



UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR INFORMATIONSSYSTEME

Özgür L. Özçep

Logic, Logic, and Logic

Lecture 1: Motivation and Overview
9 April, 2020

Informationssysteme CS4130
(Summer 2020)

Organizational Stuff

Organization

- ▶ Lectures with integrated wake-up exercises
- ▶ Lectures provided as slides and recordings
- ▶ Lecture slot (Thursdays, 12:15–13:45) used as QA slot
- ▶ Exercise labs: online in the beginning as WebEx Meeting
 - ▶ Time: Thursdays 14:15–15:45 (after lecture)
 - ▶ Start: Thursday, 9 April 2020
 - ▶ Discuss solutions presented by students
- ▶ Sign in for the course in Moodle
- ▶ Lecture and exercise related material in Moodle “Informationssysteme”
- ▶ **Written exam** at the end of the semester
 - ▶ Preliminary dates: 23 July 2020 (Second Chance: 2 October 2020)
 - ▶ Prerequisite for exam
 - ▶ Presentation of a solution for one exercise sheet
 - ▶ One-page-summary of lecture

Exercise Lab

- ▶ Weekly exercise sheets in Moodle
- ▶ Each group of (up to 2 or 3) students gets responsible for one exercise sheet and must present its solution (WebEx/blackboard or beamer)
- ▶ Other students solve the exercise on their own: No solutions to be handed in/no corrections
- ▶ Sign in for an exercise sheet in moodle

One-page summary

- ▶ Each group of students (=the same as exercise group) is responsible for summarizing one lecture (= the one associated with the exercise)
- ▶ Summary to be handed in within 2 weeks after exercise presentation (possibly revision required)
- ▶ meant as preparation for written exams for all students
- ▶ One page in any format you like / you find useful for preparing for the written exam
- ▶ Format may be as in
 - ▶ Theorem of the day (<https://www.theoremoftheday.org/>)
 - ▶ or a Poster format (but such that it is readable when printed in DIN A4)
 - ▶ See e.g. \LaTeX formats at overleaf templates (<https://www.theoremoftheday.org/>)

Sometimes English Becomes Less Important

Prologue

La loi 101 (Charte de la langue française)

Principe du deux pour un □ le texte français doit être écrit en caractères **deux fois plus gros** que ceux de la version en langue étrangère.

Two for one principle □ an english (for clarity) text should be written in characters *twice smaller* than its french counterpart.

Exception □ the english version of the text of the Law itself can be written in characters **five times bigger** than the french original.

Slide example by Bruno Poizat from a conference talk

- ▶ Model Theorist
- ▶ Has a wonderful (unconventional) book on model theory

Lit: B. Poizat. A Course in Model Theory. Universitext. Springer Verlag, 2000.

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- ▶ Model Theorist
- ▶ Has a wonderful (unconventional) book on model theory
 - ▶ Was not well received (for some years)
 - ▶ until he translated it into English

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Colour Coding and formatting for the Slides

- ▶ Formulae will be encoded in this greenish color
- ▶ Newly introduced terminology will be given in this light blue color and definitions will be given in light blue boxes
- ▶ Important results (observations, theorems) will be given in red boxes
- ▶ Emphasizing some aspects will done in red
- ▶ Examples will be given with orange boxes
- ▶ Bibliographic notes are given in this darker blue.

Agenda (overview)

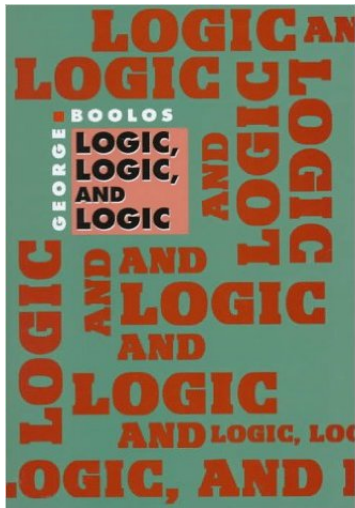
- ▶ Logic, Logic, Logic (2 lectures)
- ▶ Logical Foundations of Database Systems: Finite Model Theory (2 lectures)
- ▶ Data Integration and Data Exchange (3 lectures)
- ▶ Semantic Integration with OBDA: Bridging the DB and Ontology World (2 lectures)
- ▶ Semantic Integration on Ontology Level: Ontology Integration (2 lectures)
- ▶ Database Repairs (1 lecture)
- ▶ Stream Processing (1 lecture)

First-Order Logic

“Logic, Logic, and Logic”

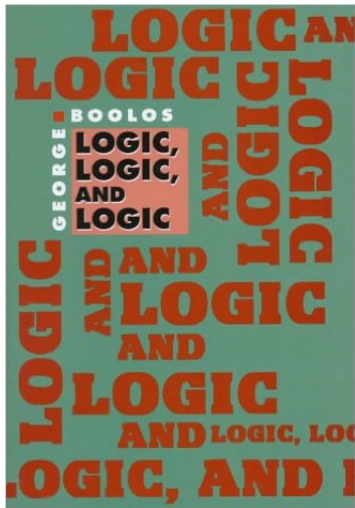
- ▶ Interesting collection of essays
- ▶ Rather “philosophical logic”
- ▶ But we adopt the motto:

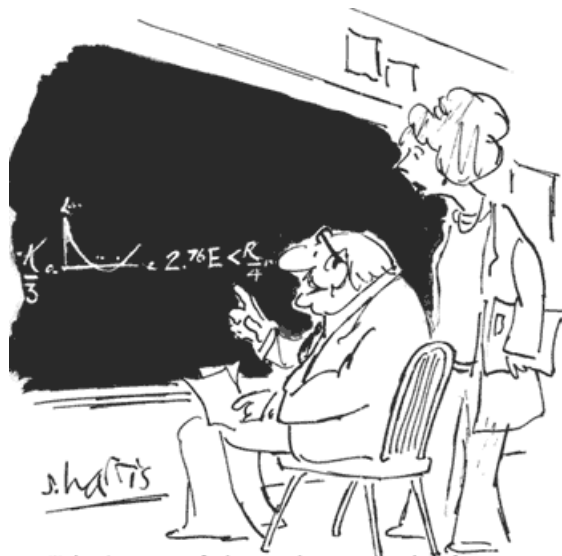
Logic everywhere !



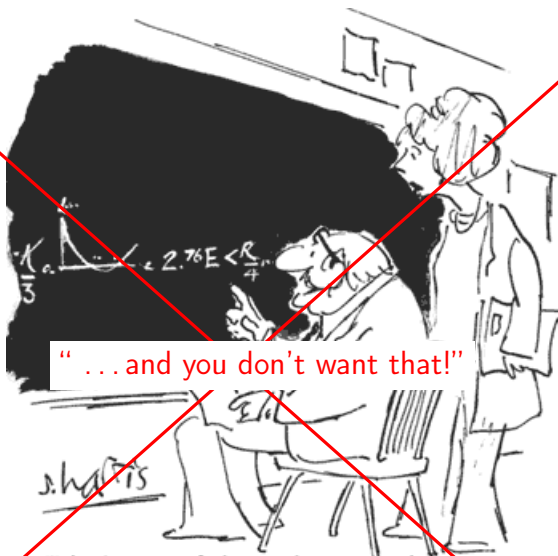
“Logic, Logic, and Logic”

- ▶ Interesting collection of essays
- ▶ Rather “philosophical logic”
- ▶ But we adopt the motto:
Logic everywhere !
- ▶ We are interested not only in logics per se but
- ▶ (Knowledge on) logics useful for computer science



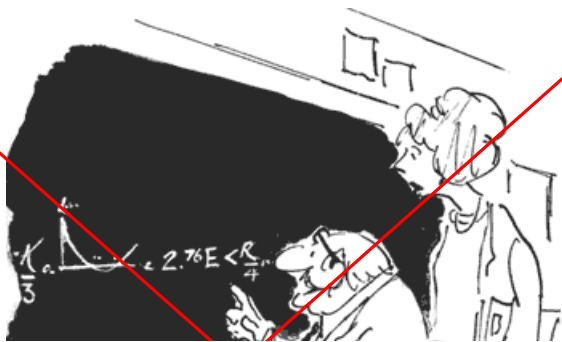


"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."



"...and you don't want that!"

"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."



But: "Nothing is more practical than a good theory"



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."

Logic and Logics

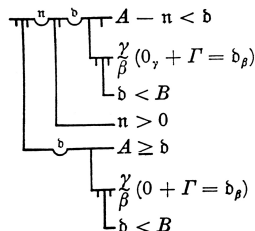
- ▶ Science of logic
 - ▶ investigates mathematical **structures** (static and dynamic)
 - ▶ and formal **languages** to describe them
 - ▶ distinguishing between **syntax**
 - ▶ and **semantics** (truth conditions for sentences)
 - ▶ providing notions of **satisfaction, entailment** (from semantics)
 - ▶ and of **provability, inference (calculus)**

- ▶ A **logic**: A language with syntax, semantics (and possibly calculus)

- ▶ There are many different logics (within computer science)
- ▶ But in any case somehow related to first-order logic

First-Order Logic (FOL)

- ▶ Also called predicate logic (or quantification logic)
- ▶ Aristotelian syllogisms already incorporate restricted FOL
 - ▶ All Philosophers are wise men. All wise men are nice. Hence all Philosophers are nice men.
 - ▶ Restricted to unary predicates
- ▶ Modern FOL started with Frege's "Begriffsschrift"
 - ▶ language constructs based on constants, variables, predicates, functions, boolean connectives, quantifiers
 - ▶ Formal axioms and inference rules
 - ▶ His 2-dimensional representation format aesthetic but not practical



FOL Structures

- ▶ A formalism to investigate (mathematical) **structures**

$$\mathfrak{A} = (A, R_1^{\mathfrak{A}}, \dots, R_n^{\mathfrak{A}}, f_1^{\mathfrak{A}}, \dots, f_m^{\mathfrak{A}}, c_1^{\mathfrak{A}}, \dots, c_l^{\mathfrak{A}})$$

- ▶ (Non-logical) Vocabulary
 - ▶ Relation symbols/predicates R_i with arities
 - ▶ Function symbols f_j (with arities)
 - ▶ Constant symbols c_i
- ▶ Components of the structure
 - ▶ Universe/Domain A
 - ▶ Interpretations/denotations of nonlogical symbols
 - ▶ Relation $R^{\mathfrak{A}} \subseteq A^n$ (for n -ary relation symbol R)
 - ▶ Function $f^{\mathfrak{A}} \in A^n \rightarrow A$ (for n -ary function symbol f)
 - ▶ Individuals $c^{\mathfrak{A}} \in A$ (for constants c)

Example FOL Structures

- ▶ Graphs $\mathfrak{G} = (V, E^{\mathfrak{G}})$
 1. V = nodes of the graph
 2. $E^{\mathfrak{G}} \subseteq V^2$ = edges of the graph

- ▶ Undirected, loopless graphs $\mathfrak{G} = (V, E^{\mathfrak{G}})$
 1. as above
 2. as above
 3. Additionally: edge relation is symmetric and a-reflexive

- ▶ Need an appropriate language to formulate constraints such as in 3.

FOL Syntax

- ▶ Allow variables (x_1, x_2, \dots) and logical constructors
- ▶ Terms
 - ▶ variables and constants are terms
 - ▶ if t_1, \dots, t_n are terms, so is $f(t_1, \dots, t_n)$ (for n -ary function symbol f)

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- ▶ Formulae
 - ▶ $t_i = t_j$ and $R(t_1, \dots, t_n)$ (for terms t_i and n -ary relation) R
 - ▶ If ϕ is a formula, so are
 - ▶ $\neg\phi$ (“Not ϕ ”)
 - ▶ $\forall x \phi$ (“For all x it holds that ϕ ”)
 - ▶ $\exists x \phi$ (“There is an x s.t. ϕ ”)
 - ▶ If ϕ, ψ are formula, so are
 - ▶ $(\phi \wedge \psi)$ (“ ϕ and ψ ”)
 - ▶ $(\phi \vee \psi)$ (“ ϕ or ψ ”)
 - ▶ $(\phi \rightarrow \psi)$ (“If ϕ then ψ ”)
 - ▶ $(\phi \leftrightarrow \psi)$ (“ ϕ iff ψ ”)

FOL Semantics

- ▶ Interpretation $\mathcal{I} = (\mathfrak{A}, \nu)$
 - ▶ ν assigns to all variables elements from domain A
 - ▶ Needed to deal with open formulae
e.g. $\forall y R(y, x)$ open/free in variable x

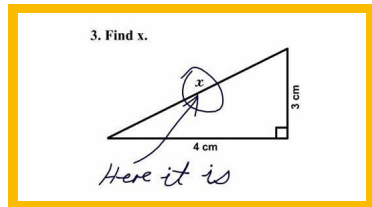
- ▶ x -Variant $\mathcal{I}_{[x/d]}$
same as \mathcal{I} but with $d \in A$ assigned to x

- ▶ Interpretation of terms
 - ▶ $\mathcal{I}(c) = c^{\mathfrak{A}}$
 - ▶ $\mathcal{I}(x) = \nu(x)$
 - ▶ $\mathcal{I}(f(t_1, \dots, t_n)) = f^{\mathfrak{A}}(\mathcal{I}(t_1), \dots, \mathcal{I}(t_n))$

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Because dealing with variables is non-trivial...



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- ▶ Satisfaction relation \models
 - ▶ $\mathcal{I} \models t_1 = t_2$ iff $\mathcal{I}(t_1) = \mathcal{I}(t_2)$
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 - ▶ $\mathcal{I} \models (\phi \wedge \psi)$ iff $\mathcal{I} \models \phi$ and $\mathcal{I} \models \psi$
 - ▶ $\mathcal{I} \models (\phi \vee \psi)$ iff $\mathcal{I} \models \phi$ or $\mathcal{I} \models \psi$
 - ▶ $\mathcal{I} \models (\phi \rightarrow \psi)$ iff: If $\mathcal{I} \models \phi$ then $\mathcal{I} \models \psi$
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 - ▶ $\mathcal{I} \models \forall x \phi$ iff for all $d \in A$: $\mathcal{I}_{[x/d]} \models \phi$
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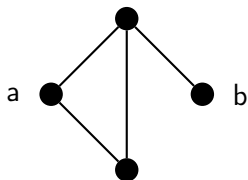
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 - ▶ $\mathcal{I} \models \forall x \phi$ iff for all $d \in A$: $\mathcal{I}_{[x/d]} \models \phi$
 - ▶ $\mathcal{I} \models \exists x \phi$ iff there is $d \in A$ s.t. $\mathcal{I}_{[x/d]} \models \phi$

- ▶ Known result: ν can be assumed to be defined only for the free variables in the formula.
- ▶ Terminology: \mathcal{I} satisfies ϕ , \mathcal{I} makes ϕ true, \mathcal{I} is a model for/of ϕ
- ▶ We also write $\mathfrak{A} \models \phi(\vec{x}/\nu)$

Examples

- ▶ Consider loopless, symmetric graphs $\mathfrak{G} = (G, E^{\mathfrak{G}})$

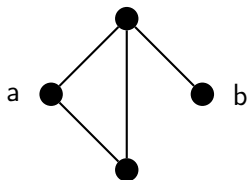


- ▶ $\phi_1 := \exists x \exists y E(x, y)$

$\mathfrak{G} \models \phi_1?$

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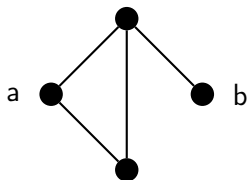


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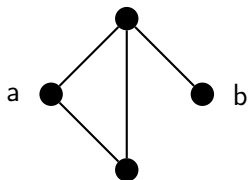
- ▶ Consider loopless, symmetric graphs $\mathfrak{G} = (G, E^{\mathfrak{G}})$



- ▶ $\phi_1 := \exists x \exists y E(x, y)$ $\mathfrak{G} \models \phi_1$ Yes!
- ▶ $\phi_2(x) := \exists y \exists z E(x, y) \wedge E(x, z) \wedge E(y, z)$ $\mathfrak{G} \models \phi_2(x/a)?$

Examples

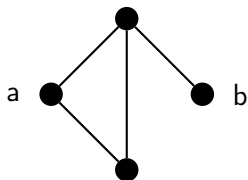
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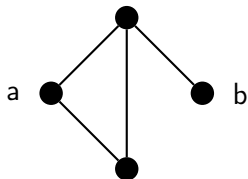
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- ▶ $\phi_3(x, y) := E(x, y)$ $\mathfrak{G} \models \phi_3(x/a, y/b)$?

Examples

- ▶ Consider loopless, symmetric graphs $\mathfrak{G} = (G, E^{\mathfrak{G}})$



- ▶ $\phi_1 := \exists x \exists y E(x, y)$ $\mathfrak{G} \models \phi_1$ **Yes!**
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- ▶ $\phi_3(x, y) := E(x, y)$ $\mathfrak{G} \models \phi_3(x/a, y/b)$ **NO!**

Entailment

- ▶ $X \models \phi$ iff all models of X are models of ϕ
 - ▶ We say: X entails ϕ or ϕ follows from X
 - ▶ X : set of sentences
 - ▶ ϕ : sentence

- ▶ Note: entailment definition (per se) not easy implementable
 \implies : Notion of derivability/inference in a calculus (see later lectures)

Algorithmic Problems in First-Order Logic

▶ Model Checking:

- ▶ Input: graph (or generally structure) \mathcal{G} , formula $\phi(x_1, \dots, x_n)$ and assignment $[x_1/a_1, \dots, x_n/a_n]$
- ▶ Output: Is $\mathcal{G} \models \phi(x_1/a_1, \dots, x_n/a_n)$ the case?

▶ Satisfiability Problem

- ▶ Input: sentence ϕ
- ▶ Output: Does there exist a structure \mathcal{G} s.t. $\mathcal{G} \models \phi$?

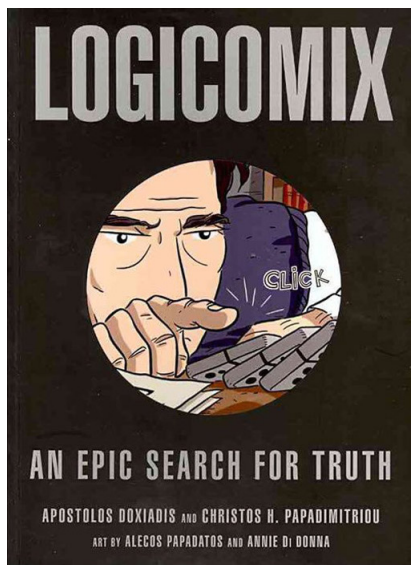
▶ Complexity of problems

- ▶ Model checking problem is decidable and PSPACE complete (in combined complexity)
- ▶ Satisfiability is undecidable

Role of Logic for/in Computer Science

The Burden of Logic in the 19-20th Century

- ▶ Role of logic as a foundation for all of mathematics
- ▶ Literature hint: Logicomix
 - ▶ fantastic graphic novel
 - ▶ Narrator: Philosopher and logician B. Russell
 - ▶ About the illusions, disillusion, and landmarking results at the end of the 19th century



Foundations of Mathematics with Mathematical Logic

- ▶ Attempts to find formal foundation for mathematical logic
- ▶ Hilberts Program (1900-1928)
 - ▶ Mathematics is consistent
 - ▶ Mathematics is (semantically) complete
 - ▶ Mathematics is decidable

Awakening

- ▶ Young Gödel proves (1931-33)
 - ▶ arithmetics not complete
 - ▶ consistency of set theory not provable

- ▶ Church/Turing (1936/37)
 - ▶ First-order logic is not decidable
 - ▶ Valid sentences not recursive
 - ▶ Sentences true in arithmetic not recursively enumerable (semi-decidable)

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- ▶ Nonetheless there are the following positive insights
 - ▶ Syntactically completeness for FOL (Gödel, 1930)
 - ▶ ZFC (Zermelo-Fraenkel Set Theory) can be used to formalize all contemporary mathematics

The Unusual Effectiveness of Logic

- ▶ Logic (Research) and Computer Science had fruitful effects onto each other
- ▶ Logic even more w.r.t. CS (than w.r.t. mathematics)
- ▶ “Logic is the calculus of CS”

Lit: M. Y. Vardi. From philosophical to industrial logics. In Proceedings of the 3rd Indian Conference on Logic and Its Applications, ICLA'09, pages 89–115, Berlin, Heidelberg, 2009. Springer-Verlag.

Lit: J. Y. Halpern, R. Harper, N. Immerman, P. G. Kolaitis, M. Y. Vardi, and V. Vianu. On the unusual effectiveness of logic in computer science. Bull. Symbolic Logic, 7(2):213–236, 2001.

Why is this the Case?

- ▶ Logic is so general that it allows to
 - ▶ talk precisely about the objects within a computer/computation model
 - ▶ specify and reason about the properties of runs in the model
- ▶ Even more: One can characterize complexity classes with logics (Descriptive Complexity)

So ...

As an upcoming computer scientist (in academia or industry) you should train in formal models, in particular **logics**, because:

- ▶ you want to **apply successfully** for a job
- ▶ But more importantly: you want to **keep** your job

Computer Science Areas Effectuated by Logic Research

- ▶ Database Systems
- ▶ Ontology-Based Information Systems
- ▶ Semantic Integration
- ▶ Computer-Aided Verification (Model Checking)
- ▶ Computational Complexity
- ▶ High-Level Stream Processing
- ▶ Multi-Agent Systems
- ▶ Machine Learning (e.g. probabilistic graph models and logics)
- ▶ Semantic Web
- ▶ Logic Programming
- ▶ Knowledge Representation
- ▶ Semantics of Programms
- ▶ Digital Design ...

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This course

Other courses of module “Web and Data Science” (CS4513)

This semester: “Web-Mining-Agenten”

Effects of Computer Science to Logic Research

- ▶ Focus/Intensive research on finite structures
 - ▶ Objects of computation are finite (Finite Model Theory)
 - ▶ But: potentially infinite structures (such as infinite DBs or streams) are useful as well
- ▶ Need for extensions of FOL
 - ▶ Higher-order logics (quantification over sets/relations)
 - ▶ Recursion (Datalog)
- ▶ Feasibility of reasoning services \implies restrictions of FOL
 - ▶ Modal and temporal logics
 - ▶ Description Logics
- ▶ Connections of logic and automata models
 - ▶ Regular expressions, finite automata, sequential logics
 - ▶ Buechi automata
- ▶ Logic engineering
- ▶ Different forms of inference ...

Overview of Course With Examples

Example: Logic in DB Research (Lectures 3-4)

- ▶ Travel DB with direct connection flights
- ▶ Reachability query
- ▶ SQL allows for recursion (CONNECT key word)
- ▶ But is it really necessary?

Table Flight	
Start	End
Hamburg	Berlin
Hamburg	New York
New York	Berlin
...	...

Query Q_{reach} : List all cities reachable from Hamburg!

Intuitively without recursion:

$$\begin{aligned}Q_{reach}(x) &= Flight(Hamburg, x) \vee \\ &\quad \exists x_1 Flight(Hamburg, x_1) \wedge Flight(x_1, x) \vee \\ &\quad \exists x_1, x_2 Flight(Hamburg, x_2) \wedge Flight(x_2, x_1) \wedge Flight(x_1, x) \vee \\ &\quad \dots\end{aligned}$$

Example: Logic In DB Research

- ▶ **Finite Model Theory (FMT)** gives a proof for the impossibility to use FOL for recursive queries
- ▶ FMT models DBs as finite relational FOL structures

Example

- ▶ Flight table becomes structure
 $\mathfrak{A} = (D, \text{Flight}^{\mathfrak{A}}, \text{Hamburg}^{\mathfrak{A}}, \text{Berlin}^{\mathfrak{A}}, \dots)$
- ▶ Domain D : all constants in DB
- ▶ Constants named by themselves, e.g., $\text{Hamburg}^{\mathfrak{A}} = \text{Hamburg}$
- ▶ $\text{Flight}^{\mathfrak{A}} = \{(\text{Hamburg}, \text{Berlin}), (\text{Hamburg}, \text{NewYork}), \dots\}$

Example: Logic In DB Research

- ▶ Investigate all relevant reasoning problems w.r.t. finite models
 - ▶ Many properties for classical FOL do not hold
 - ▶ Also w.r.t. complexity
 - ⇒ Calls for new techniques
- ▶ In particular: Investigate properties that all FOL queries have.

Theorem

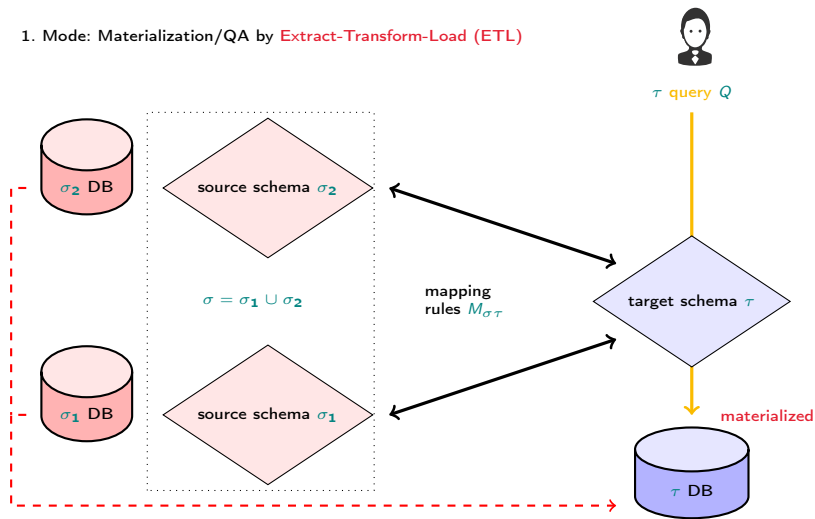
*All FOL formulas are **local**.*

(Holds even for FOL extended with aggregation)

- ▶ Recursive queries are not local!

Data Integration and Data Exchange (lectures 5-7)

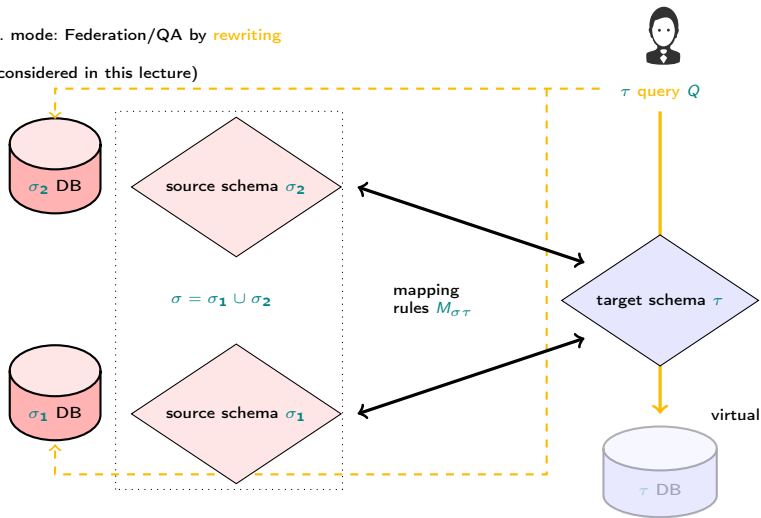
1. Mode: Materialization/QA by **Extract-Transform-Load (ETL)**



Data Integration and Data Exchange (lectures 5-7)

2. mode: Federation/QA by **rewriting**

(considered in this lecture)



Example: Querying via Ontologies (Lectures 8-9)

- ▶ Ontologies
 - ▶ Formal means to represent and reason over data
 - ▶ Specify constraints/completeness rules
 - ▶ May have many models (*open world assumption*)
 - ▶ May be used for access of heterogeneous data sources

- ▶ Appropriate ontology languages: **Description Logics** (OWL and variants)
 - ▶ Constants, concepts (unary predicates), roles (binary predicates)
 - ▶ Terminological axioms, e.g., *Students* \sqsubseteq *Humans*
 - ▶ Assertions axioms, e.g., *Student(Frege)*
 - ▶ Description logics are **feasible fragments** of FOL

Example

- ▶ No university known for *Goedel*
- ▶ Completeness:
 $Student \sqsubseteq \exists hasUniv.University$
- ▶ Functionality constraint:
(*func hasUniv*)

<i>Student</i>	<i>Univ</i>
<i>Frege</i>	<i>U – Jena</i>
<i>Russell</i>	<i>U – London</i>
<i>Goedel</i>	<i>NULL</i>
...	...

Example: Ontology Integration (Lectures 10-11)

- ▶ There exist many ontologies out there
- ▶ For some applications need to integrate ontologies
- ▶ Problem: Joining ontologies may lead to incoherences/inconsistencies

Example

Ontology A

- ▶ $Article \equiv \exists publ. Journal$
- ▶ $Journal \sqsubseteq \neg Proceedings$
- ▶ $(func\ publ)$

Ontology B

- ▶ $Article \equiv \exists publ. Journal$
 $\sqcup Proceedings$
- ▶ $publish(ab, procXY)$

- ▶ $O_A \cup O_B$ is inconsistent
- ▶ How to repair this?

Belief Revision

- ▶ Belief Revision deals with operators for revising theories under possible inconsistencies
- ▶ Investigates concrete revision operators
- ▶ Principles that these must fulfill (minimality etc.)
- ▶ Representation theorems

- ▶ Recent research how to adapt these for non-classical logics/ontologies

Database repairs (Lecture 12)

- ▶ Databases may become inconsistent (falsify some integrity constraint)
 - ▶ Lack of support for ICs
 - ▶ due to integration ...
- ▶ Database repair
 - ▶ repair (only) virtually
 - ▶ Allow query answering over inconsistent DB
 - ▶ Formalize repair notion (minimality of repair), complexity issues

Streams (Lecture 13)

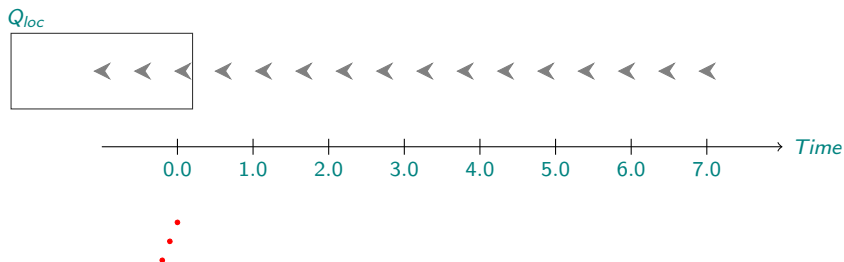
- ▶ “It’s a streaming world” (Ubiquity)
 - ▶ Many data are temporal (sensor, event data)
 - ▶ Big data is mostly temporal data
- ▶ “Streams are forever” (Potential Infinity)
 - ▶ Streams are potentially infinite
 - ▶ One has to tame the infinite
 - ▶ Streams call for *continuous querying (monitoring)*
- ▶ “Order Matters” (Sequentiality)
 - ▶ Stream elements have an arriving order next to temporal order
 - ▶ Re-ordering or special sequencing may be needed

Lit: E. Della Valle. et al. It’s a streaming world! Reasoning upon rapidly changing information. *Intelligent Systems*, IEEE, 24(6):83–89, nov.-dec. 2009.

Lit: J. Endrullis, D. Hendriks, and J. W. Klop. Streams are forever. *Bulletin of the EATCS*, 109:70–106, 2013.

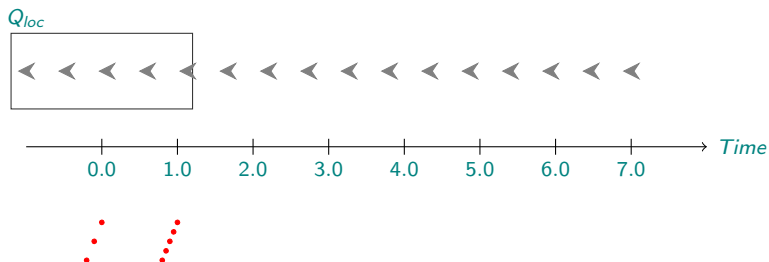
Lit: E. D. Valle et al. Order matters! Harnessing a world of orderings for reasoning over massive data. *Semantic Web*, 4(2):219–231, 2013.

Query With Sliding Window



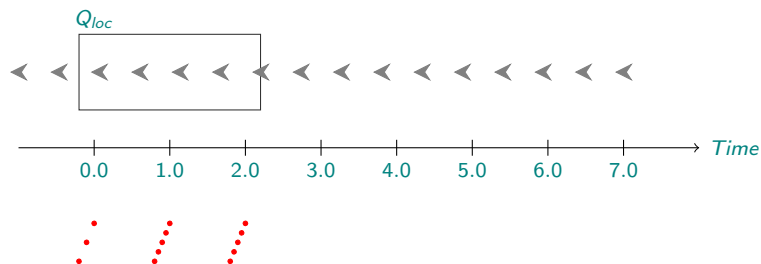
- ▶ The role of windows
- ▶ Stringology
- ▶ Bounded-memory computing
- ▶ High-level stream processing

Query With Sliding Window



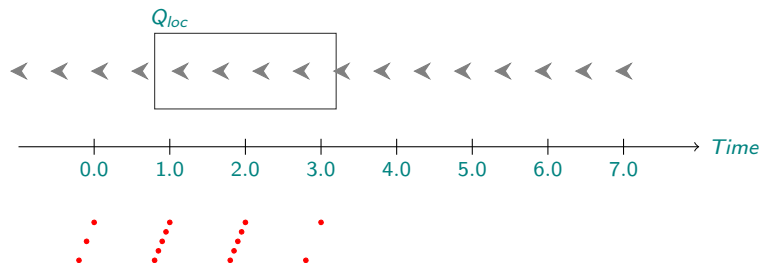
- ▶ The role of windows
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Query With Sliding Window



- ▶ The role of windows
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Query With Sliding Window



- ▶ The role of windows
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