Web-Mining Agents
Stream Mining

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Mining Data Streams (Part 1)

Mining of Massive Datasets

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New Topic: Infinite Data

High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	PageRank, SimRank	Filtering data streams	SVM	Recommen der systems
Clustering	Community Detection	Queries on streams	Decision Trees	Association Rules
Dimensional ity reduction	Spam Detection	Web advertising	Perceptron, kNN	Duplicate document detection

Data Streams

- **In many data mining situations, we do not know the entire data set in advance**

- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

Static Data vs. Stream Data

Static data	Stream of data
Fixed number of data elements (unlimited memory usage)	Potentially infinite number of data elements (memory limitation problem)
Stationary distribution of data	Changing data distribution (concept drift)
All data available at any time – multiple passes of data	One pass of data
Unlimited processing time	Processing time depends on rate of incoming data

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Ex: Leszek Rutkowski WCCI' 2016 Tutorial, Vancouver, July 24, 2016

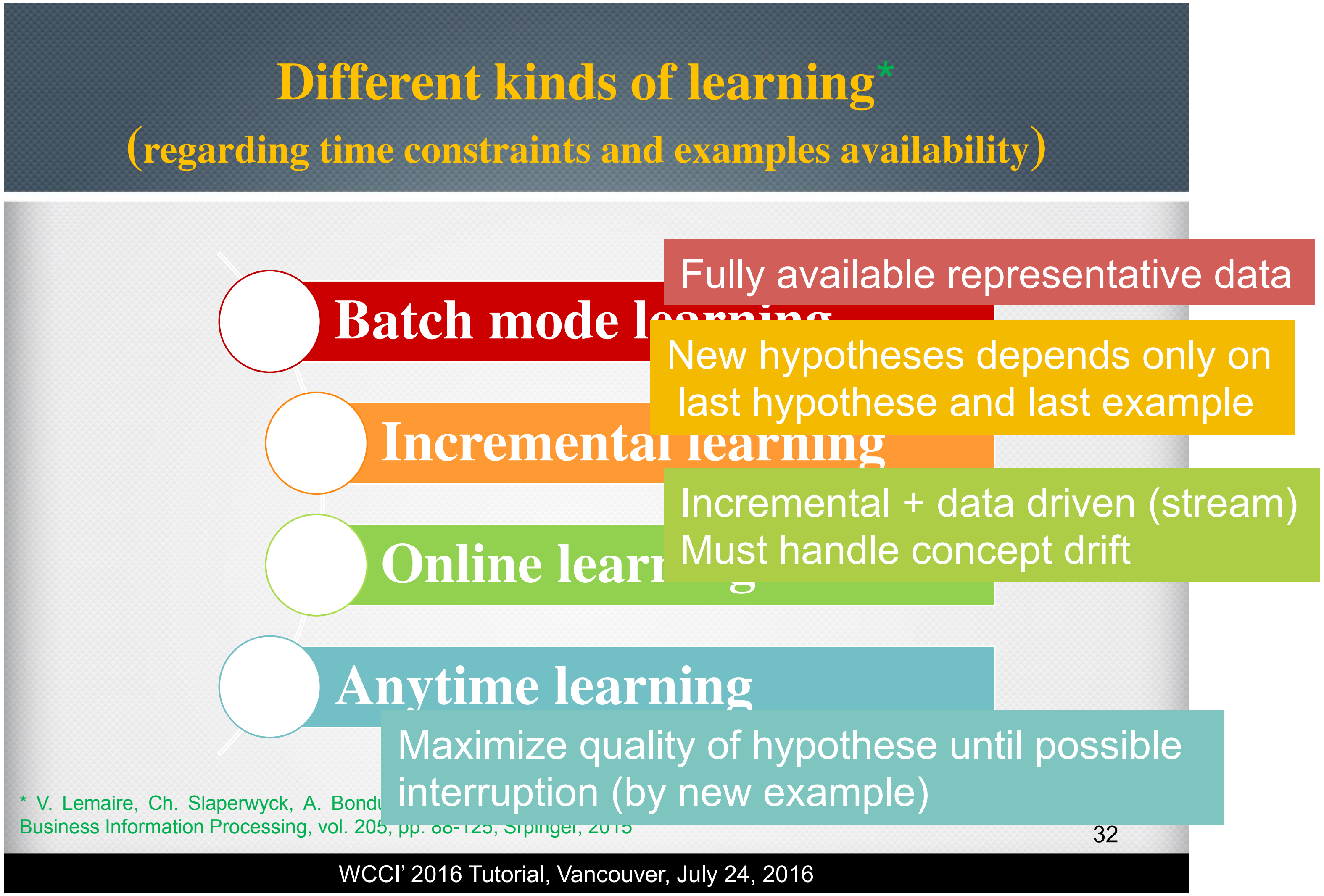
The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

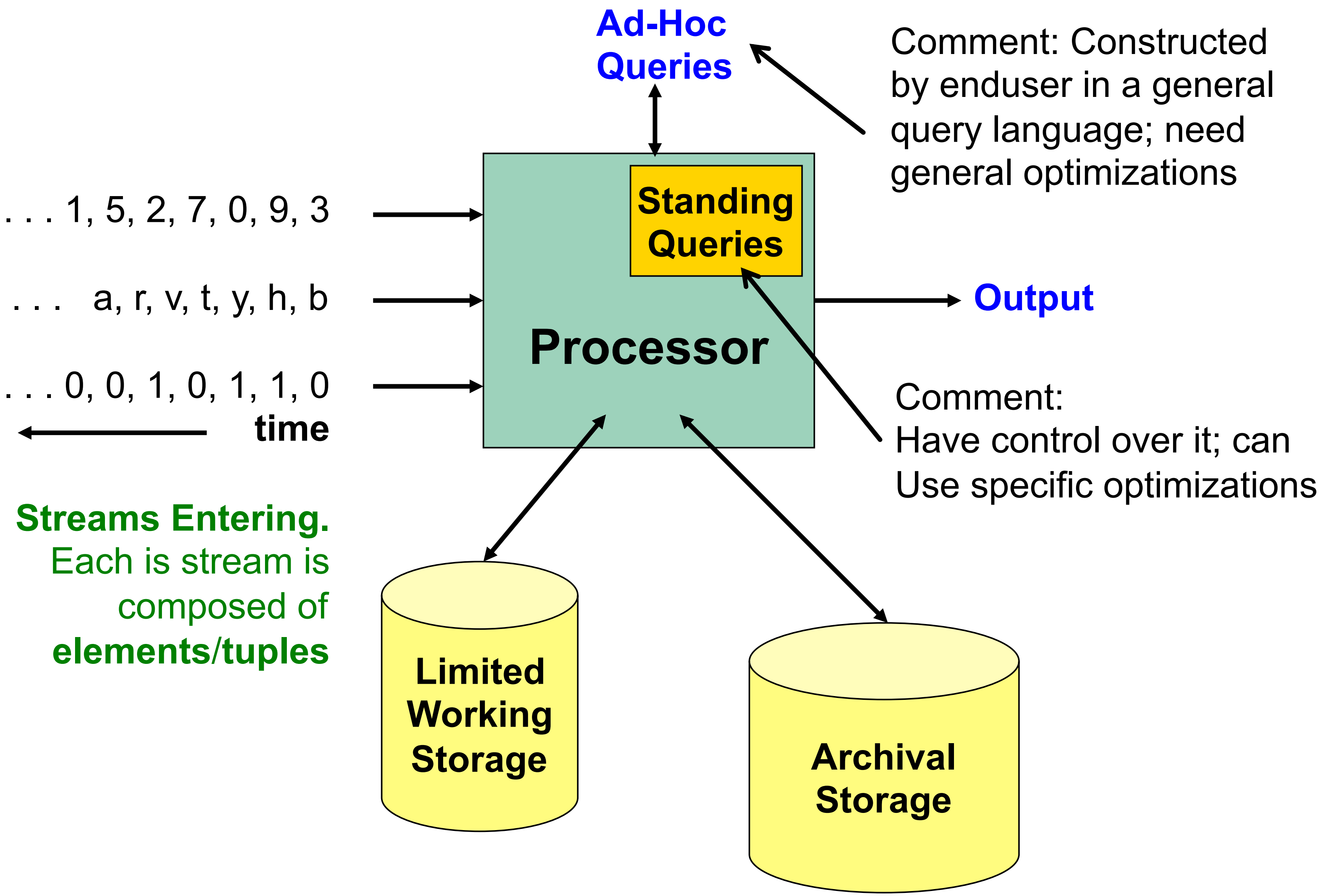
Side note: SGD is a Streaming Alg.

- **Stochastic Gradient Descent (SGD) is an example of a stream algorithm**
- **In Machine Learning we call this: Online Learning**
  - Allows for modeling problems where we have a continuous stream of data
  - We want an algorithm to learn from it and slowly adapt to the changes in data
- **Idea: Do slow updates to the model**
  - **SGD** (SVM, Perceptron) makes small updates
  - **So:** First train the classifier on training data.
  - **Then:** For every example from the stream, we slightly update the model (using small learning rate)

Learning Modes



General Stream Processing Model



Problems on Data Streams

- **Types of queries one wants to answer on a data stream:** (we'll do these today)
  - **Sampling data from a stream**
    - Construct a random sample

    Comment:
    Challenging because we need
    reliable & sufficient statistics
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

  (Comment: This is only one example among others)

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:**
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of last ***k*** elements
  - **Finding frequent elements**

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday
- **Mining click streams**
  - Yahoo (well…) wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as:** **How often did a user run the same query in a single day**
  - Have space to store **1/10th** of query stream
- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

- **Simple question: What fraction of queries by an average search engine user are duplicates?**
  - Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** queries)
    - **Correct answer: *d*/(*x*+*d*)**
  - **Proposed solution: We keep 10% of the queries**
    - Sample will contain ***x*/10** of the singleton queries and **2*d*/10** of the duplicate queries at least once
    - But only ***d*/100** pairs of duplicates
      - **$d/100 = 1/10 \cdot 1/10 \cdot d$**
    - Of ***d*** "duplicates" ***18d/100*** appear exactly once
      - **$18d/100 = ((1/10 \cdot 9/10)+(9/10 \cdot 1/10)) \cdot d$**
  - **So the sample-based answer is:**

$$d/100 /(x/10 + 18d/100 + d/100)= \mathbf{d/(10x + 19d)}$$

$$\neq d/(x+d)$$

Solution: Sample Users

Solution:

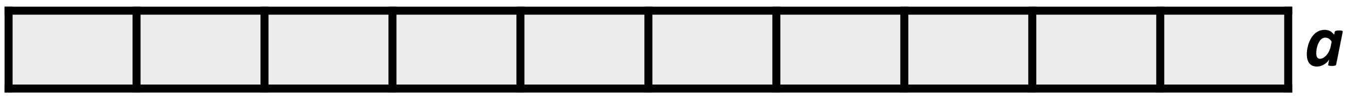
- Pick **1/10th** of **users** and take all their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); and here: key is **user**
  - Choice of key depends on application

- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose at time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the n^{th} element arrives ($n > s$)
    - With probability ***s/n***, keep the n^{th} element, else discard it
    - If we picked the n^{th} element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

- **We prove this by induction:**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing element ***n+1*** the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now element *n+1* arrives**
- **Inductive step:** For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1,*** tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1* is**

n/(n+1)* s/n = s/(n+1)

Queries over a (long) Sliding Window

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received
- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want answer queries, how many times have we sold **X** in the last **k** sales

Sliding Window: 1 Stream

- **Sliding window on a single stream:**

N = 6

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Counting Bits (1)

- **Problem:**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form
    How many 1s are in the last *k* bits? where $k \leq N$
- **Obvious solution:**
  Store the most recent ***N*** bits
  - When new bit comes in, discard the **1**st (from left to right) bit in old window

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2, Addendum)

- **You can not get an exact answer without storing the entire window**
- **Proof (by contradiction):**
  - Assume R(w) is representation of NumberOfOnes(w) with < N bits
  - There are 2^N different windows
  - #{ R(w) | w an N-window} < 2^N
  - Hence, there is w,v with R(w) = R(v)
  - Assume (k-1)-suffixes of w and v are same but w and v differ at k (from the right)
  - But NumberofOnes(w) ≠ NumberOfOnes(v)

Counting Bits (3)

- **You can not get an exact answer without storing the entire window**

- **Real Problem:**
  What if we cannot afford to store *N* bits?
  - **E.g.**, we're processing 1 billion streams and
    ***N* = 1 billion**

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 ~~1 1 0 1 1 0~~

← Past   Future →

- **But we are happy with an approximate answer**

An attempt: Simple solution

- **Q: How many 1s are in the last *N* bits?**
- A simple solution that does not really solve our problem: **Uniformity assumption**

← ***N*** →

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

← Past Future →

- **Maintain 2 counters:**
  - ***S***: number of 1s from the beginning of the stream
  - ***Z***: number of 0s from the beginning of the stream
- **How many 1s are in the last N bits?**
- **But, what if stream is non-uniform?**
  - What if distribution changes over time?

  (-> concept drift)

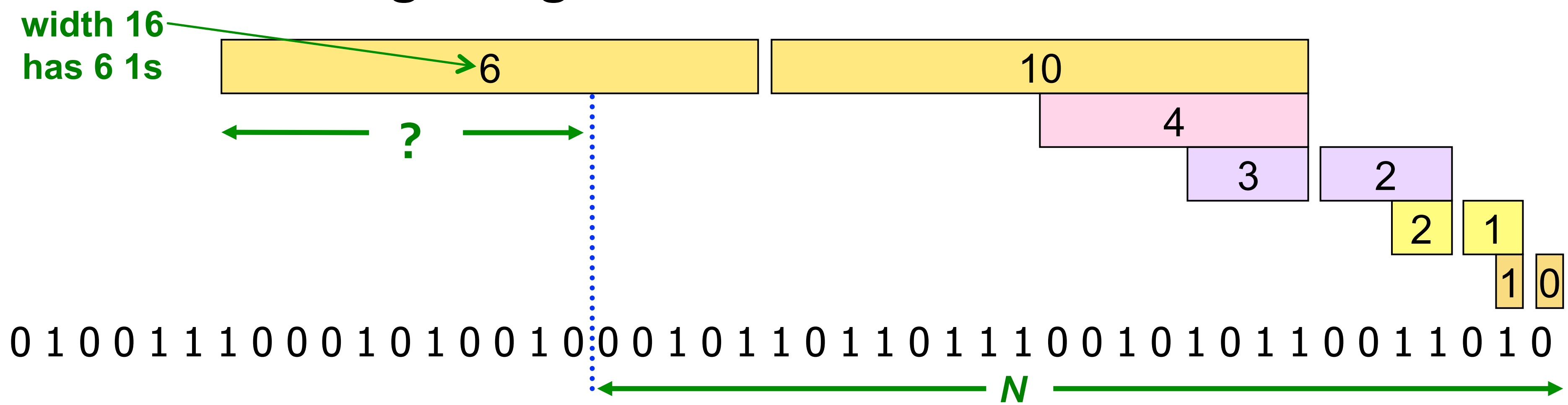
DGIM Method

[Datar, Gionis, Indyk, Motwani]

- **DGIM solution that does <u>not</u> assume uniformity**
- We store $O(\log^2 N)$ bits per stream
- **Solution gives approximate answer, never off by more than 50%**
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits

Idea: Exponential Windows

- **Solution that doesn't (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



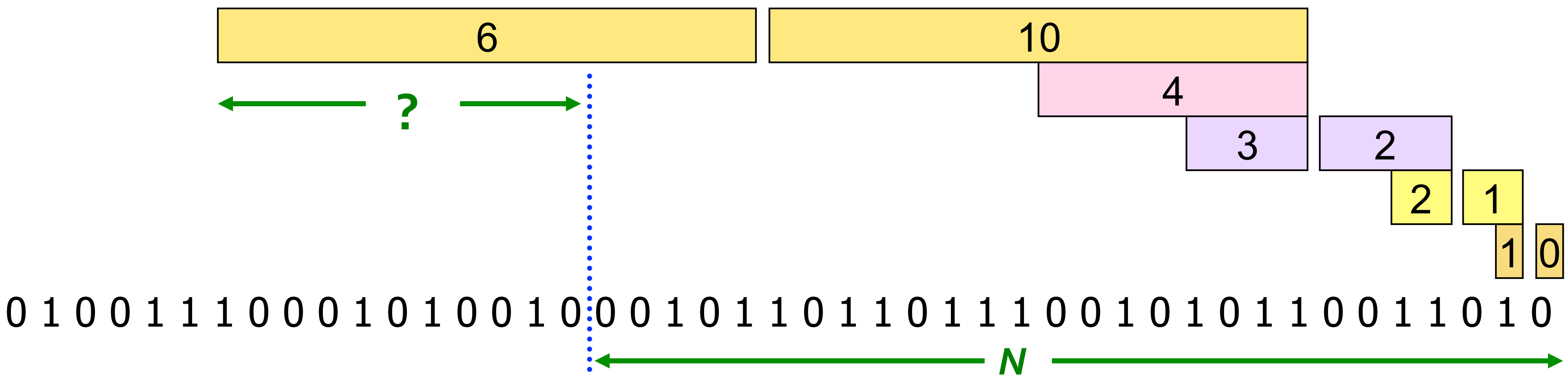
We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

- **Stores only O(log^2*N*) bits**
  - $O(\log N)$ counts O(log N) bits each
- **Easy update as more bits enter**
- Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the error is unbounded!**

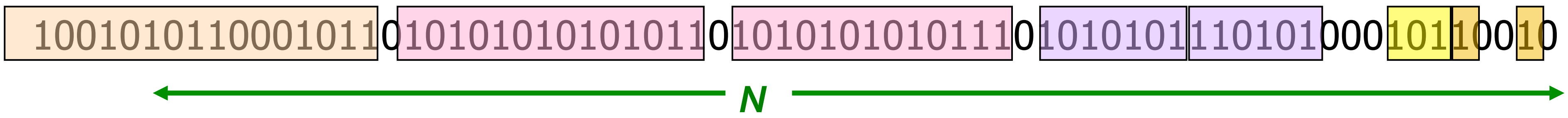


Fixup: DGIM method

[Datar, Gionis, Indyk, Motwani]

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block ***sizes*** (number of **1s**) increase exponentially

- **When there are few 1s in the window, block sizes stay small, so errors are small**

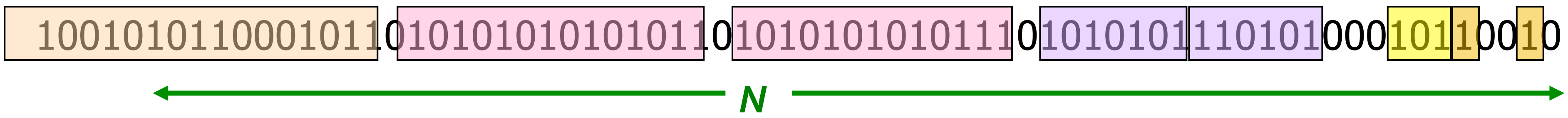


DGIM: Timestamps

- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …
- Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in $O(\log_2 N)$ bits

DGIM: Buckets

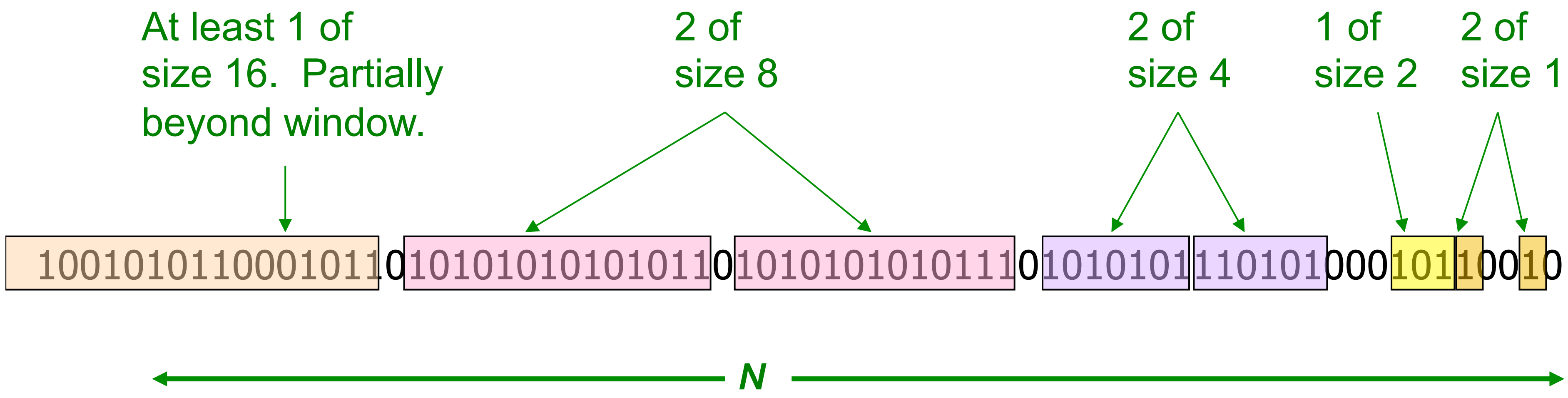
- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its right end** [O(log *N*) bits]
  - **(B) The number of 1s between its beginning and end** [O(log log *N*) bits]
- **Constraint on buckets:**
  Number of **1s** must be a power of **2**
  - That explains the **O(log log *N*)** in **(B) above**



Representing a Stream by Buckets

- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**
- **Buckets do not overlap in timestamps**
- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is **>** ***N*** time units in the past
- Addendum: Right end of bucket always a 1

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time

- **2 cases:** Current bit is **0** or **1**

- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

- **If the current bit is 1:**
  - **(1)** Create a new bucket of size **1**, for just this bit
    - **End timestamp = current time**
  - **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
  - **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
  - **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

1001010110001011010101010101011010101010101110101010111010100010110010

Bit of value 1 arrives

0010101100010110101010101010110101010101011101010101110101000101100101

Two orange buckets get merged into a yellow bucket

0010101100010110101010101010110101010101011101010101110101000101100101

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

0101100010110101010101010110101010101011101010101110101000101100101101

Buckets get merged…

0101100010110101010101010110101010101011101010101110101000101100101101

State of the buckets after merging

0101100010110101010101010110101010101011101010101110101000101100101101

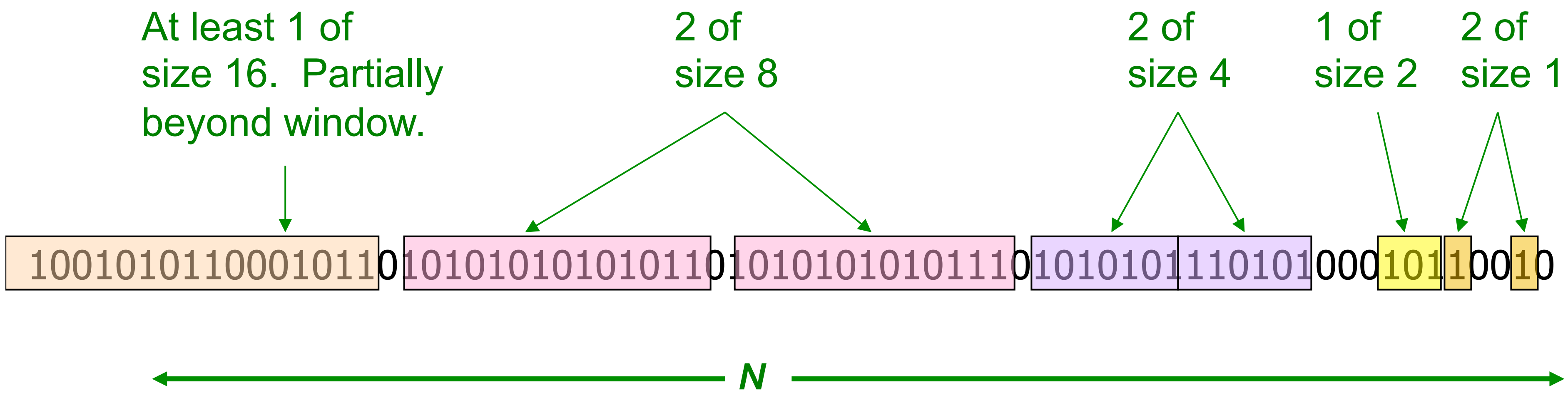
How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets (from right) but the last**

     (note "size" means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

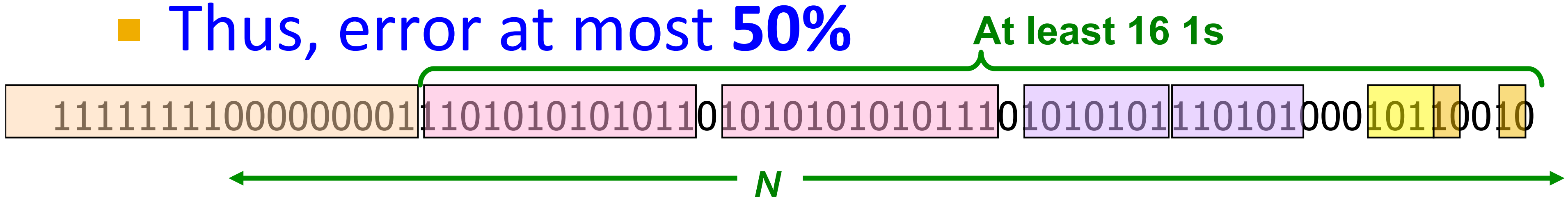
- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

Example: Bucketized Stream



Error Bound: Proof

- **Why is error 50%? Let's prove it!**
- Suppose the last bucket has size $\mathbf{2^r}$
- Then by assuming $\mathbf{2^{r-1}}$ (i.e., half) of its **1s** are still within the window, we make an error of at most $\mathbf{2^{r-1}}$
- Since there is at least one bucket of each of the sizes less than $\mathbf{2^r}$, the true sum is at least $\mathbf{1 + 2 + 4 + .. + 2^{r-1} = 2^r - 1}$
- Thus, error at most **50%**



Further Reducing the Error

- Instead of maintaining **1** or **2** of each size bucket, we allow either ***r*-1** or ***r*** buckets (***r* > 2**)
  - Except for the largest size buckets; we can have any number between **1** and ***r*** of those
- **Error is at most *O*(1/*r*)**
- By picking ***r*** appropriately, we can tradeoff between number of bits we store and the error

Extensions

- Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k* < *N***?
  - **A:** Find earliest bucket **B** that at overlaps with ***k***. Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**

1001010110001011 0 1010101010101011 0 10101010101111 0 10101011 101011 000 101 1 00 1 0

← *k* →

- **Can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?**

Extensions

- **Stream of positive integers**
- **We want the sum of the last *k* elements**
  - **Amazon:** Avg. price of last **k** sales
- **Solution:**
  - **(1) If you know all have at most *m* bits**
    - Treat ***m*** bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer
    - The sum is $=\sum_{i=0}^{m-1} c_i 2^i$

c_i …estimated count for **i-th** bit

  - **(2) Use buckets to keep partial sums**
    - **Sum of elements in size *b* bucket is at most 2^b**

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 3 5 7 1 3 3 1 2 2 6
2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 3 5 7 1 3 3 1 2 2 6 3
2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 3 5 7 1 3 3 1 2 2 6 3 2
2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 3 5 7 1 3 3 1 2 2 6 3 2 5

Idea: Sum in each bucket is at most **2^b** (unless bucket has only **1** integer)

Bucket sizes:

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Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last N elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements

- **The following is based on slides of Ullmann: “More Clustering”**

infolab.stanford.edu/~ullman/mining/pdf/cs345-cl2new.pdf

Clustering a Stream (New Topic)

- Assume points enter in a stream.
- Maintain a sliding window of points.
- Queries ask for clusters of points within some suffix of the window.
- Only important issue: where are the cluster centroids.
  - There is no notion of "all the points" in a stream.

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BDMO Approach

- BDMO = Babcock, Datar, Motwani, O'Callaghan.
- *k* –means based.
- Can use less than O(*N*) space for windows of size *N*.
- Generalizes trick of DGIM: buckets of increasing "weight."

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Recall DGIM

- Maintains a sequence of buckets B_1, B_2, …
- Buckets have timestamps (most recent stream element in bucket).
- Sizes of buckets nondecreasing.
  - In DGIM size = power of 2.
- Either 1 or 2 of each size.

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Buckets for Clustering

- In place of "size" (number of 1's) we use (an approximation to) the sum of the distances from all points to the centroid of their cluster.
- Merge consecutive buckets if the "size" of the merged bucket is less than the sum of the sizes of all later buckets.

Consequence of Merge Rule

- In a stable list of buckets, any two consecutive buckets are "bigger" than all smaller buckets.
- Thus, "sizes" grow exponentially.
- If there is a limit on total "size," then the number of buckets is O(log N).
    - N = window size.
  - E.g., all points are in a fixed hypercube.

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Outline of Algorithm

1. What do buckets look like?
   - Clusters at various levels, represented by centroids.
2. How do we merge buckets?
   - Keep # of clusters at each level small.
3. What happens when we query?
   - Final clustering of all clusters of all relevant buckets.

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Organization of Buckets

- Each bucket consists of clusters at some number of levels.
  - 4 levels in our examples.
- Clusters represented by:
  1. Location of centroid.
  2. *Weight* = number of points in the cluster.
  3. *Cost* = upper bound on sum of distances from member points to centroid.

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Processing Buckets --- (2)

- Initialize a new bucket with *k* new points.
  - Each is a cluster at level 0.
- If the timestamp of the oldest bucket is outside the window, delete that bucket.

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Level-0 Clusters

- A single point *p* is represented by (*p*, 1, 0).
- That is:
  1. A point is its own centroid.
  2. The cluster has one point.
  3. The sum of distances to the centroid is 0.

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[redacted]

Merging Buckets --- (1)

- Needed in two situations:
  1. We have to process a query, which requires us to (temporarily) merge some tail of the bucket sequence.
  2. We have just added a new (most recent) bucket and we need to check the rule about two consecutive buckets being "bigger" than all that follow.

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Merging Buckets --- (2)

- Step 1: Take the union of the clusters at each level.
- Step 2: If the number of clusters (points) at level 0 is now more than $N^{1/4}$, cluster them into k clusters.
  - These become clusters at level 1.
- Steps 3,…: Repeat, going up the levels, if needed.

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Representing New Clusters

- Centroid = weighted average of centroids of component clusters.
- Weight = sum of weights.
- Cost = sum over all component clusters of:
  1. Cost of component cluster.
  2. Weight of component times distance from its centroid to new centroid.

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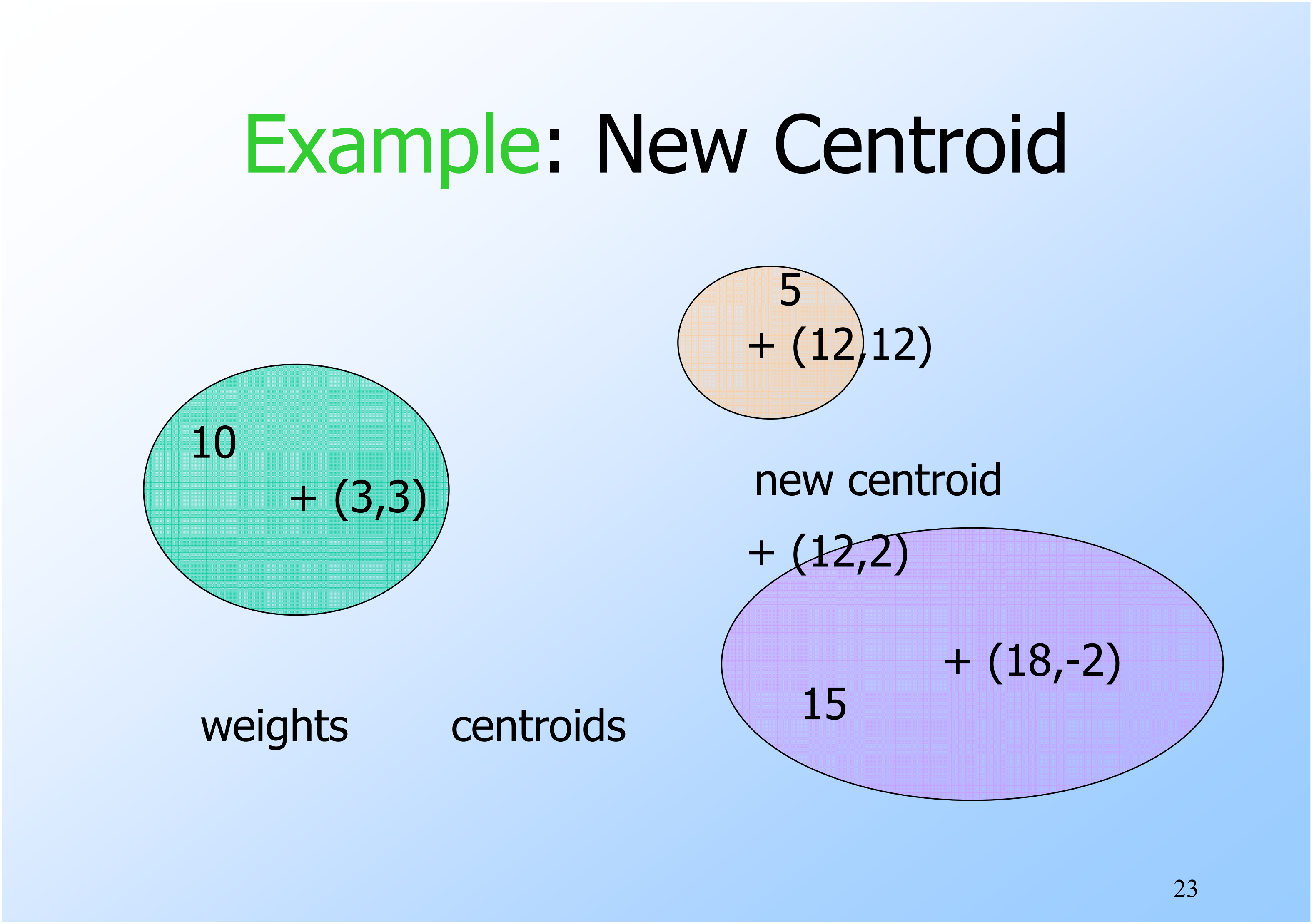
Addendum: Cost required in sitations where clusters in consecutive buckets change rapidly


Example: New Centroid
5
+ (12,12)
10
+ (3,3)
new centroid
+ (12,2)
+ (18,-2)
15
weights
centroids
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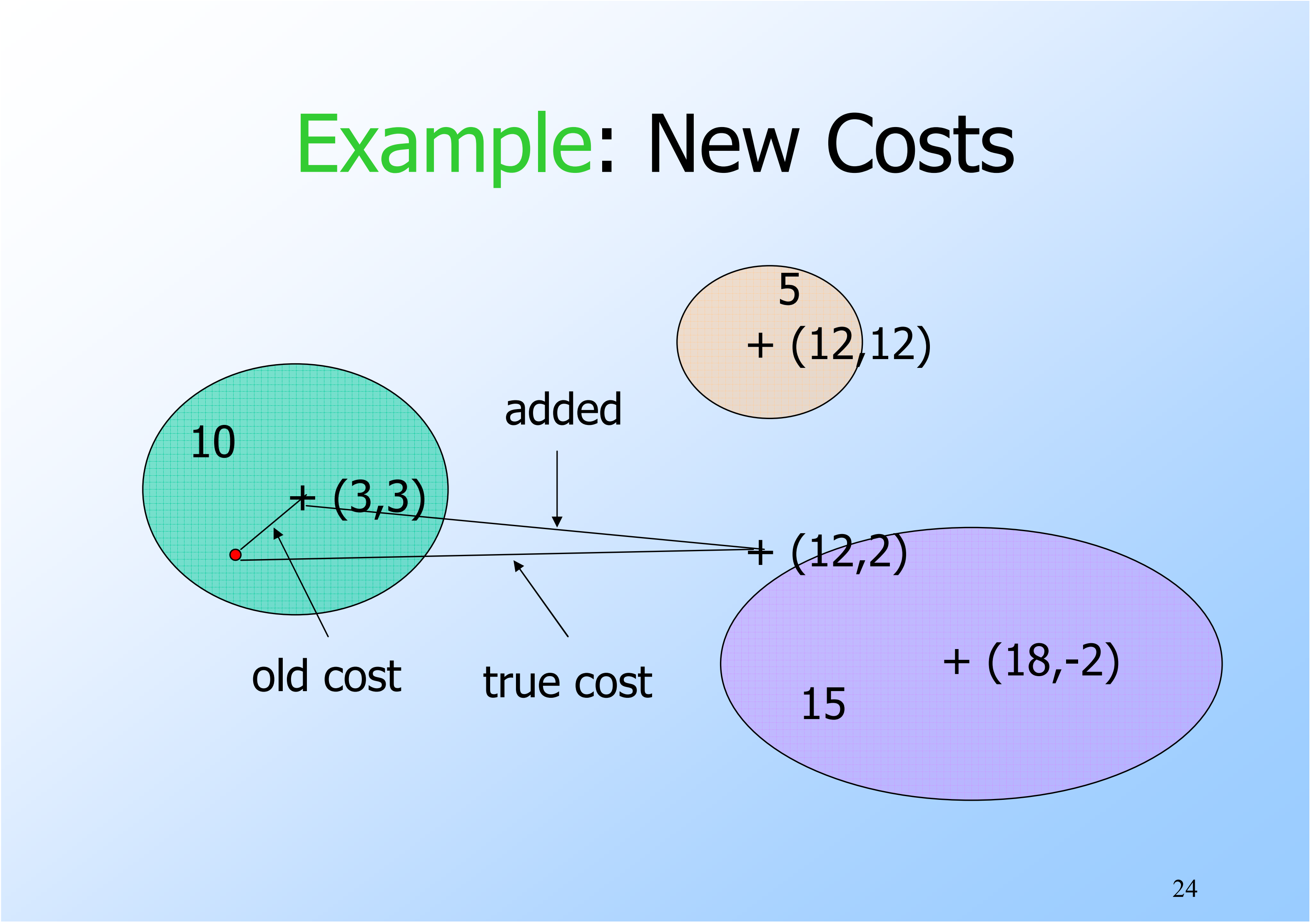
Example: New Costs
5
+ (12,12)
added
10
+ (3,3)
+ (12,2)
+ (18,-2)
old cost
true cost
15
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Addendum: E.g., Give k clusters w.r.t. last m points

Queries

- Find all the buckets within the range of the query.
  - The last bucket may be only partially within the range.
- Cluster all clusters at all levels into *k* clusters.
- Return the *k* centroids.

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Error in Estimation

- Goal is to pick the *k* centroids that minimize the *true* cost (sum of distances from each point to its centroid).
- Since recorded "costs" are inexact, there can be a factor of 2 error at each level.
- Additional error because some of last bucket may not belong.
  - But fraction of spurious points is small (why?).

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Addendum : because of (triangle inequality)

Effect of Cost-Errors

1. Alter when buckets get combined.
   - Not really important.
2. Produce suboptimal clustering at any stage of the algorithm.
   - The real measure of how bad the output is.

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