Non-Standard Databases and Data Mining

Introduction to Causal Modeling and Reasoning

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Structural Causal Models

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Part I: Basic Notions

(SCMs, d-separation)

Literature

- J. Pearl, M. Glymour, N. P. Jewell: Causal inference in statistics – A primer, Wiley, 2016.
  (Main Reference)

- J. Pearl: Causality, CUP, 2000.
  (The book on causality from the perspective of probabilistic graphical models)

- J. Pearl, D. Mackenzie: The Book of Why, Basic Books, 2018.
  (Popular science level, but worth reading)

Motivation

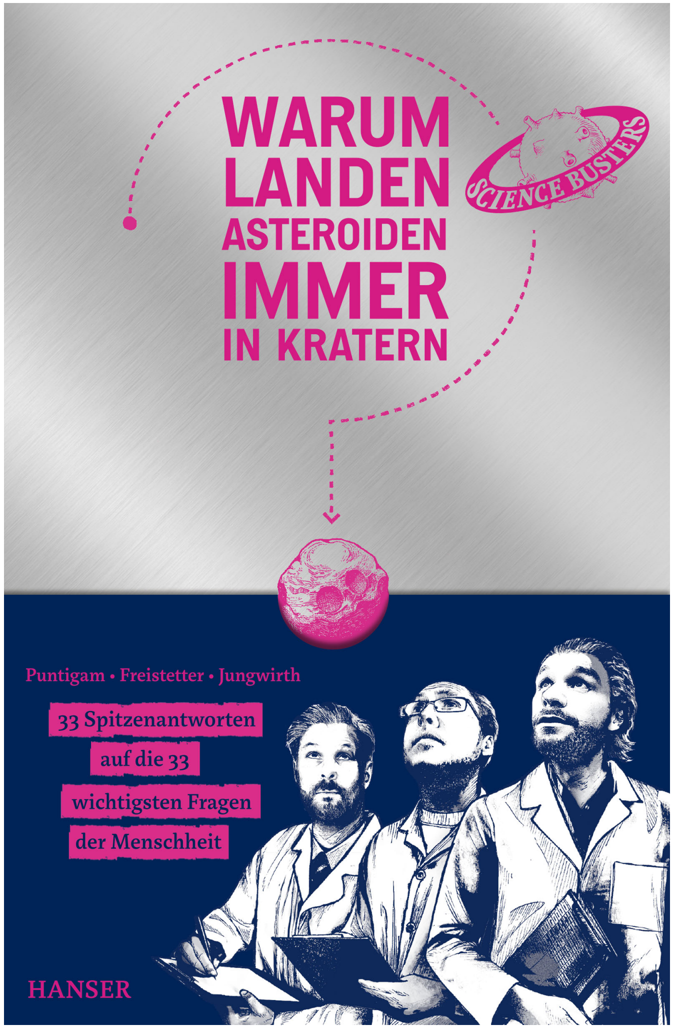
- Usual warning:

  „Correlation is not causation“

- Bulk of data mining methods is about correlation

- But sometimes (if not very often) one needs causation to understand statistical data

A remarkable correlation? A simple causality!



Simpson's Paradox (Example)

- Record recovery rates of 700 patients given access to a drug

	Recovery rate with drug	**Recovery rate without drug**
Men	81/87 (93%)	234/270 (87%)
Women	192/263 (73%)	55/80 (69%)
Combined	273/350 (78%)	289/350 (83%)

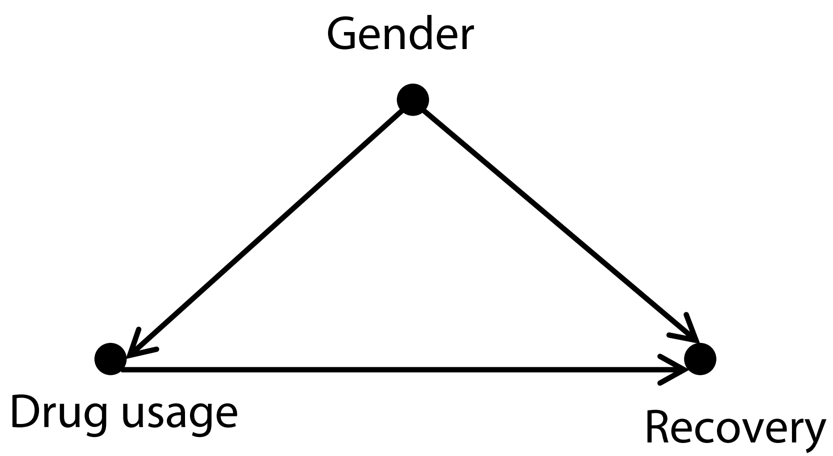
- Paradox:
  - For men, taking the drug has benefit
  - For women, taking the drug has benefit, too.
  - But: for all persons taking the drug seems to have no benefit

Resolving the Paradox (Informally)

- We need to understand the causal mechanisms that lead to the data in order to resolve the paradox
- In drug example
  - Why has taking the drug less benefit for women?
    Answer: Estrogen has negative effect on recovery
  - Data: Women more likely to take drug than men
  - So: Choosing randomly any person will rather give a woman – and for these, recovery is less beneficial
- In this case: Need to consider segregated data
  (not aggregated data)

Resolving the Paradox Formally (Look Ahead)

- We need to understand the causal mechanisms that lead to the data in order to resolve the paradox



- Drug usage and recovery have common cause
- Gender is a confounder

Simpson Paradox (Again)

- Record recovery rates of 700 patients given access to a drug w.r.t. blood pressure (BP) segregation

	Recovery rate with drug	**Recovery rate without drug**
Low BP	234/270 (87%)	81/87 (93%)
High BP	55/80 (69%)	192/263 (73%)
Combined	289/350 (83%)	273/350 (78%)

- BP recorded at end of experiment
- This time segregated data recommends not using drug whereas aggregated does

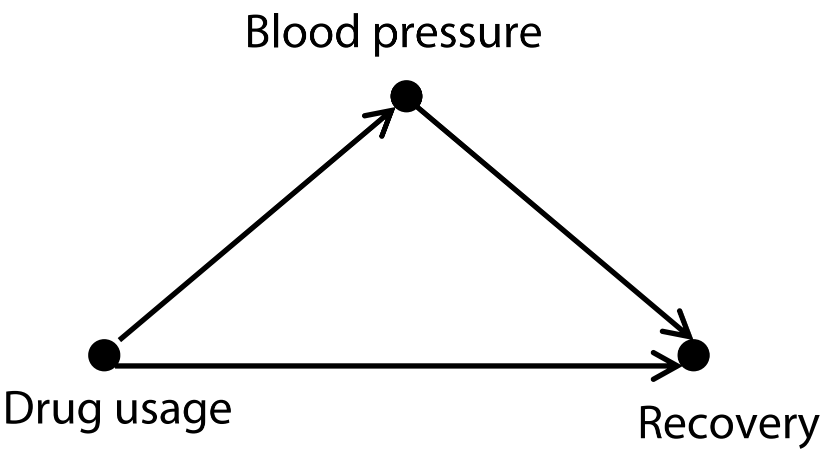
Resolving the Paradox (Informally)

- We need to understand the causal mechanisms that lead to the data in order to resolve the paradox

- In this example
  - Drug effect: lowering blood pressure (but may have toxic effects)
  - Hence: In aggregated population drug usage recommended
    - In segregated data one sees only toxic effects

Resolving the Paradox Formally (Look Ahead)

- We need to understand the causal mechanisms that lead to the data in order to resolve the paradox



Ingredients of a Statistical Theory of Causality

- Working definition of causation
- Method for creating causal models
- Method for linking causal models with features of data
- Method for reasoning over model and data

Working Definition

A (random) variable X is a cause of a (random) variable Y if Y - in any way - relies on X for its value

Structural Causal Model: Definition

Definition

A structural causal model (SCM) consists of

- A set U of exogenous variables
- A set V of endogenous variables
- A set F of functions assigning each variable in V a value based on values of other variables from V ∪ U

- Only endogenous variables V are those that are descendants of other variables
- Exogenous variables U are roots of model.
- Value instantiations of exogenous variables completely determine values of all variables in SCM

Causality in SCMs

Definition

1. X is a direct cause of Y iff $Y = f(\ldots, X, \ldots)$ for some f.
2. X is a cause of Y iff it is a direct cause of Y or there is Z s.t. X is a direct cause of Z and Z is a cause of Y.

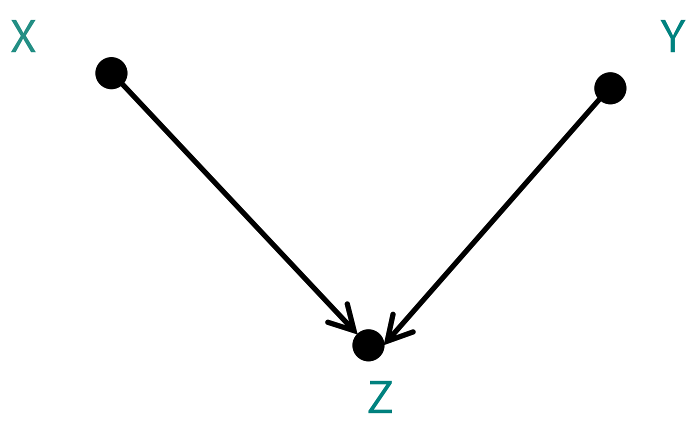
Graphical Causal Model

- Graphical causal model associated with SCM
  - Nodes = variables
  - Edges = from A to B if B = f(…,A , ….)

- Example SCM
  - U = {X,Y}
  - V = {Z}
  - F = $\{f_Z\}$
  - f_Z : Z = 2X + 3Y

(Z = salary, X = years of experience,
Y = years of profession)

- Associated graph



Graphical Models

- Graphical models capture SCMs only partially
- But they are very intuitive and still allow for conserving much of the causal information of an SCM
- Convention: Consider only Directed Acyclic Graphs (DAGs)

SCMs and Probabilities

- Consider SCMs where all variables are random variables (RVs)
- Full specification of functions f not always possible
- Instead: Use conditional probabilities as in BNs
  - $f_X(\ldots Y \ldots)$ becomes $P(X \mid \ldots Y \ldots)$
  - Technically: Non-measurable RVs U model (probabilistic) indeterminism:

$$P(X \mid \ldots . Y \ldots .) = f_X(\ldots Y \ldots, U)$$

U not mentioned here

SCMs and Probabilities

- Product rule as in BNs used for full specification of joint distribution of all RVs $X_1, \ldots, X_n$

$$P(X_1 = x_1, \ldots, X_n = x_n) = \prod_{1 \leq i \leq n} P(x_i \mid parents(x_i))$$

- Can make same considerations on (probabilistic) (in)dependence of RVs

- Will be done in the following systematically

Bayesian Networks vs. SCMs

- BNs model statistical (in)dependencies
  - Directed, but not necessarily cause-relation
  - Inherently statistical
  - Very often used for RVs with discrete domains
- SCMs model causal relations
  - SCMs with random variables (RVs) induce BNs
  - Assumption: There is hidden causal (deterministic) structure behind statistical data
  - More expressive than BNs: Every BN can be modeled by SCMs but not vice versa
  - Default application: continuous variables

Reminder: Conditional Independence

- Event A *independent* of event B iff $P(A \mid B) = P(A)$

- RV X is *independent* of RV Y iff
  $P(X \mid Y) = P(X)$ iff
  for every x-value of X and for every y-value Y
  event $X = x$ is independent of event $Y = y$
  Notation: $(X \perp\!\!\!\perp Y)_P$ or even shorter: $(X \perp\!\!\!\perp Y)$

- X is *conditionally independent* of Y given Z
  iff $P(X \mid Y, Z) = P(X \mid Z)$
  Notation: $(X \perp\!\!\!\perp Y \mid Z)_P$ or even shorter: $(X \perp\!\!\!\perp Y \mid Z)$

Independence in SCM graphs

- Almost all interesting independences of RVs in an SCM can be identified in its associated graph

- Relevant graph theoretical notion: d-separation

Property
X is independent of Y (conditioned on Z) iff
X is d-separated from Y (by Z)

- D-separation in turn rests on 3 basic graph patterns
  - Chains
  - Forks
  - Colliders

We will develop a syntactic d-separation criterion that can be checked algorithmically

Independence in SCM graphs

Property
X is independent of Y (conditioned on Z) iff
X is d-separated from Y by Z

There are two conditions here due to "iff":

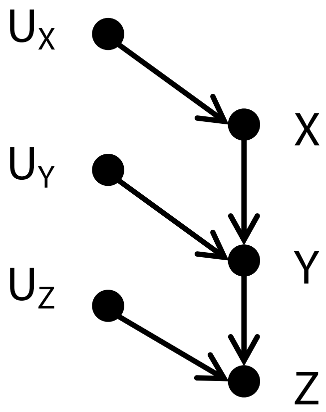
- Markov condition:
  If X is d-separated from Y (by Z)
  then X is independent of Y (conditioned on Z)
- Faithfulness:
  If X is independent of Y (conditioned on Z)
  then X is d-separated from Y (by Z)

Chains

Example (SCM 1)

(X = school funding of high school , Y = its average satisfaction score, Z = average college acceptance)

- $V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{f_X,f_Y,f_Z\}$
- f_X: $X = U_X$ f_Y: $Y = x/3 + U_Y$ f_Z: $Z = y/16 + U_Z$

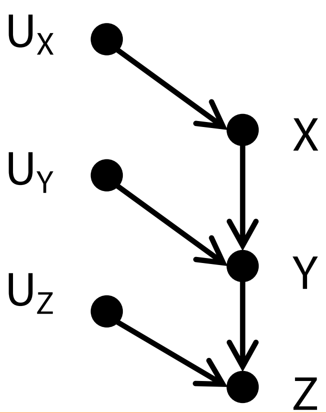


Chains

Example (SCM 2)

(X = switch, Y = circuit, Z = light bulb)

- $V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{f_X,f_Y,f_Z\}$
- f_X: $X = U_X$
- $f_Y: Y = \begin{cases} \text{closed} & \text{if } (X = \text{up} \;\&\; U_Y = 0) \text{ or } (X = \text{down} \;\&\; U_Y = 1) \\ \text{open} & \text{otherwise} \end{cases}$
- $f_Z: Z = \begin{cases} \text{on} & \text{if } (Y = \text{closed} \;\&\; U_Z = 0) \text{ or } (Y = \text{open} \;\&\; U_Z = 1) \\ \text{off} & \text{otherwise} \end{cases}$

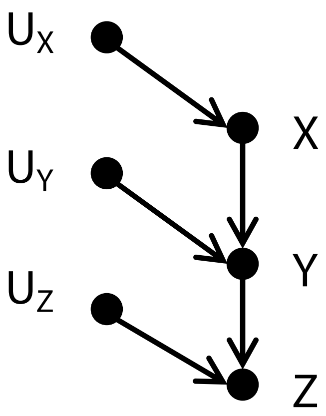


Chains

Example (SCM 3)

(X = work hours, Y = training, Z = race time)

- $V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{f_X,f_Y,f_Z\}$
- f_X: $X = U_X$
- f_Y: $Y = 84 - x + U_Y$
- f_Z: $Z = 100/y + U_Z$



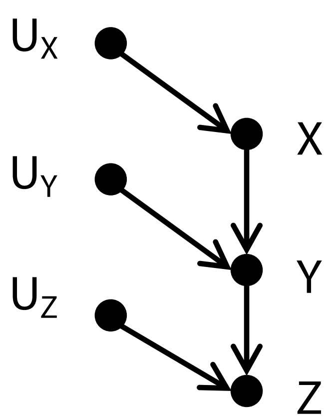
(In)Dependences in Chains

- Z and Y are likely dependent

  (For some z,y: $P(Z=z \mid Y = y) \neq P(Z = z)$)
- Y and X are likely dependent

  (…)
- Z and X are likely dependent
- Z and X are independent, conditioned on Y

  (For all x,z,y: $P(Z=z \mid X=x, Y = y) = P(Z = z \mid Y = y)$)

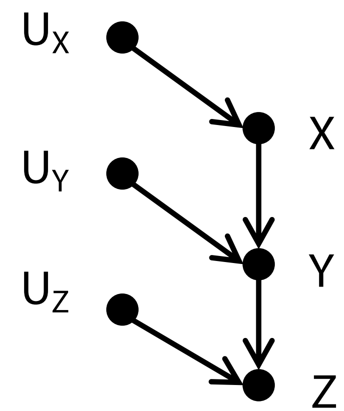


Dependence not Transitive

Example (SCM 4)

$V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{ f_X,f_Y,f_Z \}$

- f_X: $X = U_X$
- f_Y: $Y = \begin{cases} a & \text{if } X = 1 \,\&\, U_Y = 1 \\ b & \text{if } X = 2 \,\&\, U_Y = 1 \\ c & \text{if } U_Y = 2 \end{cases}$
- f_Z: $Z = \begin{cases} i & \text{if } Y = c \text{ or } U_Z = 1 \\ j & \text{if } Y \neq c \,\&\, U_Z = 2 \end{cases}$



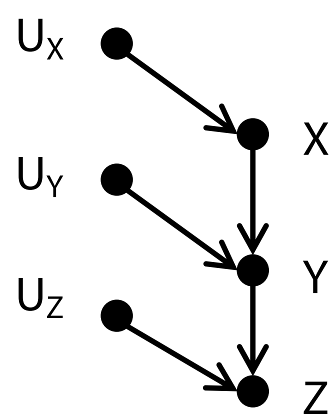
Typo in book of Pearl et al.

- Y depends on X, Z depends on Y **but** Z does not depend on X
- "Variable level" graph hides **in**dependence

Independence Rule in Chains

Rule 1 (Conditional Independence in Chains)

Variables X and Z are independent given set of variables Y
iff
there is only one path between X and Z and this path is unidirectional and Y intercepts that path

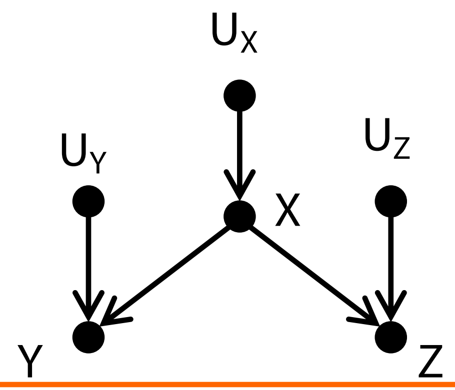


Forks

Example (SCM 5)

(X = Temperature, Y = Ice cream sale, Z = Crime)

- $V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{f_X,f_Y,f_Z\}$
- f_X: $X = U_X$
- f_Y: $Y = 4x + U_y$
- f_Z: $Z= x/10 + U_Z$

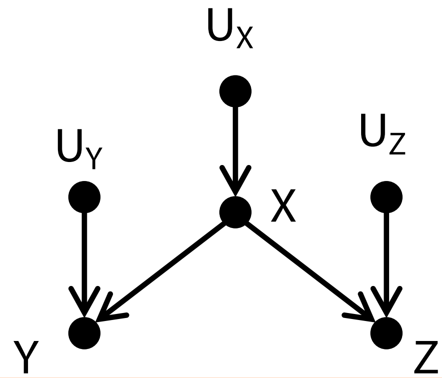


Forks

Example (SCM 5)

(X = switch, Y = light bulb 1, Z = light bulb 2)

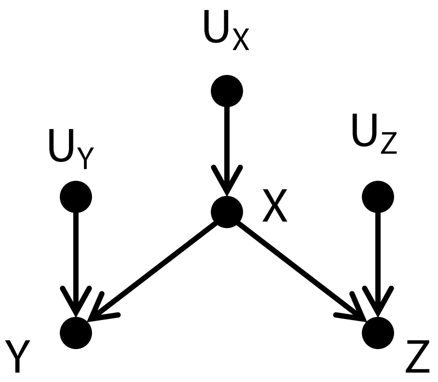
- $V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{f_X,f_Y,f_Z\}$
- f_X: $X = U_X$
- $f_Y: Y = \begin{cases} \text{on} & \text{if } (X = \text{up} \,\&\, U_Y = 0) \text{ or } (X = \text{down} \,\&\, U_Y = 1) \\ \text{off} & \text{otherwise} \end{cases}$
- $f_Z: Z = \begin{cases} \text{on} & \text{if } (X = \text{up} \,\&\, U_Z = 0) \text{ or } (X = \text{down} \,\&\, U_Z = 1) \\ \text{off} & \text{otherwise} \end{cases}$



(In)Dependences in Forks

- X and Z are likely dependent
  ($\exists x,z: P(X=x \mid Z = z) \neq P(X = x)$)
- X and Y are likely dependent

  …
- Z and Y are likely dependent
- Y and Z are independent, conditional on X
  ($\forall x,y,z: P(Y=y \mid Z=z, X = x) = P(Y = y \mid X = x)$)

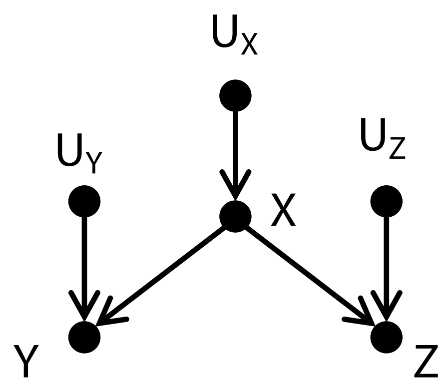


Independence Rule in Forks

Rule 2 (Conditional Independence in Forks)

If variable X is a common cause of variables Y and Z and there is only one path between Y, Z

then Y and Z are independent conditional on X.

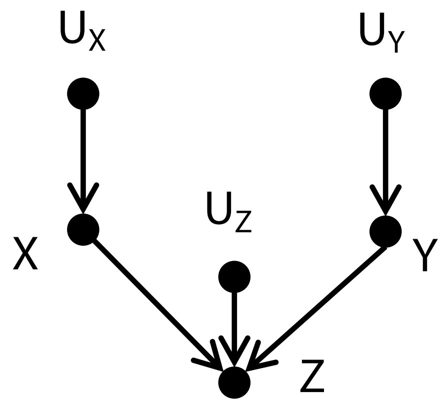


Colliders

Example (SCM 6)

(X = musical talent, Y = grade point, Z = scholarship)

- $V = \{X,Y,Z\}$ $U = \{U_X,U_Y,U_Z\}$ $F = \{f_X,f_Y,f_Z\}$
- f_X: $X = U_X$
- f_Y: $Y = U_Y$
- f_Z: $Z = \begin{cases} \text{yes} & \text{if } X = \text{yes or } Y > 80\% \\ \text{no} & \text{otherwise} \end{cases}$

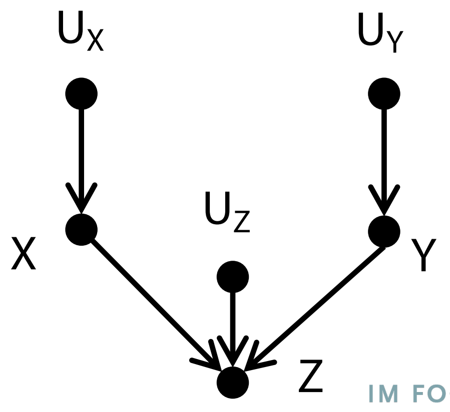


(In)dependence in Colliders

- X and Z are likely dependent
  ($\exists z,y: P(X=x \mid Z = z) \neq P(X = x)$)
- Y and Z are likely dependent
- X and Y are independent
- X and Y are likely dependent, conditional on Z
  ($\exists x,z,y: P(X= x \mid Y=y, Z = z) \neq P(X = x \mid Z = z)$)

If scholarship received (Z)
but low grade (Y),
then must be musically talented (X)

X-Y dependence (conditional on Z)
is statistical but not causal

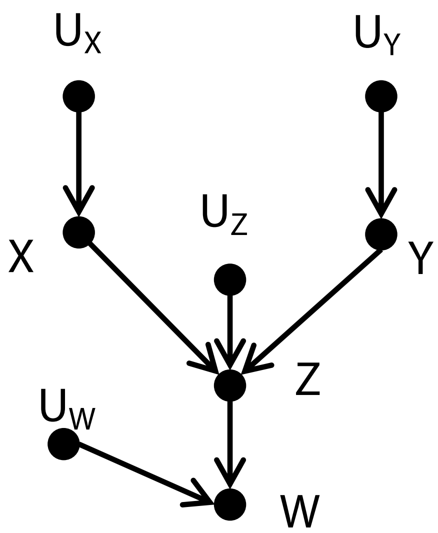


(In)dependence in Colliders (Extended)

Example (SCM 7)

(X = coin flip, Y = second coin flip,
Z = bell rings, W = bell witness)

- $V = \{X,Y,Z,W\}$ $U = \{U_X,U_Y,U_{Z,} U_W\}$ $F = \{f_X,f_{Y,}, f_W\}$
- f_X: $X = U_X$
- f_Y: $Y = U_Y$
- $f_{Z:}$ $Z =$ $\begin{cases} \text{yes} & \text{if } X = \text{head or } Y = \text{head} \\ \text{no} & \text{otherwise} \end{cases}$
- f_W: $W =$ $\begin{cases} \text{yes} & \text{if } Z = \text{yes or } (Z=\text{no and } U_W = ½) \\ \text{no} & \text{otherwise} \end{cases}$



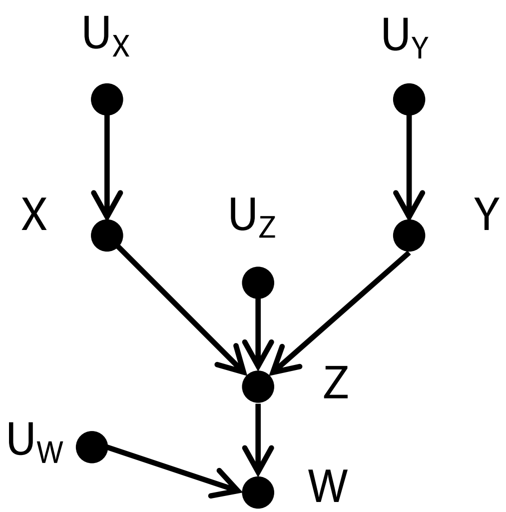
X and Y are dependent conditional on Z and on W.

Independence Rule in Colliders

Rule 3 (Conditional Independence in Colliders)

If a variable Z is the collision node between variables X and Y and there is only one path between X, Y,

then X and Y are unconditionally independent, but are dependent conditional on Z and any descendant of Z



D-separation

Recap: Property

X independent of Y (conditional on Z) w.r.t. a probability distribution iff

X d-separated from Y (by Z) in graph

Definition (informal)

X is d-separated from Y by Z iff

Z blocks every possible path between X and Y

- Z (possibly a set of variables) prohibits the ``flow'' of statistical effects/dependence between X and Y
  - Must block every path
  - Need only one blocking variable for each path

Pipeline metaphor

Blocking Conditions

Definition (formal)

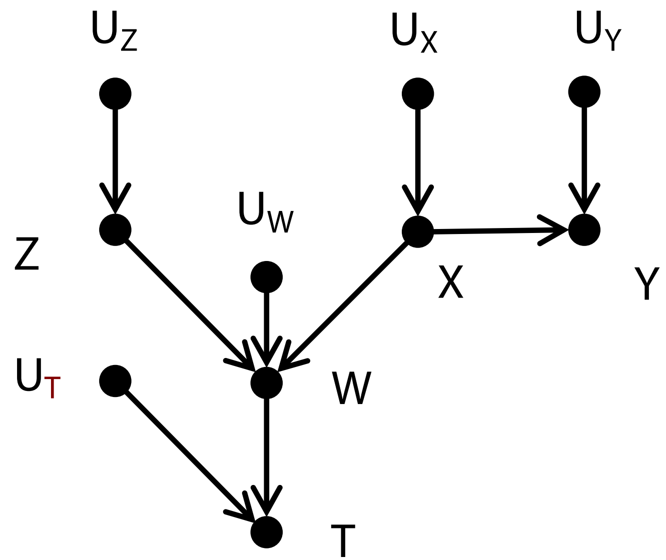
A path p in G (between X and Y) is blocked by Z iff

1. p contains chain $A \longrightarrow B \longrightarrow C$ or fork $A \longleftarrow B \longrightarrow C$ s.t. $B \in Z$ or
2. p contains collider $A \longrightarrow B \longleftarrow C$ s.t. $B \notin Z$ and all descendants of B are $\notin Z$

If Z blocks every path between X and Y, then X and Y are d-separated conditional on Z, for short: $(X \perp\!\!\!\perp Y \mid Z)_G$

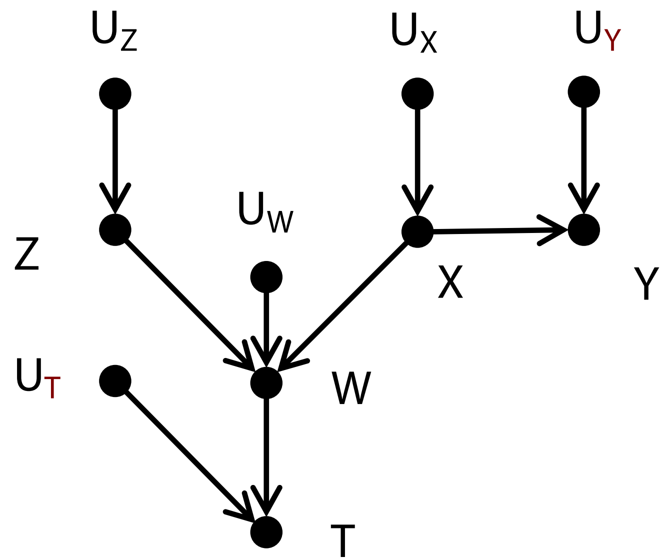
In particular: X and Y are unconditionally independent iff all X-Y paths contain colliders.

Example 1 (d-separation)



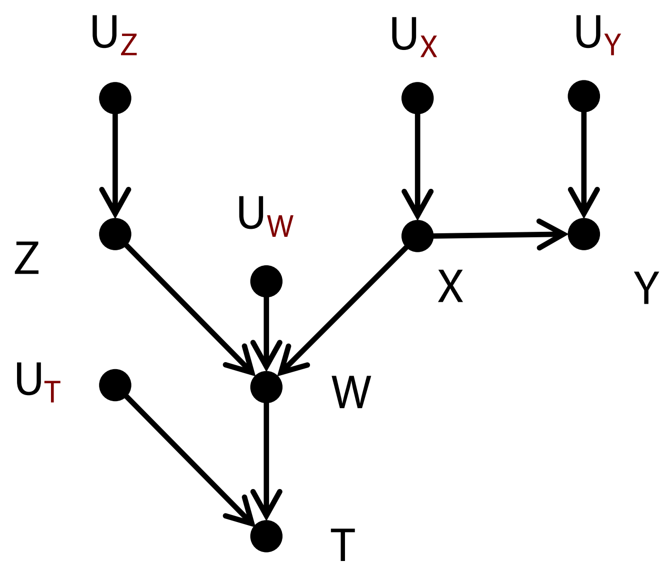
- Unconditional relation between Z and Y ?
  - D-separated because of collider on single Z-Y path.
  - Hence unconditionally independent

Example 1 (d-separation)



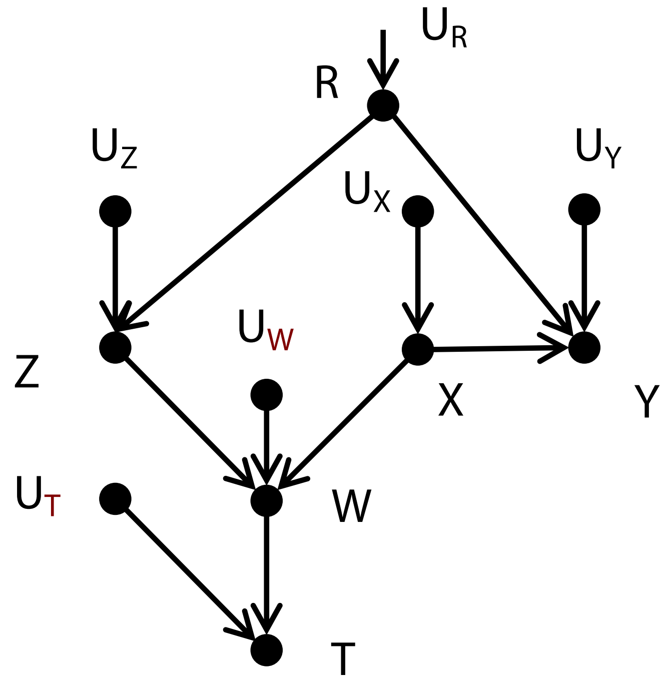
- Relation between Z and Y conditional on {W}?
  - Not d-separated
    - because fork X ∉ {W}
    - and collider ∈ {W}
  - Hence conditionally dependent on {W} (and {T})

Example 1 (d-separation)



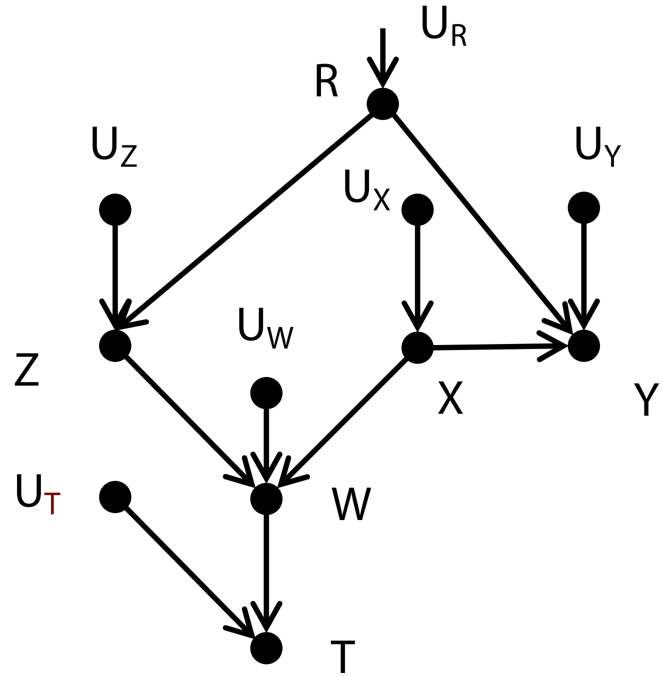
- Relation between Z and Y conditional on {W,X}?
  - d-separated
    - Because fork X blocks
  - Hence conditionally independent on {W,X}

Example 2 (d-separation)



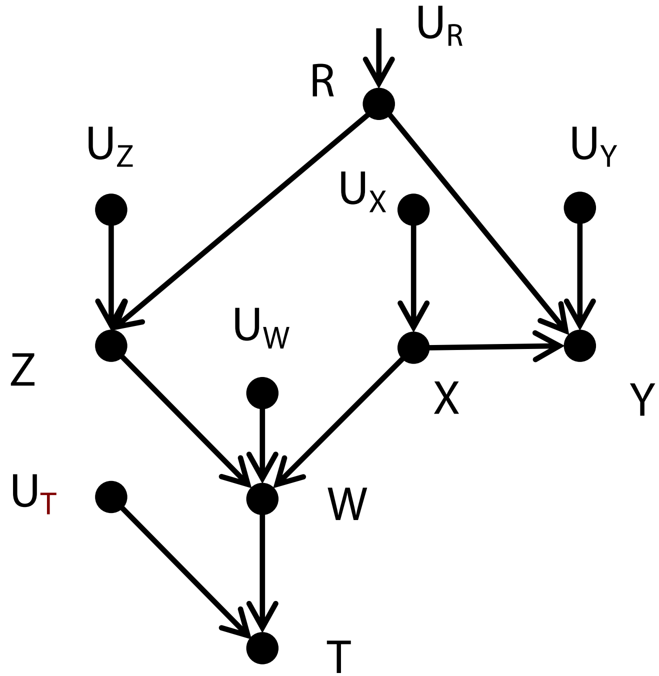
- Relation between Z and Y?
  - Not d-separated because second path not blocked (no collider)
  - Hence not unconditionally independent

Example 2 (d-separation)



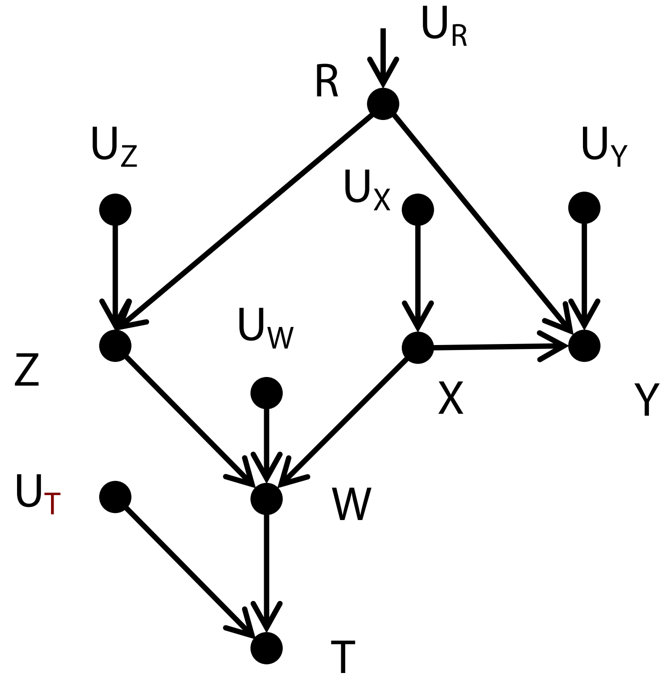
- Relation between Z and Y conditional on {R}?
  - d-separated by {R} because
    - First path blocked by fork R
    - Second path blocked by collider $W \notin \{R\}$)
  - Hence independent conditional on {R}

Example 2 (d-separation)



- Relation between Z and Y conditional on {R,W}?
  - Not d-separated by {R,W} because W unblocks second path
  - Hence not independent conditional on {R,W}

Example 2 (d-separation)



- Relation between Z and Y conditional on {R,W,X}?
  - d-separated by {R,W,X} because
    - Now second path blocked by fork X
  - Hence independent conditional on {R,W,X}

Using D-separation

- Verifying/falsifying causal models on observational data
  1. G = SCM to test for
  2. Calculate independencies I_G entailed by G using d-separation
  3. Calculate independencies I_D from data (by counting and estimating probabilities) and compare with I_G
  4. If $I_G = I_D$, SCM is a good solution. Otherwise identify problematic $I \in I_G$ and change G locally to fit corresponding $I' \in I_D$

Using D-separation

- This approach is local
  - If I_G not equal I_D, then can manipulate G w.r.t. RVs only involved in incompatibility
  - Usually seen as benefit w.r.t. global approaches via likelihood with scores, say
- Approach is qualitative and constraint-based
- Known algorithms:
  - PC (Peter Spirtes & Clark Glymour)
  - IC (Verma & Pearl)

Equivalent Graphs

- One learns graphs that are (observationally) equivalent w.r.t. entailed independence assumptions
- Formalization
  - v(G) = v-structure of G = set of colliders in G of form $A \longrightarrow B \longleftarrow C$ where A and C not adjacent
  - sk(G) = skeleton of G = undirected graph resulting from G

Definition
G_1 is equivalent to G_2 iff $v(G_1) = v(G_2)$ and $sk(G_1) = sk(G_2)$

Equivalent Graphs

Theorem
Equivalent graphs entail same set of d-separations

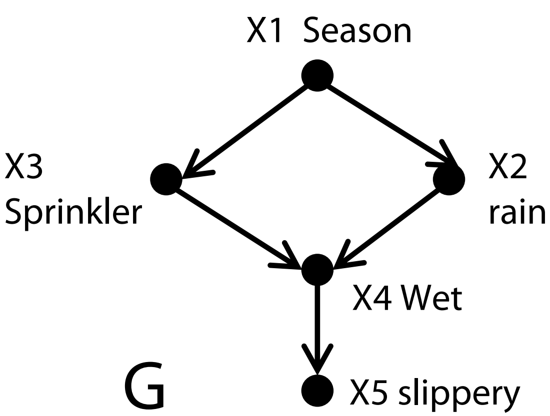
Proof sketch:

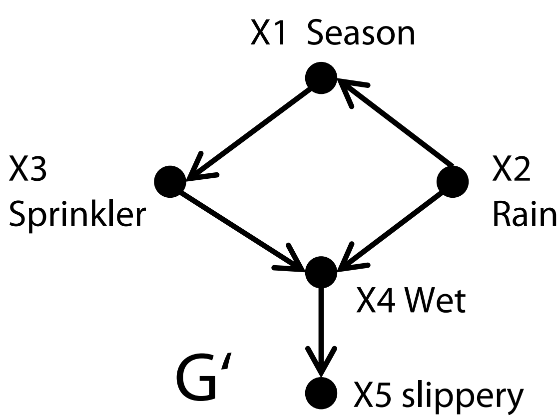
- Forks and chains have similar role w.r.t. independence (Hence forgetting about the direction in skeleton does not lead to loss of information)
- Collider has different role (hence need v-structure)

Equivalent Graphs

- v(G) = v-structure of G = set of colliders in G of form A⟶B⟵C where A and C not adjacent
- sk(G) = skeleton of G = undirected graph resulting from G

Definition
G_1 is equivalent to G_2 iff $v(G_1) = v(G_2)$ and $sk(G_1) = sk(G_2)$





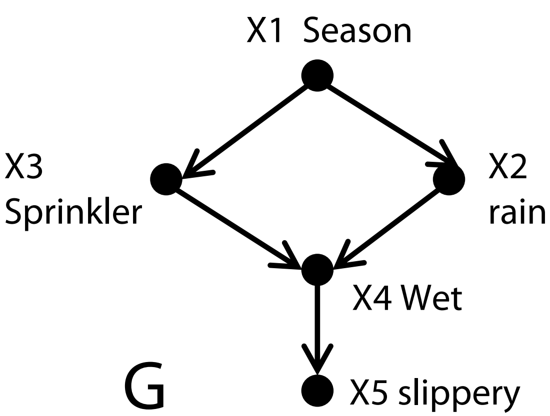
- v(G) = v(G')
- sk(G) = sk(G')
- Hence equivalent

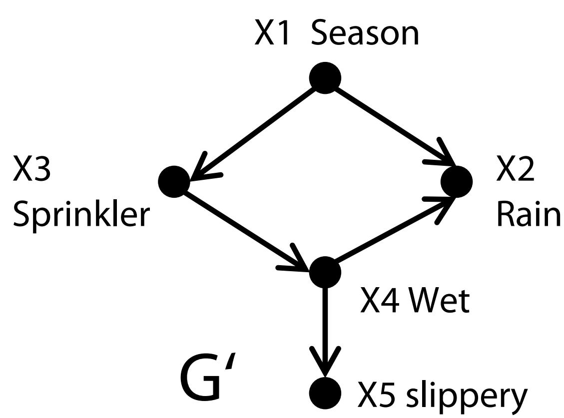
Equivalent Graphs

- v(G) = v-structure of G = set of colliders in G of form A→B←C where A and C not adjacent
- sk(G) = skeleton of G = undirected graph resulting from G

Definition

G_1 is equivalent to G_2 iff $v(G_1) = v(G_2)$ and $sk(G_1) = sk(G_2)$





- $v(G) \neq v(G')$
- $sk(G) = sk(G')$
- Hence not equivalent

IC-Algorithm (Verma & Pearl, 1990)

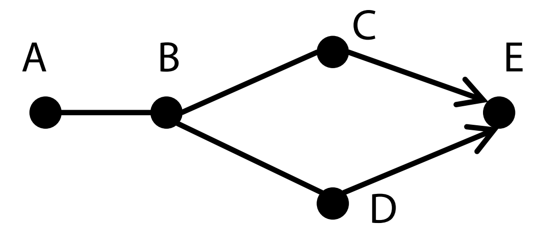
Input
P resp.
P-independencies

(C ⫫ A | B)
(C ⫫ D | B)
(D ⫫ A | B)
(E ⫫ A | B)
(E ⫫ B | C,D)

Algorithm

Steps 1-3

Output
Pattern
(represents compatible class of equivalent DAGs)



Definition

Pattern = partially directed DAG
= DAG with directed and non-directed edges

Directed edge A-> B in pattern: In any of the DAGs the edge is A->B
Undirected edge A-B in pattern: There exists (equivalent) DAGs with A->B in one and B ->A in the other

Verma, T. & Pearl, J: Equivalence and synthesis of causal models. Proceedings of the 6. conference on Uncertainty in AI, 220-227, 1990.

IC-Algorithm (Informally)

1. Find all pairs of variables that are dependent of each other (applying standard statistical methods on the database) and eliminate indirect dependencies

2. + 3. Determine directions of dependencies

Note: „Possible“ in step 3 means: if you can find two patterns such that in the first the edge A-B becomes A->B but in the other A<-B, then do not orient.

IC-Algorithm (schema)

1. Add (undirected) edge A-B iff there is no set of RVs **Z** such that $(A \perp\!\!\!\perp B \mid \mathbf{Z})_{P}$. Otherwise let Z_{AB} denote some set **Z** with $(A \perp\!\!\!\perp B \mid \mathbf{Z})_{P}$.
2. If A–B–C and not A-C, then $A \rightarrow B \leftarrow C$ iff $B \notin Z_{AC}$
3. Orient as many of the undirected edges as possible, under the following constraints:
   - Orientation should not create a new v-structure and
   - Orientation should not create a directed cycle.

Steps 1 and step 3 leave out details of search

- Hierarchical refinement of step 1 gives PC algorithm (next slide)
- A refinement of step 3 possible with 4 rules (thereafter)

PC algorithm (Spirtes & Glymour, 1991)

- Remember Step 1 of IC
  1. Add (undirected) edge A-B iff there is no set of RVs **Z** such that $(A \perp\!\!\!\perp B \mid \mathbf{Z})_P$. Otherwise let Z_{AB} denote some set **Z** with $(A \perp\!\!\!\perp B \mid \mathbf{Z})_P$.
- Have to search all possible sets **Z** of RVs for given nodes A,B
  - Done systematically by sets of cardinality 0,1,2,3…
  - Remove edges from graph as soon as independence found
  - Polynomial time for graphs of finite degree (because can restricted search for **Z** to nodes adjacent to A,B)

P.Spirtes, C. Glymour: An algorithm for fast recovery of sparse causal graphs. Social Science Computer Review 9: 62-72, **1991**.

IC-Algorithm (with rule-specified last step)

1. as before
2. as before
3. Orient undirected edges as follows
   - B — C into B→C if there is an arrow A→B s.t. A and C are not adjacent;
   - A — B into A→B if there is a chain A→C→B;
   - A — B into A→B if there are two chains A—C→B and A—D→B such that C and D are nonadjacent;
   - A — B into A→B if there are two chains A—C→D and C→D→B s.t. C and B are nonadjacent;

IC algorithm

Theorem

The 4 rules specified in step 3 of the IC algorithm are necessary (Verma & Pearl, 1992) and sufficient (Meek, 95) for getting a maximally oriented DAG compatible with the input-independencies.

T. Verma and J. Pearl. An algorithm for deciding if a set of observed independencies has a causal explanation. In D. Dubois and M. P. Wellman, editors, UAI '92: Proceedings of the Eighth Annual Conference on Uncertainty in Artificial Intelligence, 1992, pages 323–330. Morgan Kaufmann, **1992**.

Christopher Meek: Causal inference and causal explanation with background knowledge. UAI 1995: 403-410, **1995**.

Stable Distribution

- The IC algorithm accepts stable distributions P (over set of variables) as input, i.e., distribution P s.t. there is DAG G giving exactly the P-independencies

- Extension IC* works also for sampled distributions generated by so-called latent structures
  - A latent structure (LS) additionally specifies a (subset) of observation variables for a causal structure
  - A LS not determined by independencies
  - For IC* please refer to, e.g., J. Pearl: Causality, CUP, 2001, reprint, p. 52-54.

Criticism and further developments

Definition

The problem of ignorance denotes the fact that there are RVs A, B and sets of RVs $\mathbf{Z}$ such that it is not known whether $(A \perp\!\!\!\perp B \mid \mathbf{Z})_P$ or not $(A \perp\!\!\!\perp B \mid \mathbf{Z})_P$

- Problem of ignorance ubiquitous in science practice
- IC faces the problem of ignorance (Leuridan 2009)
- (Leuridan 2009) approaches this with adaptive logic
  - An adaptive logic supposes that all formulas behave normally unless and until proven otherwise

B. Leuridan. Causal discovery and the problem of ignorance: an adaptive logic approach. Journal of Applied Logic, 7(2):188–205, **2009**.