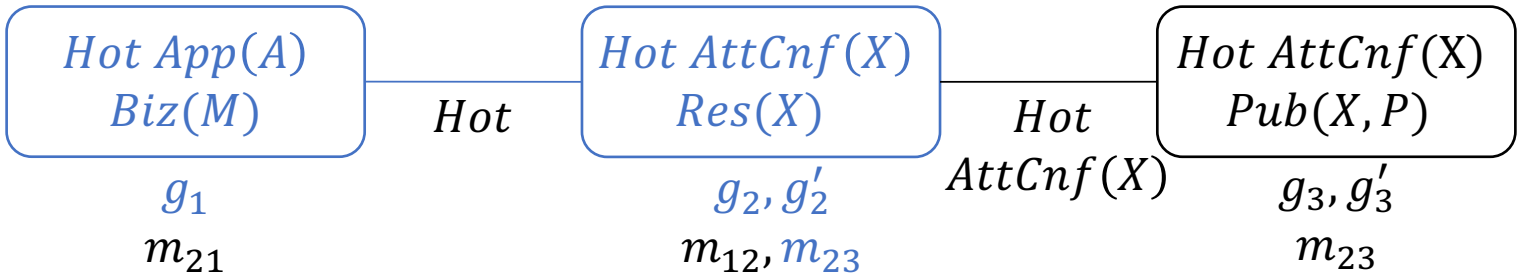
$$\forall X, P : Res(X) = 1, AttCnf(X) = 1, Pub(X, P) = 1$$



LJT: MAP

Braun and Möller (under review)

- As before: construction, evidence entering, message passing (with summing out)
- Answer MAP query: get submodel (as before)
  - Sum out non-query variables
  - Max out query variables
  - E.g., $Res.eve, Biz.m_1$
    - Sum out $Hot, App(A), AttCnf(X), Res(X), X \neq eve, Biz(M), M \neq m_1$
    - Max out $Res.eve, Biz.m_1$

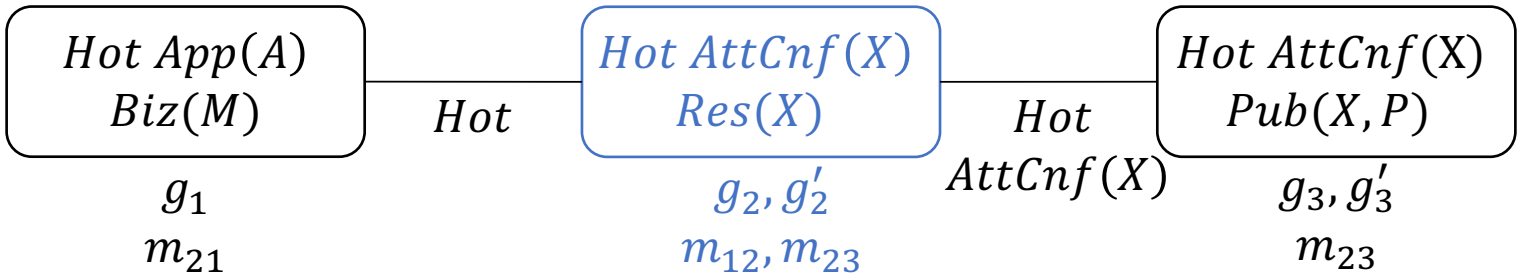


LJT: MAP

Braun and Möller (under review)

- Assignment to complete parclusters safe
  - After message pass, max out parcluster variables
  - E.g., MAP over C_2
    - Max out $AttCnf(X), Res(X), Hot$

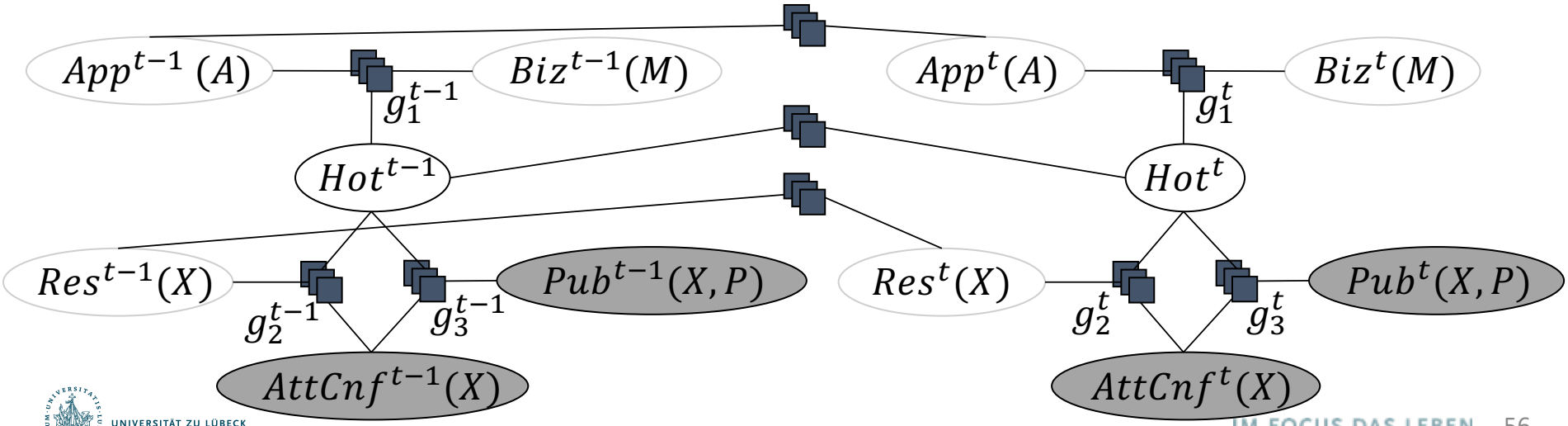
$$Hot = 0$$
$$\forall X' \in \{alice, eve\}: Res(X') = 1, AttCnf(X') = 1$$
$$\forall X : Res(X) = 1, AttCnf(X) = 1$$



Dynamic Parfactor Graphs

Gehrke et al. (under review)

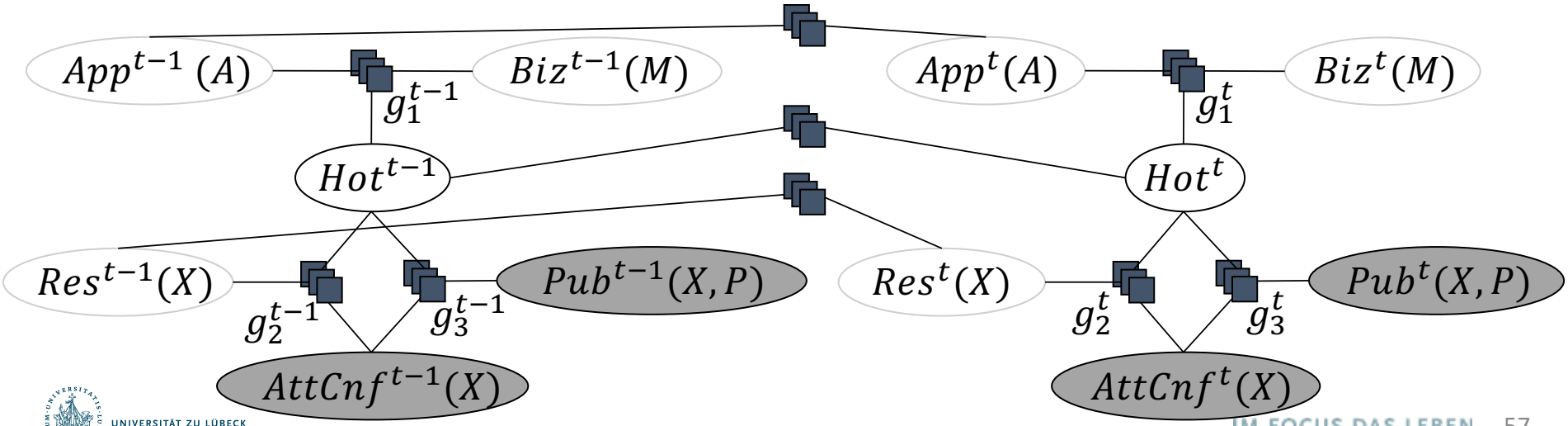
- As before: duplicate model for time slices
  - Connect PRVs from one slice to next
- Inference tasks
  - Filtering $P(X^t|E^{0:t})$, Prediction $P\left(X^{t+k}\middle|E^{0:t}\right)$
  - Smoothing $P\left(X^k\middle|E^{0:t}\right), k < t$
  - MAP, MPE (Viterbi)



Lifted Dynamic Junction Tree Alg.

Gehrke et al. (under review)

- Build FO jtrees
  - For model with t=0 (one slice)
  - For 1.5 time-slice model (one slice plus predecessors)
  - Ensure PRVs with successors in next slice in one parcluster
- Within time-slice: reasoning with LJT
- Message to transport current information to next slice



Outlook

- Optimising LJT
  - Parallelisation
  - Caching
- From discrete over interval to continuous ranges
- Learning?
  - Structure
  - Potentials
  - Symmetries
- Open world?
  - Unknown domains
  - Unknown behaviour

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Own Work

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