Intelligent Agents
Game Theory and Social Choice

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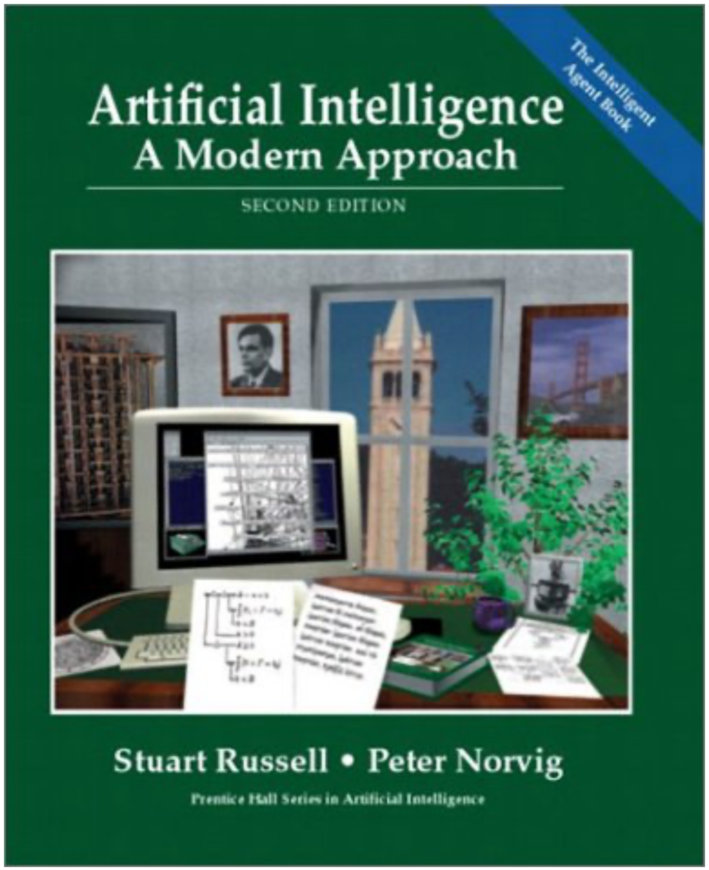
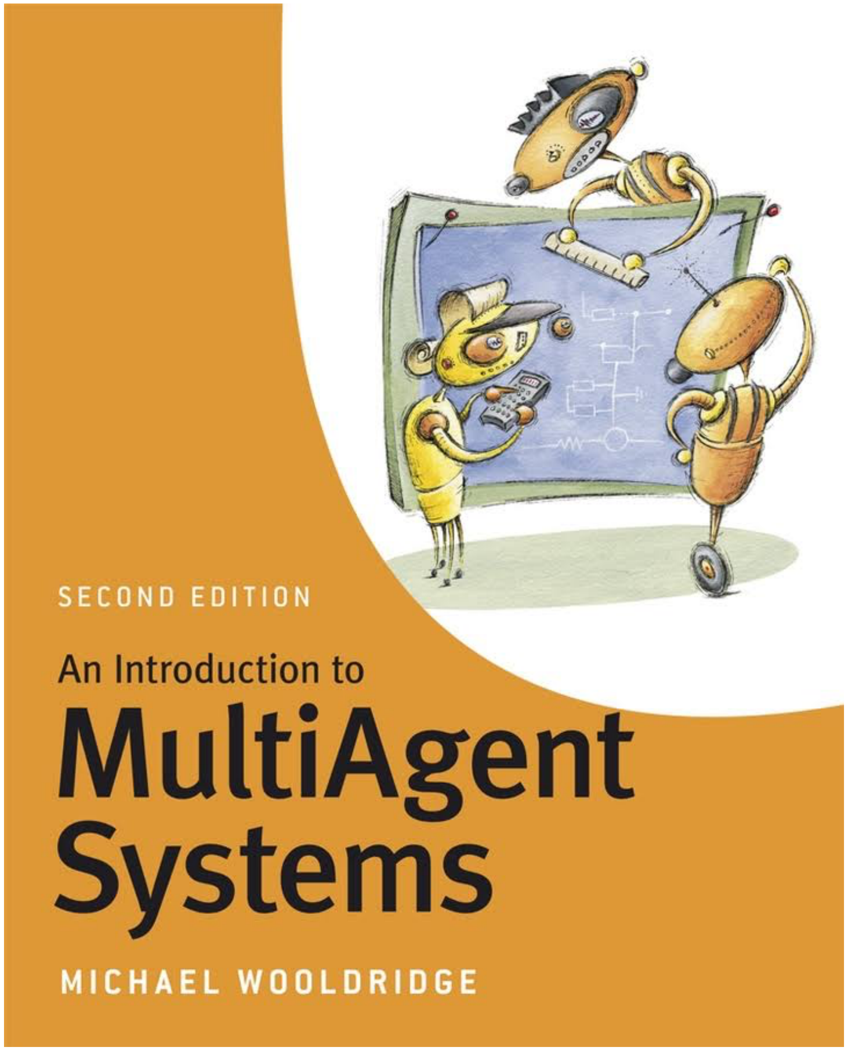
Literature



Chapter 17

Presentations from CS 886
Advanced Topics in
AI Electronic Market Design
Kate Larson
Waterloo Univ.

Acknowledgements also to…

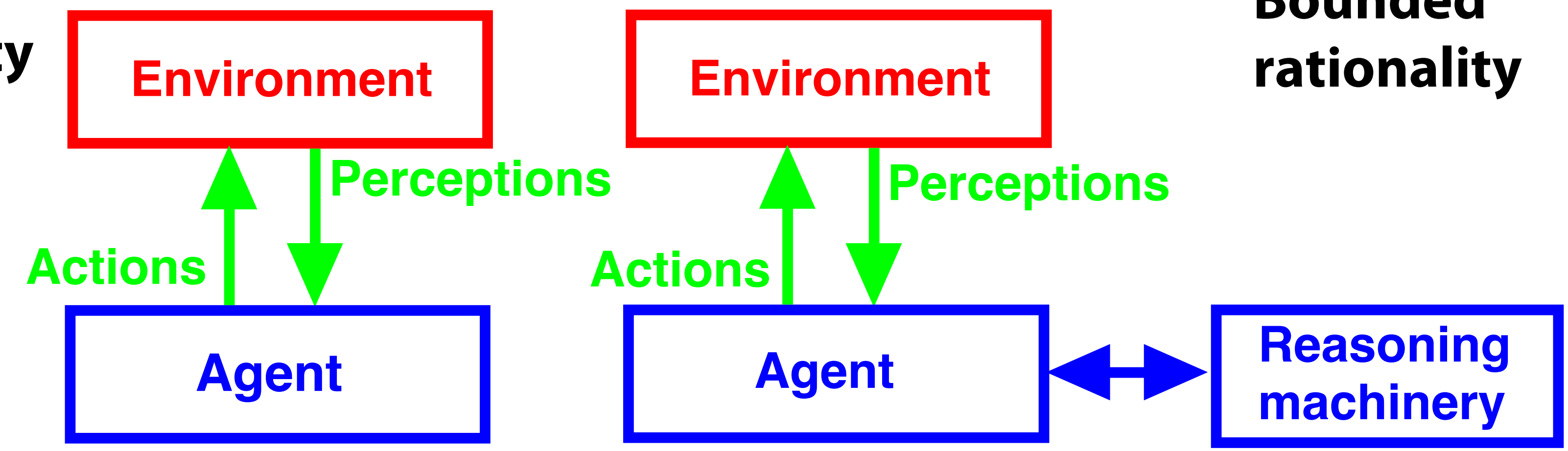


… and to many other lecturers who have shared their slides on the web!

Full vs bounded rationality

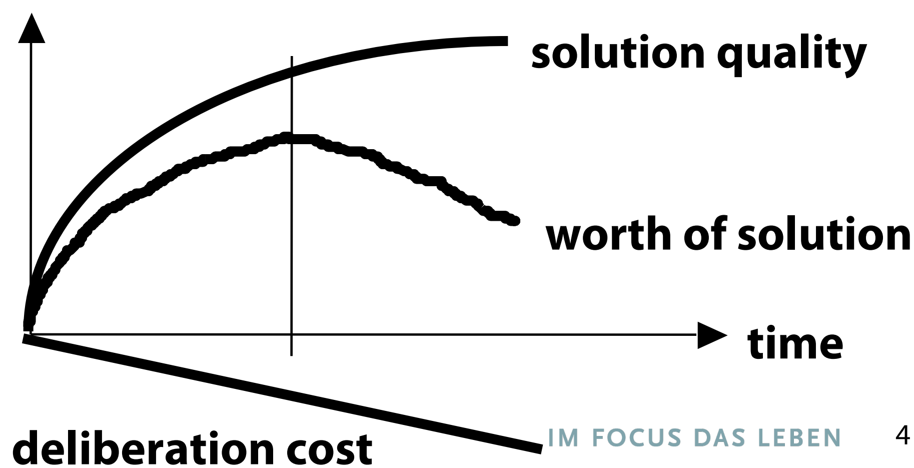
Full rationality

Bounded rationality



Descriptive vs. prescriptive theories of bounded rationality

Notion of Utility: u_i(outcome)



Impact of Reasoning Machinery

- Everything else being equal, an agent that has better algorithms and heuristics could make "more rational" (more optimal) decisions than one that has poorer heuristics and algorithms
- An agent should be able to learn heuristics
  - Possibly as important as learning models
- Rather than trying to solve a (too) difficult problem alone, an agent might decide to collaborate with others
- Need to analyze multiagent systems

Mechanisms, Protocols, and Strategies

- A mechanism defines the "rules of encounter" or protocol between agents
- Mechanism design is the theory about designing mechanisms so that they have certain desirable properties
- Given a particular protocol, how can a particular strategy be designed that individual agents can use?
- Notion of a dominant strategy
  - Best strategy to be determined w/o knowing the (best) strategies of other agents

Multiagent Systems: Criteria

- Social welfare: $\max_{outcome} \sum_i u_i(outcome)$
- Surplus: social welfare of outcome – social welfare of status quo
  - Constant sum games have 0 surplus.
  - Markets are not constant sum
- Pareto efficiency: An outcome o is Pareto efficient if there exists no other outcome o' s.t. some agent has higher utility in o' than in o and no agent has lower
  - Implied by social welfare maximization
- Individual rationality: Participating in a negotiation (or individual deal) is no worse than not participating
- Stability: No agents can increase their utility by changing their strategies (aka policies)
- Symmetry: No agent should be inherently preferred, e.g., as a dictator

Example Mechanisms: Auctions

- An auction takes place between an agent known as the *auctioneer* and a collection of agents known as the *bidders*
- The goal of the auction is for the auctioneer to allocate the *good* to one of the bidders
- In most settings the auctioneer desires to maximize the price; bidders desire to minimize price

Auction Parameters

- *Goods can have*
  - *private value*
  - *public/common value*
  - *correlated value*
- *Winner determination may be*
  - *first price*
  - *second price*
- *Bids may be*
  - *open cry*
  - *sealed bid*
- *Bidding may be*
  - *one shot*
  - *ascending*
  - *descending*

English Auctions

- Most commonly known type of auction:
  - *First price*
  - *Open cry*
  - *Ascending*
- Dominant strategy is for agent to successively bid a small amount more than the current highest bid until it reaches their valuation, then withdraw
- Susceptible to:
  - *Winner's curse*
  - *Shill-bidding*

Dutch Auctions

Dutch auctions are examples of *open-cry descending* auctions:

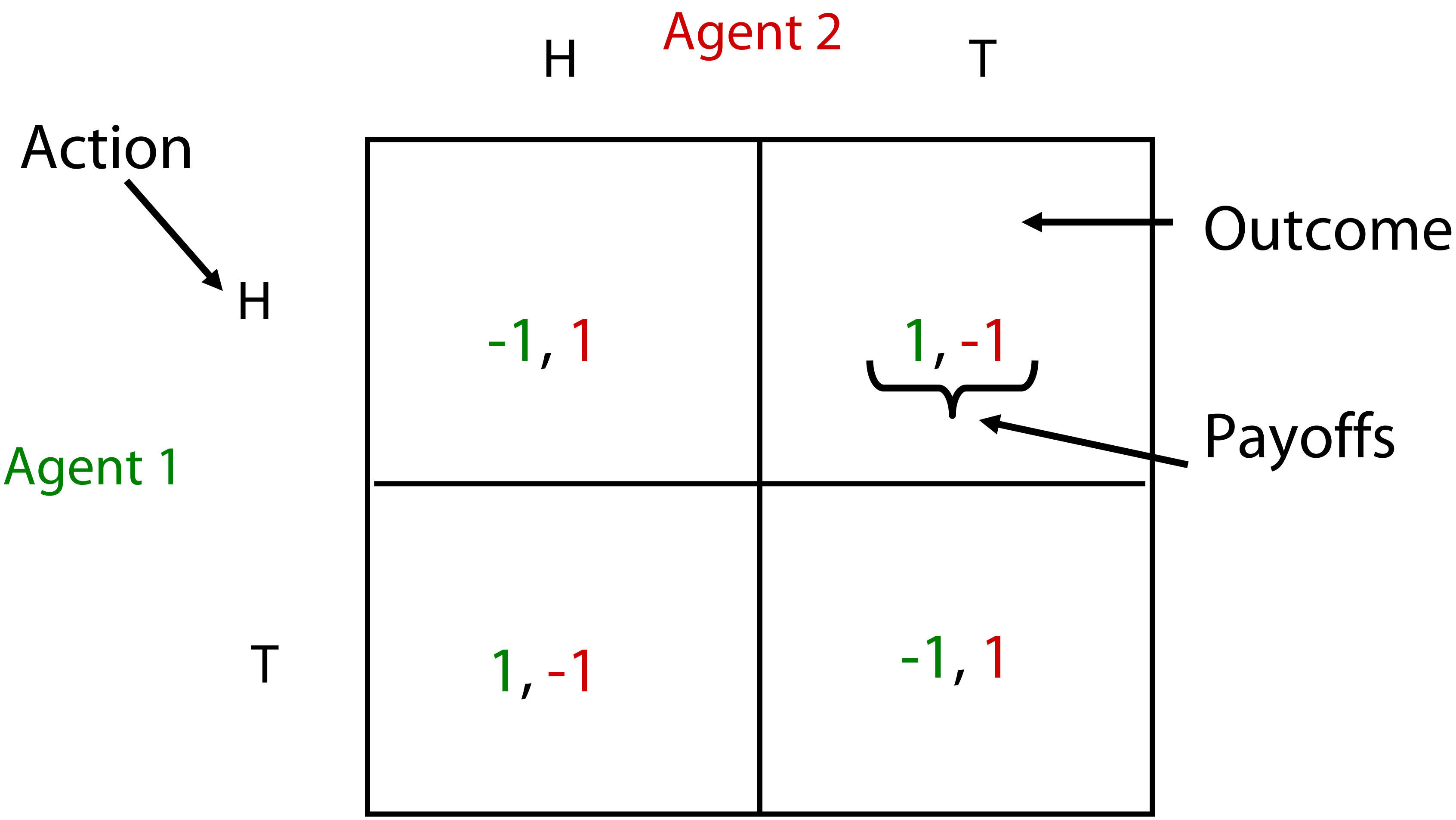
- Auctioneer starts by offering good at artificially high value
- Auctioneer lowers offer price until some agent makes a bid equal to the current offer price
- Good is then allocated to the agent that made the offer

Game Theory: Describing the Interaction of Agents

A game: Formal representation of a situation of strategic interdependence

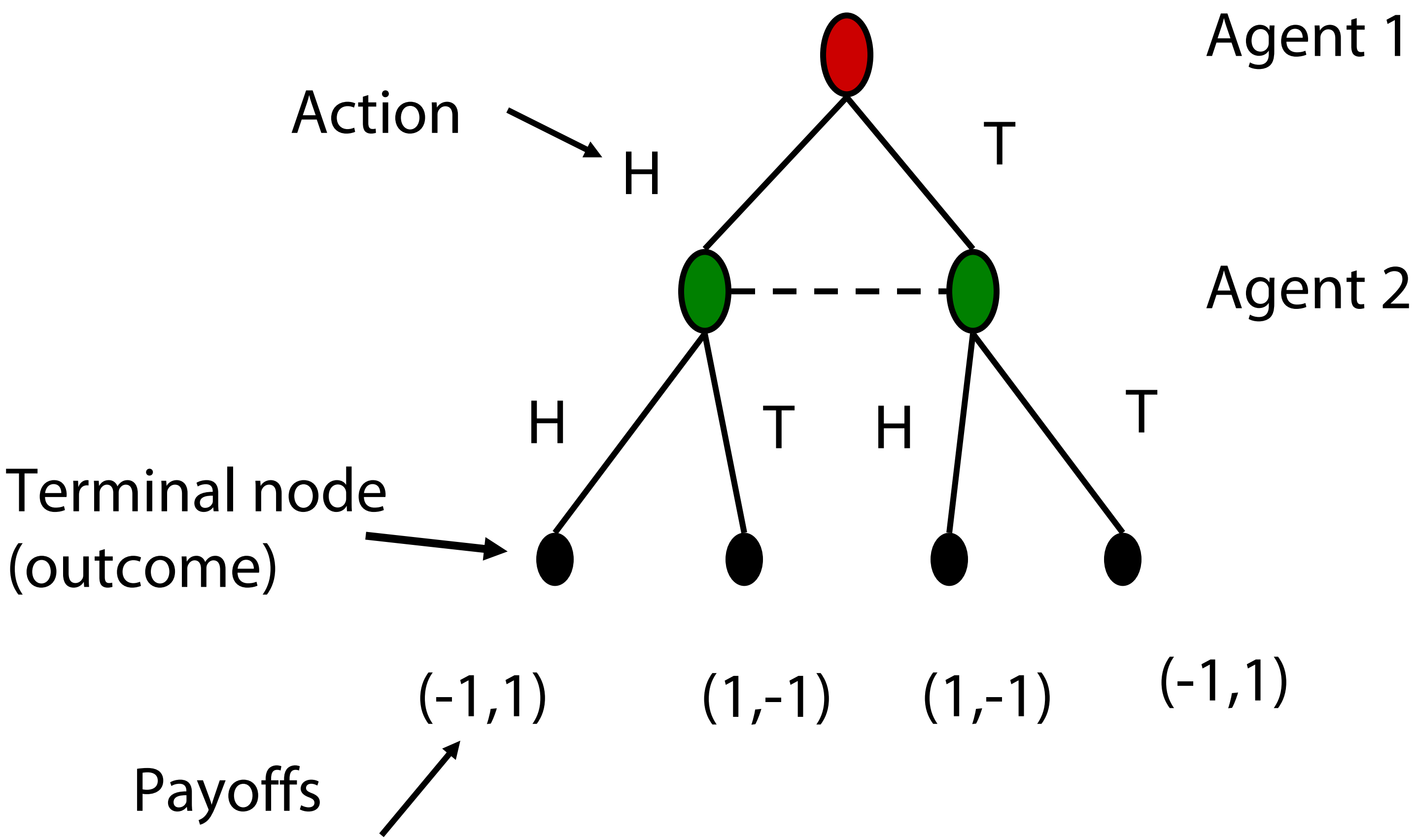
- Set of agents, I (|I|=n)
  - Aka players
- Each agent, j, has a set of actions, A_j
  - Aka moves
- Actions define outcomes
  - For each possible action there is an outcome.
- Outcomes define payoffs
  - Agents derive utility from different outcomes

Normal form game* (matching pennies)



*Aka strategic form, matrix form

Extensive form game (matching pennies)



Not necessarily executed sequentially

Strategies (aka Policies)

- Strategy:
  - A strategy, s_j, is a complete contingency plan; defines the actions agent j should take for all possible states of the world
- Strategy profile: $s=(s_1,\ldots,s_n)$
  - $s_{-i} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$
- Utility function: $u_j(s)$
  - Note that the utility of an agent j depends on the strategy profile, not just its own strategy
  - We assume agents are expected utility maximizers

Normal form game* (matching pennies)

Agent 2

Agent 1	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

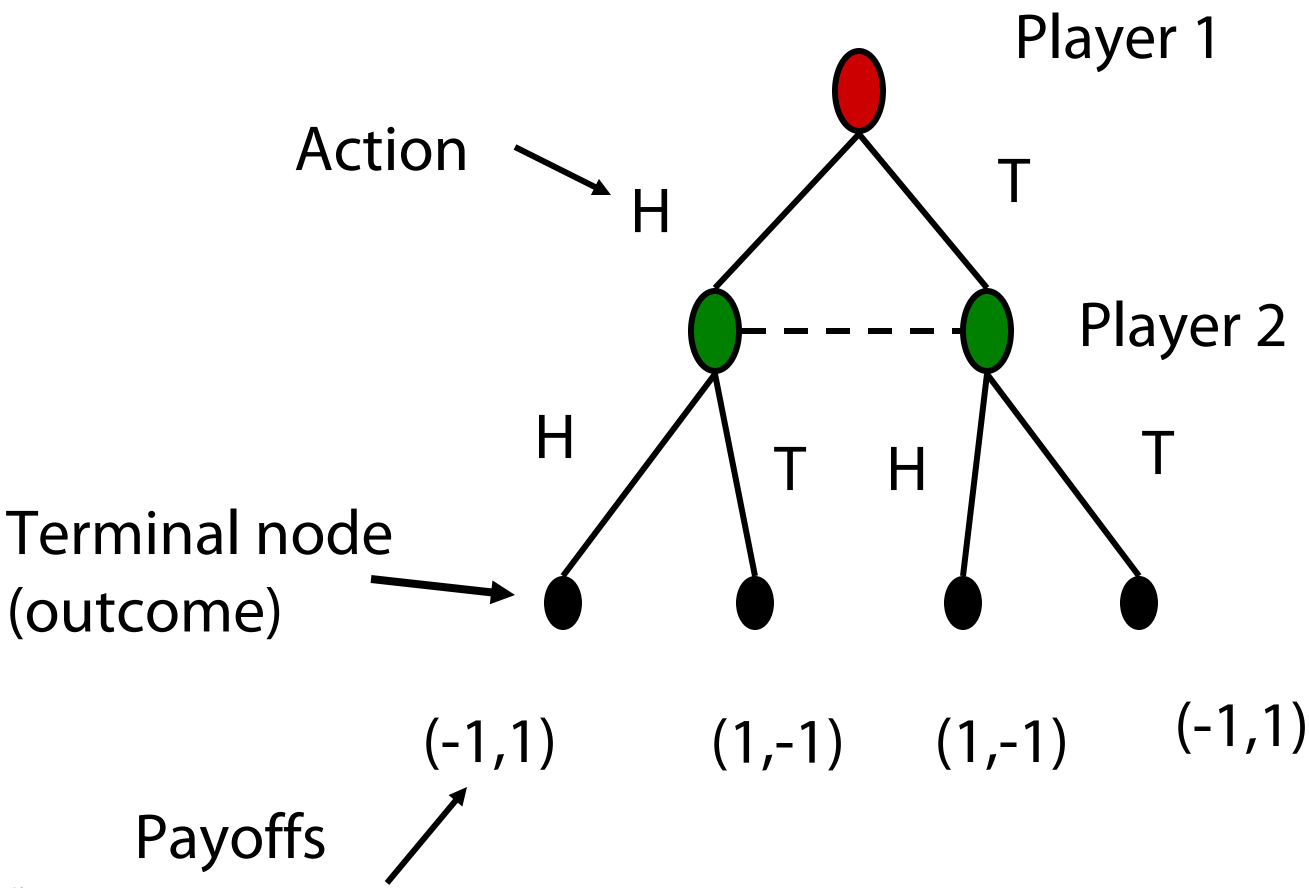
Strategy for
Agent 1: H
Agent 2: T

Strategy profile (H,T)

U1((H,T))=1
U2((H,T))=-1

*aka strategic form, matrix form

Extensive form game (matching pennies)



Strategy for
Agent 1: T
Agent 2: T

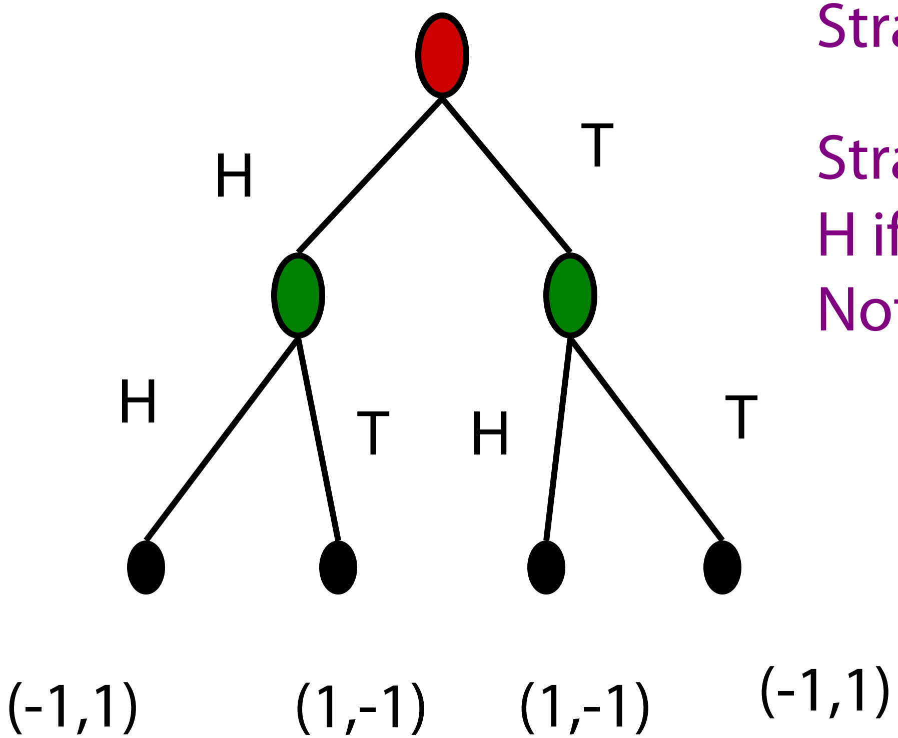
Strategy profile:
(T,T)

U1((T,T))=-1

U2((T,T))=1

Extensive form game (matching pennies, seq moves)

Recall: A strategy is a contingency plan for all states of the game



Strategy for Agent 1: T

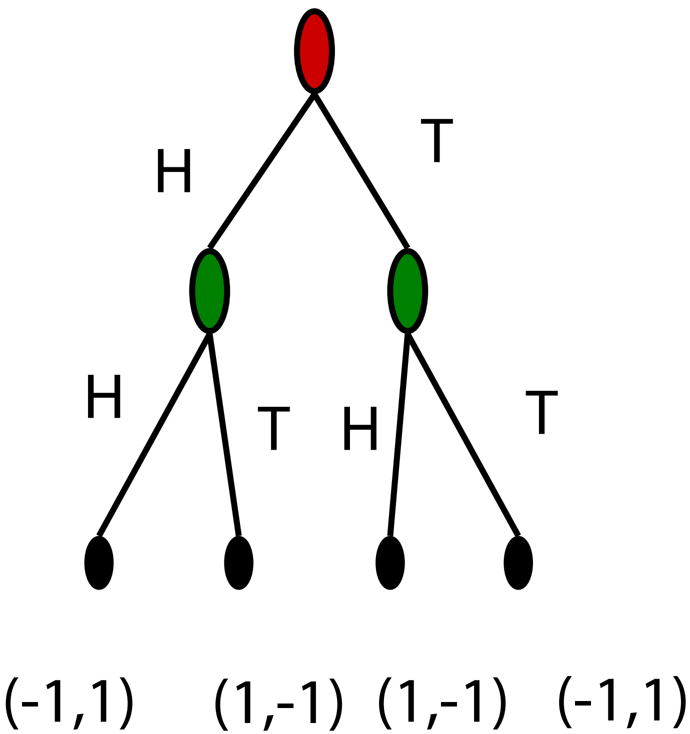
Strategy for Agent 2:
H if 1 plays H, T if 1 plays T
Notation: (H,T)

Strategy profile: (T,(H,T))

U1((T,(H,T)))=-1

U2((T,(H,T)))=1

Game Representation



	H,H	H,T	T,H	T,T
H	-1,1	-1,1	1,-1	1,-1
T	1,-1	-1,1	1,-1	-1,1

Potential combinatorial explosion →

Ascending Auction

- State of the world is defined by (x,p)
  - $x \in \{0,1\}$ indicates if the agent has the object
  - p is the current next price
- Strategy $s_j((x,p))$

$$s_j((x,p)) = \begin{cases} p, \text{ if } v_j \geqq p \text{ and } x=0 \\ \text{No bid otherwise} \end{cases}$$

Dominant Strategies

- Recall that
  - Agents' utilities depend on what strategies other agents are playing
  - Agents are expected utility maximizers
- Agents will play best-response strategies

s_i^* is a best response if $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i})$ for all s_i'

- A dominant strategy is a best-response for all s_{-i}
  - They do not always exist
  - Inferior strategies are called dominated

Dominant Strategy Equilibrium

- A dominant strategy equilibrium is a strategy profile where the strategy for each player is dominant
  - $s^*=(s_1^*,\ldots,s_n^*)$
  - $u_i(s_i^*,s_{-i})\geq u_i(s_i',s_{-i})$ for all i, for all s_i', for all s_{-i}
- **GOOD**: Agents do not need to counterspeculate!

Example: Prisoner's Dilemma

Two people are arrested for a crime, and they are interrogated separately

- If neither suspect confesses, both are released (but still, the interrogation is nasty)
- If both confess (to have carried out the crime together), then they get sent to jail
- If one confesses and the other does not, then the confessor gets a light sentence and the other gets a heavy sentence.

	A: Confess	A: Don't Confess
B:Confess	B=−5, A=−5	B=−1, A=−10
B:Don't Confess	B=−10, A=−1	B=−2, A=−2

Dom. Str. Equil (B=−5, A=−5)

Pareto Optimal Outcome (B=−2, A=−2)

Dominant strategy exists but is not Pareto efficient

Example: Split or Steal

Does initial communication help?

Only if agents do not lie to the other

Dom. Str. Eq

	A: Steal	A: Split
B:Steal	B=0, A=0	B=100, A=0
B:Split	B=0, A=100	B=50, A=50

Pareto Optimal Outcome

Example from British Game Show „Golden Balls“
See http://blogs.cornell.edu/info2040/2012/09/21/split-or-steal-an-analysis-using-game-theory/

Vickrey *) Auctions

- Vickrey auctions are:
  - *Second-price*
  - *Sealed-bid*
- Good is awarded to the agent that made the highest bid; at the price of the *second highest* bid
- *Bidding to your true valuation is dominant strategy in Vickrey auctions*
- Vickrey auctions susceptible to *antisocial* behavior

*) Named after William Vickrey (1914–1996), who won the 1996 Nobel Prize in economics for this work and died of a heart attack three days later

Example: Vickrey Auction (2nd price sealed bid)

- Each agent i has value v_i
- Strategy $b_i(v_i) \in [0,\infty)$
- $b^* :=$ 2[nd] best bid.

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b^* & \text{if } b_i > b^* \\ 0 & \text{otherwise} \end{cases}$$

Claim: Given value v_i, $b_i(v_i)=v_i$ is dominant.

Let $b'=\max_{j\neq i} b_j$. If $b'<v_i$ then any bid $b_i(v_i)\geq b'$ is optimal. If $b'\geq v_i$, then any bid $b_i(v_i)\leq v_i$ is optimal. Bid $b_i(v_i)=v_i$ satisfies both constraints.

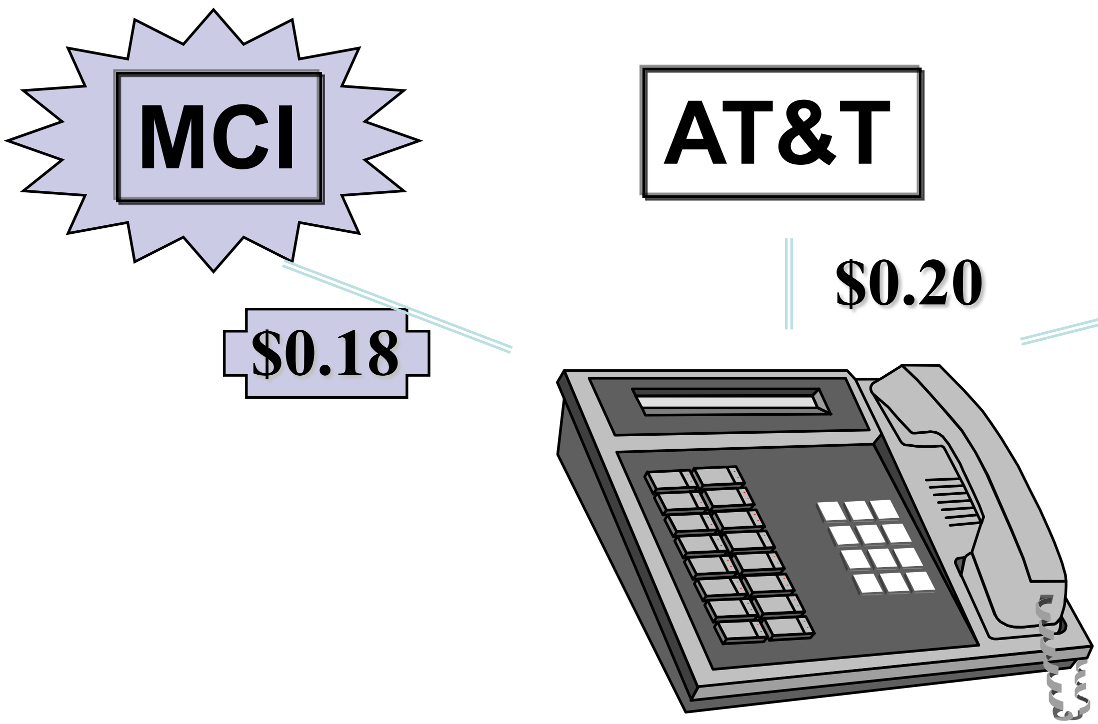
Dominant strategy is Pareto efficient

Phone Call Competition Example

- Customer wishes to place long-distance call
- Carriers simultaneously bid, sending proposed prices
- Phone automatically chooses the carrier (dynamically)

Best Bid Wins

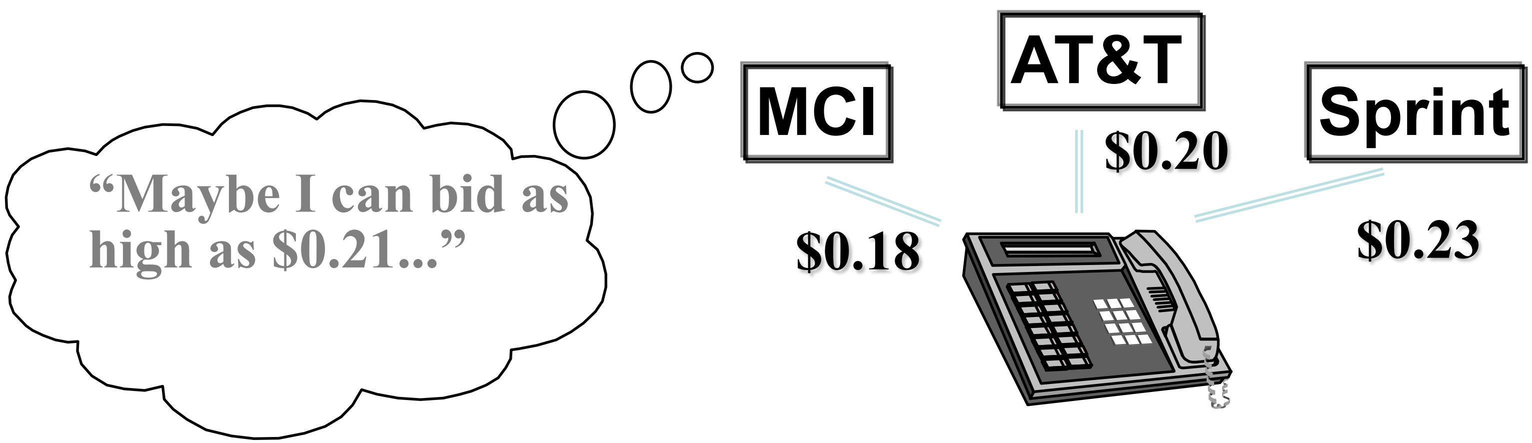
- Phone chooses carrier with lowest bid
- Carrier gets amount that it bid



Attributes of the Mechanism

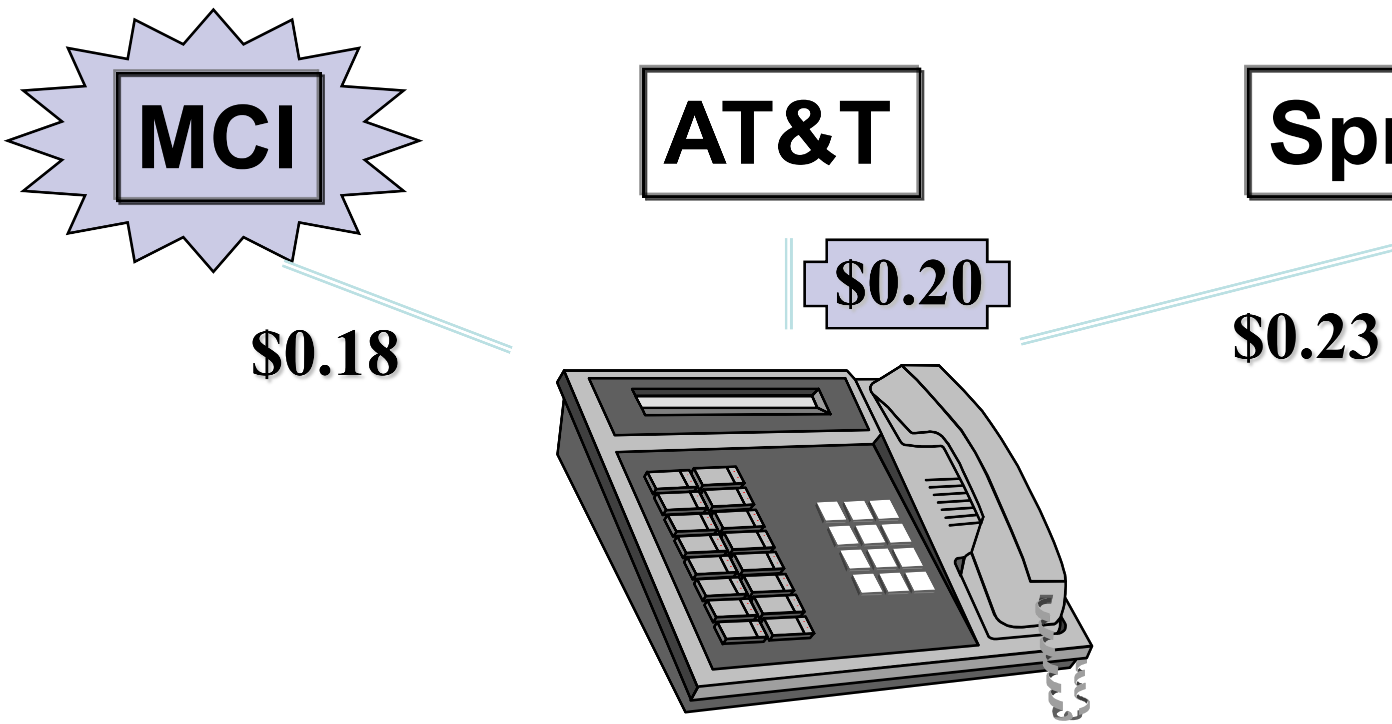
- ✓ *Distributed*
- ✓ *Symmetric*
- ✗ *Stable*
- ✗ *Efficient*

Carriers have an incentive to invest effort in strategic behavior



Best Bid Wins, Gets Second Price (Vickrey Auction)

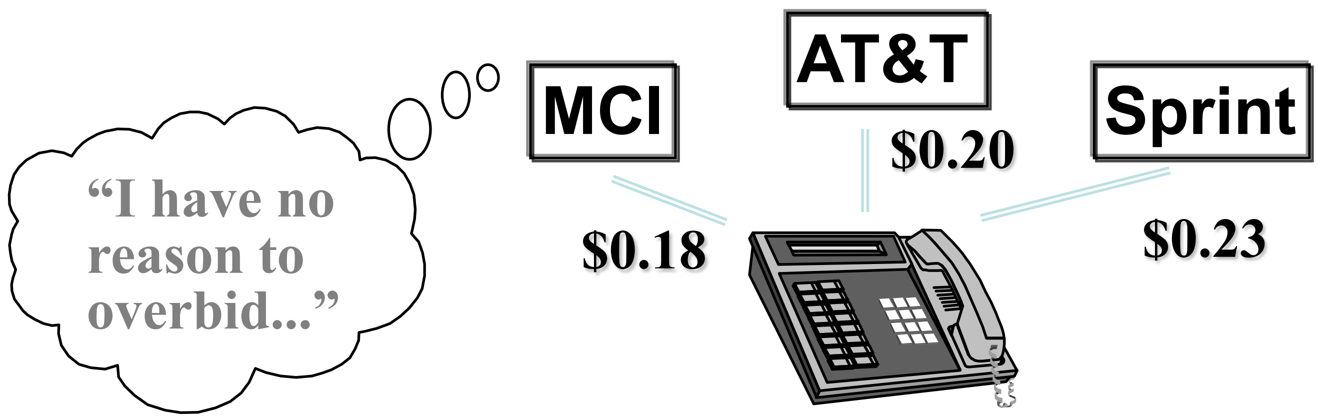
- Phone chooses carrier with lowest bid
- Carrier gets amount of second-best price



Attributes of the Vickrey Mechanism

- ✓ *Distributed*
- ✓ *Symmetric*
- ✓ *Stable*
- ✓ *Efficient*

Carriers have *no* incentive to invest effort in strategic behavior



Lies and Collusion

- The various auction protocols are susceptible to lying on the part of the auctioneer, and collusion among bidders, to varying degrees
- All four auctions (English, Dutch, First-Price Sealed Bid, Vickrey) can be manipulated by bidder collusion
- A dishonest auctioneer can exploit the Vickrey auction by lying about the 2nd-highest bid
- *Shill-bids* can be introduced to inflate bidding prices in English auctions

Does a dom. str. equil always exist?

- A couple likes going to concerts together. One loves Bach but not Stravinsky. The other loves Stravinsky but not Bach. However, they prefer being together than being apart.
- What should be the strategy profile?

Best action depends on best action of other agent

	B	S
B	2,1	0,0
S	0,0	1,2

No dom. str. equil.

Nash Equilibrium

- Sometimes an agent's best-response depends on the strategies other agents are playing
  - No dominant strategy equilibria

- A strategy profile is a Nash equilibrium if no player has incentive to deviate from his strategy given that others do not deviate:

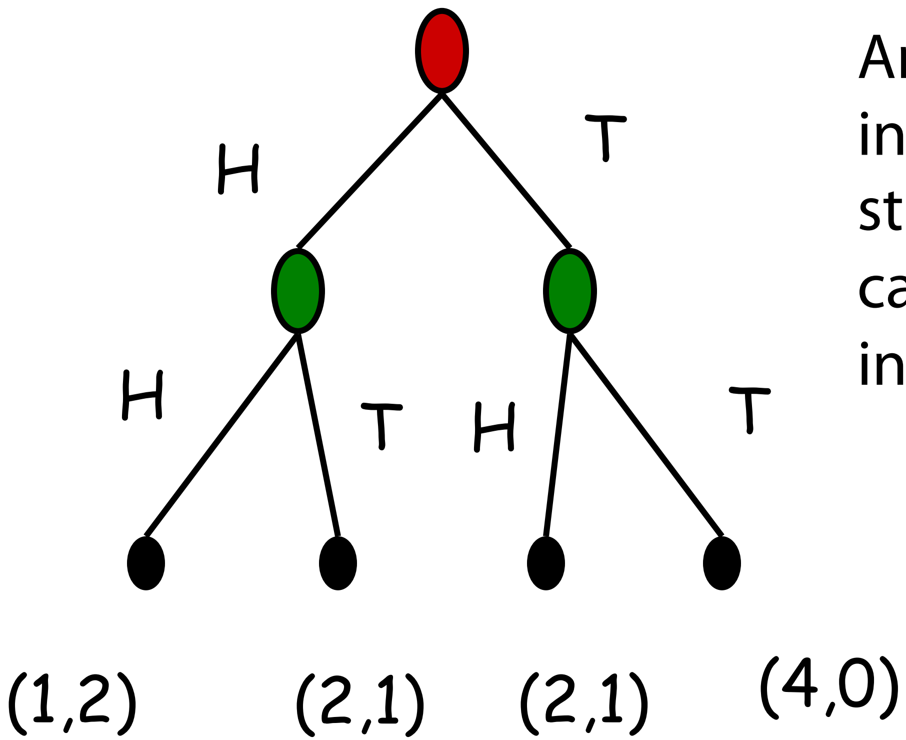
  - For every agent i, $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*)$ for all s_i'

	B	S
B	2,1	0,0
S	0,0	1,2

Nash Equilibrium

- Interpretations:
  - Focal points, self-enforcing agreements, stable social convention, consequence of rational inference
- Criticisms
  - They may not be unique (Bach or Stravinsky)
    - Ways of overcoming this
      - Refinements of equilibrium concept, mediation, learning
  - Do not exist in all games (in the form defined above)
  - They may be hard to find
  - People don't always behave based on what equilibria would predict (ultimatum games and notions of fairness,…)

Extensive Form Games



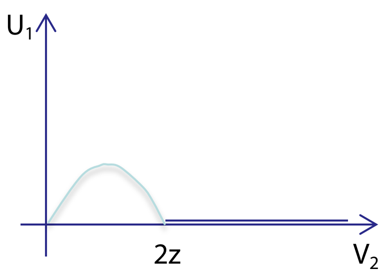
Any finite game of perfect information has a pure strategy Nash equilibrium. It can be found by backward induction.

Pure strategy: no elements of chance involved

Chess is a finite game of perfect information. Therefore, it is a “trivial” game from a game-theoretic point of view.

Example: 1^{st} price sealed-bid auction

- 2 agents (1 and 2) with values v_1, v_2 drawn uniformly from [0,1].
- Utility of agent i if it bids b_i and wins the item is $u_i = v_i - b_i$.
- Assume that agent 2's bidding strategy is $b_2(v_2) = v_2/2$ (but we do not know $\theta_2 = v_2$)
- How should 1 bid? (i.e., what is $b_1(v_1) = z$?)
- Note: given $b_2(v_2) = v_2/2$, agent 1 only wins if $v_2 < 2z$ otherwise U_1 is 0, assume uniform distribution on [0, 2z]



Expected $U_1 = \int_{x=0}^{2z} (v_1 - x)dx = [v_1 x - \frac{1}{2}x^2]_0^{2z} = 2zv_1 - 2z^2$

- $\text{argmax}_z[2zv_1 - 2z^2]$ when $z = b_1(v_1) = v_1/2$
- Similar argument for agent 2, assuming $b_1(v_1) = v_1/2$. We have an equilibrium

Example: Matching Pennies

	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

No equil. exists

So far, we have talked only about **pure** (deterministic) strategy equilibria.

Not all games have pure strategy equilibria. Some equilibria are **mixed** (randomzied) strategy equilibria.

Mixed strategy equilibria

- Let $\sum_i$ be the set of probability distributions over S_i
- σ_i in $\sum_i$
- Strategy profile: $\sigma=(\sigma_1,\ldots, \sigma_n)$
- Expected utility: $u_i(\sigma)=\sum_{s\in S_i} \sigma_i(s)u_i(s)$
- Nash Equilibrium:
  - σ^* is a (mixed) Nash equilibrium iff

    $u_i(\sigma^*_i, \sigma^*_{-i})\geq u_i(\sigma_i, \sigma^*_{-i})$ for all $\sigma_i\in\sum_i$, for all i

Example: Matching Pennies

	q H	1-q T
p H	−1, 1	1, −1
1-p T	1, −1	−1, 1

Want to play each strategy with a certain probability so that the competitor is indifferent between its own strategies.

$1p+(-1)(1-p)=(-1)p+1(1-p) \Rightarrow p=1/2$

$q-(1-q)=-q+(1-q) \Rightarrow q=1/2$

Mixed Nash Equilibrium

- **Theorem (Nash 50):**
  - Every game in which the strategy sets, $S_1,\ldots,S_n$ have a finite number of elements has a mixed strategy equilibrium
- Complexity of finding Nash Equilibria
  - "Together with prime factoring, the complexity of finding a Nash Equils is, in my opinion, the most important concrete open question on the boundary of P today." (Papadimitriou)
  - (Daskalakis, Goldberg/Papadimitriou, 2005): Finding Nash equilibrium is very hard (though not NP complete): PPAD complete (Polynomial Parity Arguments on Directed graphs)

Imperfect Information about Strategies and Payoffs

- So far, we have assumed that agents have complete information about each other (including payoffs)
  - Very strong assumption!
- Assume agent i has type $\theta_i \in \Theta_i$, which defines the payoff $u_i(s, \theta_i)$
- Agents have common prior over distribution of types $p(\theta)$
  - Conditional probability $p(\theta_{-i} | \theta_i)$ (obtained by Bayes Rule)

Bayesian-Nash Equil

- **Strategy**: $\sigma_i(\theta_i)$ is the (mixed) strategy agent i plays if its type is θ_i
- **Strategy profile**: $\sigma=(\sigma_1,\ldots,\sigma_n)$
- **Expected utility:**
  - $EU_i(\sigma_i(\theta_i),\sigma_{-i}(),\theta_i)=\sum_{\theta_{-i}} p(\theta_{-i}|\theta_i)u_i(\sigma_i(\theta_i),\sigma_{-i}(\theta_{-i}),\theta_i)$

- **Bayesian Nash Equil**: Strategy profile σ^* is a Bayesian-Nash Equil iff for all i, for all θ_i,

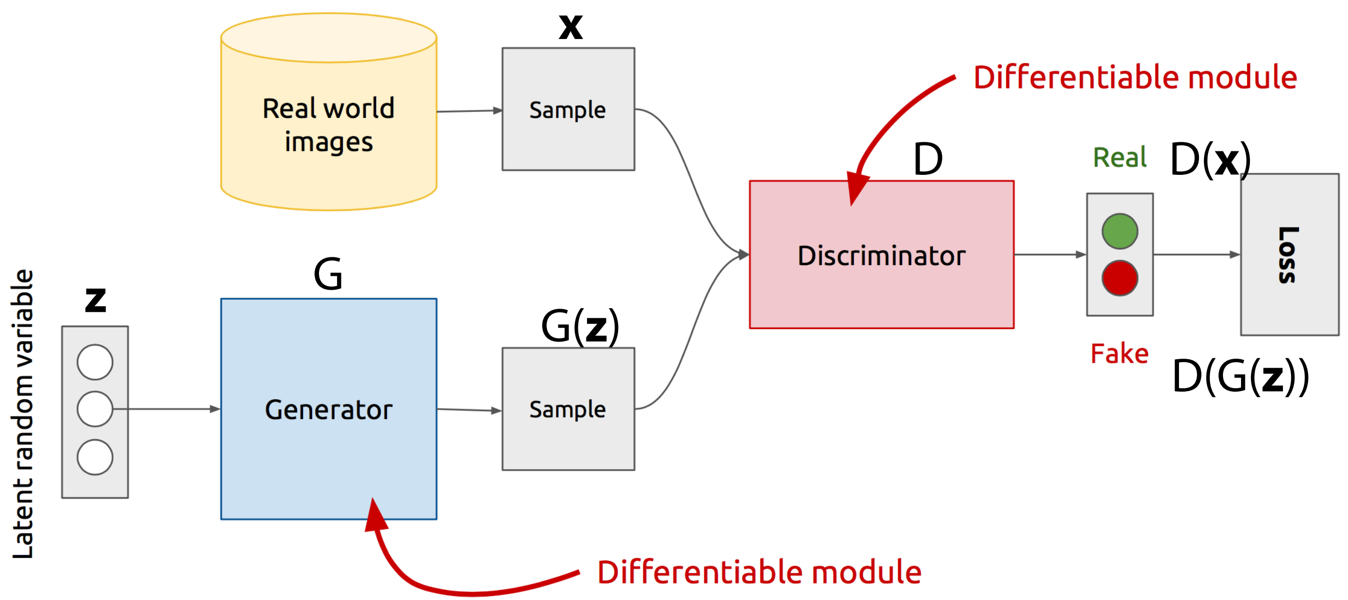
  $EU_i(\sigma^*_i(\theta_i),\sigma^*_{-i}(),\theta_i)\geq EU_i(\sigma_i(\theta_i),\sigma^*_{-i}(),\theta_i)$

  (best responding w.r.t. its beliefs about the types of the other agents, assuming they are also playing a best response)

Harsanyi, John C., "Games with Incomplete Information Played by Bayesian Players, I-III." Management Science 14 (3): 159-183 (Part I), 14 (5): 320-334 (Part II), 14 (7): 486-502 (Part III) (**1967/68**)

John Harsanyi was a co-recipient along with John Nash and Reinhard Selten of the 1994 Nobel Memorial Prize in Economics

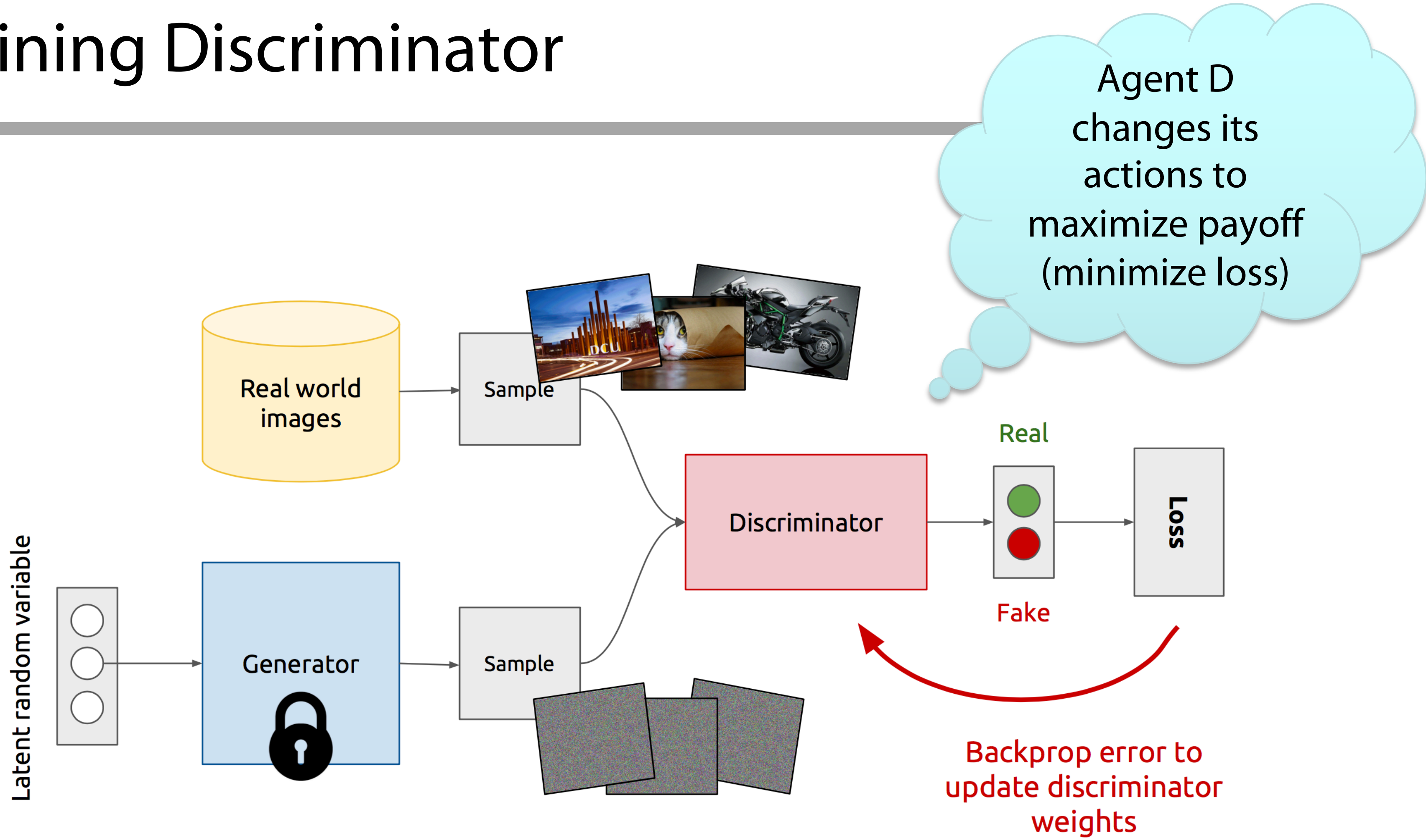
Example: GAN Architecture



- **Z** is some multidimensional random noise (Gaussian/Uniform).
- **Z** can be thought as the latent representation of the image.

https://www.slideshare.net/xavigiro/deep-learning-for-computer-vision-generative-models-and-adversarial-training-upc-2016

Training Discriminator

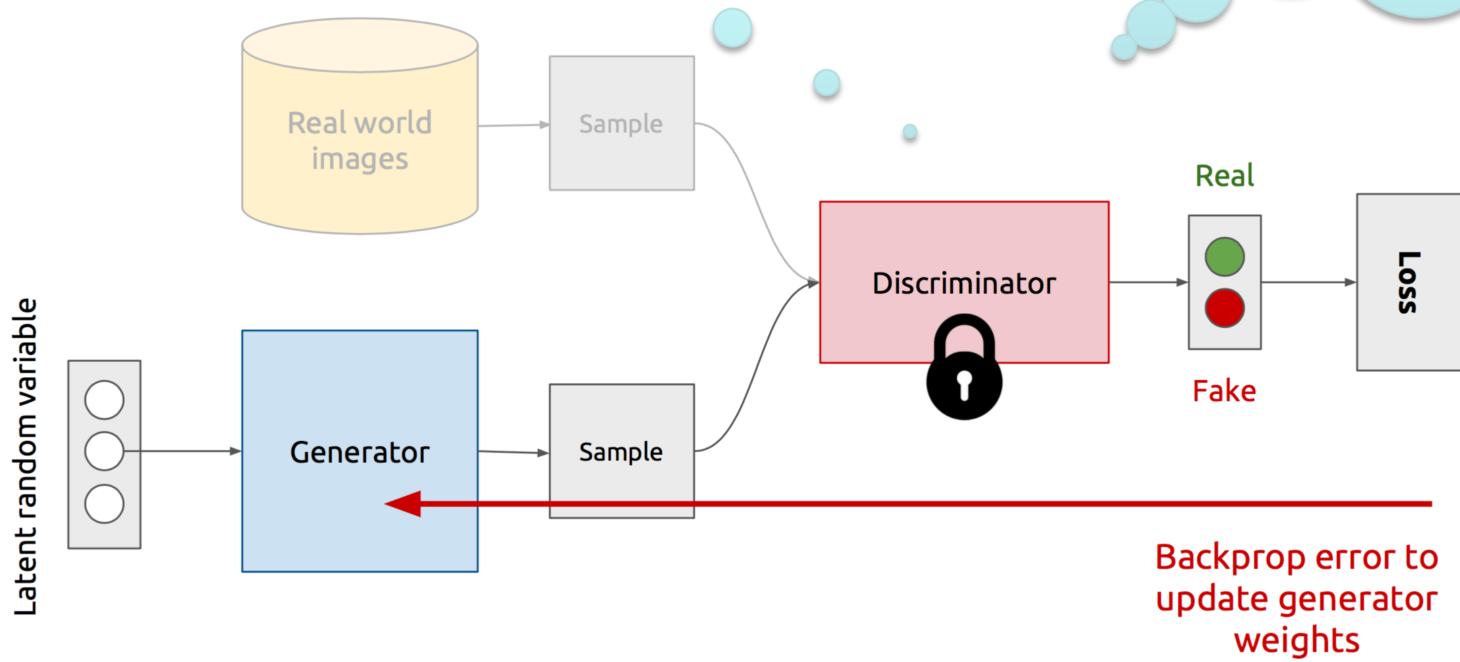


https://www.slideshare.net/xavigiro/deep-learning-for-computer-vision-generative-models-and-adversarial-training-upc-2016

Training Generator

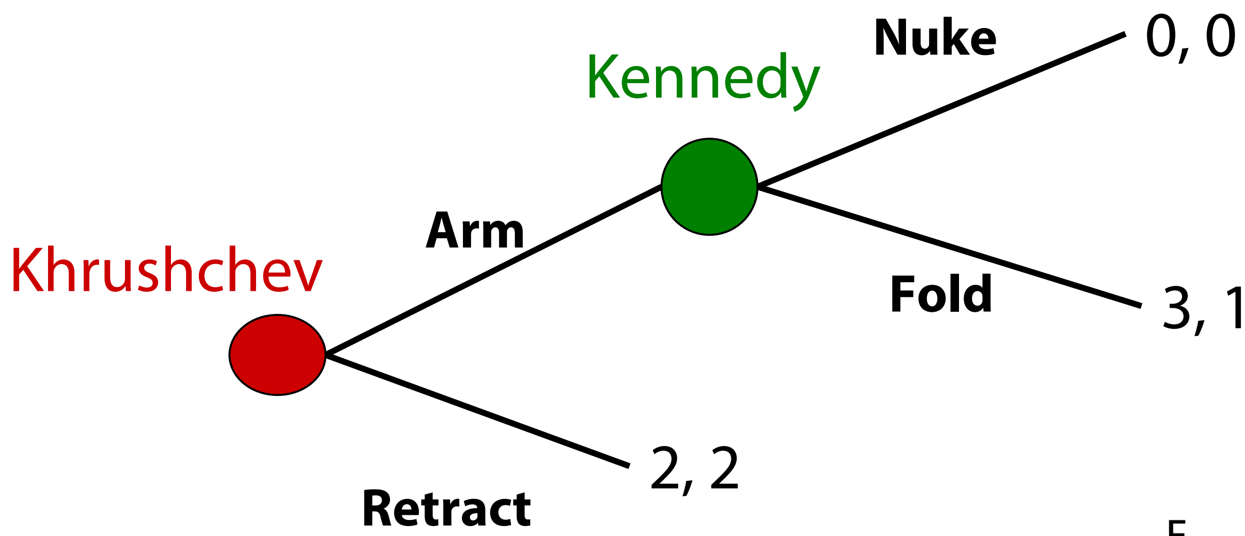
Agent G changes its actions to maximize payoff (minimize loss)

Do not change action, if other agent does not change action: Nash Equil. computed



https://www.slideshare.net/xavigiro/deep-learning-for-computer-vision-generative-models-and-adversarial-training-upc-2016

Example: Cuban Missile Crisis – Credible Threats



	F	N
A	3,1	0,0
R	2,2	2,2

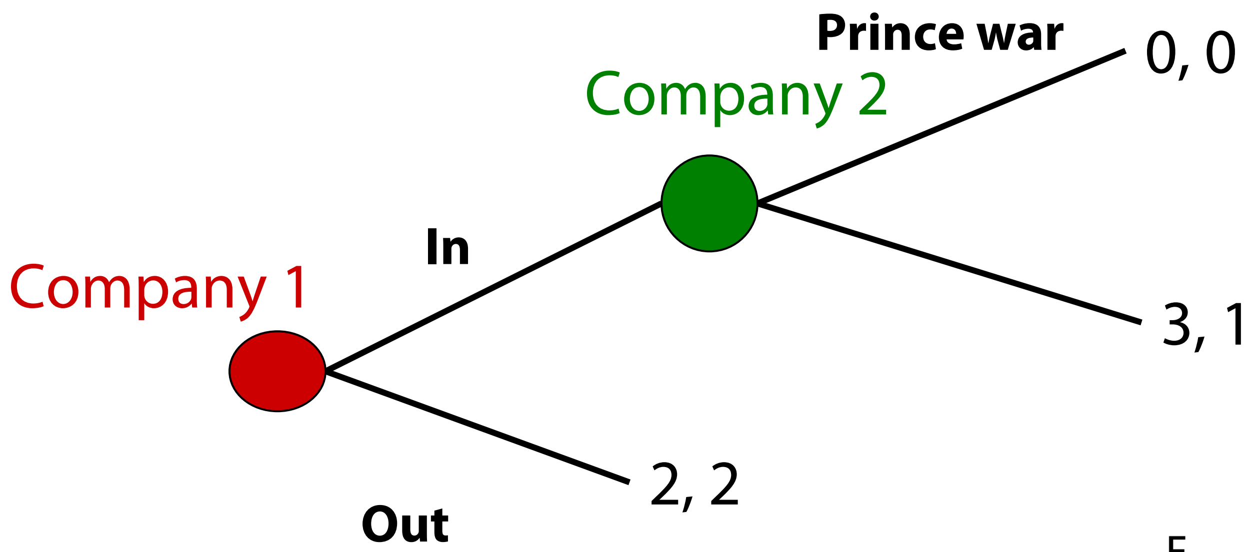
Pure strategy Nash equilibria:
(Arm, Fold) and (Retract, Nuke) if
Kennedy ever was in a position to react

Proper case distinction: If Krushchev did arm,
it would not be a good idea for Kennedy to nuke
Pure strategy subgame perfect equilibria: (Arm, Fold)

In case of Khrushchev doing a proper case distinction:
Kennedy's Nuke threat is not credible

[Reinhard Selten 72]

Example: Markets – Credible Threats



	F	N
A	3,1	0,0
R	2,2	2,2

Extract effort of case distinctions pays off

[Reinhard Selten 72]

Intelligent Agents
Game Theory and Social Choice

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Social Choice Theory

Assume a group of agents make a decision

1. Agents have preferences over alternatives
   - Agents can rank order the outcomes:

     a>b>c=d is read as "a is preferred to b
     which is preferred to c which is equivalent to d"

2. Voters are sincere
   - They truthfully tell their preferences

3. Outcome is enforced on all agents

The problem

- Majority decision:
  - If more agents prefer a to b, then a should be chosen
- Two-outcome setting is easy
  - Choose outcome with more votes!
- What happens if you have
  3 or more possible outcomes?

Case 1: Agents specify their top preference

Election System

- Plurality Voting
  - One name is ticked on a ballot
  - One round of voting
  - One candidate is chosen

Is this a "good" system?

What do we mean by good?

Example: Plurality

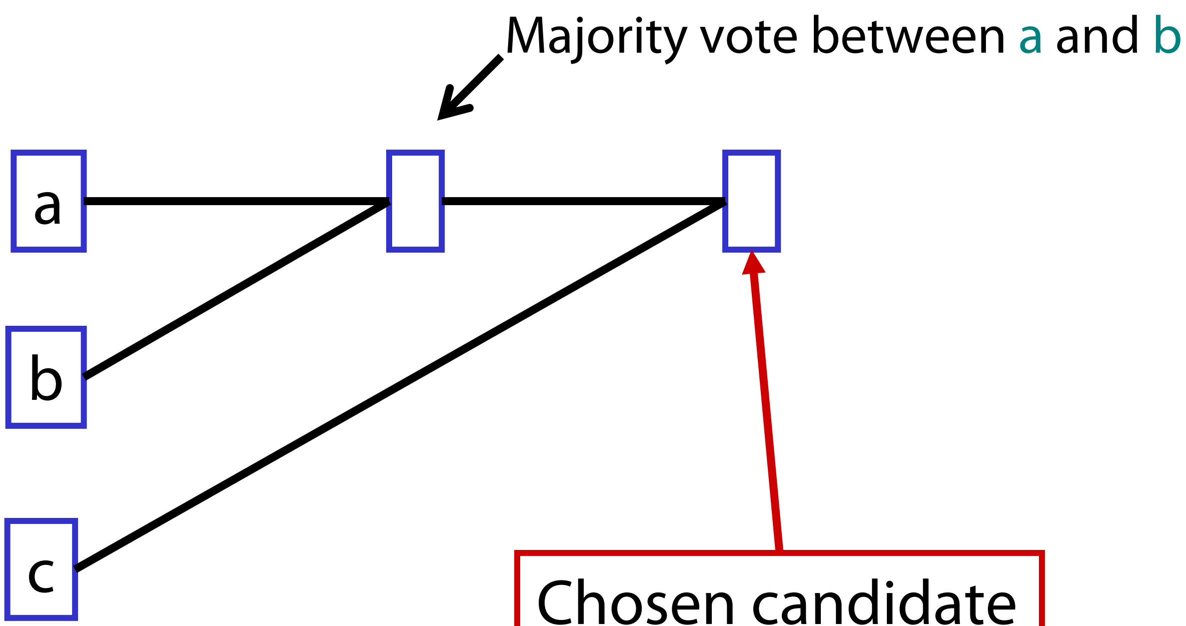
- 3 candidates
  - Lib, NDP, C
- 21 voters with the preferences
  - 10 Lib>NDP>C
  - 6 NDP>C>Lib
  - 5 C>NDP>Lib
- Result: Lib 10, NDP 6, C 5
  - But a majority of voters (11) prefer all other parties more than the Libs!

What can we do?

- Majority system
  - Works well when there are 2 alternatives
  - Not great when there are more than 2 choices
- Proposal:
  - Organize a series of votes between 2 alternatives at a time
  - How this is organized is called an agenda
    - Or a cup (often in sports)

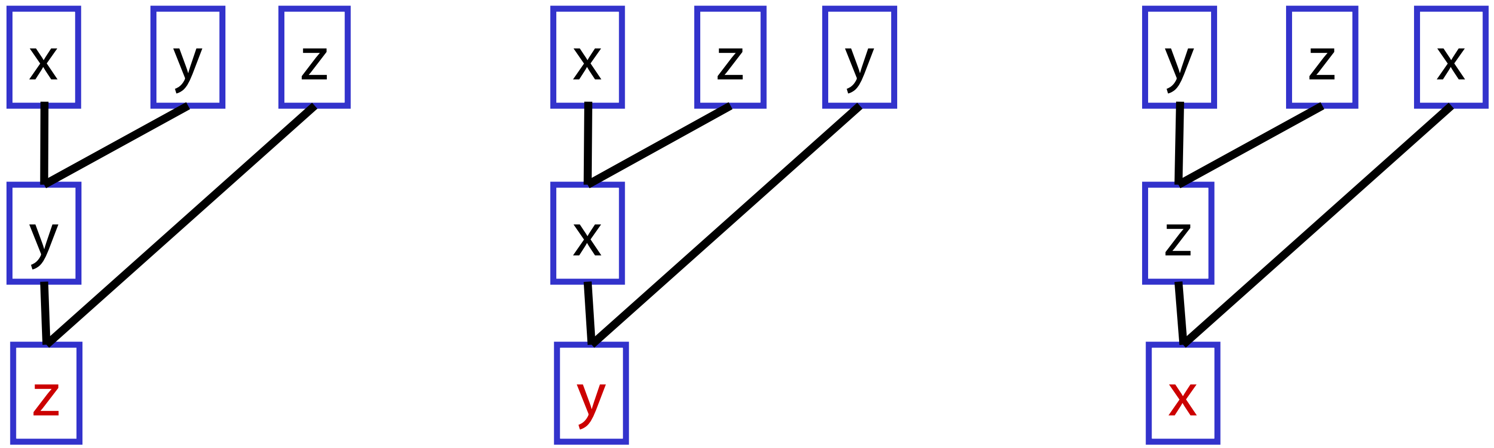
Agendas

- 3 candidates {a,b,c}
- Agenda a,b,c



Agenda paradox

- *Binary protocol (majority rule) = cup*
- Three types of agents:
  1. x > z > y (35%)
  2. y > x > z (33%)
  3. z > y > x (32%)



Power of agenda setter (e.g., chairman)

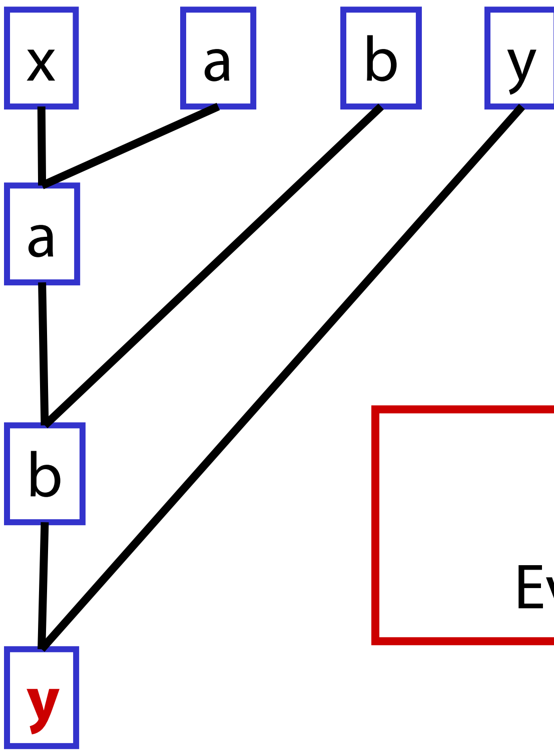
Vulnerable to irrelevant alternatives (z)

- x vs. y only lets y win
- But adding z may lead to y winning (last agenda)

Another problem: Pareto dominated winner paradox

Agents:

1. x > y > b > a
2. a > x > y > b
3. b > a > x > y



BUT

Everyone prefers x to y!

Case 2: Agents specify their complete preferences

Maybe the problem was with the ballots!

Ballot

X>Y>Z

Now have more information

Condorcet

- Proposed the following
  - Compare each pair of alternatives
  - Declare “a” is socially preferred to “b” if more voters strictly prefer a to b

- Condorcet Principle: If one alternative is preferred to all other candidates then it should be selected

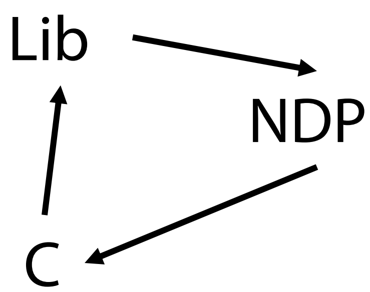
Wikipedia: Condorcet voting methods are named for the 18th-century French mathematician and philosopher Marie Jean Antoine Nicolas Caritat, the Marquis de Condorcet, who championed such voting systems. However, Ramon Llull devised the earliest known Condorcet method in 1299.

Example: Condorcet

- 3 candidates
  - Lib, NDP, C
- 21 voters with the preferences
  - 10 Lib>NDP>C
  - 6 NDP>C>Lib
  - 5 C>NDP>Lib
- Result:
  - NDP win! (11/21 prefer them to Lib, 16/21 prefer them to C)

A Problem

- 3 candidates
  - Lib, NDP, C
- 3 voters with the preferences
  - Lib>NDP>C
  - NDP>C>Lib
  - C>Lib>NDP
- Result:
  - No Condorcet Winner



Borda Count

- Each ballot is a list of ordered alternatives
- On each ballot compute the rank of each alternative
- Rank order alternatives based on sum of their ranks (lowest rank preferred)

A>B>C		A: 4
A>C>B	⇒	B: 8
C>A>B		C: 6

Wikipedia: Jean-Charles de Borda devised the system in June 1770, as a fair way to elect members to the French Academy of Sciences. The ideas were also propsed earlier.

Borda Count

- Simple
- Always a Borda Winner
- BUT does not always choose Condorcet winner!
- 3 voters
  - 2: b>a>c>d
  - 1: a>c>d>b

Borda scores:

a:5, b:6, c:8, d:11

Therefore a wins

BUT b is the Condorcet winner

Inverted-order paradox

- Borda rule with 4 alternatives
  - Each agent gives 1 point to best option, 2 to second best...
- Agents:
  1. x > c > b > a
  2. a > x > c > b
  3. b > a > x > c
  4. x > c > b > a
  5. a > x > c > b
  6. b > a > x > c
  7. x > c > b > a
- x=13, a=18, b=19, c=20
- Remove x: c=13, b=14, a=15

Borda rule vulnerable to irrelevant alternatives

- Three types of agents:

1. x > z > y (35%)
2. y > x > z (33%)
3. z > y > x (32%)

- Borda winner is x
- Remove z: Borda winner is y

Desirable properties for a voting protocol

- No dictators
- Universality (unrestricted domain)
  - It should work with any set of preferences
- Non-imposition (citizen sovereignty)
  - Every possible societal preference order should be achievable
- Independence of irrelevant alternatives (IIA)
  - The comparison of two alternatives should depend only on their standings among agents' preferences, not on the ranking of other alternatives
- Monotonicity
  - An individual should not be able to hurt an option by ranking it higher.
- Paretian
  - If all all agents prefer x to y then in the outcome x should be preferred to y

Arrow's Theorem (1951)

If there are 3 or more alternatives and a finite number of agents then there is **no** protocol which satisfies all desired properties

Take-home Message

- Despair?
  - No ideal voting method
  - That would be boring!

- A group is more complex than an individual
- Weigh the pro's and con's of each system and understand the setting they will be used in

- Do not believe anyone who says they have the best voting system out there!