UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR INFORMATIONSSYSTEME

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Ontology-Based Data Access

Lecture 8: Motivation, Description Logics
4 June 2020

Informationssysteme CS4130
(Summer 2020)

References

- ESSLLI 2010 Course by Calvanese and Zakharyaschev

  http://www.inf.unibz.it/~calvanese/teaching/2010-08-ESSLLI-DL-QA/

- Reasoning Web Summer School 2014 course by Kontchakov on Description Logics

  http://rw2014.di.uoa.gr/sites/default/files/slides/An_Introduction_to_Description_Logics.pdf

- Lecture notes by Calvanese in 2013/2014 course on Ontology and Database Systems

  https://www.inf.unibz.it/~calvanese/teaching/14-15-odbs/lecture-notes/

- Course notes by Franz Baader on Description Logics
- Parts of Reasoning Web Summer School 2014 course by Ö. on Ontology-Based Data Access on Temporal and Streaming Data

  http://rw2014.di.uoa.gr/sites/default/files/slides/Ontology_Based_Data_Access_on_Temporal_and_Streaming_Data.pdf

Ontology-Based Data Access as Information Integration

- Data Integration can be considered as information integration purely on DB level

- OBDA can be considered as information integration using an ontology
- Bridges DB world (closes world assumption) and ontology world (open world assumption)

Closed World Assumption

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  - All and only those facts mentioned in DB hold.

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- In DE incompleteness generated by different schemata
  Flight scenario: Source DB had no flight number, whilst target DB has
  $\Longrightarrow$ introduction of NULLs for flight number attribute

Closed World Assumption

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  - For one incomplete DB there are many completions
  - Nonetheless: Type information on attribute constrains the possible attribute instances
- In DE incompleteness generated by different schemata
  Flight scenario: Source DB had no flight number, whilst target DB has
  $\Longrightarrow$ introduction of NULLs for flight number attribute
- Logical theories (ontologies) adhere to open world assumption (OWA)
  - If something is not told, then we do not know
  - Logical theories (ontologies) may have many models

Close-World Assumption (CWA) for DBs

- "The world described by DBs is complete"

Example

University employee	
ID	Name
1	Sokrates
2	Platon
3	Aristotle

Professor
ID
1
2

"3" (= ID of Aristotle) not in table Professor
$\Longrightarrow$ Aristotle is not a professor

Close-World Assumption (CWA) for DBs

- "The world described by DBs is complete"

Example

Patient

ID	Name
1	Sokrates
2	Platon
3	Aristotle

Blood sugar

ID	value
1	90
2	120

"3" not in blood sugar table

$\Longrightarrow$? Aristotle has not blood sugar value?

NULLs

- NULLs intended to model incompleteness
- but semantics not clear and hence highly criticized

Lit: L. Libkin. SQL's three-valued logic and certain answers. ACM Trans. Database Syst., 41(1):1:1–1:28, 2016.

Example

Patient

ID	Name
1	Sokrates
2	Platon
3	Aristotle

Blood sugar

ID	value [30-600]
1	90
2	120
3	NULL

Aristotle has a blood sugar value (30 or 31 or . . .)

NULLs

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- but semantics not clear and hence highly criticized

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Example

Patient

ID	Name
1	Sokrates
2	Platon
3	Aristotle
4	Xanthippe
5	Leda

Pregnancy

ID	HCG value
1	NULL
2	NULL
3	NULL
4	NULL
5	130

- Male patient with NULL: no HCG test
- Female patient with NULL: not HCG test (but she has HCG value) or HCG test & not known

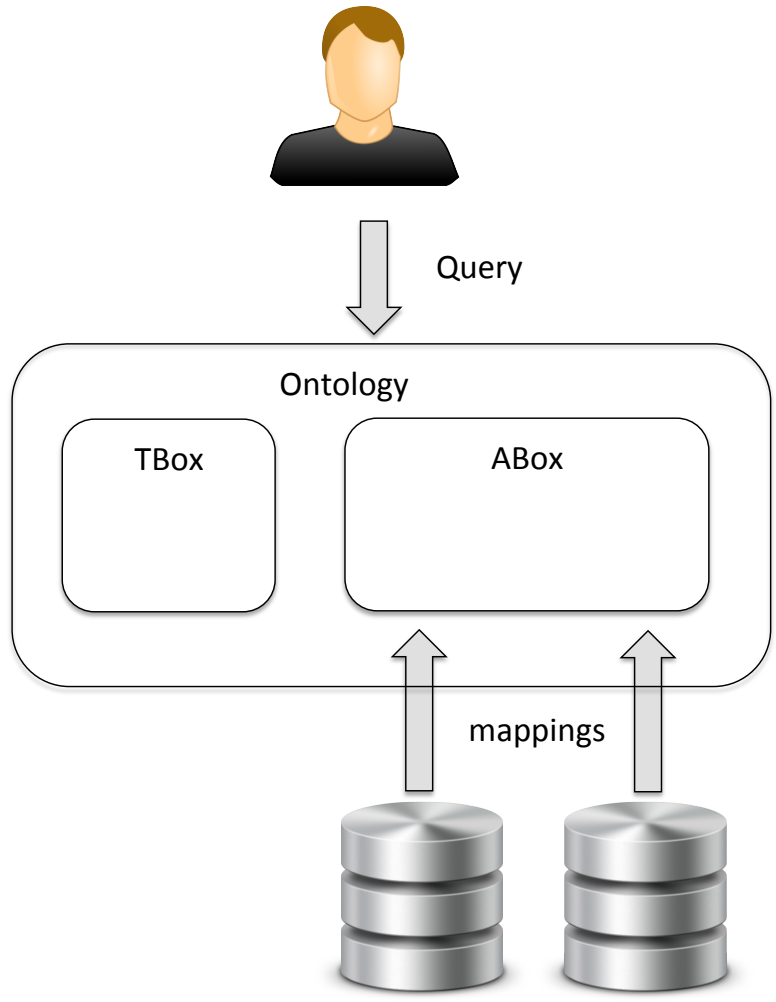
OBDA: Motivation and Overview

Ontology-Based Data Access

- Use ontologies as interface

  ...
- to access (here: query)
- data stored in some format

  ...
- using mappings



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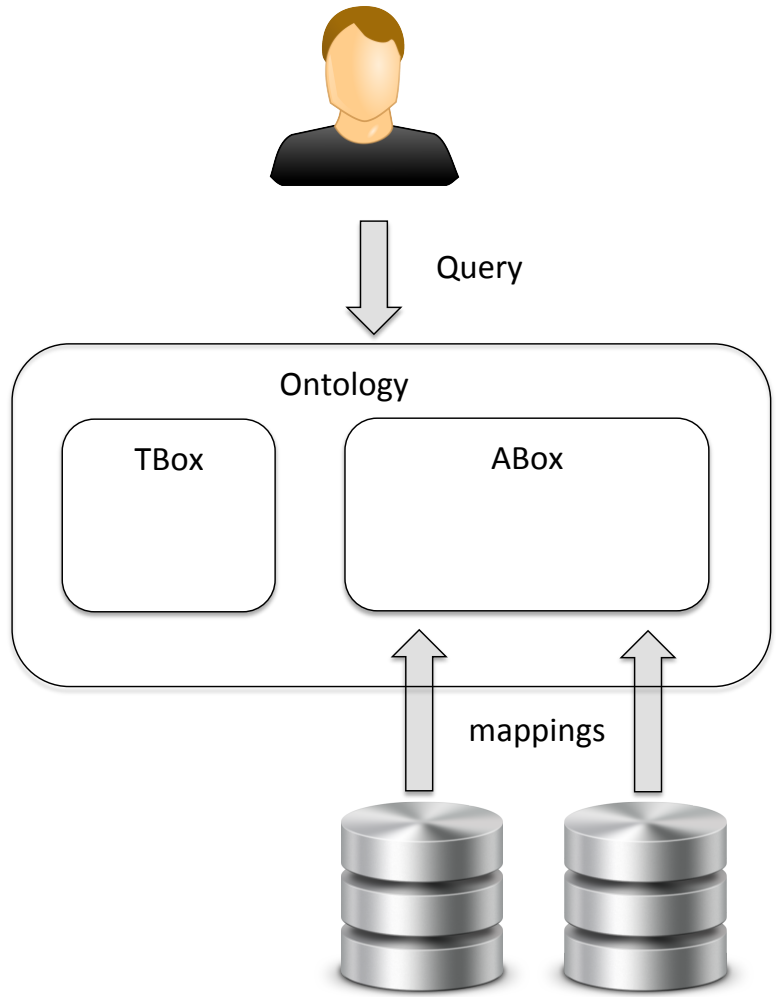

Ontologies

- Ontologies are triples of the form $\mathcal{O} = (\sigma, \mathcal{T}, \mathcal{A})$
  - Signature σ: Non-logical vocabulary $\sigma = N_I \cup N_C \cup N_R$
  - Tbox $\mathcal{T}$: set of σ-axioms in some logic to capture terminological knowledge
    This lecture: ontologies represented in Description Logics (DLs)
  - Abox $\mathcal{A}$: set of σ-axioms in (same logic) to capture assertional/contingential knowledge

- Note: Sometimes only tbox termed ontology

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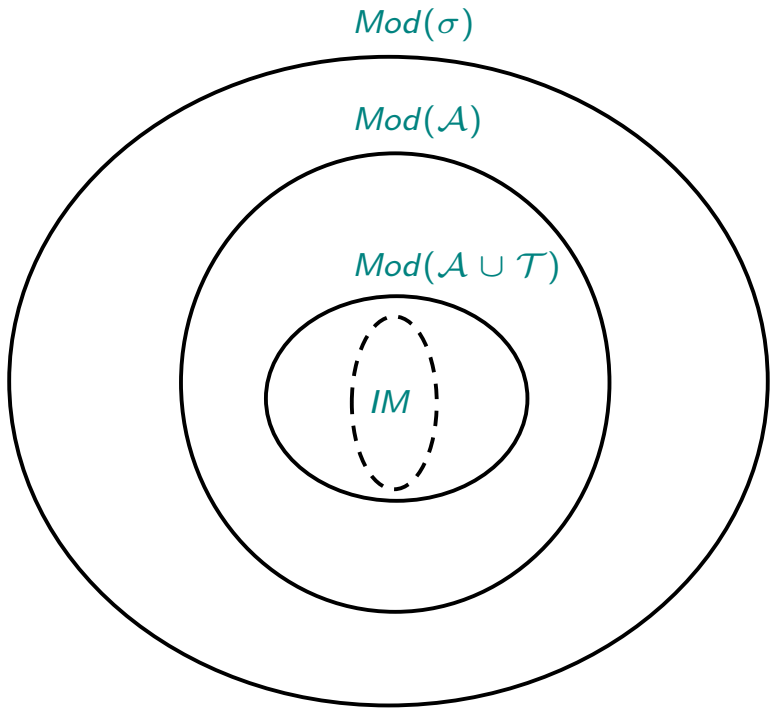
- Semantics defined on the basis of σ-interpretations $\mathcal{I}$
  - $\mathcal{I} \models Ax$ iff $\mathcal{I}$ makes all axioms in Ax true
  - $Mod(Ax) = \{\mathcal{I} \models Ax\}$

General Idea

- $\mathcal{A}$: Represents facts in domain of interest
- Open world assumption: $Mod(\mathcal{A})$ is not a singleton
- $\mathcal{T}$: Constrains $Mod(\mathcal{A})$ with intended σ readings
- Usually one has only approximations of intended models IM



General Idea

- $\mathcal{A}$: Represents facts in domain of interest
- Open world assumption: $Mod(\mathcal{A})$ is not a singleton
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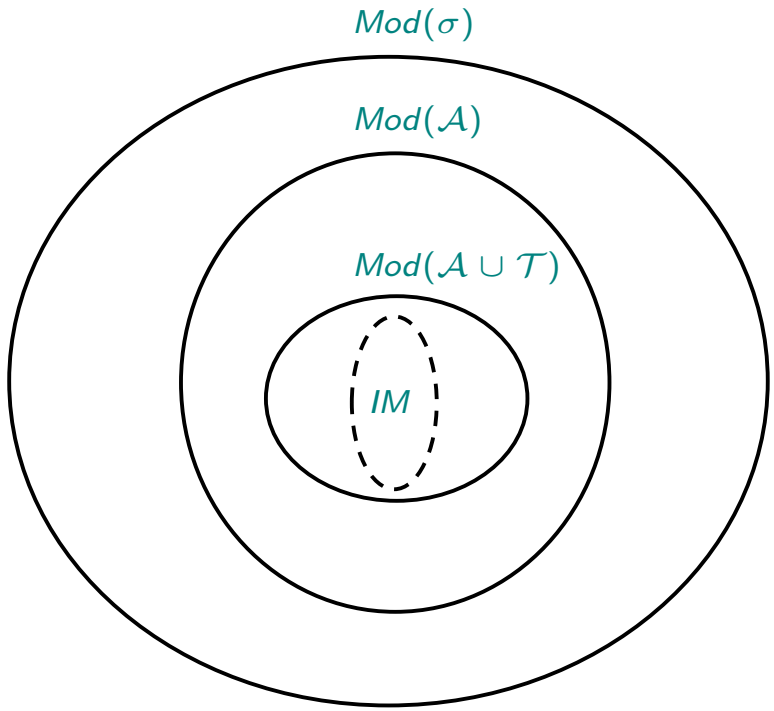
- Realize inference services on the basis of the constrained interpretations



WARNING: A Misconception

- With ontologies one does not declare data structures

- Abox data in most cases show pattern of data structures
- One does not have to re-model patterns/constraints in the abox data
  - Knowing "All A are B" in the abox is different from stipulating $A \sqsubseteq B$ (the former is known as integrity constraint)
  - Add $A \sqsubseteq B$, if you need to handle this relation for objects not mentioned in the abox

- Motto: Keep the tbox simple

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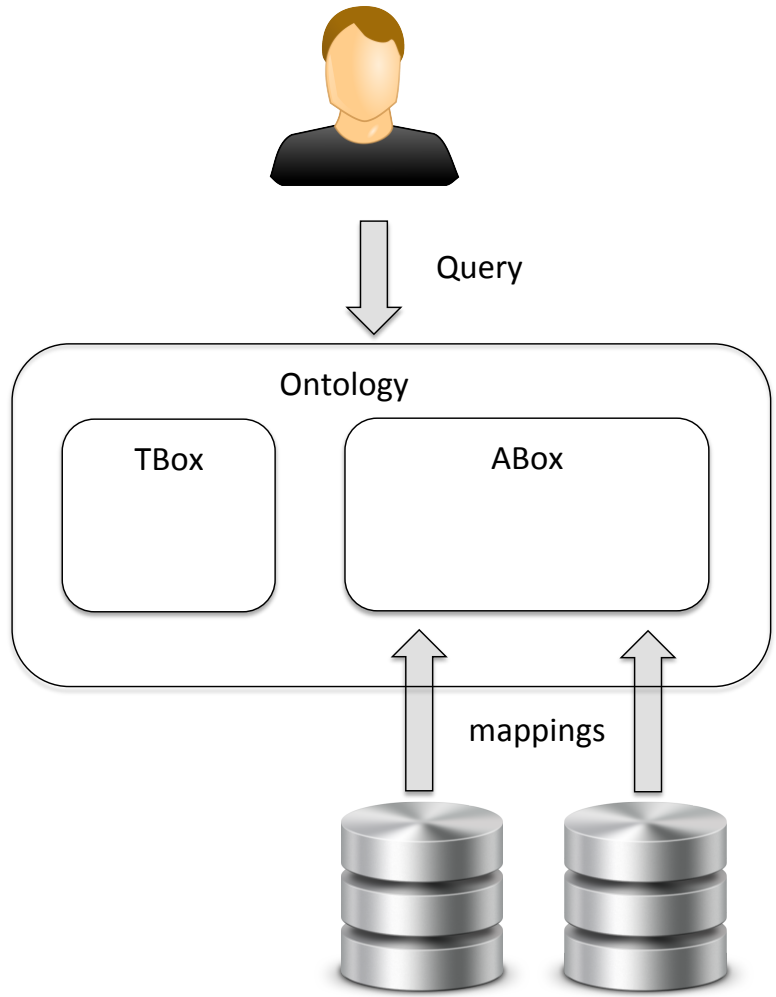

Reasoning Services

- Different standard and nonstandard reasoning services exist
- May be reducible to each other

Example (Reasoning Services)

consistency check, subsumption check, taxonomy calculations, most specific subsumer, most specific concept, matching, ...

- In classical OBDA focus on
  - Consistency checking: $Mod(\mathcal{A} \cup \mathcal{T}) \neq \emptyset$.
  - Query answering

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consistency check, subsumption check, taxonomy calculations, most specific subsumer, most specific concept, matching, ...

- In classical OBDA focus on
  - Consistency checking: $Mod(\mathcal{A} \cup \mathcal{T}) \neq \emptyset$.
  - Query answering
- Next to abox and tbox language query language QL over σ is a relevant factor for OBDA
- Certain query answering

$$cert(\psi(\vec{x}), \mathcal{T} \cup \mathcal{A}) \quad = \quad \{\vec{a} \in (N_i)^n \mid \mathcal{T} \cup \mathcal{A} \models \psi[\vec{x}/\vec{a}]\}$$

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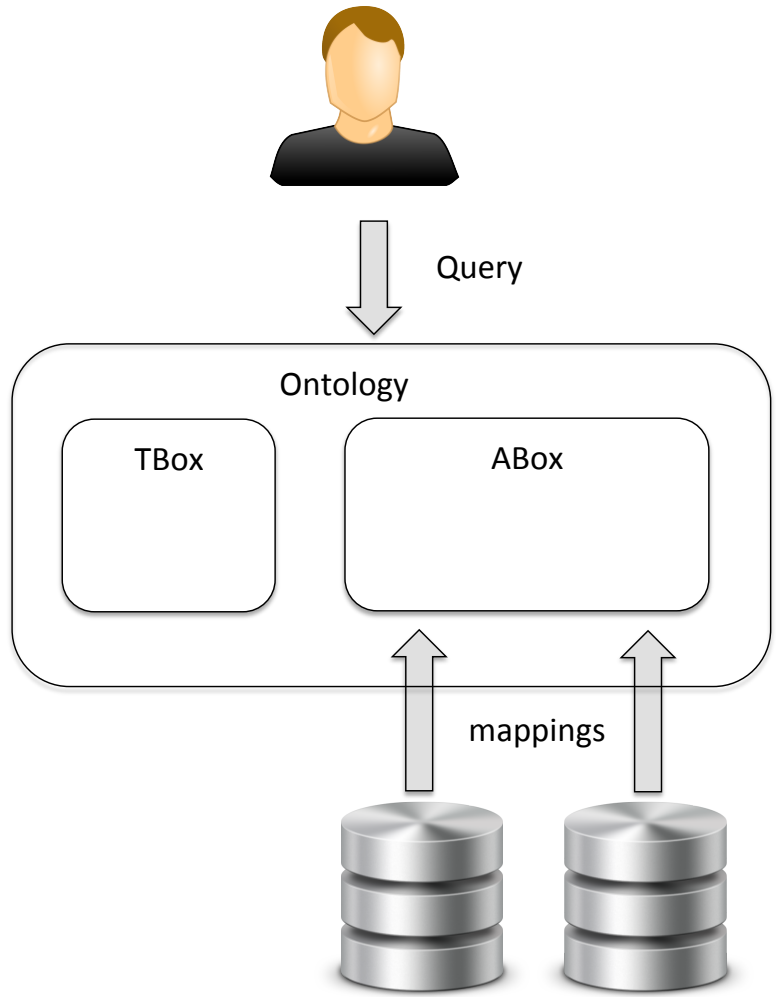

Backend Data Sources

- Classically: relational SQL DBs with static data
- Possible extensions: non-SQL DBs
  - datawarehouse repositories for statistical applications
  - pure logfiles
  - RDF repositories
- Non-static data
  - historical data (stored in temporal DB)
  - dynamic data coming in streams
- Originally intended for multiple DBs but ...

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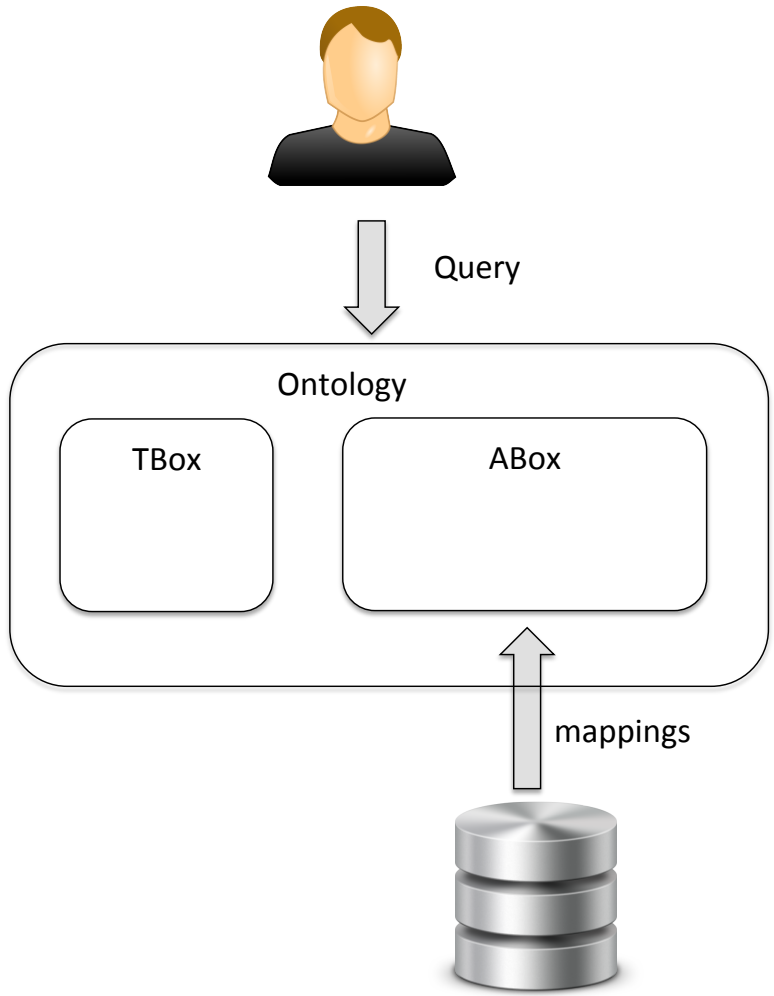

Mappings

- Mappings have an important crucial role in OBDA
- Lift data to the ontology level
  - Data level: (nearly) close world
  - Ontology Level: open world

Definition (Schema of Mappings)

$$m : \psi(\vec{f}(\vec{x})) \longleftarrow Q(\vec{x}, \vec{y})$$

- $\psi(\vec{f}(\vec{x}))$: Template (query) for generating abox axioms
- $Q(\vec{x}, \vec{y})$: Query over the backend sources
- Function $\vec{f}$ translates backend instantiations of $\vec{x}$ to constants

- Mappings M over backend sources generates abox $\mathcal{A}(M, DB)$.

Example Scenario: Measurements

- Example schema for measurement and event data in DB

```
SENSOR(SID, CID, Sname, TID, description)
SENSORTYPE(TID, Tname)
COMPONENT(CID, superCID, AID, Cname)
ASSEMBLY(AID, AName, ALocation)
MEASUREMENT(MID, MtimeStamp, SID, Mval)
MESSAGE(MesID, MesTimeStamp, MesAssemblyID,  catID, MesEventText)
CATEGORY(catID, catName)
```

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- For mapping

  m: $Sens(x) \wedge name(x, y) \longleftarrow$

```
SELECT f(SID) as x,   Sname as y FROM SENSOR
```

- the row data in SENSOR table

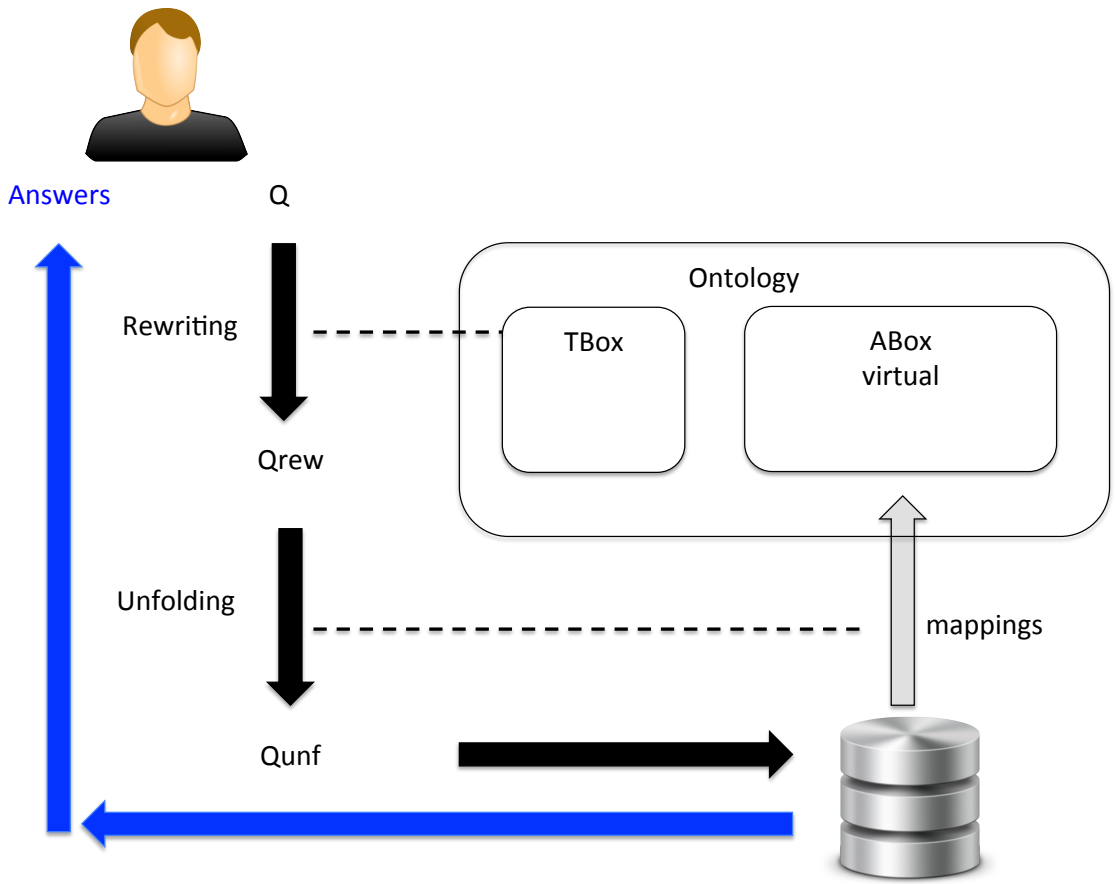
SENSOR
(123, comp45, TempSens, TC255, 'A temperature sensor')

- generates facts

  $Sens(f(123)), name(f(123), TempSens) \in \mathcal{A}(m, DB)$

OBDA in the Classical Sense

- Keep the data where they are because of large volume
- Abox is virtual (no materialization)
- First-order logic (FOL) perfect rewriting + unfolding



Ontologies and Description Logics

Description Logics

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Description logics (DLs) are logics for use in knowledge representation with special attention on a good balance of expressibility and feasibility of reasoning services

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  - as ontology representation language for conceptual modeling
  - in particular in the semantic web
  - Formal counterpart of standard web ontology language (OWL)
  - and in particular for ontology-based data access (OBDA)

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  - as ontology representation language for conceptual modeling
  - in particular in the semantic web
  - Formal counterpart of standard web ontology language (OWL)
  - and in particular for ontology-based data access (OBDA)
- Have been investigated for ca. 30 years now
  - Many theoretical insights on various different purpose DLs
  - General-purpose reasoners (RacerPro, Fact++, ...) and specific reasoners (Quest,...)
  - Various editing tools (most notably Protege)

Family of DLs

- Variable-free logics centered around concepts
- concepts = one-ary predicates in FOL = classes in OWL

Example (Concepts)

- $\mathit{Students}$ ("students")
- $\mathit{Students} \sqcap \mathit{Male}$ (" Male students")
- $\exists \mathit{attends}.\mathit{MathCourse}$ ("Those attending a math course")
- $\forall \mathit{hasFriends}.\mathit{Freaks}$ ("Those having only freaks as friends")
- $\mathit{Person} \sqcap \forall \mathit{attends}.(\mathit{Course} \sqcap \neg \mathit{easy})$ ("Persons attending only non-easy courses")

An (Semi-)Expressive Logic: $\mathcal{ALC}$

- Vocabulary: constants N_i, atomic concepts N_C, roles N_R

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$$C \quad ::= \quad A \quad (\text{for } A \in N_C) \mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \\ \forall r.C \mid \exists r.C \ (\text{for } r \in N_R) \mid \bot \mid \top$$

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- Concept(description)s: semantics

- Interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{\text{domain}}, \overbrace{\cdot^{\mathcal{I}}}^{\text{denotation function}})$
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_C$
- $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $c \in N_i$
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$

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- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $\neg C = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \text{for all } e \in \Delta^{\mathcal{I}}: \text{If } (d,e) \in r^{\mathcal{I}} \text{ then } e \in C^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \text{there is } e \in \Delta^{\mathcal{I}} \text{ s.t. } (d,e) \in r^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\}$

Tbox and Abox

- Terminological Box (tbox) $\mathcal{T}$
  - Finite set of general concept inclusions (GCIs)
  - GCI: axioms of form $C \sqsubseteq D$ (for arbitrary concept descriptions)
    $C \equiv D$ abbreviates $\{C \sqsubseteq D, D \sqsubseteq C\}$
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- Assertional Box (abox) $\mathcal{A}$
  - Finite set of assertions
  - Assertion: $C(a)$ (concept assertion), $r(a, b)$ (role assertion)
  - Semantics:
    $\mathcal{I} \models C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$
    $\mathcal{I} \models r(a, b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$.

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- Ontology: $(\sigma, \mathcal{T}, \mathcal{A})$ or just $\mathcal{T} \cup \mathcal{A}$

Example (University)

$$\begin{aligned}\mathcal{T} &= \{\ \mathit{GradStudent} \sqsubseteq \mathit{Student},\\ &\quad\ \mathit{GradStudent} \sqsubseteq \exists \mathit{takesCourse}.\mathit{GradCourse}\ \}\\ \mathcal{A} &= \{\ \mathit{GradStudent}(\mathit{john})\ \}\end{aligned}$$

Consider the following interpretations

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Consider the following interpretations

- $\mathcal{I}_1$:
  - $\mathit{john}^{\mathcal{I}_1} = j$
  - $\mathit{GradStudent}^{\mathcal{I}_1} = \{j\}$
  - $\mathit{Student}^{\mathcal{I}_1} = \{j\}$
  - $\mathit{GradCourse}^{\mathcal{I}_1} = \{s\}$
  - $\mathit{takesCourse}^{\mathcal{I}_1} = \{(j, s)\}$
- $\mathcal{I}_1 \models \mathcal{T} \cup \mathcal{A}$

- $\mathcal{I}_2$:
  - $\mathit{john}^{\mathcal{I}_2} = j$
  - $\mathit{GradStudent}^{\mathcal{I}_2} = \{j\}$
  - $\mathit{Student}^{\mathcal{I}_2} = \{j\}$
  - $\mathit{GradCourse}^{\mathcal{I}_2} = \{j\}$
  - $\mathit{takesCourse}^{\mathcal{I}_2} = \{(j, j)\}$
- $\mathcal{I}_2 \models \mathcal{T} \cup \mathcal{A}$

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Consider the following interpretations

- $\mathcal{I}_3$:
  - $\mathit{john}^{\mathcal{I}_1} = j$
  - $\mathit{GradStudent}^{\mathcal{I}_1} = \{j\}$
  - $\mathit{Student}^{\mathcal{I}_1} = \{j\}$
  - $\mathit{GradCourse}^{\mathcal{I}_1} = \emptyset$
  - $\mathit{takesCourse}^{\mathcal{I}_1} = \emptyset$
- $\mathcal{I}_3 \not\models \mathcal{T} \cup \mathcal{A}$

Stricter notion of Tbox

- Above definition of tbox very general
  - "Meanings" of concept names determined only implicitly in the whole ontology
  - No guarantee for unique extensions

- Early notion of tbox more related to idea of explicitly defining concept names
- $C \equiv D$ used as abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$
- Concept definition: $A \equiv D$ (where A atomic)

Definition

A TBox in a strict sense is a finite set of concept definitions not defining a concept multiple times or in a cyclic manner. Defined concepts occur on the lhs, primitive concept on the rhs of definitions.

Digression: Implicit vs. Explicit Definability

- Sometimes a general tbox may fix the denotation of a concept name w.r.t. denotations of the others $\Longrightarrow$ implicit definability
- Maybe then it can also be defined explicitly?

Definition

Given an FOL theory Ψ over signature σ and a predicate symbol R.

- R is implicitly defined in Ψ iff
  for any two models $\mathfrak{A} \models \Psi$ and $\mathfrak{B} \models \Psi$ agreeing on $\sigma \setminus \{R\}$
  one has $R^{\mathfrak{A}} = R^{\mathfrak{B}}$.

- R is explicitly defined in Ψ by a formula $\phi(\vec{x})$ not containing R
  iff $\Psi \models \forall \vec{x} R(\vec{x}) \leftrightarrow \phi(\vec{x})$

Digression: Beth Definability Theorem

- For FOL both implicit and explicit definability coincide

Theorem

An FOL theory defines a predicate implicitly iff it defines it explicitly

- Though DLs can be embedded into FOL, the equivalence of implicit and explicit definability does not transfer necessarily to DLs
- At least it does for $\mathcal{ALC}$ theories

  Lit: B. ten Cate, E. Franconi, and I. Seylan. Beth definability in expressive description logics. J. Artif. Int. Res., 48(1): 347–414, Oct. 2013.

Reasoning services

- Semantical notions as in FOL but additional notions due to focus on concepts
- Let $\mathcal{O} = (\sigma, \mathcal{T}, \mathcal{A})$

Definition (Basic Semantical Notions)

- Model: $\mathcal{I} \models \mathcal{O}$ iff $\mathcal{I} \models \mathcal{T} \cup \mathcal{A}$
- Satisfiability: $\mathcal{O}$ is satisfiable iff $\mathcal{T} \cup \mathcal{A}$ is satisfiable
- Coherence: $\mathcal{O}$ is coherent iff $\mathcal{T} \cup \mathcal{A}$ has a model $\mathcal{I}$ s.t. for all concept names $A^{\mathcal{I}} \neq \emptyset$
- Concept satisfiability: C is satisfiable w.r.t. $\mathcal{O}$ iff there is $\mathcal{I} \models \mathcal{O}$ s.t. $C^{\mathcal{I}} \neq \emptyset$
- Subsumption: C is subsumed by D w.r.t. $\mathcal{O}$ iff $\mathcal{O} \models C \sqsubseteq D$ iff $\mathcal{T} \cup \mathcal{A} \models C \sqsubseteq D$
- Instance check: a is an instance of C w.r.t. $\mathcal{O}$ iff $\mathcal{O} \models C(a)$

Reduction Examples

- Many of the semantical notions are reducible to each other
- We give only one example

Exercise

Show that subsumption can be reduced to satisfiability tests (allowing the introduction of new constants). More concretely:

$C \sqsubseteq D$ w.r.t. $\mathcal{O}$ iff $(\sigma \cup \{b\}, \mathcal{T}, \mathcal{A} \cup \{C(b), \neg D(b)\})$ is not satisfiable (where b is a fresh constant).

Extended Reasoning Services

Definition

- Instance retrieval: Find all constants x s.t. $\mathcal{O} \models C(x)$
- Query answering: Certain answers
  $cert(\phi(x), \mathcal{O}) = \{\vec{a} \in N_i \mid \mathcal{O} \models \phi[\vec{x}/\vec{a}]\}$
- Classification: Compute the subsumption hierarchy of all concept names
- Realization: Compute the most specific concept name to which a given constant belongs
- Pinpointing, matching, ...

Example (Certain Answers for Conjunctive Queries)

$$
\begin{aligned}
\mathcal{T} &= \{\, \top \sqsubseteq \mathit{Male} \sqcup \mathit{Female},\ \mathit{Male} \sqcap \mathit{Female} \sqsubseteq \bot \,\} \\
\mathcal{A} &= \{\, \mathit{friend}(\mathit{john}, \mathit{susan}),\ \mathit{friend}(\mathit{john}, \mathit{andrea}),\ \mathit{female}(\mathit{susan}), \\
&\qquad \mathit{likes}(\mathit{susan}, \mathit{andrea}),\ \mathit{likes}(\mathit{andrea}, \mathit{bill}),\ \mathit{Male}(\mathit{bill}) \,\}
\end{aligned}
$$

$$
Q(x) = \exists y, z(\mathit{friend}(x, y) \wedge \mathit{Female}(y) \wedge \mathit{likes}(y, z) \wedge \mathit{Male}(z))
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- We have to consider all possible models of the ontology
- But here there are actually two classes:
  Andrea is male vs. Andrea is not male.

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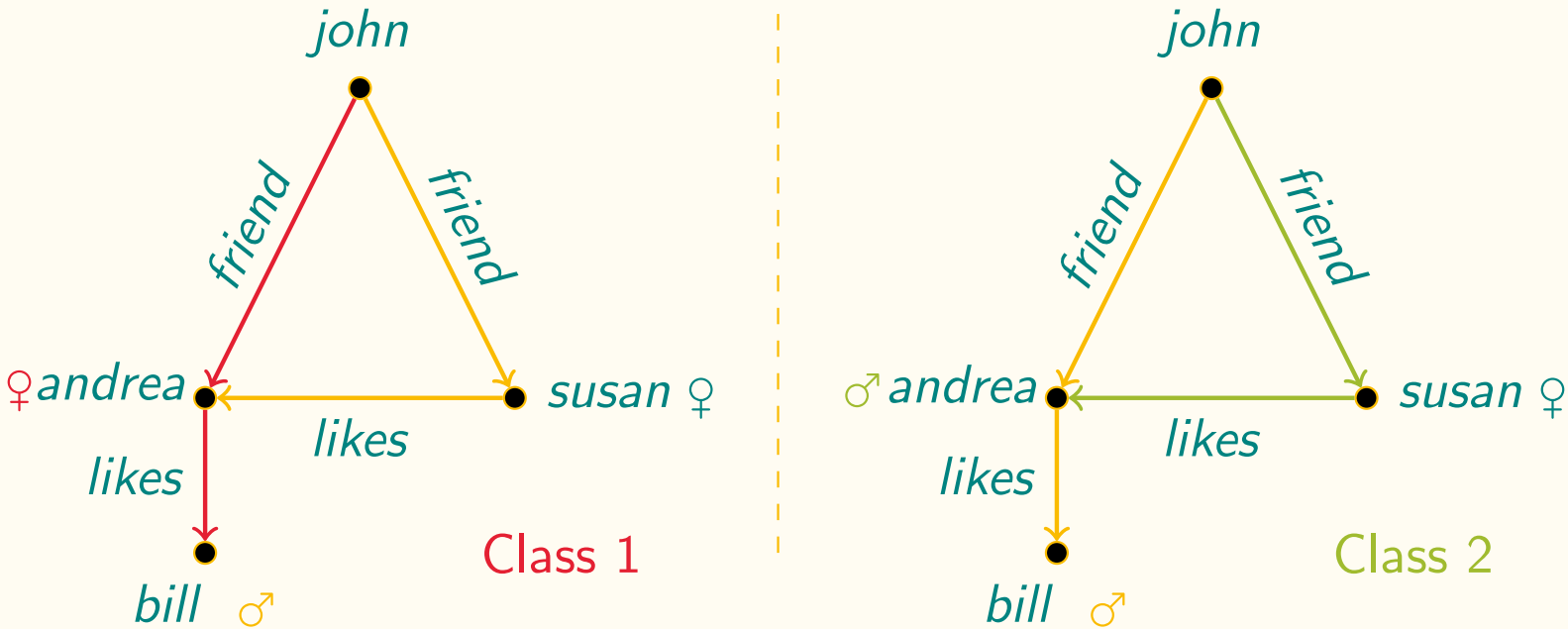

$$\mathit{cert}(Q(x), \mathcal{O}) = \{\mathit{john}\}$$

Embedding into FOL

- Most DLs (such as $\mathcal{ALC}$) can be embedded into FOL
- Notion of embedding is well-defined as FOL structures are used for semantics of DLs.

- Correspondence idea
  Concept names = unary predicates, roles = binary predicates,
  GCI = $\forall$ rules

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- Define for any concept description and variable x its corresponding x-open formula $\tau_x(C)$
  - $\tau_x(A) = A(x)$
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- Tbox axioms: $C \sqsubseteq D$ becomes $\forall x(\tau_x(C) \rightarrow \tau_x(D))$

Embedding into FOL

- For translation two variables are sufficient ("2 finger movement")
- Hence: DLs embeddable into known 2-variable fragment of FOL
- Also the fragment is a guarded fragment: one quantifies over variables fixed within atom.

DL Family

- Different DLs for different purposes
  - What is more important: Expressivity or feasibility?
  - Which kinds of reasoning services does one have to provide?

- Differences regarding
  - the allowed set of concept constructors
  - the allowed set of role constructors
  - the allowed types of tbox axioms
  - the allowed types of abox axioms
  - the allowance of concrete domains and attributes (such as *hasAge* with range the domain of integers)

Family of DLs and their Namings

- $\mathcal{AL}$: attributive language
- $\mathcal{C}$: (full) complement/negation
- $\mathcal{I}$: inverse roles $((r^{-1})^{\mathcal{I}} = \{(d, e) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid (e, d) \in r^{\mathcal{I}}\})$
- $\mathcal{H}$: role inclusions $(\mathit{hasFather} \sqsubseteq \mathit{hasParent})$

- $\mathcal{S}$: $\mathcal{ALC}$ + transitive roles $(\mathit{trans}\ \mathit{isReachable})$
- $\mathcal{N}$: unqualified number restrictions
  $((\geq n\ r)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \#(\{e \mid (d, e) \in r^{\mathcal{I}}\}) \geq n))$
- $\mathcal{O}$: nominals $\{b\}^{\mathcal{I}} = \{b^{\mathcal{I}}\}$
- $\mathcal{Q}$: qualified number restrictions
  $((\geq n\ r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \#(\{e \mid (d, e) \in r^{\mathcal{I}}\}$ and $e \in C^{\mathcal{I}}) \geq n))$
- $\mathcal{F}$: functionality constraints $\mathcal{I} \models (\mathit{func}\ R)$ iff $R^{\mathcal{I}}$ is a function
- $\mathcal{R}$: role chains and $\exists R.\mathit{Self}$ $(\mathit{hasFather} \circ \mathit{hasMother} \sqsubseteq \mathit{hasgrandMa})$
  $(\mathit{narcist} \equiv \exists \mathit{likes}.\mathit{Self})$

- OWL 2 is $\mathcal{SROIQ}$

Lightweight DLs

- Lightweight DLs favor feasibility over expressibility by, roughly, disallowing disjunction
- In principle three lightweight logics that have corresponding OWL 2 profiles (https://www.w3.org/TR/owl2-profiles/)

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- RL (OWL 2 RL)
  - Tbox restriction: "Only concept names on the rhs"
  - Polynomial time algorithms leveraging rule-extended database technologies operating directly on RDF triples

Comparison

	RL	EL	QL
inverse roles	+	-	+
rhs qual. exist	-	+	+
lhs qual. exist.	+	+	-

Complexity

- A nearly complete picture of reasoning services for DLs
- Research in DL community as of now resembles complexity farming
- DL complexity navigator:
  http://www.cs.man.ac.uk/~ezolin/dl (Last update 2013)

Tableaux Calculus for $\mathcal{ALC}$

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- Refutation calculus based on disjunctive normal form
- We demonstrate it here at an example for $\mathcal{ALC}$ tboxes
- For a full description and proofs see handbook article by Baader

  Lit: F. Baader and W. Nutt. Basic description logics. In F. Baader et al., editors, The Description Logic Handbook, pages 43–95. Cambridge University Press, 2003.

Tableaux Example

- $\mathcal{ALC}$ tableau gives tests for satisfiability of abox
- by checking whether obvious contradictions (clashes with complementary literals) are contained
- An abox that is complete (no rules applicable anymore) and open (no clashes) describes a model
- Algorithm applies tableau rules to extend abox

Rules

- Starts with an abox $\mathcal{A}_0$ which is in negation normal form (NNF): all $\neg$ appear only in front of concept names)

- Apply rules to construct new aboxes; indeterminism due to $\sqcup$ rule

Rule	Condition	$\rightsquigarrow$	Effect
$\rightsquigarrow_{\sqcap}$	$(C \sqcap D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x), D(x)\}$
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$\rightsquigarrow_{\exists}$	$(\exists r.C)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{r(x, y), C(y)\}$ for fresh y
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- Within each abox a tree-like structure is established (tree-model property)

Example

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- Expansion step assumes a tbox in a strict sense, but GCIs can be transformed to definitions (i.e., axioms of the form $A \equiv C$) using additional symbols

Example (A Tableau Derivation)

- $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
- Abbreviation: $\{\exists r.A \sqcap \exists r.B \sqcap \forall r.(\neg A \sqcup \neg B)(a)\}$

$$\mathcal{A}_0 = \exists r.A \sqcap \exists r.B \sqcap \forall r.(\neg A \sqcup \neg B)(a)$$

$\rightsquigarrow_{\sqcap}$ (2 times)

$$\mathcal{A}_1 = \mathcal{A}_0 \cup \{(\exists r.A)(a), (\exists r.B)(a), (\forall r.(\neg A \sqcup \neg B))(a)\}$$

$\rightsquigarrow_{\exists}$ (2 times)

$$\mathcal{A}_2 = \mathcal{A}_1 \cup \{r(a,b), A(b), r(a,c), B(c)\}$$

$\rightsquigarrow_{\forall}$ (2 times)

$$\mathcal{A}_3 = \mathcal{A}_2 \cup \{(\neg A \sqcup \neg B)(b), (\neg A \sqcup \neg B)(c)\}$$

$\rightsquigarrow_{\sqcup}$ $\rightsquigarrow_{\sqcup}$

$\mathcal{A}_{4.1} = \mathcal{A}_3 \cup \{\neg A(b)\}$ $\mathcal{A}_{4.2} = \mathcal{A}_3 \cup \{\neg B(b)\}$

$\rightsquigarrow_{\sqcup}$ $\rightsquigarrow_{\sqcup}$ $\rightsquigarrow_{\sqcup}$ $\rightsquigarrow_{\sqcup}$

$\mathcal{A}_{5.11} = \mathcal{A}_{4.1} \cup \{\neg A(c)\}$ clash

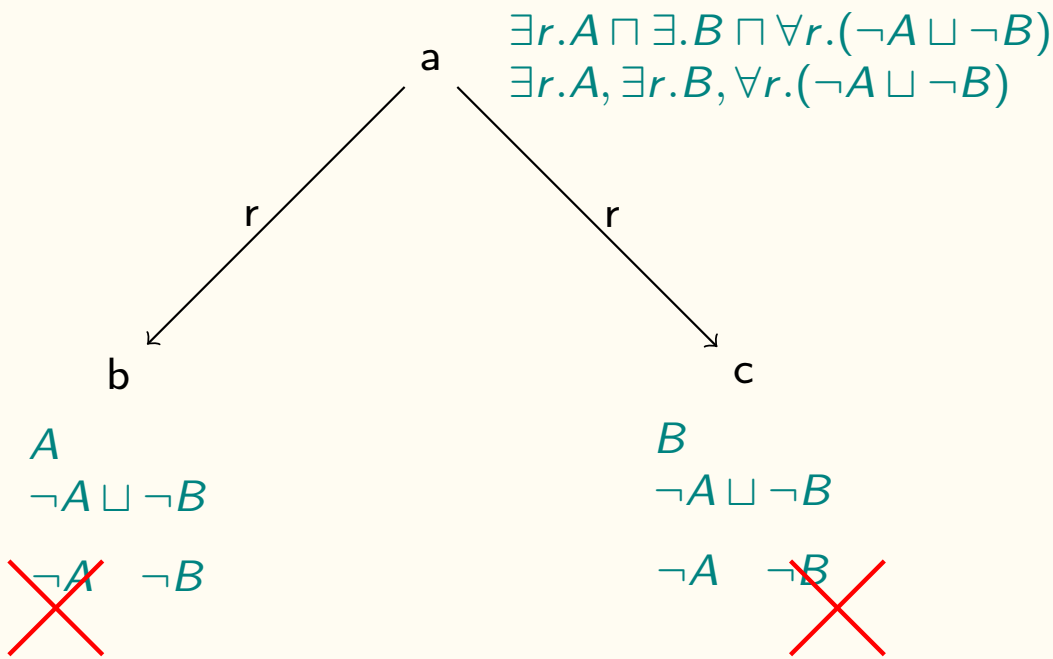
$\mathcal{A}_{5.12} = \mathcal{A}_{4.1} \cup \{\neg B(c)\}$ clash

$\mathcal{A}_{5.21} = \mathcal{A}_{4.2} \cup \{\neg A(c)\}$

$\mathcal{A}_{5.22} = \mathcal{A}_{4.2} \cup \{\neg B(c)\}$ clash

Example (The partial tree model in the aboxes)

- $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
- Abbreviation: $\{\exists r.A \sqcap \exists r.B \sqcap \forall r.(\neg A \sqcup \neg B)(a)\}$



- Canonical tree model(s) can be directly read off (here from abox $\mathcal{A}_{5.21}$ which contains no clash):
  $\mathcal{I} = (\{a, b, c\}, \cdot^{\mathcal{I}})$ with
  $r^{\mathcal{I}} = \{(a, b), (a, c)\}$ $A^{\mathcal{I}} = \{b\}$ $B^{\mathcal{I}} = \{c\}$

Tableaux Calculus

- The tableau calculus for $\mathcal{ALC}$ is complete, correct, and terminates.
- Hence, the following properties hold

Theorem

- $\mathcal{ALC}$ *abox satisfiability (concept satisfiability, subsumption...) is decidable*
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