UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR INFORMATIONSSYSTEME

Özgür L. Özçep

Ontology-Based Data Access

Lecture 7: Motivation, Description Logics

6 December, 2017

Foundations of Ontologies and Databases for Information Systems

CS5130 (Winter 17/18)

Recap of Lecture 6

Data Exchange

- Specific semantic integration scenario for two data sources with possibly different schemata
- Mapping $\mathcal{M} = (\sigma, \tau, M_{\sigma\tau}, M_\tau)$
  - σ: source schema
  - τ: target schema
  - $M_{\sigma\tau}$: source target dependencies (mostly: st-tgds)
  - M_τ: target dependencies
- Ultimate aim: For given σ instance find appropriate τ instance (solution) to do query answering on it
- Chase construction gave universal model: model with weakest assumptions
- Universal model may contain redundancies: considered cores; but as universal models are sufficient and cores may be costly, sticked to universal models
- Looked at certain answering and the use of rewriting to yield certain answers

End of Recap

References

- ESSLLI 2010 Course by Calvanese and Zakharyaschev

  http://www.inf.unibz.it/~calvanese/teaching/2010-08-ESSLLI-DL-QA/
- Reasoning Web Summer School 2014 course by Kontchakov on Description Logics

  http:

  //rw2014.di.uoa.gr/sites/default/files/slides/An_Introduction_to_Description_Logics.pdf
- Lecture notes by Calvanese in 2013/2014 course on Ontology and Database Systems

  https://www.inf.unibz.it/~calvanese/teaching/14-15-odbs/lecture-notes/
- Course notes by Franz Baader on Description Logics
- Parts of Reasoning Web Summer School 2014 course by Ö. on Ontology-Based Data Access on Temporal and Streaming Data

  http://rw2014.di.uoa.gr/sites/default/files/slides/Ontology_Based_Data_Access_on_

  Temporal_and_Streaming_Data.pdf

Ontology-Based Data Access as Integration

- Data Exchange can be considered as semantic integration purely on DB level

- OBDA can be considered as integration using an ontology
- Bridges DB world (closes world assumption) and ontology world (open world assumption)

Closed World Assumption

- DB theory: closed-world assumption (CWA)
  - All and only those facts mentioned in DB hold.
- Simple form of uncertain knowledge expressed by NULLs
  - For one incomplete DB there are many completions
  - Nonetheless: Type information on attribute constrains the possible attribute instances
- In DE incompleteness generated by different schemata
  Flight scenario: Source DB had no flight number, whilst target DB has
  $\Longrightarrow$ introduction of NULLs for flight number attribute
- Logical theories (ontologies) adhere to open world assumption (OWA)
  - If something is not told, then we do not know
  - Logical theories (ontologies) may have many models

Closed World Assumption

- DB theory: closed-world assumption (CWA)
  - All and only those facts mentioned in DB hold.
- Simple form of uncertain knowledge expressed by NULLs
  - For one incomplete DB there are many completions
  - Nonetheless: Type information on attribute constrains the possible attribute instances

- In DE incompleteness generated by different schemata
  Flight scenario: Source DB had no flight number, whilst target DB has
  $\Longrightarrow$ introduction of NULLs for flight number attribute

- Logical theories (ontologies) adhere to open world assumption (OWA)
  - If something is not told, then we do not know
  - Logical theories (ontologies) may have many models

Closed World Assumption

- DB theory: closed-world assumption (CWA)
  - All and only those facts mentioned in DB hold.
- Simple form of uncertain knowledge expressed by NULLs
  - For one incomplete DB there are many completions
  - Nonetheless: Type information on attribute constrains the possible attribute instances
- In DE incompleteness generated by different schemata
  Flight scenario: Source DB had no flight number, whilst target DB has
  $\Longrightarrow$ introduction of NULLs for flight number attribute
- Logical theories (ontologies) adhere to open world assumption (OWA)
  - If something is not told, then we do not know
  - Logical theories (ontologies) may have many models

Closed World Assumption

- DB theory: closed-world assumption (CWA)
  - All and only those facts mentioned in DB hold.
- Simple form of uncertain knowledge expressed by NULLs
  - For one incomplete DB there are many completions
  - Nonetheless: Type information on attribute constrains the possible attribute instances
- In DE incompleteness generated by different schemata
  Flight scenario: Source DB had no flight number, whilst target DB has
  $\Longrightarrow$ introduction of NULLs for flight number attribute
- Logical theories (ontologies) adhere to open world assumption (OWA)
  - If something is not told, then we do not know
  - Logical theories (ontologies) may have many models

http://www.datasciencecentral.com/profiles/blogs/data-modeling-and-the-open-world-assumption

Data Science and the Open World Assumption

Posted by Kurt Cagle on May 7, 2015 at 8:32pm View Blog

A funny thing happened in the last few years. We began to lose the Closed World Assumption.

Now I can understand that this is not exactly huge, earth-shattering news; most people do not in fact realize that they've been using the Closed World Assumption to begin with. However, I'd contend that this event is having a transformative effect upon the way that we interact with data, one that may very well change the perspective about information in ways perhaps as profound as Ted Codd's introduction of the relational model in the 1970s.

Open the Closed World Doors

In basic terms, the closed world assumption can be stated as "When we model something, our model is complete." Most people who have had to define a data model recognize that this statement is at best a convenient fiction - any effort to completely define almost any object ultimately comes down to identifying which attributes of that object are relevant to the particular business domain - yet even with this observation, the necessity of restricting attributes is so fundamental to the way that models are designed and built that it is seldom challenged.

Close-World Assumption (CWA) for DBs

- "The world described by DBs is compete"

Example

University employee

ID	Name
1	Sokrates
2	Platon
3	Aristotle

Professor

ID
1
2

"3" (= ID of Aristotle) not in table Professor
$\Longrightarrow$ Aristotle is not a professor

Close-World Assumption (CWA) for DBs

- "The world described by DBs is compete"

Example

Patient

ID	Name
1	Sokrates
2	Platon
3	Aristotle

Blood sugar

ID	value
1	90
2	120

"3" not in blood sugar table
$\Longrightarrow$? Aristotle has not blood sugar value?

NULLs

- NULLs intended to model incompleteness
- but semantics not clear and hence highly criticized

Lit: L. Libkin. SQL's three-valued logic and certain answers. ACM Trans. Database Syst., 41(1):1:1–1:28, 2016.

Example

Patient

ID	Name
1	Sokrates
2	Platon
3	Aristotle

Bloo sugar

ID	value [30-600]
1	90
2	120
3	NULL

Aristotle has a blood sugar value (30 or 31 or . . .)

NULLs

- NULLs intended to model incompleteness
- but semantics not clear and hence highly criticized

Lit: L. Libkin. SQL's three-valued logic and certain answers. ACM Trans. Database Syst., 41(1):1:1–1:28, 2016.

Example

Patient

ID	Name
1	Sokrates
2	Platon
3	Aristotle
4	Xanthippe
5	Leda

Pregnancy

ID	HCG value
1	NULL
2	NULL
3	NULL
4	NULL
5	130

- Male patient with NULL: no HCG test
- Female patient with NULL: not HCG test (but she has HCG value) or HCG test & not known

Semi-Open-World in DBs and Certain answers

- NULLs require considering many models (completions of incomplete DB)

  (compare lectures on DE)

Definition (Certain answers over incomplete DB (informally))

$$cert(Q, \mathfrak{T}) \quad = \quad \text{intersection of answers over all complete DBs represented by } \mathfrak{T}$$

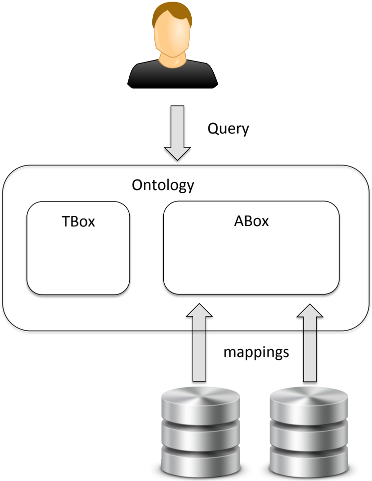
OBDA: Motivation and Overview

Ontology-Based Data Access

- Use ontologies as interface

  ...
- to access (here: query)
- data stored in some format

  ...
- using mappings

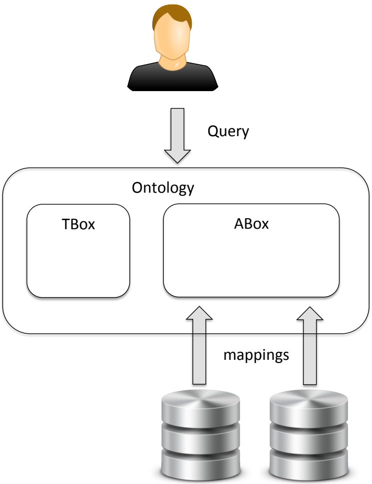



Ontology (Australia) Paradise

Backend DB Dungeon

Ontology-Based Data Access

- Use ontologies as interface
  ...
- to access (here: query)
- data stored in some format
  ...
- using mappings



Ontologies

- Ontologies are triples of the form $\mathcal{O} = (\sigma, \mathcal{T}, \mathcal{A})$
  - Signature σ: Non-logical vocabulary
    $\sigma = \mathit{Const}_\sigma \cup \mathit{Conc}_\sigma \cup \mathit{Role}_\sigma$
  - TBox $\mathcal{T}$: set of σ-axioms in some logic to capture terminological knowledge
    This lecture: ontologies represented in Description Logics (DLs)
  - ABox $\mathcal{A}$: set of σ-axioms in (same logic) to capture assertional/contingential knowledge

- Note: Sometimes only TBox termed ontology

- Semantics defined on the basis of σ-interpretations $\mathcal{I}$
  - $\mathcal{I} \models \mathit{Ax}$ iff $\mathcal{I}$ makes all axioms in Ax true
  - $\mathit{Mod}(\mathit{Ax}) = \{\mathcal{I} \models \mathit{Ax}\}$

Ontologies

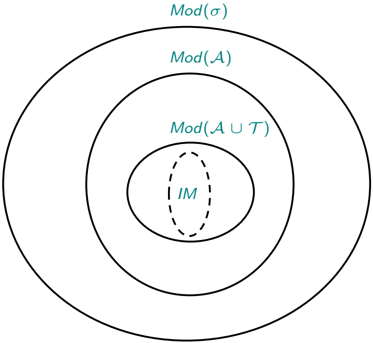
- Ontologies are triples of the form $\mathcal{O} = (\sigma, \mathcal{T}, \mathcal{A})$
  - Signature σ: Non-logical vocabulary
    $\sigma = Const_\sigma \cup Conc_\sigma \cup Role_\sigma$
  - TBox $\mathcal{T}$: set of σ-axioms in some logic to capture terminological knowledge
    This lecture: ontologies represented in Description Logics (DLs)
  - ABox $\mathcal{A}$: set of σ-axioms in (same logic) to capture assertional/contingential knowledge

- Note: Sometimes only TBox termed ontology

- Semantics defined on the basis of σ-interpretations $\mathcal{I}$
  - $\mathcal{I} \models Ax$ iff $\mathcal{I}$ makes all axioms in Ax true
  - $Mod(Ax) = \{\mathcal{I} \models Ax\}$

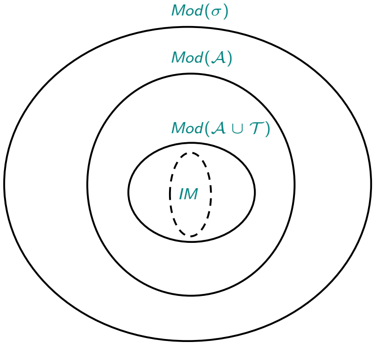
General Idea

- $\mathcal{A}$: Represents facts in domain of interest
- Open world assumption: $Mod(\mathcal{A})$ is not a singleton
- $\mathcal{T}$: Constrains $Mod(\mathcal{A})$ with intended σ readings
- Usually one has only approximations of intended models IM
- Realize inference services on the basis of the constrained interpretations



General Idea

- $\mathcal{A}$: Represents facts in domain of interest
- Open world assumption: $Mod(\mathcal{A})$ is not a singleton
- $\mathcal{T}$: Constrains $Mod(\mathcal{A})$ with intended σ readings
- Usually one has only approximations of intended models IM
- Realize inference services on the basis of the constrained interpretations



WARNING: A Misconception

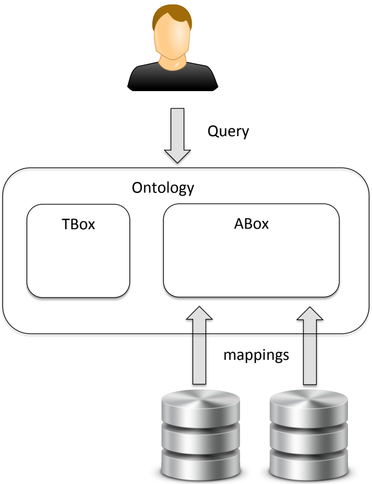
- With ontologies one does not declare data structures

- ABox data in most cases show pattern of data structures
- One does not have to re-model patterns/constraints in the ABox data
  - Knowing "All A are B" in the ABox is different from stipulating $A \sqsubseteq B$ (the former is known as integrity constraint)
  - Add $A \sqsubseteq B$, if you need to handle this relation for objects not mentioned in the ABox

- Motto: Keep the TBox simple

Ontology-Based Data Access

- Use ontologies as interface
  ...
- to access (here: query)
- data stored in some format
  ...
- using mappings



Reasoning Services

- Different standard and nonstandard reasoning services exists
- May be reducible to each other

Example

Reasoning Services consistency check, subsumption check, taxonomy calculations, most specific subsumer, most specific concept, matching, ...

- In classical OBDA focus on
  - Consistency checking: $Mod(\mathcal{A} \cup \mathcal{T}) \neq \emptyset$.
  - Query answering
- Next to ABox and TBox language query language QL over σ is a relevant factor for OBDA
- Certain query answering

$$cert(\psi(\vec{x}), \mathcal{T} \cup \mathcal{A}) \quad = \quad \{\vec{a} \in (Const_\sigma)^n \mid \mathcal{T} \cup \mathcal{A} \models \psi[\vec{x}/\vec{a}]\}$$

Reasoning Services

- Different standard and nonstandard reasoning services exists
- May be reducible to each other

Example

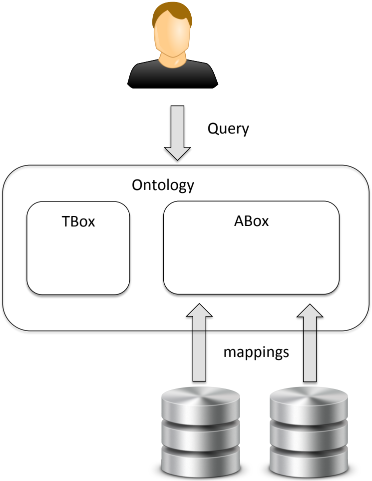
Reasoning Services consistency check, subsumption check, taxonomy calculations, most specific subsumer, most specific concept, matching, ...

- In classical OBDA focus on
  - Consistency checking: $Mod(\mathcal{A} \cup \mathcal{T}) \neq \emptyset$.
  - Query answering
- Next to ABox and TBox language query language QL over σ is a relevant factor for OBDA
- Certain query answering

$$cert(\psi(\vec{x}), \mathcal{T} \cup \mathcal{A}) \quad = \quad \{\vec{a} \in (Const_{\sigma})^n \mid \mathcal{T} \cup \mathcal{A} \models \psi[\vec{x}/\vec{a}]\}$$

Ontology-Based Data Access

- Use ontologies as interface
  ...
- to access (here: query)
- data stored in some format
  ...
- using mappings

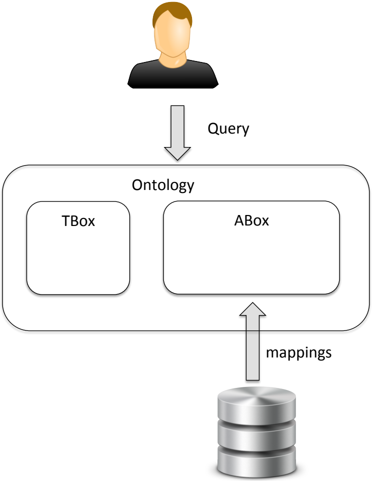


Backend Data Sources

- Classically: relational SQL DBs with static data
- Possible extensions: non-SQL DBs
  - datawarehouse repositories for statistical applications
  - pure logfiles
  - RDF repositories
- Non-static data
  - historical data (stored in temporal DB)
  - dynamic data coming in streams
- Originally intended for multiple DBs but ...

Federation

- ... we would have to deal with federation
- not trivial in classical OBDA ...
- because one has to integrate data from different DBs
- Ignore federation aspect: we have one DB but possibly many tables

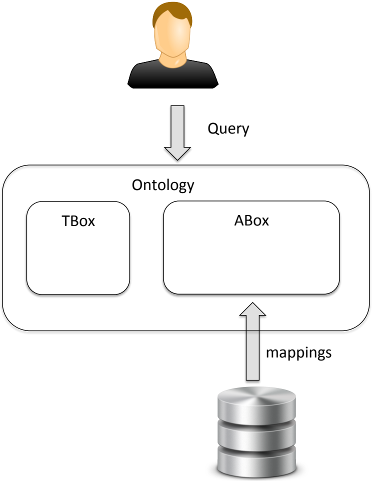


Ontology-Based Data Access

- Use ontologies as interface

  ...
- to access (here: query)
- data stored in some format

  ...
- using mappings



Mappings

- Mappings have an important crucial role in OBDA
- Lift data to the ontology level
  - Data level: (nearly) close world
  - Ontology Level: open world

Definition (Schema of Mappings)

$$m : \psi(\vec{f}(\vec{x})) \longleftarrow Q(\vec{x}, \vec{y})$$

- $\psi(\vec{f}(\vec{x}))$: Template (query) for generating ABox axioms
- $Q(\vec{x}, \vec{y})$: Query over the backend sources
- Function $\vec{f}$ translates backend instantiations of $\vec{x}$ to constants

- Mappings M over backend sources generates ABox $\mathcal{A}(M, DB)$.

Example Scenario: Measurements

- Example schema for measurement and event data in DB

```
SENSOR(SID, CID, Sname, TID, description)
SENSORTYPE(TID, Tname)
COMPONENT(CID, superCID, AID, Cname)
ASSEMBLY(AID, AName, ALocation)
MEASUREMENT(MID, MtimeStamp, SID, Mval)
MESSAGE(MesID, MesTimeStamp, MesAssemblyID,  catID, MesEventText)
CATEGORY(catID, catName)
```

- For mapping

m: $Sens(x) \wedge name(x, y) \longleftarrow$

```
SELECT f(SID) as x,  Sname as y FROM SENSOR
```

- the row data in SENSOR table

SENSOR
(123, comp45, TC255, TempSens, 'A temperature sensor')

- generates facts

$Sens(f(123)), name(f(123), TempSens) \in \mathcal{A}(m, DB)$

Example Scenario: Measurements

- Example schema for measurement and event data in DB

```
SENSOR(SID, CID, Sname, TID, description)
SENSORTYPE(TID, Tname)
COMPONENT(CID, superCID, AID, Cname)
ASSEMBLY(AID, AName, ALocation)
MEASUREMENT(MID, MtimeStamp, SID, Mval)
MESSAGE(MesID, MesTimeStamp, MesAssemblyID,  catID, MesEventText)
CATEGORY(catID, catName)
```

- For mapping

m: $\mathit{Sens}(x) \wedge \mathit{name}(x, y) \longleftarrow$

```
SELECT f(SID) as x,  Sname as y FROM SENSOR
```

- the row data in `SENSOR` table

SENSOR
(123, comp45, TC255, TempSens, ‘A temperature sensor’)

- generates facts

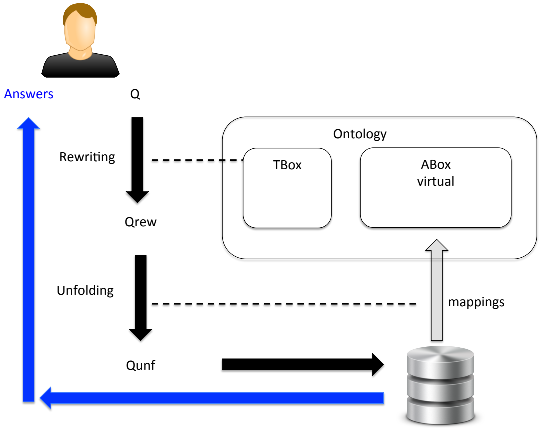
$\mathit{Sens}(f(123)), \mathit{name}(f(123), \mathit{TempSens}) \in \mathcal{A}(m, DB)$

(Strange) Maps of a Different Kind

- Jacobs strange maps: http://bigthink.com/articles?blog=strange-maps

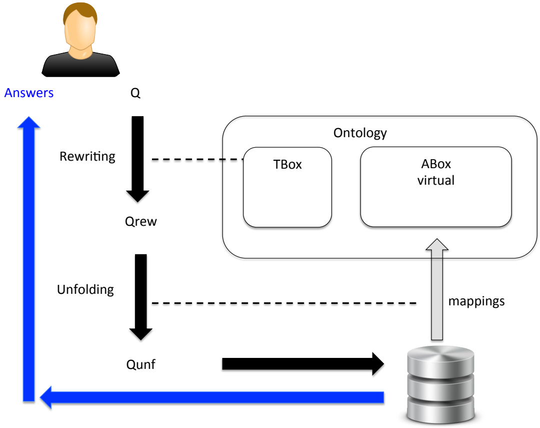
OBDA in the Classical Sense

- Keep the data where they are because of large volume
- ABox is virtual (no materialization)



OBDA in the Classical Sense

- First-order logic (FOL) perfect rewriting + unfolding for realizing reasoning services



OBDA in the Classical Sense

- $\mathcal{T}$ language: Some logic of the DL-Lite family
- $\mathcal{A}$ language: assertions of the form $A(a), R(a, b)$
- QL : Unions of conjunctive queries (UCQs)
- Language of $Qrew$: safe FOL
- Allows for perfect rewriting (of consistency checking and) UCQ answering

$$cert(Q, (\sigma, \mathcal{T}, \mathcal{A})) = ans(Qrew, DB(\mathcal{A}))$$

- and unfolding

$$cert(Q, (\sigma, \mathcal{T}, \mathcal{A}(M, DB))) = ans(Qunf, DB)$$

- Note that query language over DB is relevant for possibility of unfolding

OBDA in the Classical Sense

- $\mathcal{T}$ language: Some logic of the DL-Lite family
- $\mathcal{A}$ language: assertions of the form $A(a), R(a,b)$
- QL : Unions of conjunctive queries (UCQs)
- Language of $Qrew$: safe FOL
- Allows for perfect rewriting (of consistency checking and) UCQ answering

$$cert(Q, (\sigma, \mathcal{T}, \mathcal{A})) = ans(Qrew, DB(\mathcal{A}))$$

- and unfolding

$$cert(Q, (\sigma, \mathcal{T}, \mathcal{A}(M, DB))) = ans(Qunf, DB)$$

- Note that query language over DB is relevant for possibility of unfolding

OBDA in the Classical Sense

- $\mathcal{T}$ language: Some logic of the DL-Lite family
- $\mathcal{A}$ language: assertions of the form $A(a), R(a, b)$
- QL : Unions of conjunctive queries (UCQs)
- Language of $Qrew$: safe FOL
- Allows for perfect rewriting (of consistency checking and) UCQ answering

$$cert(Q, (\sigma, \mathcal{T}, \mathcal{A})) = ans(Qrew, DB(\mathcal{A}))$$

- and unfolding

$$cert(Q, (\sigma, \mathcal{T}, \mathcal{A}(M, DB))) = ans(Qunf, DB)$$

- Note that query language over DB is relevant for possibility of unfolding

Extended OBDA

- Use more expressive TBox language
  - ABDEO (Accessing very big data using expressive ontologies)
  - Rewritability for UCQs not guaranteed
  - Materialize ABox and use ABox modularization to answer queries
- Use different (more expressive) QL
  - E.g. SPARQL instead of UCQ; but no full existentials in combination with DL-Lite
  - OWL2QL + SPARQL used in Optique platform
- Use different reasoning/rewriting paradigm
  - e.g. combined rewriting: First enhance ABox with TBox information and then rewrite
  - Streaming

Extended OBDA

- Use more expressive TBox language
  - ABDEO (Accessing very big data using expressive ontologies)
  - Rewritability for UCQs not guaranteed
  - Materialize ABox and use ABox modularization to answer queries
- Use different (more expressive) QL
  - E.g. SPARQL instead of UCQ; but no full existentials in combination with DL-Lite
  - OWL2QL + SPARQL used in Optique platform
- Use different reasoning/rewriting paradigm
  - e.g. combined rewriting: First enhance ABox with TBox information and then rewrite
  - Streaming

Extended OBDA

- Use more expressive TBox language
  - ABDEO (Accessing very big data using expressive ontologies)
  - Rewritability for UCQs not guaranteed
  - Materialize ABox and use ABox modularization to answer queries
- Use different (more expressive) QL
  - E.g. SPARQL instead of UCQ; but no full existentials in combination with DL-Lite
  - OWL2QL + SPARQL used in Optique platform
- Use different reasoning/rewriting paradigm
  - e.g. combined rewriting: First enhance ABox with TBox information and then rewrite
  - Streaming

Ontologies and Description Logics

Description Logics

Definition

Description logics (DLs) are logics for use in knowledge representation with special attention on a good balance of expressibility and feasibility of reasoning services

- Can be mapped to fragments of FOL
- Use
  - as ontology representation language for conceptual modeling
  - in particular in the semantic web
  - Formal counterpart of standard web ontology language (OWL)
  - and in particular for ontology-based data access (OBDA)
- Have been investigated for ca. 30 years now
  - Many theoretical insights on various different purpose DLs
  - General-purpose reasoners (RacerPro, Fact++, ...) and specific reasoners (Quest,...)
  - Various editing tools (most notably Protege)

Description Logics

Definition

Description logics (DLs) are logics for use in knowledge representation with special attention on a good balance of expressibility and feasibility of reasoning services

- Can be mapped to fragments of FOL
- Use
  - as ontology representation language for conceptual modeling
  - in particular in the semantic web
  - Formal counterpart of standard web ontology language (OWL)
  - and in particular for ontology-based data access (OBDA)
- Have been investigated for ca. 30 years now
  - Many theoretical insights on various different purpose DLs
  - General-purpose reasoners (RacerPro, Fact++, ...) and specific reasoners (Quest,...)
  - Various editing tools (most notably Protege)

Description Logics

Definition

Description logics (DLs) are logics for use in knowledge representation with special attention on a good balance of expressibility and feasibility of reasoning services

- Can be mapped to fragments of FOL
- Use
  - as ontology representation language for conceptual modeling
  - in particular in the semantic web
  - Formal counterpart of standard web ontology language (OWL)
  - and in particular for ontology-based data access (OBDA)
- Have been investigated for ca. 30 years now
  - Many theoretical insights on various different purpose DLs
  - General-purpose reasoners (RacerPro, Fact++, ...) and specific reasoners (Quest,...)
  - Various editing tools (most notably Protege)

Family of DLs

- Variable-free logics centered around concepts
- concepts = one-ary predicates in FOL = classes in OWL

Example (Concepts)

- $Students$ ("students")
- $Students \sqcap Male$ (" Male students")
- $\exists attends.MathCourse$ ("Those attending a math course")
- $\forall hasFriends.Freaks$ ("Those having only freaks as friends")
- $Person \sqcap \forall attends.(Course \sqcap \neg easy)$ ("Persons attending only non-easy courses")

An (Semi-)Expressive Logic: $\mathcal{ALC}$

- Vocabulary: constants N_i, atomic concepts N_C, roles N_R

- Concept(description)s: syntax

$$C \quad ::= \quad A \quad (\text{for } A \in N_C) \mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \forall r.C \mid \exists r.C \ (\text{for } r \in N_R) \mid \bot \mid \top$$

- Concept(description)s: semantics

- Interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{domain}, \overbrace{\cdot^{\mathcal{I}}}^{denotation\ function})$
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_C$
- $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $c \in N_i$
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$

- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $\neg C = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ for all $e \in \Delta^{\mathcal{I}}$: If $(d, e) \in r^{\mathcal{I}}$ then $e \in C^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$ s.t. $(d, e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

An (Semi-)Expressive Logic: $\mathcal{ALC}$

- Vocabulary: constants N_i, atomic concepts N_C, roles N_R
- Concept(description)s: syntax

$$C \quad ::= \quad A \quad (\text{for } A \in N_C) \mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \\ \forall r.C \mid \exists r.C \; (\text{for } r \in N_R) \mid \bot \mid \top$$

- Concept(description)s: semantics

- Interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{domain}, \overbrace{\cdot^{\mathcal{I}}}^{denotation\ function})$
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_C$
- $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $c \in N_i$
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $\neg C = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ for all $e \in \Delta^{\mathcal{I}}$: If $(d, e) \in r^{\mathcal{I}}$ then $e \in C^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$ s.t. $(d, e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

An (Semi-)Expressive Logic: $\mathcal{ALC}$

- Vocabulary: constants N_i, atomic concepts N_C, roles N_R

- Concept(description)s: syntax

$$C \quad ::= \quad A \quad (\text{for } A \in N_C) \;\mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \forall r.C \mid \exists r.C \;(\text{for } r \in N_R) \mid \bot \mid \top$$

- Concept(description)s: semantics

- Interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{domain}, \overbrace{\cdot^{\mathcal{I}}}^{denotation\ function})$
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_C$
- $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $c \in N_i$
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$

- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $\neg C = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ for all $e \in \Delta^{\mathcal{I}}$: If $(d, e) \in r^{\mathcal{I}}$ then $e \in C^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$ s.t. $(d, e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

An (Semi-)Expressive Logic: $\mathcal{ALC}$

- Vocabulary: constants N_i, atomic concepts N_C, roles N_R

- Concept(description)s: syntax

$$C \quad ::= \quad A \quad (\text{for } A \in N_C) \mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \forall r.C \mid \exists r.C \; (\text{for } r \in N_R) \mid \bot \mid \top$$

- Concept(description)s: semantics

- Interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{domain}, \overbrace{\cdot^{\mathcal{I}}}^{denotation\ function})$
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_C$
- $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $c \in N_i$
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$

- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $\neg C = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ for all $e \in \Delta^{\mathcal{I}}$: If $(d, e) \in r^{\mathcal{I}}$ then $e \in C^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$ s.t. $(d, e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

An (Semi-)Expressive Logic: $\mathcal{ALC}$

- Vocabulary: constants N_i, atomic concepts N_C, roles N_R
- Concept(description)s: syntax

$$C \quad ::= \quad A \quad (\text{for } A \in N_C) \mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \forall r.C \mid \exists r.C \ (\text{for } r \in N_R) \mid \bot \mid \top$$

- Concept(description)s: semantics

- Interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{\textit{domain}}, \overbrace{\cdot^{\mathcal{I}}}^{\textit{denotation function}})$
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all $A \in N_C$
- $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for all $c \in N_i$
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all $r \in N_r$

- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $\neg C = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ for all $e \in \Delta^{\mathcal{I}}$: If $(d, e) \in r^{\mathcal{I}}$ then $e \in C^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$ s.t. $(d, e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

TBox and ABox

- Terminological Box (TBox) $\mathcal{T}$
  - Finite set of general concept inclusions (GCIs)
  - GCI: axioms of form $C \sqsubseteq D$ (for arbitrary concept descriptions)
    $C \equiv D$ abbreviates $\{C \sqsubseteq D, D \sqsubseteq C\}$
  - Semantics: $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.
- Assertional Box (ABox) $\mathcal{A}$
  - Finite set of assertions
  - Assertion: $C(a)$ (concept assertion), $r(a, b)$ (role assertion)
  - Semantics:
    $\mathcal{I} \models C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$
    $\mathcal{I} \models r(a, b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$.
- Ontology: $(\sigma, \mathcal{T}, \mathcal{A})$

We follow the bad CS practice of calling KBs in DLs ontologies. We apologize to all philosophers for this use ;)

TBox and ABox

- Terminological Box (TBox) $\mathcal{T}$
  - Finite set of general concept inclusions (GCIs)
  - GCI: axioms of form $C \sqsubseteq D$ (for arbitrary concept descriptions)
    $C \equiv D$ abbreviates $\{C \sqsubseteq D, D \sqsubseteq C\}$
  - Semantics: $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

- Assertional Box (ABox) $\mathcal{A}$
  - Finite set of assertions
  - Assertion: $C(a)$ (concept assertion), $r(a, b)$ (role assertion)
  - Semantics:
    $\mathcal{I} \models C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$
    $\mathcal{I} \models r(a, b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$.

- Ontology: $(\sigma, \mathcal{T}, \mathcal{A})$

  We follow the bad CS practice of calling KBs in DLs ontologies. We apologize to all philosophers for this use ;)

TBox and ABox

- Terminological Box (TBox) $\mathcal{T}$
  - Finite set of general concept inclusions (GCIs)
  - GCI: axioms of form $C \sqsubseteq D$ (for arbitrary concept descriptions) $C \equiv D$ abbreviates $\{C \sqsubseteq D, D \sqsubseteq C\}$
  - Semantics: $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

- Assertional Box (ABox) $\mathcal{A}$
  - Finite set of assertions
  - Assertion: $C(a)$ (concept assertion), $r(a, b)$ (role assertion)
  - Semantics:
    $\mathcal{I} \models C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$
    $\mathcal{I} \models r(a, b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$.

- Ontology: $(\sigma, \mathcal{T}, \mathcal{A})$

We follow the bad CS practice of calling KBs in DLs ontologies. We apologize to all philosophers for this use ;)

Example (University)

$$\mathcal{T} = \{\ \mathit{GradStudent} \sqsubseteq \mathit{Student},$$
$$\mathit{GradStudent} \sqsubseteq \exists \mathit{takesCourse}.\mathit{GradCourse}\ \}$$
$$\mathcal{A} = \{\ \mathit{GradStudent}(\mathit{john})\ \}$$

Consider the following interpretations

Example (University)

$$\mathcal{T} = \{ \mathit{GradStudent} \sqsubseteq \mathit{Student}, \\ \mathit{GradStudent} \sqsubseteq \exists \mathit{takesCourse}.\mathit{GradCourse} \}$$

$$\mathcal{A} = \{ \mathit{GradStudent}(\mathit{john}) \}$$

Consider the following interpretations

- $\mathcal{I}_1$:
  - $\mathit{john}^{\mathcal{I}_1} = j$
  - $\mathit{GradStudent}^{\mathcal{I}_1} = \{j\}$
  - $\mathit{Student}^{\mathcal{I}_1} = \{j\}$
  - $\mathit{GradCourse}^{\mathcal{I}_1} = \{s\}$
  - $\mathit{takesCourse}^{\mathcal{I}_1} = \{(j, s)\}$
- $\mathcal{I}_1 \models \mathcal{T} \cup \mathcal{A}$

- $\mathcal{I}_2$:
  - $\mathit{john}^{\mathcal{I}_2} = j$
  - $\mathit{GradStudent}^{\mathcal{I}_2} = \{j\}$
  - $\mathit{Student}^{\mathcal{I}_2} = \{j\}$
  - $\mathit{GradCourse}^{\mathcal{I}_2} = \{j\}$
  - $\mathit{takesCourse}^{\mathcal{I}_2} = \{(j, j)\}$
- $\mathcal{I}_2 \models \mathcal{T} \cup \mathcal{A}$

Example (University)

$$\begin{aligned}\mathcal{T} &= \{\ GradStudent \sqsubseteq Student, \\ &\quad\ GradStudent \sqsubseteq \exists takesCourse.GradCourse\ \} \\ \mathcal{A} &= \{\ GradStudent(john)\ \}\end{aligned}$$

Consider the following interpretations

- $\mathcal{I}_3$:
  - $john^{\mathcal{I}_1} = j$
  - $GradStudent^{\mathcal{I}_1} = \{j\}$
  - $Student^{\mathcal{I}_1} = \{j\}$
  - $GradCourse^{\mathcal{I}_1} = \emptyset$
  - $takesCourse^{\mathcal{I}_1} = \emptyset$
- $\mathcal{I}_3 \not\models \mathcal{T} \cup \mathcal{A}$

Stricter notion of TBox

- Above definition of TBox very general
  - "Meanings" of concept names determined only implicitly in the whole ontology
  - No guarantee for unique extensions

- Early notion of TBox more related to idea of explicitly defining concept names
- $C \equiv D$ used as abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$
- Concept definition: $A \equiv D$ (where A atomic)

Definition

A TBox in a strict sense is a finite set of concept definitions not defining a concept multiple times or in a cyclic manner. Defined concepts occur on the lhs, primitive concept on the rhs of definitions.

Implicit vs. Explicit Definability

- Sometimes a general TBox may fix the denotation of a concept name w.r.t. denotations of the others $\Longrightarrow$ implicit definability
- Maybe then it can also be defined explicitly?

Definition

Given an FOL theory Ψ over signature σ and a predicate symbol R.

- R is implicitly defined in Ψ iff
  for any two models $\mathfrak{A} \models \Psi$ and $\mathfrak{B} \models \Psi$ agreeing on $\sigma \setminus \{R\}$
  one has $R^{\mathfrak{A}} = R^{\mathfrak{B}}$.

- R is explicitly defined in Ψ by a formula $\phi(\vec{x})$ not containing R
  iff $\Psi \models \forall\vec{x} R(\vec{x}) \leftrightarrow \phi(\vec{x})$

Beth Definability Theorem

- For FOL both notions of definition coincide

Theorem

An FOL theory defines a predicate implicitly iff it defines it explicitly

- Though DLs are embedable into FOL, this coincidence does not transfer necessarily to DLs
- At least it does for $\mathcal{ALC}$ theories

  Lit: B. ten Cate, E. Franconi, and I. Seylan. Beth definability in expressive description logics. J. Artif. Int. Res., 48(1): 347–414, Oct. 2013.

Reasoning services

- Semantical notions as in FOL but additional notions due to focus on concepts
- Let $\mathcal{O} = (\sigma, \mathcal{T}, \mathcal{A})$

Definition (Basic Semantical Notions)

- Model: $\mathcal{I} \models \mathcal{O}$ iff $\mathcal{I} \models \mathcal{T} \cup \mathcal{A}$
- Satisfiability: $\mathcal{O}$ is satisfiable iff $\mathcal{T} \cup \mathcal{A}$ is satisfiable
- Coherence: $\mathcal{O}$ is coherent iff $\mathcal{T} \cup \mathcal{A}$ has a model $\mathcal{I}$ s.t. for all concept names $A^{\mathcal{I}} \neq \emptyset$
- Concept satisfiability: C is satisfiable w.r.t. $\mathcal{O}$ iff there is $\mathcal{I} \models \mathcal{O}$ s.t. $C^{\mathcal{I}} \neq \emptyset$
- Subsumption: C is subsumed by D w.r.t. $\mathcal{O}$ iff $\mathcal{O} \vDash C \sqsubseteq D$ iff $\mathcal{T} \cup \mathcal{A} \vDash C \sqsubseteq D$
- Instance check: a is an instance of C w.r.t. $\mathcal{O}$ iff $\mathcal{O} \vDash C(a)$

Reduction Examples

- Many of the semantical notions are reducible to each other
- We give only one example—which is the content of Exercise 7.3.

Exercise

Show that subsumption can be reduced to satisfiability tests (allowing the introduction of new constants). More concretely:

$C \sqsubseteq D$ w.r.t. $\mathcal{O}$ iff $(\sigma \cup \{b\}, \mathcal{T}, \mathcal{A} \cup \{C(b), \neg D(b)\})$ is not satisfiable (where b is a fresh constant).

Extended Reasoning Services

Definition

- Instance retrieval: Find all constants x s.t. $\mathcal{O} \models C(x)$
- Query answering: Certain answers $cert(\phi(x), \mathcal{O}) = \{\vec{a} \in Const_\sigma \mid \mathcal{O} \models \phi[\vec{x}/\vec{a}]\}$
- Classification: Compute the subsumption hierarchy of all concept names
- Realization: Compute the most specific concept name to which a given constant belongs
- Pinpointing, matching, ...

Example (Certain Answers for Conjunctive Queries)

$$\mathcal{T} = \{ \top \sqsubseteq Male \sqcup Female, Male \sqcap Female \sqsubseteq \bot \}$$
$$\mathcal{A} = \{ friend(john, susan), friend(john, andrea), female(susan), loves(susan, andrea), \quad loves(andrea, bill), Male(bill) \}$$

$$Q(x) = \exists y, z(friend(x, y) \land Female(y) \land loves(y, z) \land Male(z))$$

- $cert(Q(x), \mathcal{O}) =?$

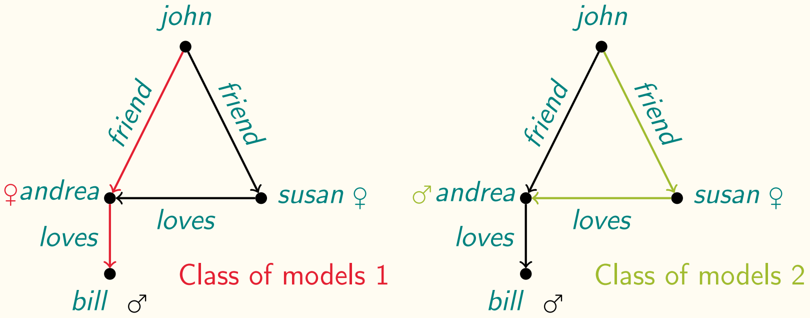
- We have to consider all possible models of the ontology
- But there actually two classes:
  Andrea is male vs. Andrea is not male.

Example (Certain Answers for Conjunctive Queries)

$$\mathcal{T} = \{ \top \sqsubseteq Male \sqcup Female, Male \sqcap Female \sqsubseteq \bot \}$$

$$\mathcal{A} = \{ friend(john, susan), friend(john, andrea), female(susan), loves(susan, andrea), \quad loves(andrea, bill), Male(bill) \}$$

$$Q(x) = \exists y, z(friend(x, y) \wedge Female(y) \wedge loves(y, z) \wedge Male(z))$$



$cert(Q(x), \mathcal{O}) = \{john\}$

Embedding into FOL

- Most DLs (such as $\mathcal{ALC}$) can be embedded into FOL
- Notion of embedding is well-defined as FOL structures are used for semantics of DLs.

- Correspondence idea
  Concept names = unary predicates, roles = binary predicates, GCI = $\forall$ rules
- Define for any concept description and variable x its corresponding x-open formula $\tau_x(C)$
  - $\tau_x(A) = A(x)$
  - $\tau_x(C \sqcap D) = \tau_x(C) \land \tau_x(D)$
  - $\tau_x(C \sqcup D) = \tau_x(C) \lor \tau_x(D)$
  - $\tau_x(\neg C) = \neg \tau_x(C)$
  - $\tau_x(\forall r.C) = \forall y(r(x,y) \to \tau_y(C))$
  - $\tau_x(\exists r.C) = \exists y(r(x,y) \land \tau_y(C))$
- ABox axioms not changed
- TBox axioms: $C \sqsubseteq D$ becomes $\forall x(\tau_x(C) \to \tau_x(D))$

Embedding into FOL

- Most DLs (such as $\mathcal{ALC}$) can be embedded into FOL
- Notion of embedding is well-defined as FOL structures are used for semantics of DLs.

- Correspondence idea
  Concept names = unary predicates, roles = binary predicates, GCI = $\forall$ rules
- Define for any concept description and variable x its corresponding x-open formula $\tau_x(C)$
  - $\tau_x(A) = A(x)$
  - $\tau_x(C \sqcap D) = \tau_x(C) \wedge \tau_x(D)$
  - $\tau_x(C \sqcup D) = \tau_x(C) \vee \tau_x(D)$
  - $\tau_x(\neg C) = \neg \tau_x(C)$
  - $\tau_x(\forall r.C) = \forall y(r(x, y) \rightarrow \tau_y(C))$
  - $\tau_x(\exists r.C) = \exists y(r(x, y) \wedge \tau_y(C))$
- ABox axioms not changed
- TBox axioms: $C \sqsubseteq D$ becomes $\forall x(\tau_x(C) \rightarrow \tau_x(D))$

Embedding into FOL

- Most DLs (such as $\mathcal{ALC}$) can be embedded into FOL
- Notion of embedding is well-defined as FOL structures are used for semantics of DLs.

- Correspondence idea
  Concept names = unary predicates, roles = binary predicates, GCI = $\forall$ rules
- Define for any concept description and variable x its corresponding x-open formula $\tau_x(C)$
  - $\tau_x(A) = A(x)$
  - $\tau_x(C \sqcap D) = \tau_x(C) \wedge \tau_x(D)$
  - $\tau_x(C \sqcup D) = \tau_x(C) \vee \tau_x(D)$
  - $\tau_x(\neg C) = \neg \tau_x(C)$
  - $\tau_x(\forall r.C) = \forall y(r(x,y) \rightarrow \tau_y(C))$
  - $\tau_x(\exists r.C) = \exists y(r(x,y) \wedge \tau_y(C))$
- ABox axioms not changed
- TBox axioms: $C \sqsubseteq D$ becomes $\forall x(\tau_x(C) \rightarrow \tau_x(D))$

Embedding into FOL

- Most DLs (such as $\mathcal{ALC}$) can be embedded into FOL
- Notion of embedding is well-defined as FOL structures are used for semantics of DLs.

- Correspondence idea
  Concept names = unary predicates, roles = binary predicates, GCI = $\forall$ rules
- Define for any concept description and variable x its corresponding x-open formula $\tau_x(C)$
  - $\tau_x(A) = A(x)$
  - $\tau_x(C \sqcap D) = \tau_x(C) \land \tau_x(D)$
  - $\tau_x(C \sqcup D) = \tau_x(C) \lor \tau_x(D)$
  - $\tau_x(\neg C) = \neg \tau_x(C)$
  - $\tau_x(\forall r.C) = \forall y(r(x,y) \rightarrow \tau_y(C))$
  - $\tau_x(\exists r.C) = \exists y(r(x,y) \land \tau_y(C))$
- ABox axioms not changed
- TBox axioms: $C \sqsubseteq D$ becomes $\forall x(\tau_x(C) \rightarrow \tau_x(D))$

Embedding into FOL

- For translation two variables are sufficient ("2 finger movement")
- Hence: DLs embeddable into known 2-variable fragment of FOL
- Also the fragment is a guarded fragment: one quantifies over variables fixed within atom.

Wake-Up Exercise

Calculate $\tau_x(\forall r.(A \sqcap \exists r.B))$ using only two variables.

Embedding into FOL

- For translation two variables are sufficient ("2 finger movement")
- Hence: DLs embeddable into known 2-variable fragment of FOL
- Also the fragment is a guarded fragment: one quantifies over variables fixed within atom.

Wake-Up Exercise

Calculate $\tau_x(\forall r.(A \sqcap \exists r.B))$ using only two variables.

Solution:

$$\forall y[r(x,y) \rightarrow (A(y) \wedge \exists x[r(y,x) \wedge B(x)])]$$

NB: There are free and bound occurrences of x

DL Family

- Different DLs for different purposes
  - What is more important: Expressivity or feasibility?
  - Which kinds of reasoning services does one have to provide?

- Differences regarding
  - the allowed set of concept constructors
  - the allowed set of role constructors
  - the allowed types of TBox axioms
  - the allowed types of ABox axioms
  - the allowance of concrete domains and attributes (such as *hasAge* with range the domain of integers)

Family of DLs and their Namings

- $\mathcal{AL}$: attributive language
- $\mathcal{C}$: (full) complement/negation
- $\mathcal{I}$: inverse roles $((r^{-1})^{\mathcal{I}} = \{(d,e) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid (e,d) \in r^{\mathcal{I}}\})$
- $\mathcal{H}$: role inclusions $(\mathit{hasFather} \sqsubseteq \mathit{hasParent})$

- $\mathcal{S}$: $\mathcal{ALC}$ + transitive roles $(\mathit{trans\ isReachable})$
- $\mathcal{N}$: unqualified number restrictions $((\geq n\ r)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \#(\{e \mid (d,e) \in r^{\mathcal{I}}\}) \geq n\})$
- $\mathcal{O}$: nominals $\{b\}^{\mathcal{I}} = \{b^{\mathcal{I}}\}$
- $\mathcal{Q}$: qualified number restrictions $((\geq n\ r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \#(\{e \mid (d,e) \in r^{\mathcal{I}}\} \text{ and } e \in C^{\mathcal{I}}) \geq n\})$
- $\mathcal{F}$: functionality constraints $\mathcal{I} \models (\mathit{func}\ R)$ iff $R^{\mathcal{I}}$ is a function
- $\mathcal{R}$: role chains and $\exists R.\mathit{Self}$ $(\mathit{hasFather} \circ \mathit{hasMother} \sqsubseteq \mathit{hasgrandMa})$ $(\mathit{narcist} \equiv \exists \mathit{likes}.\mathit{Self})$

- OWL 2 is $\mathcal{SROIQ}$

Lightweight DLs

- Lightweight DLs favor feasibility over expressibility by, roughly, dis-allowing disjunction
- In principle three lightweight logics that have corresponding OWL 2 profiles (https://www.w3.org/TR/owl2-profiles/)

- $\mathcal{EL}$ (OWL 2 EL)
  - No inverses, no negation, no $\forall$
  - polynomial time algorithms for all the standard reasoning tasks with large ontologies
- DL-Lite (OWL 2 QL)
  - TBox: No qualified existentials on lhs
  - Feasible CQ answering using rewriting and unfolding leveraging RDBS technology
- RL (OWL 2 RL)
  - TBox restriction: "Only concept names on the rhs"
  - Polynomial time algorithms leveraging rule-extended database technologies operating directly on RDF triples

Lightweight DLs

- Lightweight DLs favor feasibility over expressibility by, roughly, dis-allowing disjunction
- In principle three lightweight logics that have corresponding OWL 2 profiles (`https://www.w3.org/TR/owl2-profiles/`)

- $\mathcal{EL}$ (OWL 2 EL)
  - No inverses, no negation, no $\forall$
  - polynomial time algorithms for all the standard reasoning tasks with large ontologies
- DL-Lite (OWL 2 QL)
  - TBox: No qualified existentials on lhs
  - Feasible CQ answering using rewriting and unfolding leveraging RDBS technology
- RL (OWL 2 RL)
  - TBox restriction: "Only concept names on the rhs"
  - Polynomial time algorithms leveraging rule-extended database technologies operating directly on RDF triples

Lightweight DLs

- Lightweight DLs favor feasibility over expressibility by, roughly, dis-allowing disjunction
- In principle three lightweight logics that have corresponding OWL 2 profiles (https://www.w3.org/TR/owl2-profiles/)

- $\mathcal{EL}$ (OWL 2 EL)
  - No inverses, no negation, no $\forall$
  - polynomial time algorithms for all the standard reasoning tasks with large ontologies
- DL-Lite (OWL 2 QL)
  - TBox: No qualified existentials on lhs
  - Feasible CQ answering using rewriting and unfolding leveraging RDBS technology
- RL (OWL 2 RL)
  - TBox restriction: "Only concept names on the rhs"
  - Polynomial time algorithms leveraging rule-extended database technologies operating directly on RDF triples

Lightweight DLs

- Lightweight DLs favor feasibility over expressibility by, roughly, dis-allowing disjunction
- In principle three lightweight logics that have corresponding OWL 2 profiles (`https://www.w3.org/TR/owl2-profiles/`)

- $\mathcal{EL}$ (OWL 2 EL)
  - No inverses, no negation, no $\forall$
  - polynomial time algorithms for all the standard reasoning tasks with large ontologies
- DL-Lite (OWL 2 QL)
  - TBox: No qualified existentials on lhs
  - Feasible CQ answering using rewriting and unfolding leveraging RDBS technology
- RL (OWL 2 RL)
  - TBox restriction: "Only concept names on the rhs"
  - Polynomial time algorithms leveraging rule-extended database technologies operating directly on RDF triples

Comparison

	RL	EL	QL
inverse roles	+	-	+
rhs qual. exist	-	+	+
lhs qual. exist.	+	+	-

Complexity

- A nearly complete picture of reasoning services for DLs
- Research in DL community as of now resembles complexity farming
- DL complexity navigator:
  http://www.cs.man.ac.uk/~ezolin/dl (Last update 2013)

Tableaux Calculus for $\mathcal{ALC}$

- Efficient calculi are at the core of DL reasoners
- Tableaux calculi have been implemented successfully
- Refutation calculus based on disjunctive normal form
- We demonstrate it here at an example for $\mathcal{ALC}$ TBoxes
- For a full description and proofs see handbook article by Baader

  Lit: F. Baader and W. Nutt. Basic description logics. In F. Baader et al., editors, The Description Logic Handbook, pages 43–95. Cambridge University Press, 2003.

Tableaux Calculus for $\mathcal{ALC}$

- Efficient calculi are at the core of DL reasoners
- Tableaux calculi have been implemented successfully
- Refutation calculus based on disjunctive normal form
- We demonstrate it here at an example for $\mathcal{ALC}$ TBoxes
- For a full description and proofs see handbook article by Baader

  Lit: F. Baader and W. Nutt. Basic description logics. In F. Baader et al., editors, The Description Logic Handbook, pages 43–95. Cambridge University Press, 2003.

Tableaux Example

- $\mathcal{ALC}$ tableau gives tests for satisfiability of ABox
- by checking whether obvious contradictions (clashes with complementary literals) are contained
- An ABox that is complete (no rules applicable anymore) and open (no clashes) describes a model
- Algorithm applies tableau rules to extend ABox

Rules

- Starts with an ABox $\mathcal{A}_0$ which is in negation normal form (NNF, $\neg$ in front of concept names)
- Apply rules to construct new ABoxes; indeterminism due to $\sqcup$ rule

Rule	Condition	$\rightsquigarrow$	Effect
$\rightsquigarrow_\sqcap$	$(C \sqcap D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x), D(x)\}$
$\rightsquigarrow_\sqcup$	$(C \sqcup D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x)\}$or $\mathcal{A} \cup \{D(x)\}$
$\rightsquigarrow_\exists$	$(\exists r.C)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{r(x,y), C(y)\}$ for fresh y
$\rightsquigarrow_\forall$	$(\forall r.C)(x), r(x,y) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(y)\}$

- Rules only applicable if they lead to an addition of assertion
- One obtains a tree with ABoxes (due to indeterminism)
- Within each ABox a tree-like structure is established (tree-model property)

Rules

- Starts with an ABox $\mathcal{A}_0$ which is in negation normal form (NNF, $\neg$ in front of concept names)
- Apply rules to construct new ABoxes; indeterminism due to $\sqcup$ rule

Rule	Condition	$\rightsquigarrow$	Effect
$\rightsquigarrow_\sqcap$	$(C \sqcap D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x), D(x)\}$
$\rightsquigarrow_\sqcup$	$(C \sqcup D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x)\}$or $\mathcal{A} \cup \{D(x)\}$
$\rightsquigarrow_\exists$	$(\exists r.C)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{r(x,y), C(y)\}$ for fresh y
$\rightsquigarrow_\forall$	$(\forall r.C)(x), r(x,y) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(y)\}$

- Rules only applicable if they lead to an addition of assertion
- One obtains a tree with ABoxes (due to indeterminism)
- Within each ABox a tree-like structure is established (tree-model property)

Rules

- Starts with an ABox $\mathcal{A}_0$ which is in negation normal form (NNF, $\neg$ in front of concept names)
- Apply rules to construct new ABoxes; indeterminism due to $\sqcup$ rule

Rule	Condition	$\rightsquigarrow$	Effect
$\rightsquigarrow_\sqcap$	$(C \sqcap D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x), D(x)\}$
$\rightsquigarrow_\sqcup$	$(C \sqcup D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x)\}$or $\mathcal{A} \cup \{D(x)\}$
$\rightsquigarrow_\exists$	$(\exists r.C)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{r(x,y), C(y)\}$ for fresh y
$\rightsquigarrow_\forall$	$(\forall r.C)(x), r(x,y) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(y)\}$

- Rules only applicable if they lead to an addition of assertion
- One obtains a tree with ABoxes (due to indeterminism)
- Within each ABox a tree-like structure is established (tree-model property)

Rules

- Starts with an ABox $\mathcal{A}_0$ which is in negation normal form (NNF, $\neg$ in front of concept names)
- Apply rules to construct new ABoxes; indeterminism due to $\sqcup$ rule

Rule	Condition	$\rightsquigarrow$	Effect
$\rightsquigarrow_\sqcap$	$(C \sqcap D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x), D(x)\}$
$\rightsquigarrow_\sqcup$	$(C \sqcup D)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(x)\}$or $\mathcal{A} \cup \{D(x)\}$
$\rightsquigarrow_\exists$	$(\exists r.C)(x) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{r(x,y), C(y)\}$ for fresh y
$\rightsquigarrow_\forall$	$(\forall r.C)(x), r(x,y) \in \mathcal{A}$	$\rightsquigarrow$	$\mathcal{A} \cup \{C(y)\}$

- Rules only applicable if they lead to an addition of assertion
- One obtains a tree with ABoxes (due to indeterminism)
- Within each ABox a tree-like structure is established (tree-model property)

Example

- Given: $\mathcal{T} = \{GoodStudent \equiv Smart \sqcap Studious\}$
- Subsumption test: $\mathcal{T} \models \exists knows.Smart \sqcap \exists knows.Studious \sqsubseteq \exists knows.GoodStudent$
- Reduction to ABox satisfiability: $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.GoodStudent)(a)\}$ satisfiable?
- Expansions of definition $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.(Smart \sqcap Studious))(a)\}$ satisfiable?
- Transform to NNF $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$ satisfiable?

- GCIs can be transformed to definitions (i.e. axioms of the form $A \equiv C$) using additional symbols

Example

- Given: $\mathcal{T} = \{GoodStudent \equiv Smart \sqcap Studious\}$
- Subsumption test:
  $\mathcal{T} \models \exists knows.Smart \sqcap \exists knows.Studious \sqsubseteq \exists knows.GoodStudent$
- Reduction to ABox satisfiability:
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.GoodStudent)(a)\}$ satisfiable?
- Expansions of definition
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.(Smart \sqcap Studious))(a)\}$
  satisfiable?
- Transform to NNF
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
  satisfiable?

- GCIs can be transformed to definitions (i.e. axioms of the form $A \equiv C$) using additional symbols

Example

- Given: $\mathcal{T} = \{GoodStudent \equiv Smart \sqcap Studious\}$
- Subsumption test:
  $\mathcal{T} \models \exists knows.Smart \sqcap \exists knows.Studious \sqsubseteq \exists knows.GoodStudent$
- Reduction to ABox satisfiability:
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.GoodStudent)(a)\}$ satisfiable?
- Expansions of definition
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.(Smart \sqcap Studious))(a)\}$ satisfiable?
- Transform to NNF
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$ satisfiable?

- GCIs can be transformed to definitions (i.e. axioms of the form $A \equiv C$) using additional symbols

Example

- Given: $\mathcal{T} = \{GoodStudent \equiv Smart \sqcap Studious\}$
- Subsumption test:
  $\mathcal{T} \models \exists knows.Smart \sqcap \exists knows.Studious \sqsubseteq \exists knows.GoodStudent$
- Reduction to ABox satisfiability:
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.GoodStudent)(a)\}$ satisfiable?
- Expansions of definition
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.(Smart \sqcap Studious))(a)\}$
  satisfiable?
- Transform to NNF
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
  satisfiable?

- GCIs can be transformed to definitions (i.e. axioms of the form $A \equiv C$) using additional symbols

Example

- Given: $\mathcal{T} = \{GoodStudent \equiv Smart \sqcap Studious\}$
- Subsumption test:
  $\mathcal{T} \models \exists knows.Smart \sqcap \exists knows.Studious \sqsubseteq \exists knows.GoodStudent$
- Reduction to ABox satisfiability:
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.GoodStudent)(a)\}$ satisfiable?
- Expansions of definition
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.(Smart \sqcap Studious))(a)\}$
  satisfiable?
- Transform to NNF
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
  satisfiable?

- GCIs can be transformed to definitions (i.e. axioms of the form $A \equiv C$) using additional symbols

Example

- Given: $\mathcal{T} = \{GoodStudent \equiv Smart \sqcap Studious\}$
- Subsumption test:
  $\mathcal{T} \models \exists knows.Smart \sqcap \exists knows.Studious \sqsubseteq \exists knows.GoodStudent$
- Reduction to ABox satisfiability:
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.GoodStudent)(a)\}$ satisfiable?
- Expansions of definition
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \neg(\exists knows.(Smart \sqcap Studious))(a)\}$ satisfiable?
- Transform to NNF
  $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$ satisfiable?

- GCIs can be transformed to definitions (i.e. axioms of the form $A \equiv C$) using additional symbols

Example (A Tableau Derivation)

- $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
- Abbreviation: $\{\exists r.A \sqcap \exists r.B \sqcap \forall r.(\neg A \sqcup \neg B)(a)\}$

$$\mathcal{A}_0 = \exists r.A \sqcap \exists r.B \sqcap \forall r.(\neg A \sqcup \neg B)(a)$$

$\rightsquigarrow_\sqcap$ (2 times)

$$\mathcal{A}_1 = \mathcal{A}_0 \cup \{(\exists r.A)(a),\ (\exists r.B)(a),\ (\forall r.(\neg A \sqcup \neg B))(a)\}$$

$\rightsquigarrow_\exists$ (2 times)

$$\mathcal{A}_2 = \mathcal{A}_1 \cup \{r(a,b), A(b), r(a,c), B(c)\}$$

$\rightsquigarrow_\forall$ (2 times)

$$\mathcal{A}_3 = \mathcal{A}_2 \cup \{(\neg A \sqcup \neg B)(b), (\neg A \sqcup \neg B)(c)\}$$

$\rightsquigarrow_\sqcup$ / $\rightsquigarrow_\sqcup$

$$\mathcal{A}_{4.1} = \mathcal{A}_3 \cup \{(\neg A)(b)\} \qquad \mathcal{A}_{4.2} = \mathcal{A}_3 \cup \{(\neg B)(b)\}$$

$\rightsquigarrow_\sqcup$ / $\rightsquigarrow_\sqcup$ / $\rightsquigarrow_\sqcup$ / $\rightsquigarrow_\sqcup$

$\mathcal{A}_{5.11} = \mathcal{A}_{4.1} \cup \{\neg A(c)\}$ — clash

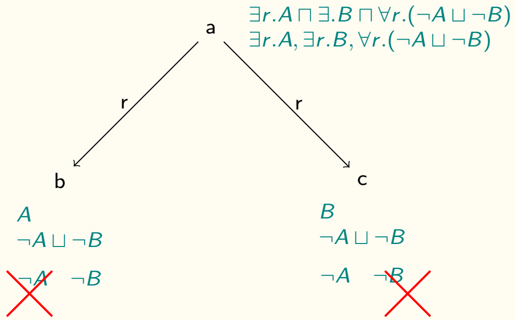
$\mathcal{A}_{5.12} = \mathcal{A}_{4.1} \cup \{\neg B(c)\}$ — clash

$\mathcal{A}_{5.21} = \mathcal{A}_{4.2} \cup \{\neg A(c)\}$

$\mathcal{A}_{5.22} = \mathcal{A}_{4.2} \cup \{\neg B(c)\}$ — clash

Example (The partial tree model in the ABoxes)

- $\{\exists knows.Smart \sqcap \exists knows.Studious \sqcap \forall knows.(\neg Smart \sqcup \neg Studious)(a)\}$
- Abbreviation: $\{\exists r.A \sqcap \exists r.B \sqcap \forall r.(\neg A \sqcup \neg B)(a)\}$



- Canonical tree model(s) can be directly read off:
  $\mathcal{I} = (\{a, b, c\}, \cdot^{\mathcal{I}})$ with
  $r^{\mathcal{I}} = \{(a, b), (a, c)\}$ $\quad A^{\mathcal{I}} = \{b\}$ $\quad B^{\mathcal{I}} = \{c\}$

Tableaux Calculus

- The tableau calculus for $\mathcal{ALC}$ is complete, correct, and terminates.
- Hence, the following properties hold

Theorem

- *$\mathcal{ALC}$ ABox satisfiability (concept satisfiability, subsumption...) is decidable*
- *$\mathcal{ALC}$ has the finite model property, i.e.*
  if an $\mathcal{ALC}$ ontology has a model, then it has a finite model.
- *$\mathcal{ALC}$ has the tree model property*

Solutions to Exercise 6

Exercise 6.1 (6 bonus points)

Show the following technical lemma:

(*) If $\mathfrak{T} \in Sol_{\mathcal{M}}(\mathfrak{S})$, then also $\mathfrak{T}' \in Sol_{\mathcal{M}}(\mathfrak{S})$, where $\mathfrak{T}'$ results from $\mathfrak{T}$ by substituting all marked nulls $\bot_1, \ldots, \bot_n$ occurring in $\mathfrak{T}$ (consistently) with new constants c_i.

Solution: See book of Murlak et al, p. 57.

- Clearly $\mathfrak{T}'$ is in $Rep(\mathfrak{T})$.
- Assume for contradiction that $\mathfrak{T}' \notin Sol_{\mathcal{M}}(\mathfrak{S})$. Hence $\mathfrak{T} \not\models \Sigma_t$. There are two cases
  1. $\mathfrak{T}'$ falsifies a tgd, say $\phi(\vec{x}) \rightarrow \exists \vec{y} \psi(\vec{x}, \vec{y})$. Hence there exists $\vec{a}$ of elements from $dom(\mathfrak{T}')$ such that $\mathfrak{T}' \models \phi(\vec{a})$ and $\mathfrak{T}' \not\models \exists \vec{y} \psi(\vec{a}, \vec{y})$. Let $\vec{a}' = \vec{a}[c_1/\bot_1, \ldots c_n/\bot_n]$. Then $\mathfrak{T} \models \phi(\vec{a}')$ and $\mathfrak{T} \not\models \exists \vec{y} \psi(\vec{a}', \vec{y})$ because there is a one-to-one homomorphism from $\mathfrak{T}$ to $\mathfrak{T}'$ sending each $\bot_i$ to a fresh c_i. But then $\mathfrak{T} \notin Sol_{\mathcal{M}}(\mathfrak{S})$. Contradiction.
  2. $\mathfrak{T}$ falsifies an egd. Similar argumentation.

Exercise 6.2 (4 points)

Show that the first definition of universal solutions $USol_1(\mathfrak{T})$ entails the second definition of universal solutions $USol_2(\mathfrak{T})$. Lemma (*) from Exercise 6.1 may be helpful.

Solution: (See book of Murlak et al. p 58.)

- Remember

$$USol_1(\mathfrak{T}) : \{\mathfrak{T}' \in SOL_{\mathcal{M}}(\mathfrak{S}) \mid \mathfrak{T}' \text{ complete}\} \subseteq Rep(\mathfrak{T})$$

$$USol_2(\mathfrak{T}) : Rep(\mathfrak{T}') \subseteq Rep(\mathfrak{T}) \quad \text{for every } \mathfrak{T}' \in SOL_{\mathcal{M}}(\mathfrak{S})$$

- Assume $\mathfrak{T}' \in SOL_{\mathcal{M}}(\mathfrak{S})$ is an arbitrary solution.
- $\perp_1, \ldots, \perp_m$ = nulls in $Dom(\mathfrak{T}')$
- $\mathfrak{T}''$ = substitution result from $\mathfrak{T}'$ as in the lemma above.
- As $\mathfrak{T}'' \in Sol_{\mathcal{M}}(\mathfrak{S})$ and does not contain nulls it follows from $Usol_1(\mathfrak{T})$ that $\mathfrak{T}'' \in Rep(\mathfrak{T})$.
- A homomorphism h witnessing $\mathfrak{T}'' \in Rep(\mathfrak{T})$ can be changed into mapping h' from $Dom(\mathfrak{T})$ into $Dom(\mathfrak{T}')$ by setting $h'(\perp) = \perp_i$ whenever $h(\perp) = c_i$ and otherwise h' is the same as h.
- h' is a homomorphism from $\mathfrak{T}$ into $\mathfrak{T}'$.
- Take arbitrary $\mathfrak{T}* \in Rep(\mathfrak{T}')$ and let h'' be homomorphism from $\mathfrak{T}'$ to $\mathfrak{T}*$. Then $h'' \circ h'$ is a homomorphism from $\mathfrak{T}$ to $\mathfrak{T}*$.

Exercise 6.3 (6 Points)

1. Prove that every finite graph has a core (2 points)
2. Prove that two cores of the same graph are isomorphic. (4 points)

Solution

1. Stepwise eliminate edges and vertices (more generally in DB setting: eliminate row entries in tables) until no sub-graph (sub-instance) can be embedded homomorphically into it. Will reach core after finite steps as graph is finite.
2. Take two cores of a graph $\mathfrak{G}1$ and $\mathfrak{G}2$. There is $h1 : \mathfrak{G} \xrightarrow{hom} \mathfrak{G}1$ and $h2 : \mathfrak{G} \xrightarrow{hom} \mathfrak{G}2$. The restriction $h1'$ of $h1$ to $\mathfrak{G}2$ must be a surjective homomorphism (otherwise the image of the restriction $h1'[\mathfrak{G}2]$ would be a proper subgraph of $\mathfrak{G}1$ into which $h1' \circ h2$ would give a homomorphic embedding of $\mathfrak{G}$). Similarly for $h2$. Hence $\mathfrak{G}1$ and $\mathfrak{G}2$ have surjective homomorphisms into each other and so they are isomorphic.

Exercise 7 (14 points)

Exercise 7.1 (4 points)

1. Give a DL formalization of the following concept description
   "Father who has only children that are doctors or managers "
2. Give a DL formalization of the following assertion:
   "A busy female lecturer is a person who teaches at least three courses"

Exercise 7.2 (7 points)

Consider the following TBox $\mathcal{T}$

$$\begin{aligned} A &\sqsubseteq B \\ B &\sqsubseteq C \\ C &\sqsubseteq \exists R.D \\ D &\sqsubseteq \neg A \end{aligned}$$

1. In which DL is $\mathcal{T}$?
2. Is $\mathcal{T}$ satisfiable? If so, give a model, else argue why it is not satisfiable.
3. Is the concept D satisfiable w.r.t. $\mathcal{T}$, i.e., is there a model of $\mathcal{T}$ in which D is not interpreted by the empty set? If yes, give such a model else argue why it is not satisfiable
4. Is $D \sqcap A$ satisfiable w.r.t. $\mathcal{T}$? If so, give a model, else argue why it is not satisfiable.

Exercise 7.2 (3 points)

Show that subsumption can be reduced to satisfiability tests (allowing the introduction of new constants). More concretely:

$C \sqsubseteq D$ w.r.t. $\mathcal{O}$ iff $(\sigma \cup \{b\}, \mathcal{T}, \mathcal{A} \cup \{C(b), \neg D(b)\})$ is not satisfiable (where b is a fresh constant).