



UNIVERSITÄT ZU LÜBECK  
INSTITUT FÜR INFORMATIONSSYSTEME

Özgür L. Özçep

# Logic, Logic, and Logic

*Lecture 1: Motivation and Overview*

*21 October, 2015*

*Foundations of Ontologies and Databases  
for Information Systems  
CS5130 (Winter 2015)*

# Organizational Stuff

# Organization

- ▶ Lectures with integrated exercises (sometimes homework)
- ▶ Exercise slot may vary: so come to the lectures
- ▶ **Start:** Today, Wed, 21 October, 2015, 16.00h
- ▶ Lecture and exercise related material in Moodle “Grundlagen von Ontologien und Datenbanken für Informationssysteme - CS5130”
- ▶ **Oral exam** at the end of the semester
  - ▶ **Register** for the course in Moodle
  - ▶ Prerequisite for exam: At least 50 percent of exercises solved successfully
- ▶ The lectures and the exercises are in English

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# Sometimes English Becomes Less Important

## Prologue

### La loi 101 (Charte de la langue française)

Principe du deux pour un □ le texte français doit être écrit en caractères **deux fois plus gros** que ceux de la version en langue étrangère.

Two for one principle □ an english (for clarity) text should be written in characters *twice smaller* than its french counterpart.

Exception □ the english version of the text of the Law itself can be written in characters **five times bigger** than the french original.

Slide example by Bruno Poizat from a conference talk

- ▶ Model Theorist
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# Plan

- ▶ Logic, Logic, Logic (2 lectures)
- ▶ Logical Foundations of Database Systems: Finite Model Theory (2 lectures)
- ▶ Semantic Integration with OBDA: Bridging the DB and Ontology World (2-3 lectures)
- ▶ Semantic Integration on Ontology Level: Ontology Integration (2-3 lectures)
- ▶ Stream Processing (2-3 lectures)
- ▶ Process Analysis and Design (2-3 lectures)
- ▶ Round Up: A Vademecum for Information Systems (0,5 - 1 Lecture)

# First-Order Logic

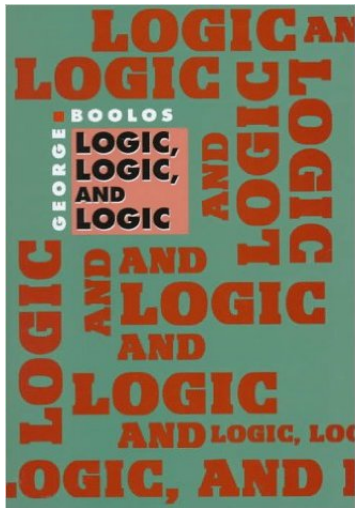


# “Logic, Logic, and Logic”

- ▶ Interesting collection of essays
- ▶ Rather “philosophical logic”
- ▶ But we adopt the motto:

Logic everywhere !

- ▶ We are interested not only in logics per se but
- ▶ (Knowledge on) logics useful for computer science

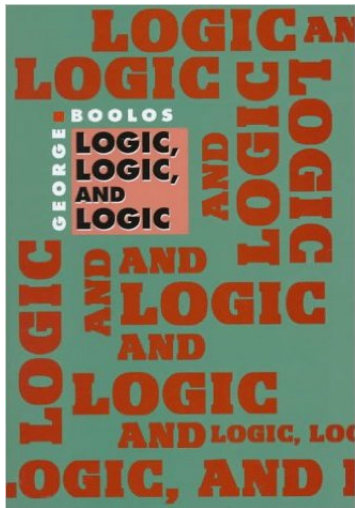


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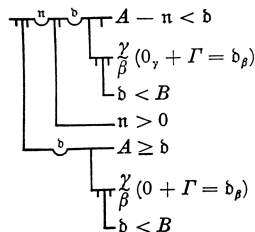


# Logic and Logics

- ▶ Science of logic
  - ▶ investigates mathematical **structures** (static and dynamic)
  - ▶ and formal **languages** to describe them
  - ▶ distinguishing between **syntax**
  - ▶ and **semantics** (truth conditions for sentences)
  - ▶ providing notions of **satisfaction, entailment** (from semantics)
  - ▶ and of **provability, inference (calculus)**
- ▶ Investigated languages with syntax and semantics called logics
- ▶ In particular there are many logics within computer science
- ▶ But in any case somehow related to first-order logic

# First-Order Logic (FOL)

- ▶ Also called predicate logic (or quantification logic)
- ▶ Aristotelian syllogisms already incorporate restricted FOL
  - ▶ All Philosophers are wise men. All wise men are nice. Hence all Philosophers are nice men.
  - ▶ restricted to unary predicates
- ▶ Modern FOL started with Frege's "Begriffsschrift"
  - ▶ language constructs based on constants, variables, predicates, functions, boolean connectives, quantifiers
  - ▶ Formal axioms and inference rules
  - ▶ His 2-dimensional representation format aesthetic but not practical



# FOL Structures

- ▶ A formalism to investigate (mathematical) **structures**

$$\mathfrak{A} = (A, R_1^{\mathfrak{A}}, \dots, R_n^{\mathfrak{A}}, f_1^{\mathfrak{A}}, \dots, f_m^{\mathfrak{A}}, c_1^{\mathfrak{A}}, \dots, c_l^{\mathfrak{A}})$$

- ▶ **(Non-logical) Vocabulary**

- ▶ Relation symbols/predicates  $R_i$  with arities
- ▶ Function symbols  $f_i$  (with arities)
- ▶ Constant symbols  $c_i$

- ▶ Components of the structure

- ▶ **Universe/Domain**  $A$
- ▶ **Interpretations/denotations** of nonlogical symbols
  - ▶ Relation  $R^{\mathfrak{A}} \subseteq A^n$  (for  $n$ -ary relation symbol  $R$ )
  - ▶ Function  $f^{\mathfrak{A}} \in A^n \rightarrow A$  (for  $n$ -ary function symbol  $f$ )
  - ▶ Individuals  $c^{\mathfrak{A}} \in A$  (for constants  $c$ )

## Example FOL Structures

- ▶ **Graphs**  $\mathfrak{G} = (V, E^{\mathfrak{G}})$ 
  1.  $V$  = nodes of the graph
  2.  $E^{\mathfrak{G}} \subseteq V^2$  = edges of the graph
  
- ▶ **Undirected, loopless graphs**  $\mathfrak{G} = (V, E^{\mathfrak{G}})$ 
  1. as above
  2. as above
  3. Additionally: edge relation is symmetric and a-reflexive
  
- ▶ Need an appropriate language to formulate constraints such as in 3.

# FOL Syntax

- ▶ Allow variables ( $x_1, x_2, \dots$ ) and logical constructors
- ▶ **Terms**
  - ▶ variables and constants are terms
  - ▶ if  $t_1, \dots, t_n$  are terms, so is  $f(t_1, \dots, t_n)$  (for  $n$ -ary function symbol  $f$ )
- ▶ **Formulae**
  - ▶  $t_i = t_j$  and  $R(t_1, \dots, t_n)$  (for terms  $t_i$  and  $n$ -ary relation)  $R$
  - ▶ If  $\phi$  is a formula, so are
    - ▶  $\neg\phi$  ("Not  $\phi$ ")
    - ▶  $\forall x \phi$  ("For all  $x$  it holds that  $\phi$ ")
    - ▶  $\exists x \phi$  ("There is an  $x$  s.t.  $\phi$ ")
  - ▶ If  $\phi, \psi$  are formula, so are
    - ▶  $(\phi \wedge \psi)$  (" $\phi$  and  $\psi$ ")
    - ▶  $(\phi \vee \psi)$  (" $\phi$  or  $\psi$ ")
    - ▶  $(\phi \rightarrow \psi)$  ("If  $\phi$  then  $\psi$ ")
    - ▶  $(\phi \leftrightarrow \psi)$  (" $\phi$  iff  $\psi$ ")

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# FOL Semantics

- ▶ **Interpretation**  $\mathcal{I} = (\mathfrak{A}, \nu)$ 
  - ▶  $\nu$  assigns to all variables elements from domain  $A$
  - ▶ Needed to deal with open formulae  
e.g.  $\forall y R(y, x)$  open/free in variable  $x$
  
- ▶  **$x$ -Variant**  $\mathcal{I}_{[x/d]}$   
same as  $\mathcal{I}$  but with  $d \in A$  assigned to  $x$
  
- ▶ **Interpretation of terms**
  - ▶  $\mathcal{I}(c) = c^{\mathfrak{A}}$
  - ▶  $\mathcal{I}(x) = \nu(x)$
  - ▶  $\mathcal{I}(f(t_1, \dots, t_n)) = f^{\mathfrak{A}}(\mathcal{I}(t_1), \dots, \mathcal{I}(t_n))$

# FOL Semantics

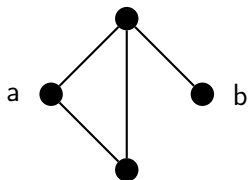
- ▶ **Satisfaction relation**  $\models$ 
  - ▶  $\mathcal{I} \models t_1 = t_2$  iff  $\mathcal{I}(t_1) = \mathcal{I}(t_2)$
  - ▶  $\mathcal{I} \models R(t_1, \dots, t_n)$  iff  $(\mathcal{I}(t_1), \dots, \mathcal{I}(t_n)) \in R^{\mathfrak{A}}$
  
  - ▶  $\mathcal{I} \models \neg\phi$  iff not  $\mathcal{I} \models \phi$
  
  - ▶  $\mathcal{I} \models (\phi \wedge \psi)$  iff  $\mathcal{I} \models \phi$  and  $\mathcal{I} \models \psi$
  - ▶  $\mathcal{I} \models (\phi \vee \psi)$  iff  $\mathcal{I} \models \phi$  or  $\mathcal{I} \models \psi$
  - ▶  $\mathcal{I} \models (\phi \rightarrow \psi)$  iff: If  $\mathcal{I} \models \phi$  then  $\mathcal{I} \models \psi$
  - ▶  $\mathcal{I} \models (\phi \leftrightarrow \psi)$  iff:  $\mathcal{I} \models \phi$  iff  $\mathcal{I} \models \psi$
  
  - ▶  $\mathcal{I} \models \forall x \phi$  iff: For all  $d \in A$ :  $\mathcal{I}_{[x/d]} \models \phi$
  - ▶  $\mathcal{I} \models \exists x \phi$  iff: There is  $d \in A$  s.t.  $\mathcal{I}_{[x/d]} \models \phi$
  
- ▶ Known result:  $\nu$  can be assumed to be defined only for the free variables in the formula.
- ▶ Terminology  $\mathcal{I}$  satisfies  $\phi$ ,  $\mathcal{I}$  makes  $\phi$  true,  $\mathcal{I}$  is a model for  $\phi$
- ▶ We also write  $\mathfrak{A} \models \phi(\vec{x}/\nu)$

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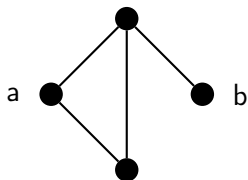
- ▶ Consider loopless, symmetric graphs  $\mathfrak{G} = (G, E^{\mathfrak{G}})$



- ▶  $\phi_1 := \exists x \exists y E(x, y)$   $\mathfrak{G} \models \phi_1?$
- ▶  $\phi_2(x) := \exists y \exists z E(x, y) \wedge E(x, z) \wedge E(y, z)$   $\mathfrak{G} \models \phi_2(x/a)$
- ▶  $\phi_3(x, y) := E(x, y)$   $\mathfrak{G} \models \phi_3(x/a, y/b)$

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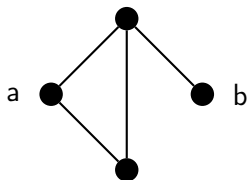
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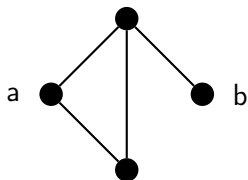
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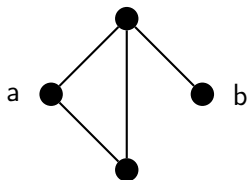
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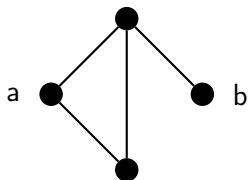


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# Algorithmic Problems in First-Order Logic

## ▶ Model Checking:

- ▶ Input: graph (or generally structure)  $\mathfrak{G}$ , formula  $\phi(x_1, \dots, x_n)$  and assignment  $[x_1/a_1, \dots, x_n/a_n]$
- ▶ Output: Is  $\mathfrak{G} \models \phi(x_1/a_1, \dots, x_n/a_n)$  the case?

## ▶ Satisfiability Problem

- ▶ Input: sentence  $\phi$
- ▶ Output: Does there exist a structure  $\mathfrak{G}$  s.t.  $\mathfrak{G} \models \phi$ ?

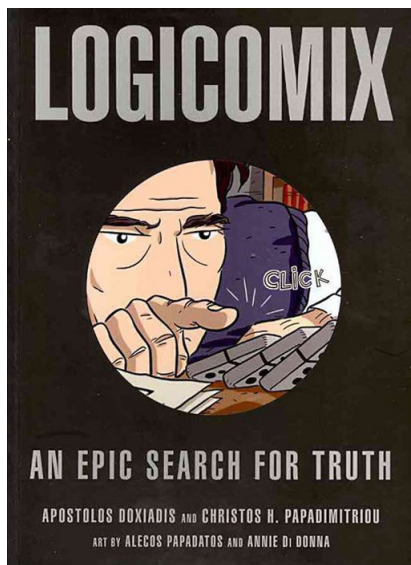
## ▶ Complexity of problems

- ▶ Model checking problem is decidable and PSPACE complete (in combined complexity)
- ▶ Satisfiability is undecidable

# Role of Logic for/in Computer Science

# The Burden of Logic in the 19-20th Century

- ▶ Role of logic as a foundation for all of mathematics
- ▶ Literature hint: Logicomix
  - ▶ fantastic graphic novel
  - ▶ Narrator: Philosopher and logician B. Russell
  - ▶ About the illusions, disillusion, and landmarking results at the end of the 19th century



# Foundations of Mathematics with Mathematical Logic

- ▶ Attempts to find formal foundation for mathematical logic
- ▶ Hilberts Program (1900-1928)
  - ▶ Mathematics is consistent
  - ▶ Mathematics is (semantically) complete
  - ▶ Mathematics is decidable

# Awakening

- ▶ Young Gödel proves (1931-33)
  - ▶ arithmetics not complete
  - ▶ consistency of set theory not provable
- ▶ Church/Turing (1936/37)
  - ▶ First-order logic is not decidable
  - ▶ Valid sentences not recursive
  - ▶ Sentences true in arithmetic not recursively enumerable (semi-decidable)
- ▶ Nonetheless following positive insights
  - ▶ Syntactically completeness for FOL (Gödel, 1930)
  - ▶ ZFC (Zermelo-Fraenkel Set Theory) can be used to formalize all contemporary mathematics

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# The Unusual Effectiveness of Logic

- ▶ Logic (Research) and Computer Science had fruitful effects onto each other
- ▶ Logic even more w.r.t. CS (than w.r.t. mathematics)
- ▶ “Logic is the calculus of CS”

**Lit:** M. Y. Vardi. From philosophical to industrial logics. In Proceedings of the 3rd Indian Conference on Logic and Its Applications, ICLA'09, pages 89–115, Berlin, Heidelberg, 2009. Springer-Verlag.

**Lit:** J. Y. Halpern, R. Harper, N. Immerman, P. G. Kolaitis, M. Y. Vardi, and V. Vianu. On the unusual effectiveness of logic in computer science. Bull. Symbolic Logic, 7(2):213–236, 2001.



# Why is this the Case?

- ▶ Logic is so general that it allows to
  - ▶ talk precisely about the objects within a computer/computation model
  - ▶ specify and reason about the properties of runs in the model
- ▶ Even more: One can characterize complexity classes with logics (Descriptive Complexity)

# Computer Science Areas Effectuated by Logic Research

- ▶ Database Systems
- ▶ Ontology-Based Information Systems
- ▶ Semantic Integration
- ▶ Computer-Aided Verification (Model Checking)
- ▶ Computational Complexity
- ▶ High-Level Stream Processing
- ▶ Multi-Agent Systems
- ▶ Machine Learning (e.g. probabilistic graph models and logics)
- ▶ Semantic Web
- ▶ Logic Programming
- ▶ Knowledge Representation
- ▶ Semantics of Programms
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This course

Other courses of module “Web and Data Science” (CS4513)

This semester: “Web-Mining-Agenten”

# Effects of Computer Science to Logic Research

- ▶ Focus/Intensive research on finite structures
  - ▶ Objects of computation are finite (Finite Model Theory)
  - ▶ But: potentially infinite structures (such as infinite DBs or streams) are useful as well
- ▶ Need for extensions of FOL
  - ▶ Higher-order logics (quantification over sets/relations)
  - ▶ Recursion (Datalog)
- ▶ Feasibility of reasoning services  $\implies$  restrictions of FOL
  - ▶ Modal and temporal logics
  - ▶ Description Logics
- ▶ Connections of logic and automata models
  - ▶ Regular expressions, finite automata, sequential logics
  - ▶ Buechi automata
- ▶ Logic engineering
- ▶ Different forms of inference ...

# Overview on Course With Examples

## Example: Logic in DB Research (Lectures 3-4)

- ▶ Travel DB with direct connection flights
- ▶ Reachability query
- ▶ SQL allows for recursion (CONNECT key word)
- ▶ But is it really necessary?

Table Flight	
Start	End
Hamburg	Berlin
Hamburg	New York
New York	Berlin
...	...

Query  $Q_{reach}$ : List all cities reachable from Hamburg!

Intuitively without recursion:

$$\begin{aligned}Q_{reach}(x) &= Flight(Hamburg, x) \vee \\ &\quad \exists x_1 Flight(Hamburg, x_1) \wedge Flight(x_1, x) \vee \\ &\quad \exists x_1, x_2 Flight(Hamburg, x_2) \wedge Flight(x_2, x_1) \wedge Flight(x_1, x) \vee \\ &\quad \dots\end{aligned}$$

## Example: Logic In DB Research

- ▶ **Finite Model Theory (FMT)** gives a proof for the impossibility to use FOL for recursive of queries
- ▶ FMT models DBs as finite relational FOL structures

### Example

- ▶ Flight table becomes structure  
 $\mathfrak{A} = (D, Flight^{\mathfrak{A}}, Hamburg^{\mathfrak{A}}, Berlin^{\mathfrak{A}}, \dots)$
- ▶ Domain  $D$ : all constants in DB
- ▶ Constants named by themselves, e.g.,  $Hamburg^{\mathfrak{A}} = Hamburg$
- ▶  $Flight^{\mathfrak{A}} = \{(Hamburg, Berlin), (Hamburg, NewYork), \dots\}$



## Example: Logic In DB Research

- ▶ Investigate all relevant reasoning problems w.r.t. finite models
  - ▶ Many properties for classical FOL do not hold
  - ▶ Also w.r.t. complexity
    - ⇒ Calls for new techniques
- ▶ In particular: Investigate properties that all FOL queries have.

### Theorem

*All FOL formulas are **local**.*

*(Holds even for FOL extended with aggregation)*

- ▶ Recursive queries are not local!

## Example: Data Exchange (Lectures 5-6)

- ▶ Deals in a specific way with the integration of DBs
- ▶ Scenario
  - ▶ You have two DBs (source and target) on the same domain but different schemata  $S$  and  $T$
  - ▶ You have some relationship specifications  $\Sigma(T, S)$  for  $T$  and  $S$
  - ▶ Aim: Answer queries over  $T$  to get answers with DBs over  $S$
  - ▶ Subaims: Find (good) instances for  $T$  corresponding to given instances over  $S$  and answer over found solution set.
- ▶ And here comes logic
  - ▶ Language for specifying  $\Sigma_{ST} \implies$  Specific FOL formulas called tuple generating dependencies (tgds)
  - ▶ Criteria for goodness of solutions  $\implies$  universal model notion
  - ▶ How specify answers?  $\implies$  Certain answer semantics

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## Example: Data Exchange

### Example

- ▶  $S$ : student(name)
- ▶  $T$ : univ(sname, uname)
- ▶  $\Sigma_{ST} : student(x) \rightarrow \exists y univ(x, y)$   
“If something is a student in a  $S$ -DB, then there is an associated university in the  $T$ -DB”
- ▶ Example  $T$ -query:  $Q(x) = \exists y.univ(x, y)$
- ▶ What should be the answers for given  $S$ -DB  $I = \{student(Frege)\}$ ?  
 $cert(Q(x), I) = \{Frege\}$

## Example: Querying via Ontologies (Lectures 7-8)

- ▶ Ontologies as formal means to represent and reason over data
- ▶ Ontologies specify constraints and completeness rules
- ▶ Ontologies may have many models (open world assumption)
- ▶ May be used for access of heterogeneous data sources
  
- ▶ Appropriate ontology languages: Description Logics (OWL and variants)
  - ▶ Constants, concepts (unary predicates), roles (binary predicates)
  - ▶ Terminological axioms, e.g.,  $Students \sqsubseteq Humans$
  - ▶ Assertions axioms, e.g.,  $Student(Frege)$
  - ▶ Description logics are **feasible fragments** of FOL

## Example

- ▶ No university known for *Goedel*
- ▶ Completeness:  
 $Student \sqsubseteq \exists hasUniv.University$
- ▶ Functionality constraint:  
(*func hasUniv*)

Student	Univ
Frege	U-Jena
Russell	U-London
Goedel	NULL
...	...

## Example: Ontology Integration (Lectures 9-10)

- ▶ There exist many ontologies out there
- ▶ For some applications need to integrate ontologies
- ▶ Problem: Joining ontologies may lead to incoherences/inconsistencies

### Example

#### Ontology A

- ▶  $Article \equiv \exists publ. Journal$
- ▶  $Journal \sqsubseteq \neg Proceedings$
- ▶  $(func\ publ)$

#### Ontology B

- ▶  $Article \equiv \exists publ. Journal$   
 $\sqcup Proceedings$
- ▶  $publish(ab, procXY)$

- ▶  $O_A \cup O_B$  is inconsistent
- ▶ How to repair this?

# Belief Revision

- ▶ Belief Revision deals with operators for revising theories under possible inconsistencies
- ▶ Investigates concrete revision operators
- ▶ Principles that these must fulfill (minimality etc.)
- ▶ Representation theorems
  
- ▶ Recent research how to adapt these for non-classical logics/ontologies



# Streams (Lectures 11-12)

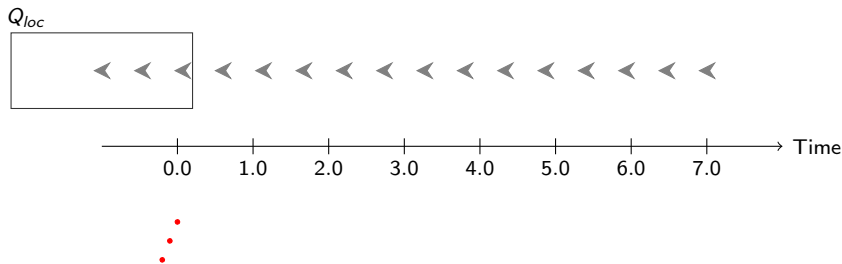
- ▶ “It’s a streaming world” (Ubiquity)
  - ▶ Many data are temporal (sensor, event data)
  - ▶ Big data is mostly temporal data
- ▶ “Streams are forever” (Potential Infinity)
  - ▶ Streams are potentially infinite
  - ▶ One has to tame the infinite
  - ▶ Streams call for continuous **querying** (as in real-time monitoring)
- ▶ “Order Matters” (Sequentiality)
  - ▶ Stream elements have an arriving order next to temporal order
  - ▶ Re-ordering or special sequencing may be needed

**Lit:** E. Della Valle. et al. It’s a streaming world! Reasoning upon rapidly changing information. *Intelligent Systems, IEEE*, 24(6):83–89, nov.-dec. 2009.

**Lit:** J. Endrullis, D. Hendriks, and J. W. Klop. Streams are forever. *Bulletin of the EATCS*, 109:70–106, 2013.

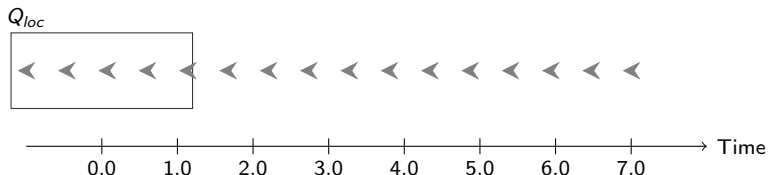
**Lit:** E. D. Valle et al. Order matters! Harnessing a world of orderings for reasoning over massive data. *Semantic Web*, 4(2):219–231, 2013.

# Query With Sliding Window



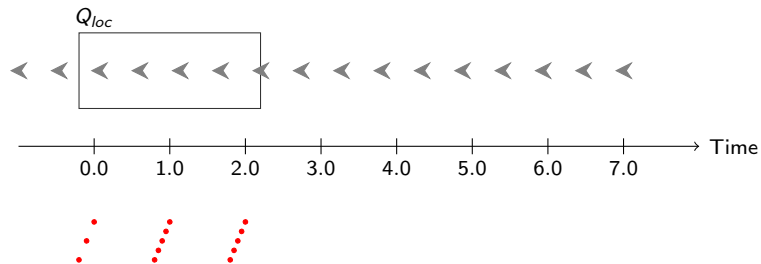
- ▶ Sliding window for taming the infinite
- ▶ **Query** window contents with local query  $Q_{loc}$
- ▶ Example:  $Q_{loc} = \text{Show all failure events in the window}$
- ▶ For High-Level Stream Processing: Incorporate background knowledge

# Query With Sliding Window



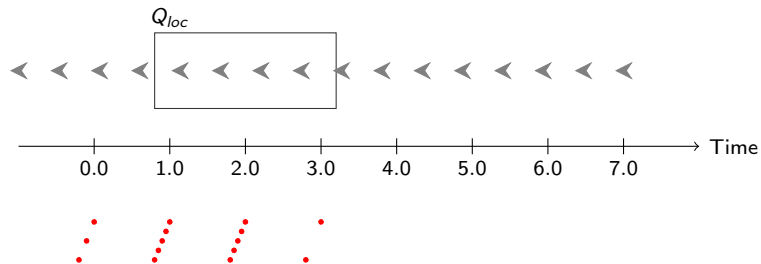
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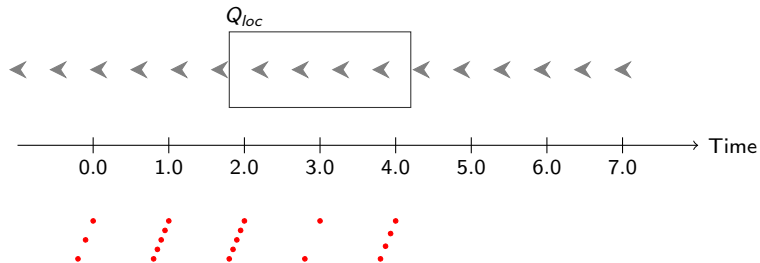
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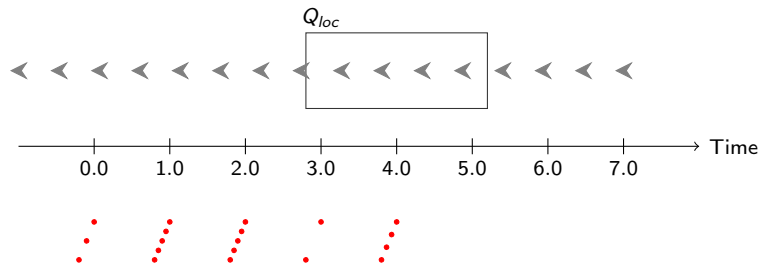
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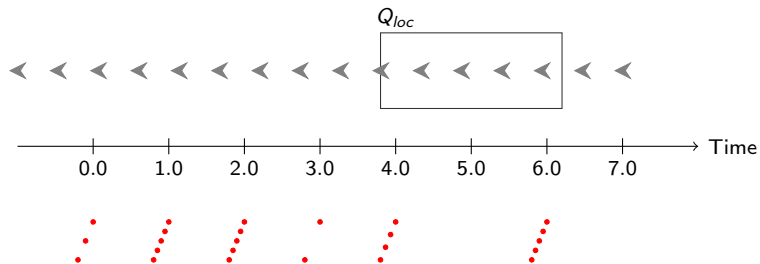
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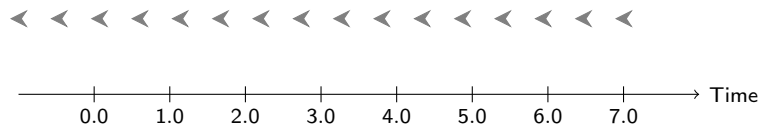
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## Process Verification (Lectures 12-13)

- ▶ Verification of system behavior very important for industrial applications
- ▶ Model Checking mature theory with well-proven software implementations used in industry
  - ▶ given a system description (model) and (desired) specifications (axioms in **temporal logics**)
  - ▶ Check whether (all runs of the) model fulfills specification

### Example (Linear Temporal Logic)

- ▶ Excluding unwanted conditions for every time point

$$\square \neg(\text{turbineTemp} > 90^\circ)$$

- ▶ Ensuring wanted conditions

$$\square(\text{startTurbine} \rightarrow \diamond \text{TurbineIsRunning})$$

# Process Verification

- ▶ Lift verification ideas/tools to verify business processes
- ▶ Challenges
  - ▶ Have to incorporate (large amounts of) data  
⇒ artifact-centric approach (early 2000)
  - ▶ Finite state models not sufficient  
⇒ finite state transducers

## Exercise 1 (6 points)

Describe an example application or a computer science sub-area from your CS studies or from your job which exemplifies the “use” of some form of logic. In particular answer the following questions (on 2-3 slides in pdf):

1. What kind of logic is used?
  2. What is its relation to FOL?
  3. How/why is it used in the area/application?
- ▶ Send your solutions in one pdf file as presentation until Tuesday evening, 27th of October 2015 to [oezcep@ifis.uni-luebeck.de](mailto:oezcep@ifis.uni-luebeck.de).
  - ▶ You may work in pairs
  - ▶ State your name, your study course (Studiengang) and your identity number (Matrikelnummer) at the title page