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INSTITUT FÜR INFORMATIONSSYSTEME

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Data Exchange 2

Lecture 6: Universal Solutions, Core, Certain Answers

25 November, 2015

Foundations of Ontologies and Databases
for Information Systems
CS5130 (Winter 2015)

Recap of Lecture 5

Data Exchange

- Specific semantic integration scenario for two data sources with possibly different schemata
- Mapping $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$
  - σ: source schema
  - τ: target schema
  - $M_{\sigma\tau}$: source target dependencies (mostly: st-tgds)
  - M_τ: target dependencies
- Ultimate aim: For given σ instance find appropriate τ instance (solution) to do query answering on it

- SOLEXISTENCE$_{\mathcal{M}}$: Is there a solution for a given $\mathcal{M}$
- Chase construction for finding solutions
- Chase construction gives sufficient and necessary condition if termination is guaranteed
- Termination with weakly acyclic dependencies

End of Recap

Universal Solutions

What are Good Solutions?

- We are seeking **universal** solutions: they represent all other ones

- A solution $\mathfrak{T}$ may contain NULLs
- A DB instance is **complete** iff it does not contain NULLs

- $Rep(\mathfrak{T})$ = all complete DBs instances that represent $\mathfrak{T}$
- Explicate "represent" by homomorphism notion
- Now formally define

$$Rep(\mathfrak{T}) = \{\mathfrak{T}' \mid \text{There is } h : \mathfrak{T} \xrightarrow{hom} \mathfrak{T}' \text{ for complete } \mathfrak{T}'\}$$

Homomorphism

- Intuitively, homomorphisms are structure preserving mappings
- Defined here for DB instances but similarly definable for arbitrary structures

Definition

A **Homomorphism** $h : \mathfrak{T} \xrightarrow{hom} \mathfrak{T}'$ is a map

$$h : Var(\mathfrak{T}) \cup CONST \rightarrow VAR(\mathfrak{T}') \cup CONST$$

s.t.

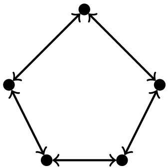
- $h(c) = c$ for all $c \in CONST$ and
- if $R(\vec{t}) \in \mathfrak{T}$, then $R(h(\vec{t})) \in \mathfrak{T}'$

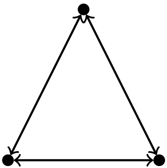
Wake-Up Exercise

Consider two instances that are graphs, namely

- $\mathfrak{G}$ = cycle on 5 nodes with marked nulls $\nu_1, \ldots, \nu_5$
- $\mathfrak{G}'$ = cycle on 3 nodes with marked nulls ν_1', ν_2', ν_3'.

Give examples of a mapping $h : \mathfrak{G} \to \mathfrak{G}'$ that is a homomorphism, resp. not a homomorphism.





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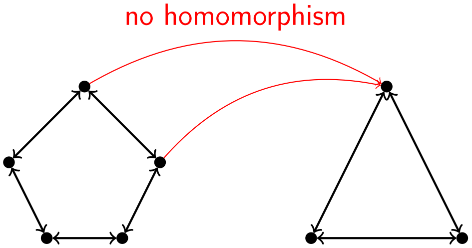

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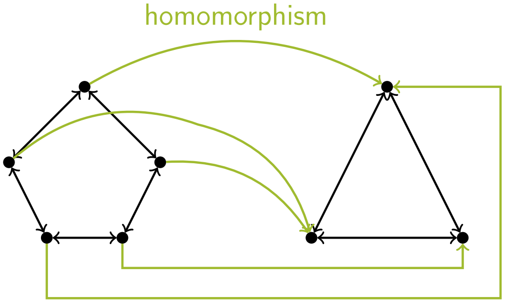

Universal Solutions

- There are three **equivalent** characterizations of universal solutions

Definition (Universal Solution)

1. **Solution $\mathfrak{T}$ describing all others**

$$\{\mathfrak{T}' \in \mathit{SOL}_{\mathcal{M}}(\mathfrak{S}) \mid \mathfrak{T}' \text{ complete}\} \subseteq \mathit{Rep}(\mathfrak{T})$$

2. **Solution $\mathfrak{T}$ general as all others**

$$\mathit{Rep}(\mathfrak{T}') \subseteq \mathit{Rep}(\mathfrak{T}) \quad \text{for every } \mathfrak{T}' \in \mathit{SOL}_{\mathcal{M}}(\mathfrak{S})$$

3. **Solution $\mathfrak{T}$ mapping homomorphically into others**

$$\text{For all } \mathfrak{T}' \in \mathit{SOL}_{\mathcal{M}}(\mathfrak{S}) \text{ there is } h : \mathfrak{T} \xrightarrow{\mathit{hom}} \mathfrak{T}'$$

Example (Universal Solution)

Source DB

Flight (	src,	dest,	airl,	dep	)
	paris	sant.	airFr	2320	

Target DB

Routes(fno, src, dest)

Info(fno, dep, arr, airl)

- **Dependencies** $M_{\sigma\tau}$

$$
\begin{aligned}
&Flight(src, dest, airl, dep) \longrightarrow \\
&\quad \exists fno\ \exists\ arr(Routes(fno, src, dest) \wedge Info(fno, dep, arr, airl))
\end{aligned}
$$

- τ **solutions**

$$
\begin{aligned}
\mathfrak{T} &= \{Routes(\perp_1, paris, sant), Info(\perp_1, 2320, \perp_2, airFr)\} \\
\mathfrak{T}' &= \{Routes(\perp_1, paris, sant), Info(\perp_1, 2320, \perp_1, airFr)\} \\
\mathfrak{T}'' &= \{Routes(123, paris, sant), Info(123, 2320, 930, airFr)\}
\end{aligned}
$$

- $\mathfrak{T}$ is a universal solution, $\mathfrak{T}'$ and $\mathfrak{T}''$ are not

Example (Non-existence of Universal Solutions)

- $M_{\sigma\tau} = \{\ E(x, y) \to G(x, y)\ \}$
- $M_{\tau} = \{\ G(x, y) \to \exists z\ L(y, z), \quad L(x, y) \to \exists z\ G(y, z)\ \}$
- Source instance $\mathfrak{S} = \{E(a, b)\}$

- $\mathfrak{T} = \{G(a, b), L(b, a)\}$ is a solution
- But there is no universal solution

Proof sketch (by contradiction)

- A universal solution must have an infinite sequence $(\mathfrak{S}, \{G(a, b), L(b, \nu_1), G(\nu_1, \nu_2), L(\nu_2, \nu_3), G(\nu_3, \nu_4) \ldots\})$
- As $\mathfrak{T}$ is finite there must be some identification of an ν_i with a or b or with another ν_j
- In any case a contradiction follows

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Undecidability of Universal Solution Existence

UNISOLEXISTENCE$_{\mathcal{M}}$

- Input: A source instance $\mathfrak{S}$
- Output: Is there a universal solution for $\mathfrak{S}$ under $\mathcal{M}$?

- Allowing arbitrary dependencies leads to undecidability
- Shown by of reduction of halting problem

Theorem

There exists a relational mapping $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ s.t. UNISOLEXISTENCE$_{\mathcal{M}}$ is undecidable

- Proof in book of Arenas et al. 5 pages long, so ... we do not show it here

By the way: There are Longer Proofs

- Recent example: A computer aided proof for the Erdős Discrepancy Problem (EDP) by Alexei Lisitsa and Boris Konev
- File containing the proof about 13 GB

- **Lit:** B. Konev and A. Lisitsa. Computer-aided proof of erdos discrepancy properties. Artif. Intell., 224(C):103? 118, July 2015.
- **Lit:** `https://rjlipton.wordpress.com/2014/02/28/practically-pnp/`

Desiderata

- Due to the undecidabiltiy result one has to constrain dependencies
- Constraints such that the following are fulfilled:

(C1) Existence of solutions entails existence of universal solutions

(C2) UNIVSOLEXISTENCE decidable and even tractable

(C3) If a solutions exists, then universal solutions should be constructible in polynomial time

Chase Helps Again

Theorem

Results of successful chase sequences are universal solutions (and sometimes called ***canonical*** *solutions).*

Proof Sketch

- Have to show only universality of chase $\mathfrak{T}$
- Use the third definition
- Let $\mathfrak{T}'$ be any solution
- Lemma: Adding facts in chase step preserves homomorphism
- Argue inductively starting from empty homomorphism
- Distinguish between tgd and egd

Nice Properties of Universal Solutions

Theorem

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping where M_τ is the union of egds and weakly acyclic tgds. Then:

- *UNISOLEXISTENCE$_\mathcal{M}$ can be solved in PTIME (C2).*
- *And if solutions exist, then a universal solutions exist (C1),*
- *and a canonical solution can be computed in polynomial time (C3).*

Example (Non-uniqueness of canonical Solutions)

- $M_{\sigma\tau} = \{\ P(x) \rightarrow \exists z \exists w (E(x,y) \wedge E(x,w)\ \}$
- $M_{\tau} = \{\ \underbrace{E(x,y) \rightarrow \exists z\ F(y,z)}_{\chi_1}, \underbrace{E(x,y) \wedge E(x,y') \rightarrow y = y'}_{\chi_2}\ \}$
- Source instance $\mathfrak{S} = \{P(a)\}$
- First step: $\mathfrak{T} = \{E(a, \perp_1), E(a, \perp_2)\}$
- Two different solutions
  - Apply χ_1, then χ_2:

    $$T_1 = \{E(a, \perp_1), F(\perp_1, \perp_3)), F(\perp_1, \perp_4)\}$$

  - Apply χ_2, then χ_1:

    $$T_2 = \{E(a, \perp_1), F(\perp_1, \perp_2)\}$$

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- Source instance $\mathfrak{S} = \{P(a)\}$

- First step: $\mathfrak{T} = \{E(a,\bot_1), E(a,\bot_2)\}$
- Two different solutions
  - Apply χ_1, then χ_2:

$$T_1 = \{E(a,\bot_1), F(\bot_1,\bot_3)), F(\bot_1,\bot_4)\}$$

  - Apply χ_2, then χ_1:

$$T_2 = \{E(a,\bot_1), F(\bot_1,\bot_2)\}$$

Non-uniqueness

- Non-uniqueness no serious problem as all universal solutions are good
- Nonetheless one can show

Proposition

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping s.t. M_τ consists of egds only. Then every source instance $\mathfrak{S}$ has a unique canonical solution $\mathfrak{T}$ (up to a reaming of NULLS) under $\mathcal{M}$.

The Core

Running Example: Flight Domain

Source DB σ

Geo(city, coun, pop)

city	coun	pop
paris,	france,	2M

Flight (src, dest, airl, dep)

src	dest	airl	dep
paris	amst.	KLM	1410
paris	amst.	KLM	2230

Canonical Solution $\mathfrak{T}$

Routes(fno, src, dest)

fno	src	dest
$\perp_1$,	paris,	amst.
$\perp_3$,	paris,	amst.

Info(fno, dep, arr, airl)

fno	dep	arr	airl
$\perp_1$,	1410,	$\perp_2$	klm
$\perp_1$,	2320,	$\perp_2$	klm

Serves(airl, city, coun, phone)

airl	city	coun	phone
klm,	paris,	france,	$\perp_5$
klm,	paris,	france,	$\perp_6$

Mapping rules $M_{\sigma\tau}$

1. $Flight(src, dest, airl, dep) \longrightarrow$
   $\exists fno \; \exists \; arr(Routes(fno, src, dest) \wedge Info(fno, dep, arr, airl))$

2. $Flight(src, dest, airl, dep) \wedge Geo(city, coun, pop) \longrightarrow$
   $\exists phone(Serves(airl, city, coun, phone))$

3. $Flight(src, city, airl, dep) \wedge Geo(city, coun, pop) \longrightarrow$
   $\exists phone \; (Serves(airl, city, coun, phone))$

Running Example: Flight Domain

Source DB σ

Geo(	city,	coun,	pop	)
	paris,	france,	2M	

Flight (	src,	dest,	airl,	dep	)
	paris	amst.	KLM	1410	
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Smallest Solution $\mathfrak{T}^*$

Routes(	<u>fno</u>,	src,	dest	)
	$\perp_1$,	paris,	amst.	
	$\perp_3$,	paris,	amst.	

Info(	<u>fno</u>,	dep,	arr,	airl	)
	$\perp_1$,	1410,	$\perp_2$	klm	
	$\perp_1$,	2320,	$\perp_2$	klm	

Serves(	airl,	city,	coun,	phone	)
	klm,	paris,	france,	$\perp_5$	
	~~klm,~~	~~paris,~~	~~france,~~	$\cancel{\perp_6}$	

Mapping rules $M_{\sigma\tau}$

1. $Flight(src, dest, airl, dep) \longrightarrow$
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Better than Universal? The Core?

- Universal solutions may still contain redundant information
- Seeking for smallest universal solutions: cores

- $\mathfrak{T}'$ is **subinstance** of $\mathfrak{T}$, for short $\mathfrak{T}' \subseteq \mathfrak{T}$, iff $R^{\mathfrak{T}'} \subseteq R^{\mathfrak{T}}$ for all relation symbols R

Definition

A subsinstance $\mathfrak{T}' \subseteq \mathfrak{T}$ is a **core** of $\mathfrak{T}$ iff there is $h : \mathfrak{T} \xrightarrow{hom} \mathfrak{T}'$ but there is not a homomorphism from $\mathfrak{T}$ to a proper subinstance of $\mathfrak{T}'$.

- Intuitively: An instance can be retracted (structure preservingly) to its core but not further

Properties of Cores

Definition

A subinstance $\mathfrak{I}' \subseteq \mathfrak{I}$ is a **core** of $\mathfrak{I}$ iff there is $h : \mathfrak{I} \xrightarrow{hom} \mathfrak{I}'$ but there is not a homomorphism from $\mathfrak{I}$ to a proper subinstance of $\mathfrak{I}'$.

Proposition

1. *Every instance has a core.*
2. *All cores of the same instance are isomorphic (same up to renaming of NULLs)* *($\Longrightarrow$ Talk of <u>the</u> core justified)*
3. *Two instances are homomorphically equivalent iff their cores are isomorphic*
4. *If $\mathfrak{I}'$ is core of $\mathfrak{I}$, then there is $h : \mathfrak{I} \xrightarrow{hom} \mathfrak{I}'$ s.t. $h(\nu) = \nu$ for all $\nu \in DOM(\mathfrak{I}')$*

Main Theorem for Cores

Theorem

1. *If* $\mathfrak{T} \in SOL_{\mathcal{M}}(\mathfrak{S})$, *then also* $core(\mathfrak{T}) \in SOL_{\mathcal{M}}(\mathfrak{S})$
2. *If* $\mathfrak{T} \in UNIVSOL_{\mathcal{M}}(\mathfrak{S})$ *then also* $core(\mathfrak{T}) \in UNIVSOL_{\mathcal{M}}(\mathfrak{S})$
3. *If* $UNIVSOL_{\mathcal{M}}(\mathfrak{S}) \neq \emptyset$, *then all* $\mathfrak{T} \in UNIVSOL_{\mathcal{M}}(\mathfrak{S})$ *have same core (up to renaming of NULLs), and the core of any universal solution is the smallest universal solution*

Computing the Core

- Easy Case: No tgds in M_τ
- Simple algorithm $\mathit{COMPUTECORE}(\mathcal{M})$
  - Assume $\mathfrak{S}$ has successful sequence with result $\mathfrak{T}$.
  - If $\mathfrak{T} = \mathit{fail}$, then also the output fail
  - Otherwise: remove facts as long as $M_{\sigma\tau}$ fulfilled.

Theorem

If chase not fails, then $\mathit{COMPUTECORE}(\mathcal{M})$ outputs core of universal solutions in polynomial time.

- Algorithm works as egds satisfactions preserved for subinstances
- More sophisticated methods needed in presence of tgds in M_τ

The Core

- Core has nice properties: Uniqueness
- But may be more costly to compute than universal canonical solution
- In the end: We want to use solution for QA—and for this canonical universal solutions suffice

Query Answering

Certain Answers

- Given mapping $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$
- Semantics of query answering specified as certain answer semantics

Definition

The **certain answers** of query Q over τ for given instance $\mathfrak{S}$ is defined as

$$cert_{\mathcal{M}}(Q, \mathfrak{S}) = \bigcap \{\, Q(\mathfrak{T}) \mid \mathfrak{T} \in SOL_{\mathcal{M}}(\mathfrak{S}) \,\}$$

- Note: $Q(\mathfrak{T})$ gives set of tuples of constants from $\mathfrak{S}$
- Definition does not tell how to actually compute the certain answers
- In many cases it is not necessary to compute all solutions to get certain answers

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Algorithmic Problems for Certain Answers

Problem: $\mathit{CERTAIN}_{\mathcal{M}}(Q)$

Input: Source instance $\mathfrak{S}$ and tuple of elements $\vec{t} \in \mathit{DOM}(\mathfrak{S})$
Output: Answer whether $\vec{t} \in \mathit{certain}_{\mathcal{M}}(Q, \mathfrak{S})$

- Again, to guarantee tractability or even decidability one has to restrict the involved components
  - Constrain query language (e.g., from FOL to CQs)
  - Constrain dependencies (e.g., to weakly acyclic TGDs)

Proposition

There is an FOL query Q and a $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau})$ s.t. $\mathit{CERTAIN}_{\mathcal{M}}(Q)$ is undecidable.

Answering Conjunctive Queries (CQs)

- Conjunctive queries (CQs)

$$Q(\vec{x}) = \exists \vec{y} \left(\alpha_1(\vec{x_1}, \vec{y_1}) \wedge \cdots \wedge \alpha_n(\vec{x_n}, \vec{y_n}) \right)$$

- Unions of conjunctive queries (UCQs)

$$Q(\vec{x}) = CQ_1(\vec{x}) \vee \cdots \vee CQ_n(\vec{x})$$

- Crucial Property: (U)CQs are preserved under homomorphisms

Proposition

Let $h : \mathfrak{S} \xrightarrow{hom} \mathfrak{S}'$ and Q be a UCQ. Then:

$$Q(\mathfrak{S}) \subseteq Q(\mathfrak{S}')$$

(In detail: for all tuples of constants: If $\vec{a} \in Q(\mathfrak{S})$, then $\vec{a} \in Q(\mathfrak{S}')$

Follows easily from homomorphism definition

As a corollary one immediately gets also preservation for certain query answering.

Proposition

Let $h : \mathfrak{S} \xrightarrow{hom} \mathfrak{S}'$ and Q be a UCQ. Then:

$$\mathit{certain}(Q, \mathfrak{S}) \subseteq \mathit{certain}(Q, \mathfrak{S}')$$

Certain Answering UCQs

Theorem

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping where M_τ is a union of egds and weakly acyclic tgds and let Q be a UCQ.

Then $CERTAIN_{\mathcal{M}}(Q)$ can be solved in PTIME.

Proof Sketch

- Consider naive evaluation strategy Q_{naive}
  - Let $\mathfrak{T}$ arbitrarily chosen universal solution
  - Treat marked NULLS in $\mathfrak{T}$ as constants.
  - Calculate $Q(\mathfrak{T})$ under this perspective
  - and then eliminate all tuples from $Q(\mathfrak{T})$ containing a NULL
- Now one can show $certain_{\mathcal{M}}(Q, \mathfrak{S}) = Q_{naive}(\mathfrak{T})$.

Showing $certain_{\mathcal{M}}(Q, \mathfrak{S}) = Q_{naive}(\mathfrak{T})$

- We know that a universal solution $\mathfrak{T}$ can be constructed in polynomial time.
- For every $\mathfrak{T}' \in SOL_{\mathcal{M}}$ there is $\mathfrak{T} \xrightarrow{hom} \mathfrak{T}'$
- NULL-free tuples in $Q(\mathfrak{T}) \subseteq$
  $\bigcap_{\mathfrak{T}' \in SOL_{\mathcal{M}}}$ NULL-free tuples in $Q(\mathfrak{T}')$
- Answering FOL queries (and so of UCQs) computable in PTIME data complexity

QA for other classes of Queries

- Proof above used a simple strategy for certain answering by naive evaluation

Naive Evaluation Strategy

$$cert(\mathfrak{S}, Q) = Q_{naive}(\mathfrak{T})$$

where $\mathfrak{T}$ is a (universal) solution

- This strategy works also for Datalog programs as constraints for the target schema τ
  - Reason: Datalog programs are preserved under homomorphisms
  - Even if one adds inequalities, naive evaluation works
  - Hence certain answering is here in PTime

Rewritability

- Naive evaluation is a form of **rewriting**
- Fundamental method that re-appears in different areas of CS
- Rewrite a query w.r.t. a given KB into a new query that "contains" the knowledge of KB

- Challenges
  - Preserve the semantics in the rewriting process
  - The language of the output query is constraint to a "simple language" (so rewritability not always guaranteed)

Rewritability for DE

Definition (FOL Rewritability)

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping and let Q be a quer over τ.

Then Q is said to be **FOL-rewritable** over the canonical universal solution under $\mathcal{M}$ if there is a FOL query Q_{rew} over τ^C such that

$$certain_{\mathcal{M}}(Q, \mathfrak{S}) = Q_{rew}(\mathfrak{T})$$

- Here $\tau^C = \tau \cup \{C\}$ where unary predicate C depicts all constants (not NULLs) in targets
- Works like a type predicate

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$$certain_{\mathcal{M}}(Q, \mathfrak{S}) = Q_{rew}(\mathfrak{T})$$

There is **one** rewriting for any given pair of source $\mathfrak{S}$ and universal solution $\mathfrak{T}$

- The known component is the mapping $\mathcal{M}$
- The unknown components are all pairs $(\mathfrak{S}, \mathfrak{T})$

Rewritability for DE

Definition (FOL Rewritability)

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping and let Q be a quer over τ.

Then Q is said to be **FOL-rewritable** over the canonical universal solution under $\mathcal{M}$ if there is a FOL query Q_{rew} over τ^C such that

$$certain_{\mathcal{M}}(Q, \mathfrak{S}) = Q_{rew}(\mathfrak{T})$$

If in definition one talk about cores $\mathfrak{T}$ instead of universal solutions then Q is said to be **FOL rewritable over cores**

Theorem

FOL rewritability over core $\vDash$ *FOL rewritability over universal sol., but not vice versa.*

Rewritability for DE

Definition (FOL Rewritability)

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping and let Q be a quer over τ.

Then Q is said to be **FOL-rewritable** over the canonical universal solution under $\mathcal{M}$ if there is a FOL query Q_{rew} over τ^C such that

$$certain_{\mathcal{M}}(Q, \mathfrak{S}) = Q_{rew}(\mathfrak{T})$$

Example

- $Q(\vec{x})$: a conjunctive query
- Q_{rew}: $Q(\vec{x}) \land C(x_1) \land \cdots \land C(x_n)$
- The rewriting is even independent of $\mathcal{M}$

Adding Negations to Query Language

- Negations in query languages lead to lose of naive rewriting technique
- Even if one allows only negation in inequalities

Definition (Conjunctive Queries with inequalities $\mathrm{CQ}^{\neq}$)

A conjunctive query with inequalities is a query of the form

$$Q(\vec{x}) = \exists \vec{y} \left(\alpha_1(\vec{x_1}, \vec{y_1}) \wedge \cdots \wedge \alpha_n(\vec{x_n}, \vec{y_n})\right)$$

where α_i is either an atomic relational formula or an inequality $z_i \neq z_j$.

Example (No Naive Evaluation Possible)

Source DB

Flight (	src,	dest,	airl,	dep	)
	paris	sant.	airFr	2320	
	paris	sant.	lan	2200	

Target DB

Routes(fno, src, dest)

Info(fno, dep, arr, airl)

- **Dependencies** $M_{\sigma\tau}$

$$Flight(src, dest, airl, dep) \longrightarrow$$
$$\exists fno\ \exists\ arr(Routes(fno, src, dest) \wedge Info(fno, dep, arr, airl))$$

- Any universal solution $\mathfrak{T}'$ contains solution τ **solutions**

$$\mathfrak{T} = \{\ Routes(\perp_1, paris, sant),\ Info(\perp_1, 2320, \perp_2, airFr),$$
$$Routes(\perp_3, paris, sant),\ Info(\perp_3, 2320, \perp_4, lan)\ \}$$

- Query $Q(x,z) = \exists y \exists y'(Routes(y,x,z) \wedge Routes(y',x,z) \wedge y \neq y')$
- $Q_{naive}(\mathfrak{T}') = \{(paris, sant)\}$ (for any universal solution$\mathfrak{T}'$)
- But: $cert(Q(x,z), \mathfrak{S})_{\mathcal{M}} = \emptyset$ because there is a solution

$$\mathfrak{T}'' = \{\ Routes(\perp_1, paris, sant),\ Info(\perp_1, 2320, \perp_2, airFr),$$
$$Info(\perp_1, 2320, \perp_2, lan)\ \}$$

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- Any universal solution $\mathfrak{T}'$ contains solution τ **solutions**

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Example (No Naive Evaluation Possible)

Source DB

Flight (	src,	dest,	airl,	dep	)
	paris	sant.	airFr	2320	
	paris	sant.	lan	2200	

Target DB

Routes(<u>fno</u>, src, dest)

Info(<u>fno</u>, dep, arr, airl)

- **Dependencies** $M_{\sigma\tau}$

$$Flight(src, dest, airl, dep) \longrightarrow$$
$$\exists fno\, \exists\, arr(Routes(fno, src, dest) \wedge Info(fno, dep, arr, airl))$$

- Any universal solution $\mathfrak{T}'$ contains solution τ **solutions**

$$\begin{aligned}\mathfrak{T} \quad = \quad \{\ & Routes(\bot_1, paris, sant),\ Info(\bot_1, 2320, \bot_2, airFr),\\ & Routes(\bot_3, paris, sant),\ Info(\bot_3, 2320, \bot_4, lan)\ \}\end{aligned}$$

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Example (No Naive Evaluation Possible)

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	paris	sant.	airFr	2320	
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  $$Flight(src, dest, airl, dep) \longrightarrow \exists fno\ \exists\ arr(Routes(fno, src, dest) \wedge Info(fno, dep, arr, airl))$$

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- $Q_{naive}(\mathfrak{T}') = \{(paris, sant)\}$ (for any universal solution$\mathfrak{T}'$)
- But: $cert(Q(x, z), \mathfrak{S})_{\mathcal{M}} = \emptyset$ because there is a solution

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Example (No Naive Evaluation Possible)

Source DB

Flight (src, dest, airl, dep)

src	dest	airl	dep
paris	sant.	airFr	2320
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Routes(fno, src, dest)

Info(fno, dep, arr, airl)

- **Dependencies** $M_{\sigma\tau}$

$$\begin{array}{l} Flight(src, dest, airl, dep) \longrightarrow \\ \quad \exists fno \, \exists \, arr(Routes(fno, src, dest) \wedge Info(fno, dep, arr, airl)) \end{array}$$

- Any universal solution $\mathfrak{T}'$ contains solution τ **solutions**

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$CQ^{\neq}$ is in coNP

- In case of $CQ^{\neq}$ one cannot even find a tractable possibility to certain answer them Answering

Theorem

Let $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$ be a mapping where M_τ is the union of egds and weakly acyclic tgds, and let Q be a $UCQ^{\neq}$ query. Then:

$CERTAIN_{\mathcal{M}}(Q)$ is in coNP

Non-rewritability

- Generally it is not possible to decide whether rewritability holds

Theorem

For mappings without target constraints one can not decide whether a given FOL query is rewritable over the canonical solutions (over the core).

- Showing Non-FOL-rewritability can be done with locality tools
- Actually: One uses Hanf-locality of FOL
- Adaptation to DE setting

Not Covered

- Different semantics for query answering
  - Combinations of open-world (certain answers) and closed-word semantics
- Whole sub-field of mapping management
  - How to compose mappings
  - How to maintain mappings (e.g., w.r.t. consistency)
  - How to invert mappings: Get back source DB from target DB
- DE for non-relational DBs
  - e.g., DE for semi-structured data (XML)
  - different techniques needed

Exercise 5

Exercise 5.1 (4 Points)

Prove the folklore proposition that conjunctive queries are preserved under homomorphisms.

Exercise 5.2 (6 Points)

Complete the proof of the non-existence of a universal solution for the example given in the lecture.

Example (Non-existence of Universal Solutions)

- $M_{\sigma\tau} = \{ \underbrace{E(x,y) \rightarrow G(x,y)}_{\theta_1} \}$

  $M_{\tau} = \{ \underbrace{G(x,y) \rightarrow \exists z\, L(y,z)}_{\chi_1}, \underbrace{L(x,y) \rightarrow \exists z\, G(y,z)}_{\chi_2} \}$
- Source instance $\mathfrak{S} = \{E(a,b)\}$

Exercise 5.3 (4 Points)

1. Prove that every finite graph has a core
2. Prove that two cores of the same graph are isomorphic