Intelligent Agents
Knowledge and Seeing

Özgür L. Özçep
Universität zu Lübeck
Institut für Informationssysteme
Today's lecture based on

- The AAMAS 2019 Tutorial „EPISTEMIC REASONING IN MULTI-AGENT SYSTEMS“, Part 2: Knowledge and Seeing
MOTIVATION
• Agents equipped with vision devices, positioned in the plane / space, e.g. robots that cooperate
• Aim: Represent and compute visual-epistemic reasoning of agents
Spatial reasoning

• Kripke models/epistemic models: abstract notion of possible world and of accessibility

• But agents usually act in space (and time)
  – Should be accounted for
  – The approach discussed here does this within the semantics of specific of form $a \text{ sees } b$
  – Leads to ability to express (qualitative spatial notions)

• Spatial Reasoning and spatial logics (temporal logics, see next lecture next to time) is a huge topic (see (Aiello et al, 2007))
MODELING
Assumptions (common knowledge)

- Agents are transparent points in the plane
- All objects of interest are agents
- Agents see infinite sectors
- Angles of vision are the same
- No obstacles (yet)

Each agent has a sector (cone) of vision
Possible Worlds

$U$ is set of unit vectors of $\mathbb{R}^2$

**Definition**

A geometrical possible world is a tuple $w = (\text{pos}, \text{dir})$ where:

- $\text{pos}: \text{Agt} \rightarrow \mathbb{R}^2$
- $\text{dir}: \text{Agt} \rightarrow U$

Remember: $\text{Agt} = \text{set of agents}$

- $\text{dir}(a)$ is the bisector of the sector of vision with angle $\alpha$
- $C_{p,u,\alpha}$: closed sector with vertex at the point $p$, angle $\alpha$ and bisector in direction $u$
- The region seen by $a$ is $C_{\text{pos}(a),\text{dir}(a),\alpha}$
An agent sees another

Definition

\[ a \text{ sees } b \text{ in } w = (\text{pos}, \text{dir}) \text{ iff } \text{pos}(b) \in C_{\text{pos}(a),\text{dir}(a),a} \]

- \(a\) sees \(a\)
- \(a\) sees \(b\)
- \(a\) does not see \(c\)
Epistemic model $\mathcal{M}_{\text{flatland}}$

**Definition**

$\mathcal{M}_{\text{flatland}} = (W, (\sim_a)_{a \in AGT}, V)$ with

- $W$ is the set of geometrical possible worlds
- $w \sim_a u$ iff agent $a$ sees the same agents in both $w$ and $u$ and these agents have the same position and directions in both $w$ and $u$

Accessibility relation $\sim_a$ is an equivalence relation. (logic: S5)
Axiomatization: Disjunctive surprises!

- $\models (K_a a \text{ sees } b) \lor K_a a \text{ sees } b$
  (Note that this is not an instance of a tautology)
- $\models K_a (b \text{ sees } c \lor d \text{ sees } e) \leftrightarrow K_a (b \text{ sees } c) \lor K_a (d \text{ sees } e)$
Example

- $K_aK_BCK_{c,d,e}(f\ sees\ g)$
  (Note that we use now $CK$ instead of $C$ to denote common knowledge operator)
In 1D, only qualitative positions matter

Observation
Qualitative positions are expressible in the language

- \( \text{sameDir}(a, b) := (a \text{ sees } b \leftrightarrow b \text{ sees } a) \)
- \( a \text{ is between } b, c := (a \text{ sees } b \leftrightarrow a \text{ sees } c) \)
Abstraction of the Kripke model in 1D

**Definition**

\[ abs(w) = \{ b \text{ sees } c \mid \mathcal{M}_{robots,1D}, w \models b \text{ sees } c \} \]

with

\[ w \equiv abs(u) \]

\[ \sim_a \]

\[ \sim_{abs_a} \]
Axiomatization in 1D

- Propositional tautologies;
- \((\text{sameDir}(a, b) \leftrightarrow \text{sameDir}(b, c)) \rightarrow \text{sameDir}(a, c)\);
- \(\neg (a \text{ isBetween } b, c) \lor \neg (b \text{ isBetween } a, c)\);
- \((K_a a \text{ sees } b) \lor (K_a a \text{ sees } b)\)
- \(a \text{ sees } b \rightarrow ((K_a b \text{ sees } c) \lor (K_a b \text{ sees } c))\)
- \(\chi \rightarrow \widehat{K}_a \psi\)
  where \(\chi, \psi\) are complete descriptions with \(\chi \sim_a^{\#_a} \psi\)
- \(K_a \phi \rightarrow \phi\)

A complete description is a conjunction that
- contains \(a \text{ sees } b\) or \(a \text{ sees } b\) for all agents \(a, b\)
- is satisfiable
In 2D, qualitative representation is open issue

Example

\[ K_b (a \text{ sees } b \land a \text{ sees } d \rightarrow a \text{ sees } c) \]

(Assuming here that cone of vision is 1-D: \( \alpha = 0 \))
Abstraction of the Kripke model in 2D

Definition

\[ \text{abs}(w) = \{ b \text{ sees } c \mid \mathcal{M}_{\text{robots,2D}}, w \models b \text{ sees } c \} \] with

Diagram:

\( w \) \( \sim_a \) \( u \)
\( \text{abs}(w) \)
\( \text{abs}(u) \)
Model checking

• Input
  – A description of a world \( w \) (not the whole model)
  – A formula \( \phi \)

• Output: yes iff \( w \models \phi \)
### Complexity

<table>
<thead>
<tr>
<th>Lineland</th>
<th>flatland</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSPACE-complete</td>
<td>PSPACE-hard, and in EXPSPACE</td>
</tr>
<tr>
<td></td>
<td>(the latter shown by reduction to $\mathbb{R}$-FOL-theory)</td>
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</tbody>
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$\mathbb{R}$-FOL-theory = elementary algebra: First-order logic (FOL) of the reals

Language:
- FOL with equality and
- Constants 0, 1
- Functions symbols $+$, $\times$
- Relation symbols $<$

- Can define, e.g., reals as solutions of polynomials
- Validity of elementary algebra is known to be in EXSPACE
Complexity

**Definition**

Standard translation from modal logic to first-order logic

- Atomic propositions $p$ are rewritten to unary predicates $P$
- $K_a p$ rewritten to $\forall u (R(w, u) \rightarrow P(u))$

(see e.g. Blackburn et al. Modal logic, 2001)

**Observation (Adapted translation to ℝ-FOL-theory)**

$K_a (b \text{ sees } c)$ rewritten into

$$\forall pos_a' \forall pos_b' \ldots \forall dir_a' \forall dir_b' \ldots$$

$$\bigwedge_{b \in \text{AGT}} \left[ (pos_b \in C_{pos(a), dir(a), a}) \rightarrow (pos_b' = pos_b \land dir_b' = dir_b) \right] \land$$

$$\left[ (pos_b \notin C_{pos(a), dir(a), a}) \rightarrow (pos_b' \notin C_{pos(a), dir(a), a}) \right]$$

$$\rightarrow (pos_c' \notin C_{pos(b), dir(b), a})$$
VARIANT WITH CAMERAS
Agents are cameras

• Cameras
  – Can turn
  – Can NOT move

• Common knowledge
  – Of the positions of agents
  – Of the abilities of perception
Semantics: restricted set of worlds

**Definition**

Given a fixed $pos': AGENTS \rightarrow \mathbb{R}^2$, worlds are $w = (pos, dir)$ s.t. $pos = pos'$
Semantics: $\mathcal{M}_{\text{cameras}}$

**Definition**

$\mathcal{M}_{\text{cameras}}$ is $\mathcal{M}_{\text{flatland}}$ where we publicly announced the current positions of the agents.
Abstraction of the Kripke model in 2D works

Definition

\[ \text{abs}(w) = \{ b \text{ sees } c \mid \mathcal{M}_{\text{cameras}}, w \models b \text{ sees } c \} \]

with

\[ \sim_a \]

\[ \sim_{\text{abs}}a \]
Example (Family of vision sets of agent a)

\[ S_a = \{\{b\}, \emptyset, \{c\}, \{d\}, \{d, f\}, \{d, f, e\}, \{f, e\}, \{e\}\} \]

A possible world \((dir)\) can be described as \(\{S_a\}_{a \in AGT}\)

(configurations on next slides by \(a\) moving counterclockwise)
Example (Family of vision sets of agent a)

\[ S_a = \{\{b\}, \emptyset, \{c\}, \{d\}, \{d, f\}, \{d, f, e\}, \{f, e\}, \{e\}\} \]
Spectrum of vision

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NB:
- each \( S_a \) is computed in \( O(k \log k) \) steps, where \( k = \#(Agt) \).
PDL (Propositional Dynamic Logic)

Definition (PDL Syntax)

\[ \phi ::= a \text{ sees } b \mid \neg \phi \mid \phi \lor \psi \mid [\pi]\phi \]

- Intended semantics for \([\pi]\phi\): after all executions of program \(\pi\), it holds that \(\phi\)
PDL Language

Definition (Syntax of programs)

\[ \pi ::= a \sim \mid \phi \mid \pi ; \pi' \mid \pi \cup \pi' \mid \pi^* \]

- Intended semantics for
  - \( a \sim \): a turns;
  - \( \phi \): the program succeeds when \( \phi \) is true
  - \( \pi ; \pi' \): \( \pi \) followed by \( \pi' \)
  - \( \pi \cup \pi' \): non-deterministically execute \( \pi \) or \( \pi' \)
  - \( \pi^* \): repeat \( \pi \) a finite, but non-deterministically, number of times
Translating epistemic operators in programs

- $K_a$ is simulated by

\[
[(a \text{ sees } b_1) \cup (a \text{ sees } b_1; b_1^\wedge)); \ldots; (a \text{ sees } b_n) \cup (a \text{ sees } b_n; b_n^\wedge))]
\]

\[\pi_a\]

($b_i = \text{all agents except for } a$)

- Each component program $[(a \text{ sees } b_i) \cup (a \text{ sees } b_i; b_i^\wedge)]$ says: can turn view of $b_i$ iff $a$ does not see $b_i$
- Thus the program may change arbitrarily all agents, other than $a$, that $a$ cannot see
- And this is exactly the semantics of $K_a$
Model checking

Observation

Model checking of PDL for cameras is PSPACE-complete

(Gasquet et al. 2014)
Summary: Visual-epistemic reasoning of agents

• Epistemic language involving atomic propositions ‘a sees b’.

• Semantics in geometric and Kripke models.

• 1D case and 2D case with cameras (spectrum of vision):
  – Finite abstraction in the 1D case and in the 2D case with cameras (spectrum of vision).
  – Optimal PSPACE model checking.

• Open problem for the full 2D case: finite abstraction?
Future work

• Obstacles (occlusion)

• Moving agents/cameras in the plane: mathematically more complex; finite abstractions may not work

• Agents/cameras in the 3D space
APPENDIX
References

Color Convention in this course

- **Formulae, when occurring inline**
- **Newly introduced terminology and definitions**
- **Important results (observations, theorems) as well as emphasizing some aspects**
- **Examples are given with standard orange with possibly light orange frame**
- **Comments and notes**
- **Algorithms**