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Ontology Change I

Lecture 10: AGM Belief Revision
18 June, 2020

Informationssysteme CS4130
(Summer 2020)

References

- Eduardo Ferme: Belief Revision from 1985 to 2013
  Slides of IJCAI 2013-Tutorial
  http://www.ijcai13.org/files/tutorial_slides/ta4.pdf
- **Lit:** P. Gärdenfors. Knowledge in Flux: Modeling the Dynamics of Epistemic States. The MIT Press, Bradford Books, Cambridge, MA, 1988.
- **Lit:** S. O. Hansson. A Textbook of Belief Dynamics. Kluwer Academic Publishers, 1999.
- New book (available as PDF in our library)
  Lit: E. Ferme, S. O. Hansson: Belief Change: Introduction and Overview, SpringerBriefs in Intelligent Systems

Motivation

Ontology-Level Integration

- So far: Two (different) types of integration
  - Data exchange: directed schema-level integration over finite DBs
  - OBDA: directed schema-level-to-ontology integration

- We consider now: ontology-level integration
- Required in different ontology change scenarios where multiple ontologies exist, such as ontology ...
  - import
  - merge
  - versioning
  - development
  - alignment etc.

  Lit: G. Flouris et al. Ontology change: classification and survey. The Knowledge Engineering Review, 23(2):117–152, 2008.

- Main problem: Ensuring coherence/consistency along change

Example (Incompatible ontologies)

$\mathcal{O}_A$

A1 $Article \equiv \exists publ.Journal$

A2 $Journal \sqsubseteq \neg Proceedings$

A3 $(func\ publ)$

$\mathcal{O}_B$

B1 $Article \equiv \exists publ.Journal \sqcup Proceedings$

B2 $publish(ab, procXY)$

B3 $Proceedings(procXY)$

- $\mathcal{O}_A \cup \mathcal{O}_B$ is inconsistent
- How to repair this?
  - Find all sets of culprits (Here one set: $\mathcal{O}_A \cup \mathcal{O}_B$)
  - If a culprit set has more than one sentence, then problem to decide which to eliminate? (Here: Eliminate A1 or ... or B3?)

$\Longrightarrow$ Research field Ontology Change (OC)

  - This lecture: Research field Belief Revision (BR)
  - Next lecture: Extensions of BR w.r.t. OC and OC in detail

Belief Revision (BR)

- About 35 years aged interdisciplinary research field in philosophy, cognitive science, CS
- Landmark paper by AGM (Alchourrón, Gärdenfors, Makinson)
  Lit: C.E. Alchourrón, P. Gärdenfors, and D. On the logic of theory change: partial meet contraction and revision functions. Journal of Symbolic Logic, 50:510–530, 1985.
- BR deals with operators for revising theories under possible inconsistencies
  - Investigates concrete revision operators
  - Principles that these must fulfill
  - Representation theorems
- Recent research how to adapt these for non-classical logics/ontologies, mappings, programs.

Terminology

- Unfortunately the field of Belief Revision is called after the particular class of revision operators
- But it handles other types of changing beliefs/theories: expansion, update, and contraction
- We stick to this folklore use and hide the name of the field behind the acronym BR

AGM Postulates

Consequence Operator

- AGM framework based on general notion of logic in polish tradition

  Lit: R. Wójcicki. Theory of Logical Calculi. Kluwer Academic Publishers, Dordrecht, 1988.

Definition (Logic in Polish Tradition)

A logic is a pair $(\mathcal{L}, Cn)$ where

- $\mathcal{L}$: Set of well-formed sentences
- Cn: Consequence operator $Pow(\mathcal{L}) \longrightarrow Pow(\mathcal{L})$

  Note: No distinction between syntax and semantics

AGM Consequence Operator

Definition (Tarskian consequence operator)

For all $X, X_1, X_2 \subseteq \mathcal{L}$:

1. $X \subseteq Cn(X)$ (Inclusion)
2. If $X_1 \subseteq X_2$, then $Cn(X_1) \subseteq Cn(X_2)$. (Monotonicity)
3. $Cn(X) = Cn(Cn(X))$ (Idempotence)

- AGM additionally has further requirements on Cn
- These lead to their well-known representation results

AGM-Requirements for Consequence Operators

- Language expressivity: Language $\mathcal{L}$ should contain all propositional connectors
- Supra-classicality: If α can be derived from X by propositional logic then $\alpha \in Cn(X)$
- Compactness: If $\alpha \in Cn(X)$ then $\alpha \in Cn(X')$ for some finite $X' \subseteq X$.
- Deduction: $\beta \in Cn(X \cup \{\alpha\})$ iff $(\alpha \to \beta) \in Cn(X)$
- Disjunction in premisses: If $\gamma \in Cn(X \cup \{\alpha\}) \cap Cn(X \cup \{\beta\})$, then $\gamma \in Cn(X \cup \{\alpha \vee \beta\})$.

Definition (Belief Set)

- Belief set (BS) for $(\mathcal{L}, Cn)$ is a set of the form $Cn(X)$ for some $X \subseteq \mathcal{L}$.
- $\mathcal{BS}_{\mathcal{L}}$ = Set of all belief sets for $(\mathcal{L}, Cn)$

- Idealization of the beliefs of a rational agent

- For post-AGM work on more realistic agents see, e.g.,

Lit: R. Wasserman. Resource bounded Belief Revision. PhD thesis, Institute for Logic Language and Information, Amsterdam, 1999.

Elements of AGM Belief Revision

- AGM consider (inter-related) operators for changing BSs into new BSs under a single trigger sentence $\in \mathcal{L}$
- Types of AGM change operators $\mathcal{BS}_{\mathcal{L}} \times \mathcal{L} \longrightarrow \mathcal{BS}_{\mathcal{L}}$
  - Expansion: add trigger and close up w.r.t. Cn
  - Contraction: delete trigger from BS
  - Revision: add trigger and eliminate inconsistencies

AGM Postulates for Expansion

(E1) $K + \alpha \in \mathcal{BS}_{\mathcal{L}}$ (Closure)

(E2) $\alpha \in K + \alpha$ (Success)

(E3) $K \subseteq K + \alpha$ (Inclusion)

(E4) If $\alpha \in K$, then $K = K + \alpha$. (Vacuity)

(E5) If $K \subseteq X$, then $K + \alpha \subseteq X + \alpha$. (monotonicity)

(E6) $K + \alpha$ is the smallest belief set fulfilling (E1)–(E5).

Note:

- Postulates defined for fixed belief set K.
- Postulates specify properties of intended change operator
- In general, many structurally different operators may fulfil the postulates, but . . .

AGM Postulates for Expansion

(E1) $K + \alpha \in \mathcal{BS}_{\mathcal{L}}$ (Closure)

(E2) $\alpha \in K + \alpha$ (Success)

(E3) $K \subseteq K + \alpha$ (Expansion 1)

(E4) If $\alpha \in K$, then $K = K + \alpha$. (Expansion 2)

(E5) If $K \subseteq X$, then $K + \alpha \subseteq X + \alpha$. (Monotonicity)

(E6) $K + \alpha$ is the smallest belief set fulfilling (E1)–(E5).

- ... (E1)–(E6) are such specific that they uniquely identify $+$

Proposition

An operator $+$ fulfils (E1)–(E6) iff for all α: $K + \alpha = Cn(K \cup \{\alpha\})$

- This is a representation result

AGM Postulates for Contraction

(C1) $K \div \alpha \in \mathcal{BS}_{\mathcal{L}}$ (Closure)

(C2) $K \div \alpha \subseteq K$ (Inclusion)

(C3) If $\alpha \notin K$, then $K = K \div \alpha$ (Vacuity)

(C4) If $\alpha \notin Cn(\emptyset)$, then $\alpha \notin K \div \alpha$. (Success)

(C5) If $\alpha \in K$, then $K \subseteq (K \div \alpha) + \alpha$. (Recovery)

(C6) If $\alpha \leftrightarrow \beta \in Cn(\emptyset)$, then $K \div \alpha = K \div \beta$. ((Right) Extensionality)

(C7) $K \div \alpha \cap K \div \beta \subseteq K \div (\alpha \wedge \beta)$ (Conjunction 1)

(C8) If $\alpha \notin K \div (\alpha \wedge \beta)$, then $K \div (\alpha \wedge \beta) \subseteq K \div \alpha$. (Conjunction 2)

AGM Postulates for Revision

(R1) $K * \alpha \in \mathcal{BS}_{\mathcal{L}}$ (Closure)

(R2) $\alpha \in K * \alpha$ (Success)

(R3) $K * \alpha \subseteq K + \alpha$ (Expansion 1/Inclusion)

(R4) If $\neg\alpha \notin K$, then $K + \alpha \subseteq K * \alpha$. (Expansion 2/Vacuity)

(R5) If $\mathcal{L} = Cn(K * \alpha)$, then $\neg\alpha \in Cn(\emptyset)$. (Consistency)

(R6) If $\alpha \leftrightarrow \beta \in Cn(\emptyset)$, then $K * \alpha = K * \beta$.
((Right) Extensionality)

(R7) $K * (\alpha \wedge \beta) \subseteq (K * \alpha) + \beta$ (Conjunction 1)

(R8) If $\neg\beta \notin K * \alpha$, then $(K * \alpha) + \beta \subseteq K * (\alpha \wedge \beta)$.
(Conjunction 2)

Levi-Identity

Revision is definable by contraction.

Definition

Given a (contraction) operator $\dot{-}$, the (revision) operator $*$ defined by the Levi Identity is:

$$K * \alpha = (K \dot{-} \neg\alpha) + \alpha$$

Theorem

The operator defined by the Levi Identity fulfils (R1)–(R8) if $\dot{-}$ *fulfils (C1)–(C8).*

Harper Identity

Contraction is definable by revision

Definition

Given a (revision) operator the (contraction) operator defined by the Harper Identity is

$$K \div \alpha = K \cap (K * \neg\alpha)$$

Theorem

The contraction operator defined by the Harper Identity fulfils (C1)–(C8) if $*$ fulfils (R1)–(R8).

AGM Operators

Operators for Revision and Contraction Postulates

- We still did not see concrete revision and contraction operators
- We seek for models of Postulates (R1)–(R8) and (C1)–(C8).

- In contrast to $+$, the postulates do not fix a single operator but a whole class
- But: Postulates are so specific that the classes can be characterised by construction principles.

- There are various construction principles leading to different classes
  - Partial meet
  - Safe/kernel
  - Epistemic entrenchment
  - Possible worlds
  - Sphere-based
  - ...

Remainder Set

- Main construct underlying partial meet operators
- Describe maximal possible scenarios that are compatible with the negation of the trigger

Definition (Remainder Set Informally)

The remainder set $X \perp \alpha$ of X by α consists of all maximal subsets of X not entailing α.

The sets in $X \perp \alpha$ are called remainders.

Remainder Set

Definition (Remainder Set formally)

The remainder set $X \perp \alpha$ of X by α consists of all sets X' s.t.:

1. $X' \subseteq X$;
2. $\alpha \notin Cn(X')$;
3. There is no X'', such that $X' \subsetneq X'' \subseteq X$ and $\alpha \notin Cn(X'')$.

Example (Hansson Dynamics of Belief, Exercise 26a,f)

- $\{p, q\} \perp (p \wedge q) = \{\{p\}, \{q\}\}$
- $\{p \vee r, p \vee \neg r, q \wedge s, q \wedge \neg s\} \perp (p \wedge q) =$
  $\{ \{p \vee r, p \vee \neg r\}, \{p \vee r, q \wedge s\}, \{p \vee r, q \wedge \neg s\},$
  $\{p \vee \neg r, q \wedge s\}, \{p \vee \neg r, q \wedge \neg s\} \}$

Selection Function

- Handle multiplicity of scenarios (remainder sets) with fairness condition
  $\Longrightarrow$ Apply selection function

Definition (Selection Function)

An AGM-selection function $\gamma : Pow(\mathcal{BS}_{\mathcal{L}}) \longrightarrow Pow(\mathcal{BS}_{\mathcal{L}})$ for K fulfills for all α:

1. If $K \perp \alpha \neq \emptyset$, then $\emptyset \neq \gamma(K \perp \alpha) \subseteq K \perp \alpha$;
2. $\gamma(\emptyset) = \{K\}$.

- Note: γ is defined for a given K

Partial Meet

Definition

For a selection function γ defined on K let:

- $K \div_\gamma \alpha = \bigcap \gamma(K \perp \alpha)$ (Partial meet contraction)
- $K *_\gamma \alpha = (K \div_\gamma \neg\alpha) + \alpha$ (Partial meet revision)
- Maxi-Choice = partial meet with $|\gamma(X)| = 1$.
- Full meet = partial meet change with $\gamma(X) = X$ (for $X \neq \emptyset$).

- Maxi-choice and full-meet are two extremes of partial meet change

Properties Maxi-Choice and Full-Meet

- Maxi-choice revision is all-too deterministic: It decides the status of any sentence

Theorem

Let $*_\gamma$ be a maxi-choice revision operator. Then, for any (!) $\beta \in \mathcal{L}$ either $\beta \in K *_\gamma \alpha$ or $\neg\beta \in K *_\gamma \alpha$

- Full-meet revision is too skeptical.

Theorem

Let $*_\gamma$ be a full-meet revision operator. Then for all α with $\neg\alpha \in K$: $K *_\gamma \alpha = Cn(\alpha)$.

Representation Theorem

- The basic axioms for AGM revision and contraction characterise the class of partial meet revision and partial meet contraction operators

Theorem

An operator $\div$ on belief set K fulfils (C1)–(C6) iff there is a selection function γ such that for all α:

$$K \div \alpha = K \div_{\gamma} \alpha$$

An operator $*$ on belief set K fulfils (R1)–(R6) iff there is a selection function γ such that for all α:

$$K * \alpha = K *_{\gamma} \alpha$$

- Partial-meet operators do not necessarily fulfil the additional postulates (R7,8), (C7,8), resp.
- For this one considers γ with additional properties

Representation Theorems

- Representation theorem in a general sense
  - Given a class A of structures satisfying a set of axioms
  - Output: A class of structures B (adhering to some simple construction) such that any A-structure is structure-equivalent to some B-structure
  - Different notions of structure equivalence
  - Example: Stone's result that every boolean algebra is isomorphic to an algebra of sets

- Representation theorems in BR are special cases
  - Domains of operators are fixed
  - Equality instead of isomorphism

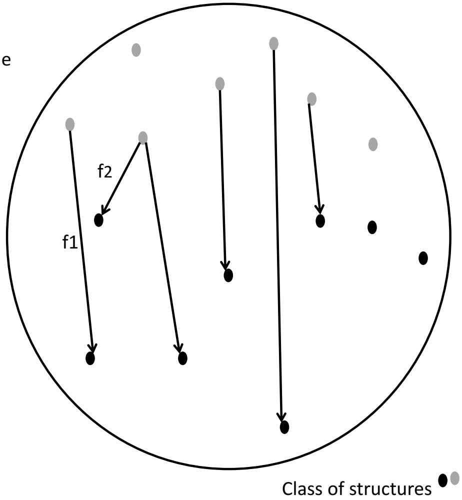
Representation (General)

Class of representing structures
With simple construction principle

fi Structure preserving
representation mappings fi

Class of represented
(Non-protypical) structures

Class of all structures



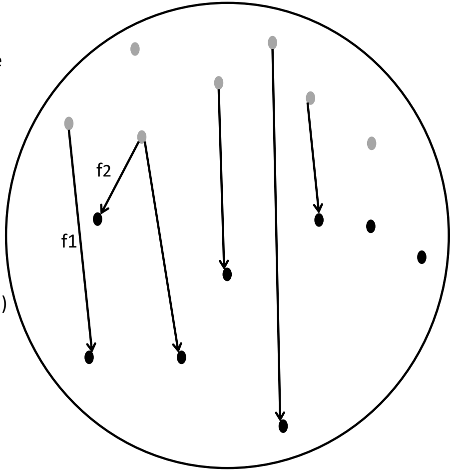
Representation (Non-Mathematical Example)

Class of representing structures with simple construction principle
(VW beetle)

fi Structure preserving mappings
(VW beetle blueprint-> Porsche blueprint)

Class of represented (non-typical) structures
(Non-VW-beetle)

Class of all structures
(VW cars)



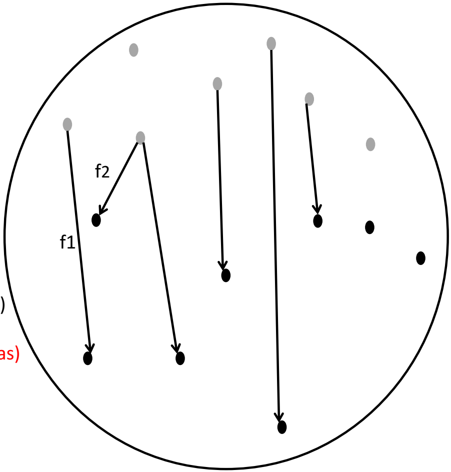
Representation (The Classical Mathematical Example)

Class of representing structures with simple construction principle
(set algebras)

fi Structure preserving mappings
(isomorphisms
set-algebra to Boolean algebra)

Class of represented (non-typical) structures
(Non-set-algebra Boolean-algebras)

Class of all structures
(Boolean algebras)



Other Constructions for Concrete Operators

- Other equally powerful constructions exist that lead to representation theorems for AGM postulates
- Kernel revision
  - Consider duals to remainder set: kernels
  - Kernel = Minimal set responsible for inconsistency (culprit set)
  - Revision: Revise by eliminating from every kernel at least one element
- Rank based revision (such as epistemic entrenchment)
  - Idea: Specify (partial) order on sentences w.r.t. a belief set
  - Revision: Eliminate the least epistemically entrenched ones
- Possible Worlds (see following slides)

AGM: criticism, extensions and more

AGM: the Core of BR Research

- AGM change operators have been criticised on different grounds—again and again
- This shows importance of AGM rather than weakness

- We discuss criticisms of AGM, extensions, and alternative operators ...
- ... mainly with respect to use of BR for CS and ontology change

General Criticism: Recovery

Remember: If $\alpha \in K$, then $K \subseteq (K \div \alpha) + \alpha$. (Recovery)

Example

- Belief set K contains
  - Cleopatra had a son. (γ)
  - Cleopatra had a daughter (β)
  - Cleopatra had a child. ($\gamma \vee \beta$)
- Contract with $\gamma \vee \beta$ (The α in recovery postulate)
- Then add $\gamma \vee \beta$.
- Why should one still believe in facts γ and β?

- Recovery somehow wrongly implements minimality

General Criticisms: No Minimality

Example

- AGM postulates allow amnestic revision of form

$$K * \alpha = Cn(\alpha)$$

- This is not minimal in a genuine sense

- Lead to invention of relevance postulates

Definition (Relevance Postulate Template)

Allow the elimination only of those sentences of a knowledge base that are relevant for the trigger.

Formalizing Relevance according to (Parikh 99)

Definition (Finest splitting of a knowledge base KB)

Finest partition $(V_i)_{i \in I}$ of the vocabulary $V(KB)$ of KB such that there are knowledge bases KB_i over V_i with $KB \equiv \bigcup_{i \in I} KB_i$.

Lit: R. Parikh. Beliefs, belief revision, and splitting languages. In Logic, Language and Computation, vol. 2, pages 266–278,1999.

Formalizing Relevance according to (Parikh 99)

Definition (Finest splitting of a knowledge base KB)

Finest partition $(V_i)_{i \in I}$ of the vocabulary $V(KB)$ of KB such that there are knowledge bases KB_i over V_i with $KB \equiv \bigcup_{i \in I} KB_i$.

Example

1. $KB = \{p \rightarrow \neg q, \neg q \rightarrow r, p \vee s, \neg s, (r \rightarrow t) \vee (t \rightarrow r)\}$
   - $V = V(KB) = \{p, q, r, s, t\}$
   - Coarsest partition: V taking $I = \{1\}$, $KB_1 = KB$.
   - Finer partition: $V_1 = \{p, q\}$, $V_2 = \{r, s, t\}$ and $KB_1 = \{p, \neg q\}$,$KB_2 = \{r, \neg s\}$
   - Finest splitting: $\{\{p\}, \{q\}, \{r\}, \{s\}, \{t\}\}$ with $KB_1 = \{p\}$, $KB_2 = \{\neg q\}$, $KB_3 = \{r\}$, $KB_4 = \{\neg s\}$, $KB_5 = \{\}$.
2. $KB = \{(p \rightarrow \neg q) \wedge (r \rightarrow s)\}$
   - Finest splitting: $\{\{p, q\}, \{r, s\}\}$ and $KB_1 = \{p \rightarrow q\}$, $KB_2 = \{r \rightarrow s\}$

Formalizing Relevance (Parikh)

Theorem (Parikh 1999)

Every (finite) KB has a unique finest splitting

Definition (Cell-relevance)

α is cell-relevant for β modulo KB iff there is a V_i in the finest splitting of KB such that α^* and β^* share a symbol with V_i (not necessarily the same).

(α^* = formula equivalent to α in least-letter set for α. Eg: $(\neg p \wedge (\neg p \vee q))^* = \neg p$)

Definition (Relevance postulate instance)

If $\beta \in KB$ but $KB * \alpha \not\models \beta$ then α is cell-relevant for β modulo KB

Relevance

- Parikh's approach generalized to infinite bases and for FOL:
  Lit: D. Makinson and G. Kourousias. Parallel interpolation, splitting, and relevance in belief change. Journal of Symbolic Logic, 72:994?1002, September 2007.

- For a consideration of different aspects of relevance see
  Lit: D. Makinson. Propositional relevance through letter-sharing: review and contribution. In: Formal Models of Belief Change in Rational Agents, volume 07351 of Dagstuhl Seminar Proceedings, 2007.

- But there are also considerations why "dogma of minimality" is not satisfiable
  Lit: H. Rott. Two dogmas of belief revision. The Journal of Philosophy, 97(9):503–522, 2000.

General Criticism: Success postulate

Example

- Child: "There was a dinosaur in our flat who broke the vase"
- One wants to trust only some parts of information (a glass was broken) but not other parts (it was a dinosaur)

- Lead to non-prioritized belief revision: no priority for trigger
- Types
  1. Revise only with credible triggers
  2. Delete elements from belief base or the trigger
  3. Delete elements from belief base or from closure of trigger
  4. Extend with trigger and then delete inconsistencies
  5. Decide which part $f(\alpha)$ to delete from trigger

Lit: S. O. Hansson. A survey of non-prioritized belief revision. Erkenntnis, 50(2-3):413–427, 1999.

Requirement of Finite Belief Sets

- CS cannot handle infinite belief sets
- Objects (data base, knowledge base, ontology etc.) are finite or finitely representable

- Three possible approaches
  1. Change operators for finitely generated belief sets $Cn(X)$ with X finite (see textbook of Hansson)
  2. Change operators for finite belief bases
     Belief base = not necessarily closed subset of $\mathcal{L}$
  3. Change operators for models of finite belief bases

Syntax-sensitive Belief Base Revision

- Hansson's approach: use syntax sensitivity in order to represent additional justification information

Example

- $B_1 = \{p, q\}$
  Belief in p and q with independent justifications for p and q
- $B_2 = \{p \wedge q\}$
  Belief in p and q but with common justification for p and q
- $B_1 \equiv B_2$

- $B_1 \div p$ may reasonably contain q
- $B_2 \div p$ leads to $\emptyset$

Syntax-sensitive Belief Base Revision

- Similar constructions and postulates as in AGM
- Main difference: expansion now reads as $B + \alpha = B \cup \{\alpha\}$
- Additional phenomena and revision operators due to handling of inconsistency
  - First prevent inconsistency then add trigger
    $B *_{internal} \alpha = (B \div \neg\alpha) + \alpha$ (as in AGM)
  - First add trigger then handle inconsistency
    $B *_{external} \alpha = (B + \alpha) \div \bot$ (New)
- In-depth treatment in textbook (Hansson 99).

Lit: S. O. Hansson. A Textbook of Belief Dynamics. Kluwer Academic Publishers, 1999.

Summary and Outlook

- Considered classical AGM theory of belief revision for handling inconsistencies
- Started discussing extensions based on criticisms of AGM in particular for use in CS applications
- Next week: continue consideration of extensions with in particular for use in ontology integration