Rescued from a Sea of Queries

Exact Inference in Probabilistic Relational Models

Colloquium

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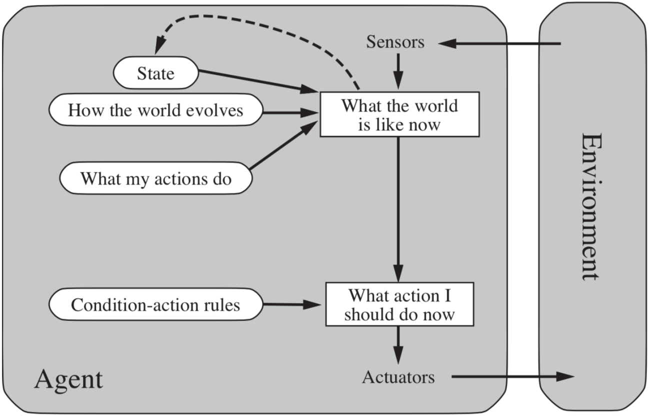
Artificial Intelligence: An Agent Perspective

Russell and Norvig (2010)



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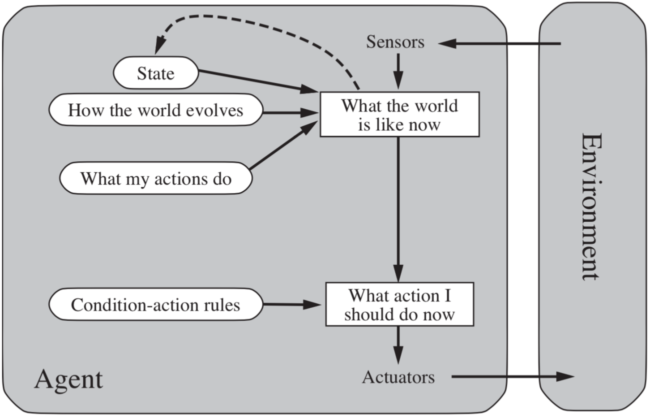
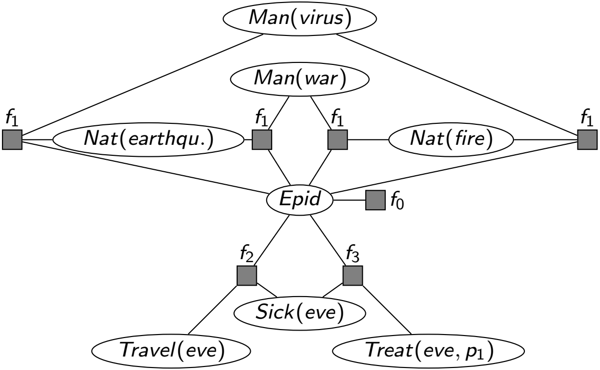


Knowledge representation and reasoning under uncertainty
→ Statistical Relational AI

Probabilistic Graphical Models

Hammersley and Clifford (1971), Kschischang et al. (2001)

Factor graph F: Compact encoding of full joint distribution $P_F = \frac{1}{Z} \prod_i f_i$



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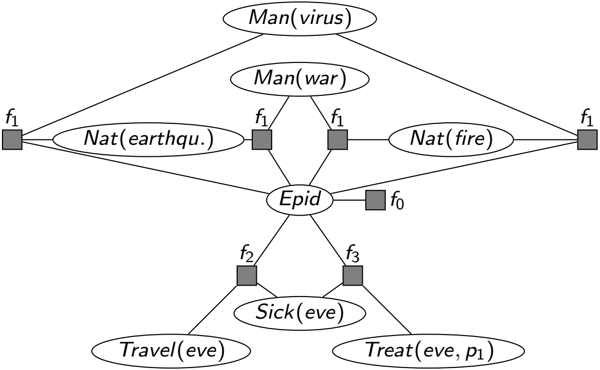

Query answering (QA): Eliminate all non-query variables **avoiding building** P_F

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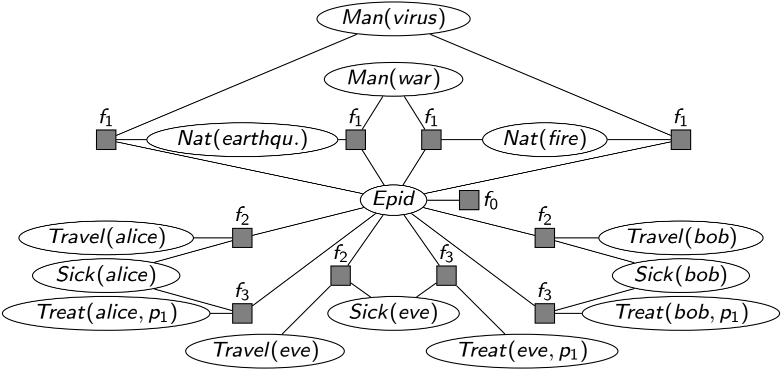
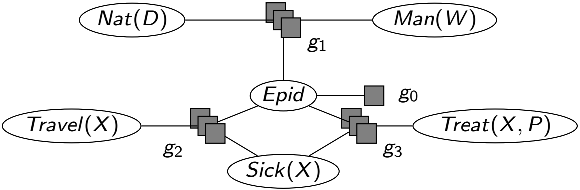


Query answering (QA): Eliminate all non-query variables **avoiding building** P_F

Probabilistic Relational and Lifted Models

Sato (1995), Poole (2003), Ahmadi et al. (2013)

Parfactor graph G: Compact encoding of full joint d. $P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$



Probabilistic Relational and Lifted Models

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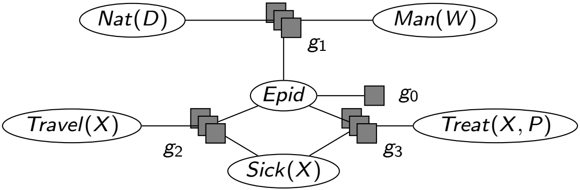
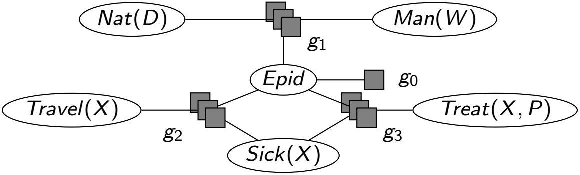


QA: Eliminate all non-query variables
while **avoiding grounding G and building P_G**

QA: Lifted Variable Elimination (LVE)

Poole (2003), de Salvo Braz et al. (2005), Milch et al. (2008), Taghipour et al. (2013b)



$P(Sick(eve))$

$\sum_V$ indicates a sum over the values of V, $|X|$ a domain size

QA: Lifted Variable Elimination (LVE)

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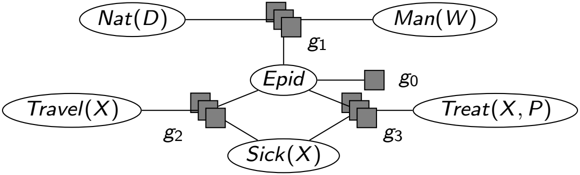


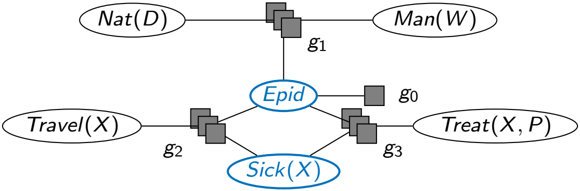
$$P(Sick(eve)) \propto \sum_{Epid} g_0 \left(\sum_{\substack{Sick(X)\\X \neq eve}} \sum_{Travel(X)} g_2 \left(\sum_{Treat(X,P)} g_3 \right)^{|P|} \right)^{|X|_{X \neq eve}}$$

$$\sum_{\#_D[Nat(D)]} \left(\sum_{Man(W)} g_1^{\#} \right)^{|W|}$$

$\sum_V$ indicates a sum over the values of V, $|X|$ a domain size

QA: Conditional Independences and Dynamic Programming

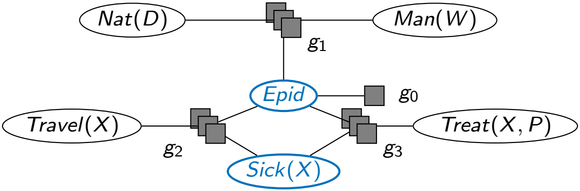
Lauritzen and Spiegelhalter (1988)



QA based on submodels
ensured to be independent

QA: Conditional Independences and Dynamic Programming

Lauritzen and Spiegelhalter (1988)



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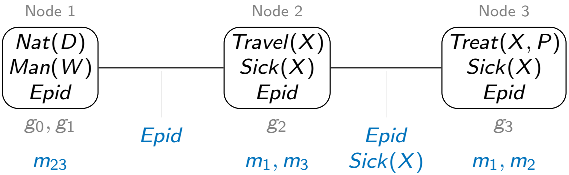
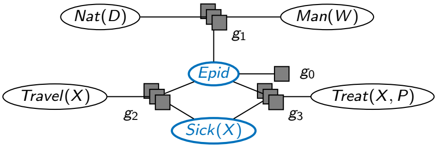


Lifting + Conditional Independences and Beyond

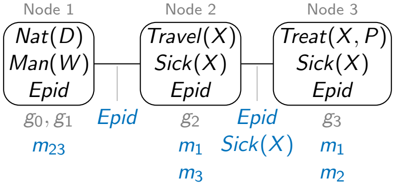


Lifted Junction Tree Alg. (LJT)

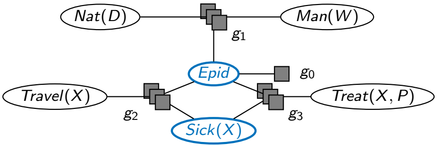
B and Möller (2016)

Answer multiple queries efficiently

QA based on submodels
ensured to be independent



Lifting + Conditional Independences and Beyond



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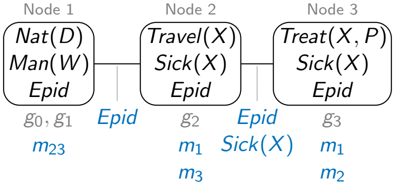


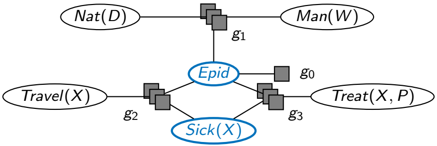
Lifted Junction Tree Alg. (LJT)

Liftability

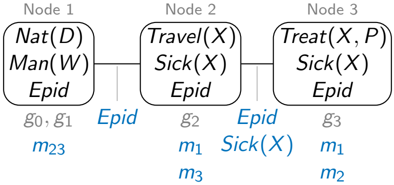
B and Möller (2017)

Avoid message-induced groundings

Lifting + Conditional Independences and Beyond



QA based on submodels ensured to be independent



Lifted Junction Tree Alg. (LJT)

Liftability

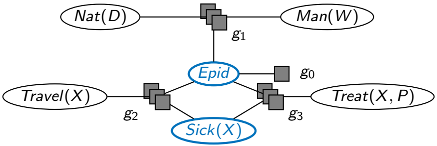
Marginal queries

B and Möller (2018a,c)

Conjunctive: $P(Sick(eve), Epid)$

Parameterised: $P(Sick(X))$

Lifting + Conditional Independences and Beyond



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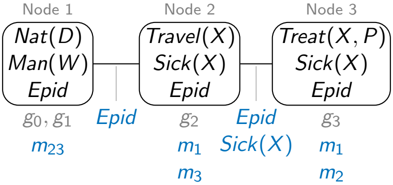


Lifted Junction Tree Alg. (LJT)

Liftability

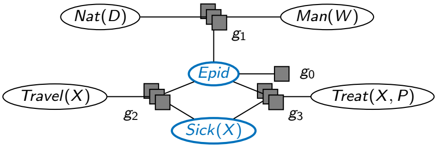
Marginal queries

Assignments queries

B and Möller (2018b)

LVE + LJT versions using arg max

Lifting + Conditional Independences and Beyond



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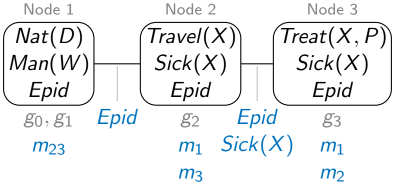


Lifted Junction Tree Alg. (LJT)

Liftability

Marginal queries

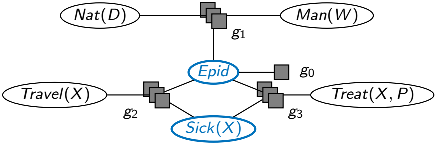
Assignments queries

Complexity & Completeness

Polynomial w.r.t. domain size

Classes of liftable queries

Lifted Inference Continued...



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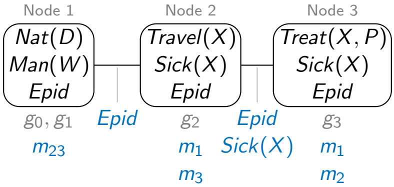
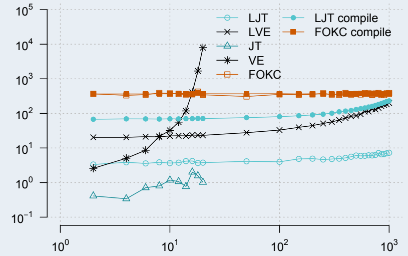
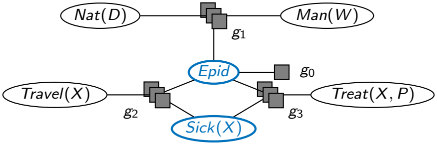


Empirical studies



Lifted Inference Continued...



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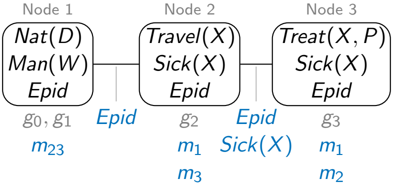


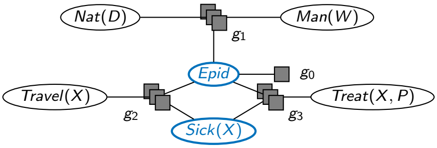
Empirical studies

Adaptive inference

B and Möller (2018e)

Adapt to local changes

Lifted Inference Continued...



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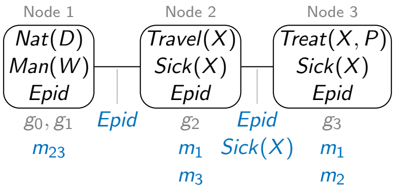


Empirical studies

Adaptive inference

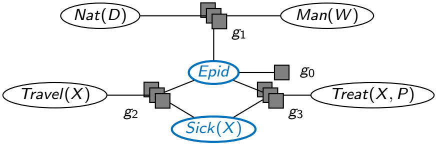
LJT inference framework

B and Möller (2018d)

Use other QA algorithms

Conditions apply

Lifted Inference Continued...



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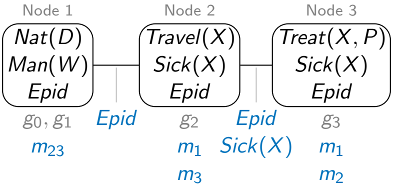


Empirical studies

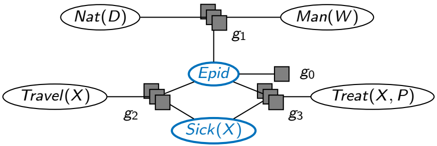
Adaptive inference

LJT inference framework

Unknown universe

B and Möller (2019)
Retain tractability

Lifted Inference Continued...



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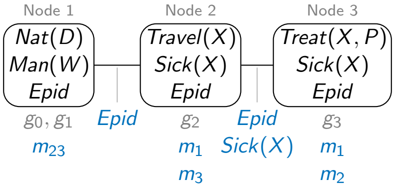


Empirical studies

Adaptive inference

LJT inference framework

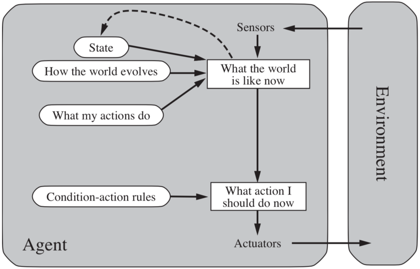
Unknown universe

Continued by/with colleagues

- lifted dynamic models
- lifted decision making
- lifted continuous models

Conclusion

Rescued from a Sea of Queries



Russell and Norvig (2010)

Knowledge representation and reasoning under uncertainty
→ Statistical Relational AI

Exact Lifted Inference

- Lifting junction trees
- Lifting marginal and assignment queries
- Classes of liftable queries

Tractable inference for a variety of queries

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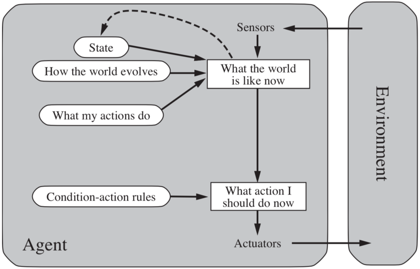

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Knowledge representation and reasoning under uncertainty
$\rightarrow$ Statistical Relational AI

Exact Lifted Inference

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Tractable inference for a variety of queries

Future Work

- Going beyond explanations
- Manoeuvring open universes
- Travelling between universes

Appendix

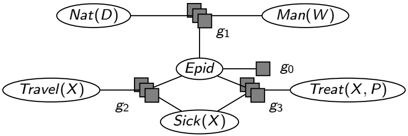
4 Construction
- First-order Decomposition Tree
- First-order Junction Tree
- Fusion

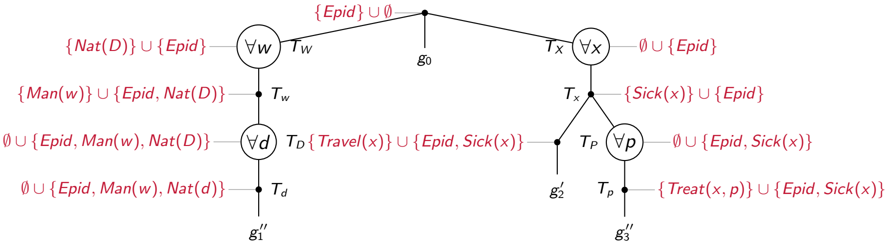
5 Queries
- Parameterised Queries
- Assigment Queries
- Variety of Queries
- Liftable Queries

6 Extensions
- Adaptive Inference
- LJT as a Backbone
- Unknown Universes

First-order Decomposition Tree (FO Dtree)

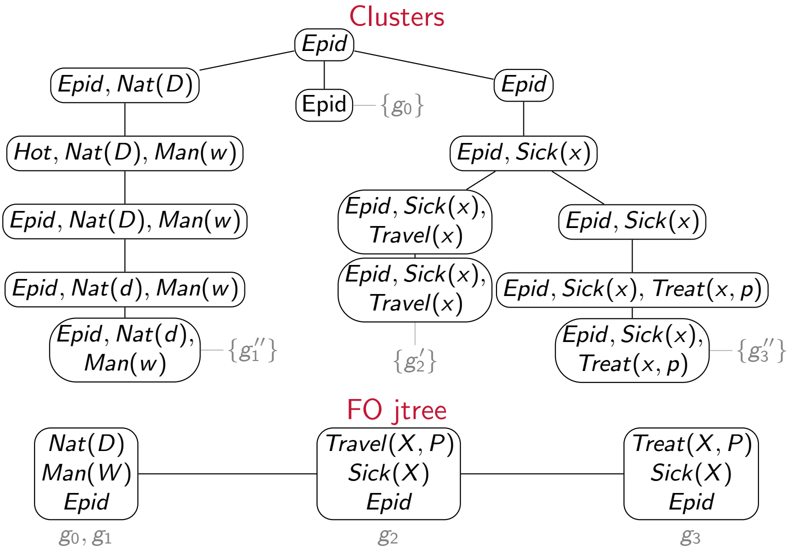
Darwiche (2001), Taghipour et al. (2013a)



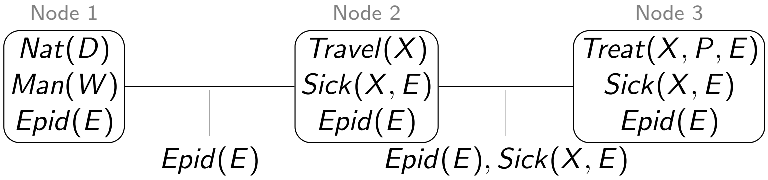


Labels: *cutset(N)* ∪ *context(N)*

First-order Junction Trees (FO Jtrees)



Fusion: Ensuring Lifted Calculations



Elimination order restricted by tree structure AND logical variables

- Lifted summing out of A in g:
  A has to contain all logical variables in g
- Message calculation:
  Terms not on edge need to be eliminated
- $Travel(X)$ has to be eliminated but does not contain X and E

$\rightarrow$ Merge nodes 2 and 3 to avoid elimination

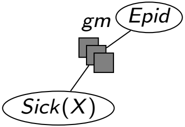
Parameterised Queries

$P(Sick(eve), Sick(alice), Sick(bob))$ vs. $P(Sick(X)_{|\top})$

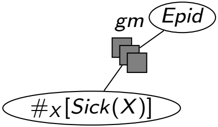
$Sick(alice)$	$Sick(eve)$	$Sick(bob)$	g'
0	0	0	1
0	0	1	2
0	1	0	2
1	0	0	2
1	1	0	3
1	0	1	3
1	1	0	3
1	1	1	4

$\#_X[Sick(X)]$	g'
$[0,3]$	1
$[1,2]$	2
$[2,1]$	3
$[3,0]$	4

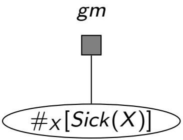
Elimination



Count conversion

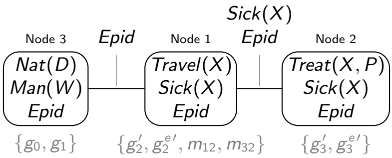


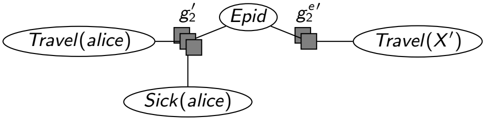
Elimination



Assignment Queries: Most Probable Explanation (MPE)

$\arg\max_{\mathbf{V}} P(\mathbf{V} | Sick(eve) = true, Sick(bob) = true)$





Node 2

LJT^{MPE}

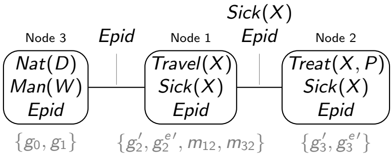
- Absorb evidence at nodes
- Pass messages (inward)
  - $1 \longrightarrow 2$
  - $3 \longrightarrow 2$
- At inner-most node
  - Eliminate variables
  - Output MPE

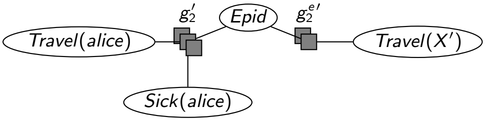
MPE

- $Epid = false$
- $\forall X' \in \{eve, bob\}, P :$ $Treat(X', P) = true,\ Travel(X') = false$
- $Sick(alice) = false,\ Travel(alice) = true,$ $\forall P : Treat(alice, P) = false$
- $\forall D : Nat(D) = false$
- $\forall W : Man(W) = false$

Assignment Queries: Maximum A Posterioi (MAP)

$\arg\max_{Travel(X')} \sum_{\mathbf{T}} P(Travel(X') | Sick(X') = true), X' \in \{eve, bob\}$





Node 1

LJT^{MAP}

- Absorb evidence at nodes
- Pass messages with LVE
- Answer query ($\mathbf{V} = \{Travel(X')\}$)
  - Find nodes covering $\mathbf{V}$
  - Eliminate remaining variables of $\mathbf{T}$
  - Eliminate $\mathbf{V}$

Answer query

- $Travel(eve)$, $Travel(bob)$: node 1
- Eliminate with LVE: $Travel(alice), Sick(alice), Epid$
- Eliminate with LVE^{MPE}: $Travel(X')$
- MAP: $\forall X' \in \{eve, bob\}$: $Travel(X') = false$

LJT for a Variety of Queries

procedure COM-LJT(Model G, Query terms and types $\{(\mathbf{Q}_k, t_k)\}_{k=1}^m$, Evidence $\mathbf{E}$)
  Construct an FO jtree J for G
  Enter $\mathbf{E}$ into J
  Pass messages on J ▷ LVE as subroutine
  for each $(\mathbf{Q}_k, t_k) \in \{(\mathbf{Q}_k, t_k)\}_{k=1}^m$ **do**
    if t_k = MPE **then** ▷ $\mathbf{Q}_k = \emptyset$
      J-MPE-LJT(J) ▷ Output or store result
    else
      Find a subtree J' s.t. $\mathbf{Q}_k \subseteq rv(J')$
      if t_k = MAP $\wedge$ $\mathbf{Q}_k = rv(J')$ **then**
        J-MPE-LJT(J') ▷ Output or store result
      else
        Extract a submodel G' from J'
        if t_k = MAP **then** ▷ $\mathbf{Q}_k \subset rv(J')$
          MAP-LVE(G', $\mathbf{Q}_k$, $\emptyset$) ▷ Output or store result
        else
          LVE(G', $\mathbf{Q}_k$, $\emptyset$) ▷ Output or store result
        end if
      end if
    end if
  end for
end procedure

Liftable Probability Queries

Conjunctive queries

For each logical variable, only one set of constants occurs.

$$P(Sick(eve), Treat(eve, p_1))$$
$$P(Sick(eve), Treat(alice, p_1))$$

Parameterised queries

Each query term contains at most one logical variable and one set of constants per logical variable.

$$P(Sick(X)_{X \in \{alice, eve\}}, Travel(X)_{X \in \{alice, eve\}})$$
$$P(Sick(X)_{X \in \{alice, eve\}}, Travel(X'))_{X' \in \{bob, eve\}})$$

Liftable Assignment Queries

Most probable explanation (MPE)

Liftability results from LVE transfer to MPE.

Maximum a posteriori (MAP)

Each MAP term contains at most one logical variable and one set of constants per logical variable.

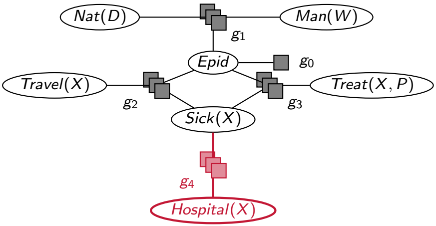
Bounded MAP queries

If the random variables of entire subtrees occur as MAP terms, then the MAP query does not lead to a higher tree width.

MAP query over $Sick(X), Travel(X), Epid$
MAP query over $Treat(X, P), Travel(X), Epid$

Adaptive Inference

Solve each query more efficiently than starting from scratch



Evidence changes (new/retracted)

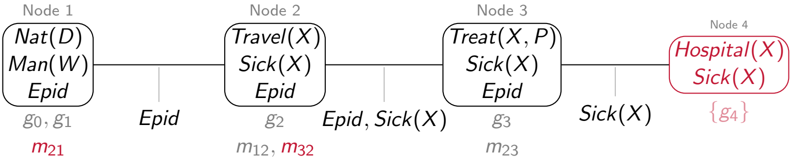
$Travel(alice) = true$

$Travel(bob) = true$

Domains change (new constants)

$X \in \{alice, eve, bob\}$

$\cup \{charlie\}$



LJT as a Backbone for Lifted Inference

Requirements for Subroutines

1. Lifted evidence handling
2. Lifted message calculation (conjunctive, parameterised query)

→ Expressiveness of the query language of the subroutine determines the expressivity of the query language of LJT.

LVE

1. Lifted absorption ✓
2. Eliminate all non-query terms with LVE ✓

→ Marginal or conditional distributions of conjunctions of random variables

FOKC (Van den Broeck, 2013)

1. Lifted conditioning ✓
2. Circuit determines queries easily answered ϟ

→ Marginal or conditional distributions of single random variables

LJT with LVE and FOKC

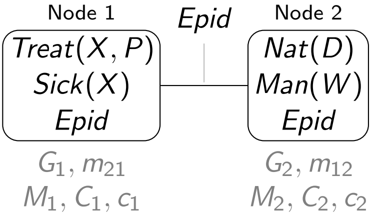
Parfactor model

$g_0(Epid)$

$g_1(Epid, Nat(D), Man(W))$

$g_2(Epid, Sick(X), Treat(X, P))$

First-order Junction Tree J



LVE in steps II + III

FOKC in step IV + V

Algorithm steps

- I. Construct J
- II. Enter evidence $\mathbf{E}$
- III. Pass messages
- IV. For each node i
  - i. Transform node into a Markov logic network M_i
  - ii. Transform M_i into d-DNNF.
  - iii. Build C_i for M_i.
  - iv. Compute WFOMC c_i in C_i.
- V. Answer queries $Q \in \mathbf{Q}$
  - i. Build C_q for $M_i \wedge q$
  - ii. Compute WFOMC c_q in C_q.
  - iii. Compute $\frac{c_q}{c_i}$

Probabilistic Inference with Unknown Universes

Syntactic components:

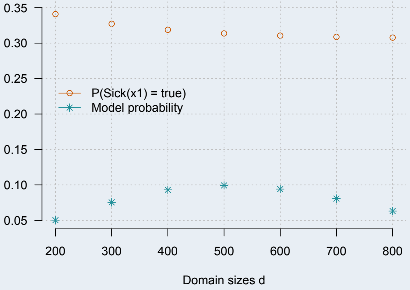
Template model, constraint program, domain worlds

→ Set of possible worlds: Expected values, runtime increases

→ Constraint meta-programming: Build oracle for algorithms

→ Transfer learning: Decoupling from specific domain

New queries emerging



Exploration and model checking, e.g., does the probability of

- an individual being sick decrease with larger domains?
- an epidemic rise if more people travel?

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