Intelligent Agents

Knowledge and Time

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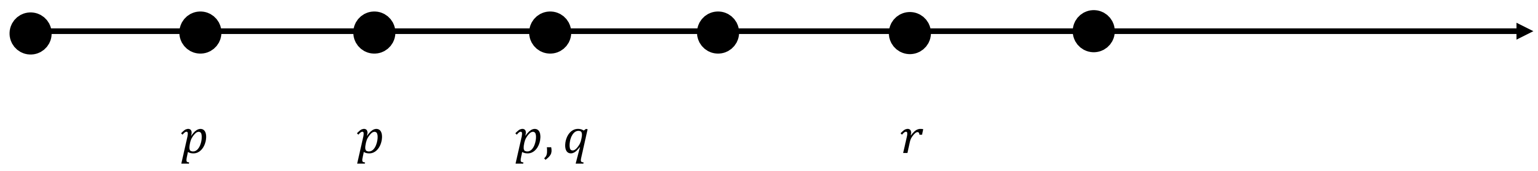
Todays lecture based on

- The AAMAS 2019 Tutorial **„EPISTEMIC REASONING IN MULTI-AGENT SYSTEMS“, Part 3: Knowledge and Time**
  http://people.irisa.fr/Francois.Schwarzentruber/2019AAMAStutorial/

LINEAR TEMPORAL LOGIC

Models



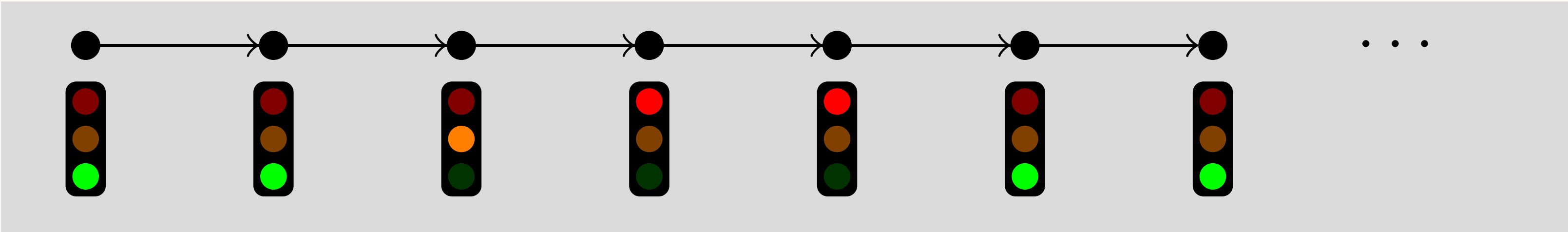
Definition

A linear temporal model is a structure $(\mathbb{N}, <, V)$ such that :

- $V: N \rightarrow 2^{AP}$
- $<$ is the natural order on $\mathbb{N}$

We sometimes do not mention the linear order $<$

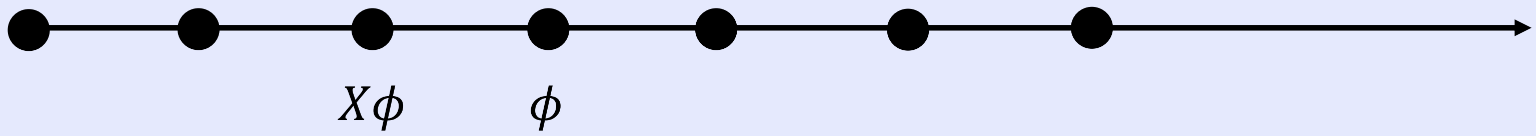
Example (traffic light)



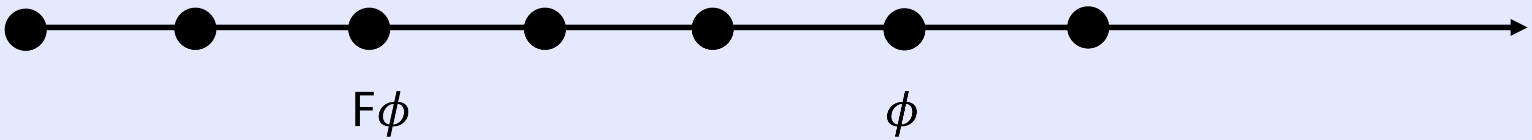
Syntax and semantics

Definition (Temporal Modalities of LTL)

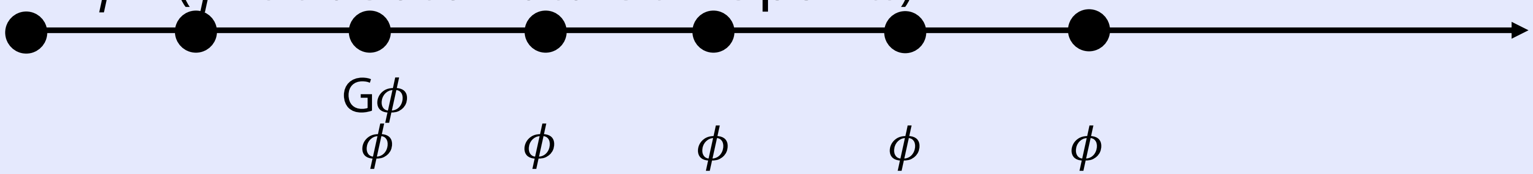
- $X\phi$ (ϕ is true at the next time)



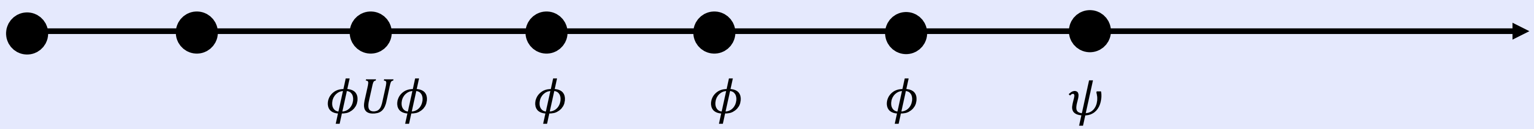
- Fϕ (ϕ is true at some point in the future)



- Gϕ (ϕ is true at all future time points)



- $\phi U \psi$ (ψ is true at some future time point and ϕ holds until ψ)



Syntax and semantics

- $(\mathbb{N}, V), t \vDash p$ if $p \in V(t)$
- $(\mathbb{N}, V), t \vDash \neg\phi$ if not $(\mathbb{N}, V), t \vDash \phi$
- $(\mathbb{N}, V), t \vDash \phi \vee \psi$ if $(\mathbb{N}, V), t \vDash \phi$ or $(\mathbb{N}, V), t \vDash \psi$
- $(\mathbb{N}, V), t \vDash X\phi$ if $(\mathbb{N}, V), t+1 \vDash \phi$
- $(\mathbb{N}, V), t \vDash F\phi$ if there is t′ $\geq t$ such that $(\mathbb{N}, V), t' \vDash \phi$
- $(\mathbb{N}, V), t \vDash G\phi$ if for all t′ $\geq t$: $(\mathbb{N}, V), t' \vDash \phi$
- $(\mathbb{N}, V), t \vDash \phi U \psi$ if there is t′ $\geq t$ such that $(\mathbb{N}, V), t' \vDash \psi$ and $(\mathbb{N}, V), t'' \vDash \phi$ for all $t'' \in [t, t'-1]$

Satisfiability problem (reminder)

Definition

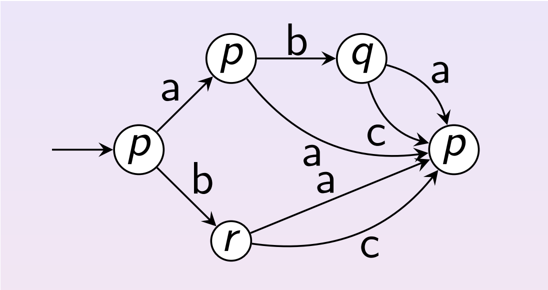
The satisfiability problem is:

- Input: a formula ϕ
- Output: yes if there is V such that $(\mathbb{N}, V), t \vDash \phi$

Theorem

The satisfiability problem is PSPACE-complete

Model checking (reminder)



Definition

The model checking problem is:

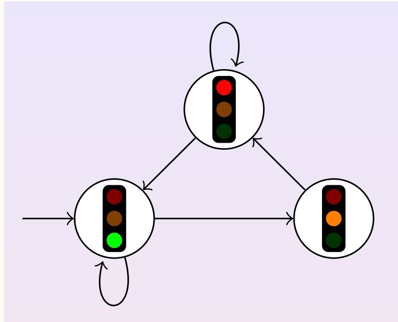
- Input: a transition system S; an LTL formula ϕ
- Output: yes if all paths of S starting from an initial state of S satisfy ϕ

Theorem

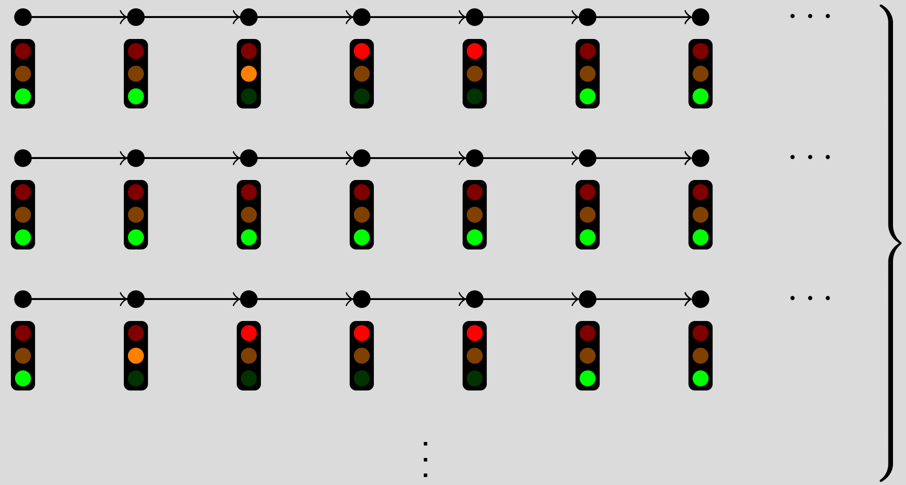
The model checking problem of LTL is PSPACE-complete

Example

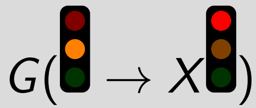
Transition system S:



Paths of S starting from initial state



they all satisfy



EPISTEMIC LINEAR TEMPORAL LOGIC

A combined logic

- Epistemic linear temporal logic (ELTL)
  - Epistemic logic (with epistemic operators K_a) combined with
  - Linear temporal logic (with temporal operators X, F, G, U)

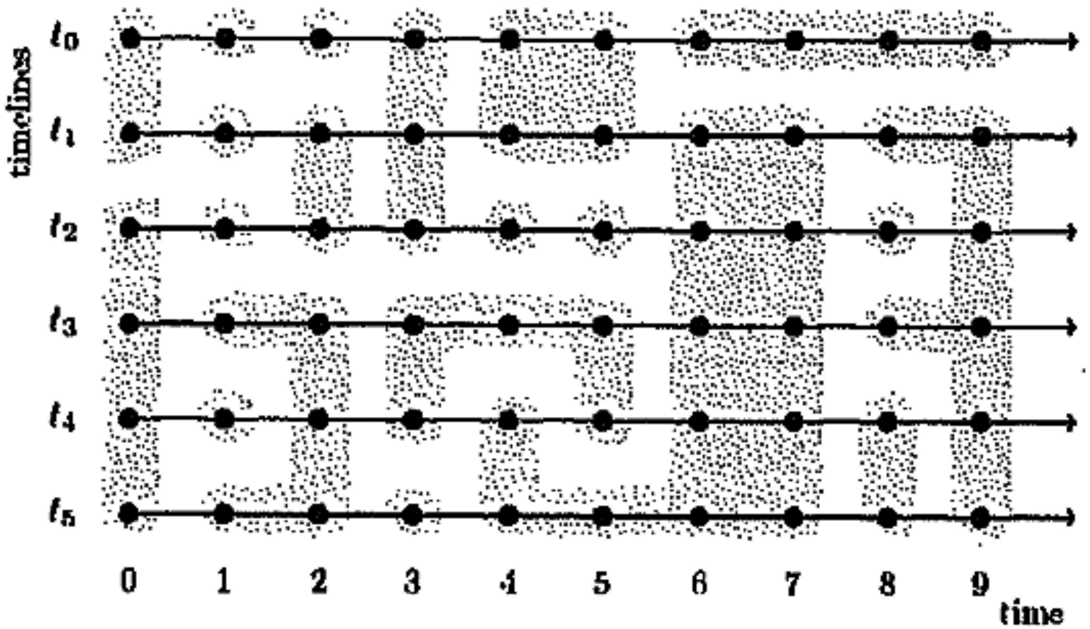
- Example of combining systems/logics
  - Conference series „Frontiers of combining systems“ (Frocos)
  - Interesting (ancient Dialogue-style) paper on combining systems : P. Blackburn and M. De Rijke., 1997
  - Overview in Stanford Encyclopedia of Philosophy: Carnielli and Coniglio: Combining Logics, 2020

Models

Definition

An ELTL model is a structure $\mathcal{M} = (TL \times \mathbb{N}, (\sim_a)_{a \in AGT}, V)$ such that

- TL is a non-empty set of timelines (runs)
- For all agents a, $\sim_a$ is an equivalence relation on $TL \times \mathbb{N}$
- $V: TL \times \mathbb{N} \rightarrow 2^{AP}$



Case of one agent a; regions denote equivalence classes of $\sim_a$

INTERACTION BETWEEN KNOWLEDGE AND TIME

Axiomation in case: no interaction -> Fusion

- All classical tautologies (and their uniform substitutions)

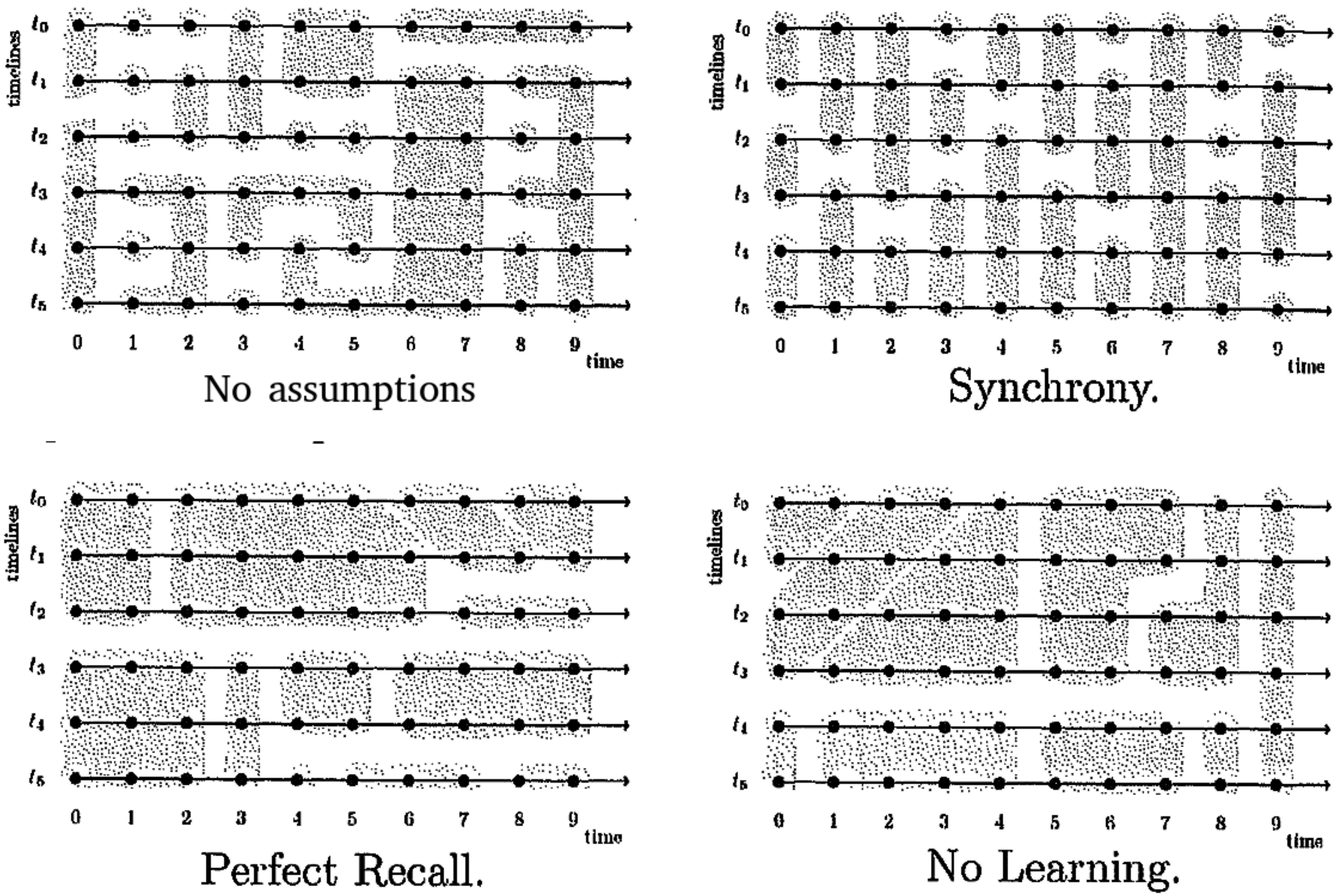
EL

- $K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$
- $K_a\phi \to \phi$
- $\widehat{K}_a \top$
- $K_a\phi \to K_aK_a\phi$
- $\neg K_a\phi \to K_a\neg K_a\phi$

LTL

- $G(\phi \to \psi) \to (G\phi \to G\psi)$
- $X(\phi \to \psi) \to (X\phi \to X\psi)$
- $X \neg\phi \leftrightarrow \neg X \phi$
- $G\phi \to (\phi \wedge XG\phi)$
- $G(\phi \to X\phi) \to (\phi \wedge G\phi)$
- $(\phi U \psi) \to F\psi$
- $(\phi U \psi) \leftrightarrow (\psi \vee X(\phi U \psi))$

Adding interaction

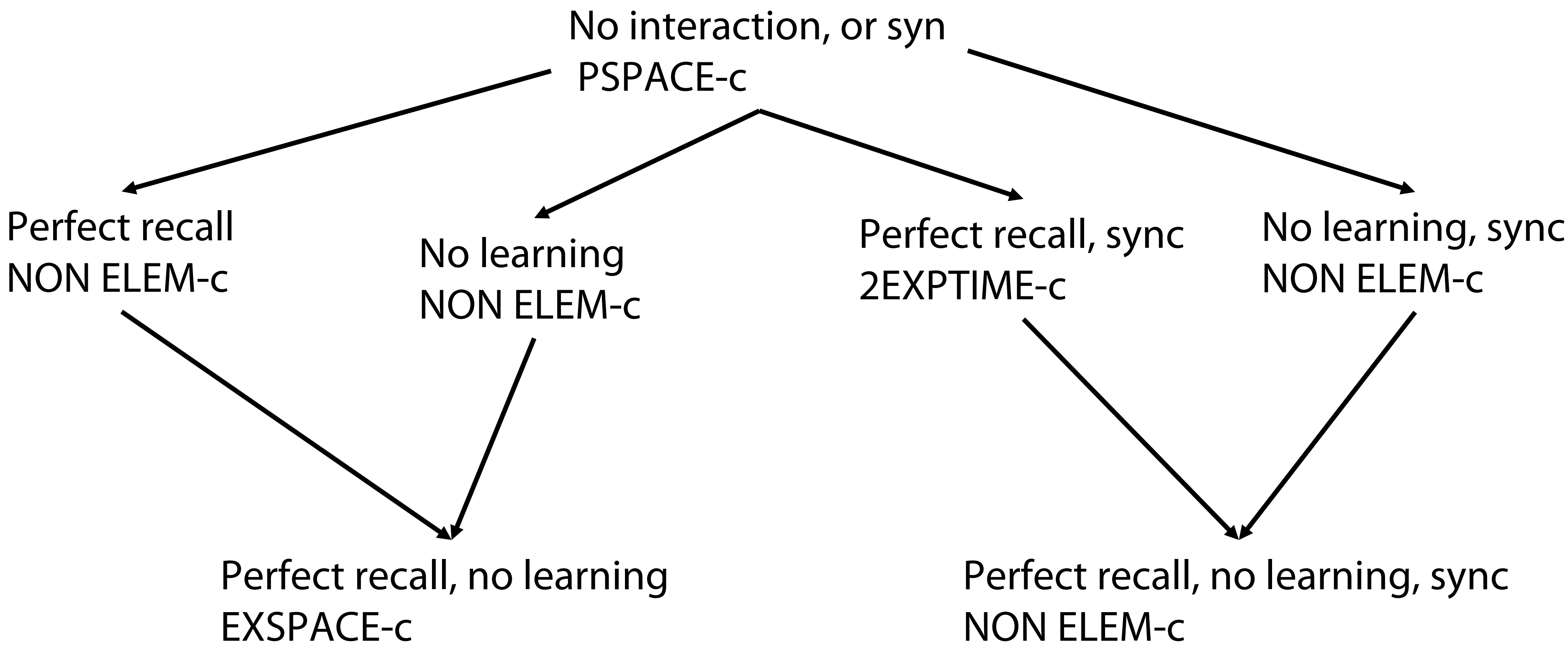


For additional criteria (resulting in 96 different epistemic temporal logics see: Halpern/Vardi, 1989)

Corresponding axioms

Synchronous	Agents know the time t (not an axiom)
Perfect recall, Synchronous	$K_a X\phi \rightarrow X K_a\phi$
Perfect recall	$K_a\phi \wedge X(K_a\psi \wedge \neg K_a\chi) \rightarrow \neg K_a\neg(K_a\phi\ U(K_a\psi U \neg\chi))$
No learning	$(K_a\phi U\ K_a\phi) \rightarrow K_a(K_a\phi U K_a\psi)$
No learning, Synchronous	$X K_a\phi \rightarrow K_a X\phi$

Complexity of the satisfiability problem

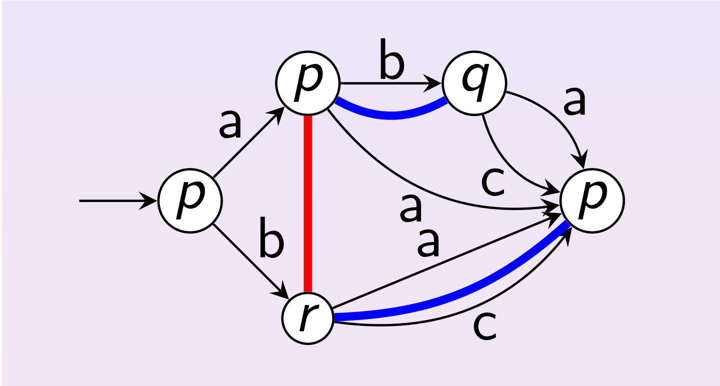


((Reminder:

Complexity Class ELEMANTARY= $\bigcup_{k\in\mathbb{N}} k - EXP = \ DTIME(2^{n}\ ) \cup DTIME\left(2^{2^{n}}\ \right) \cup \cdots$))

MODEL CHECKING

Model checking



Definition

The model checking problem is:

- Input:
  - an epistemic transition system S, i.e. a transition system augmented with epistemic relations $(R_a)_{a \in AGT}$ with a set of initial states;
  - an LTL formula ϕ
- Output: yes if "$\mathcal{M}_S, (\rho, 0) \models \phi$" for all paths ρ of S starting from an initial state of S

Possible Definition of $\mathcal{M}_S$

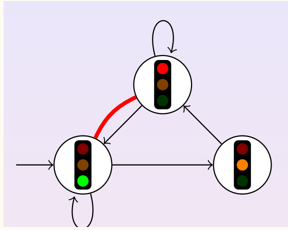
Definition

Given a transition system S, define
$\mathcal{M}_S = (\mathrm{TL} \times \mathbb{N}, (\sim_a)_{a \in AGT}, V)$ such tat

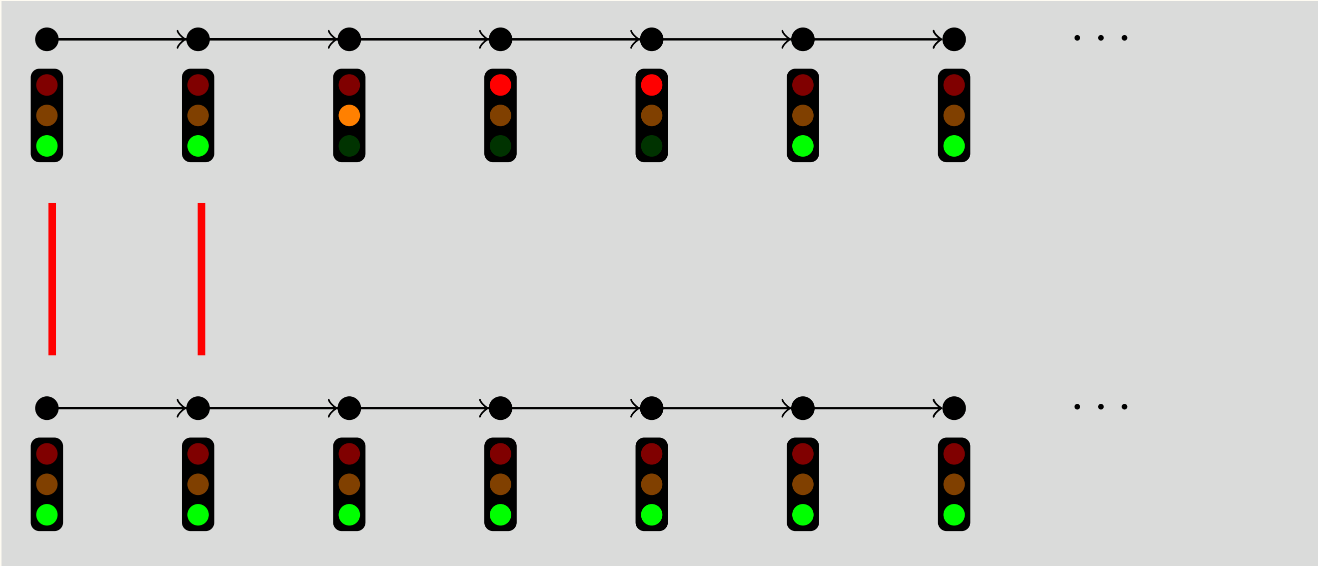
- TL is the set of paths of S starting in an initial state of S;
- For all agents a: $(\rho, t) \sim_a (\rho', t')$ if
  - $t = t'$ (synchrony)
  - $\rho[i] R_a \rho'[i]$ for all $i \in \{0, \dots, t\}$ (perfect recall)
- $V: TL \times \mathbb{N} \rightarrow 2^{AP}$ is defined by
  $V(\rho, t) =$ set of propositions true at $\rho[t]$

Example

Transition system S:



Perfect recall



Another Possible Definition of $\mathcal{M}_S$

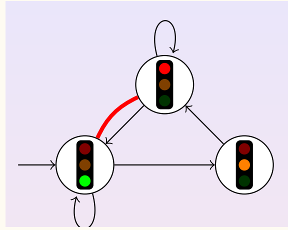
Definition

Given a transition system S, define
$\mathcal{M}_S = (\mathrm{TL} \times \mathbb{N}, (\sim_a)_{a \in AGT}, V)$ such tat

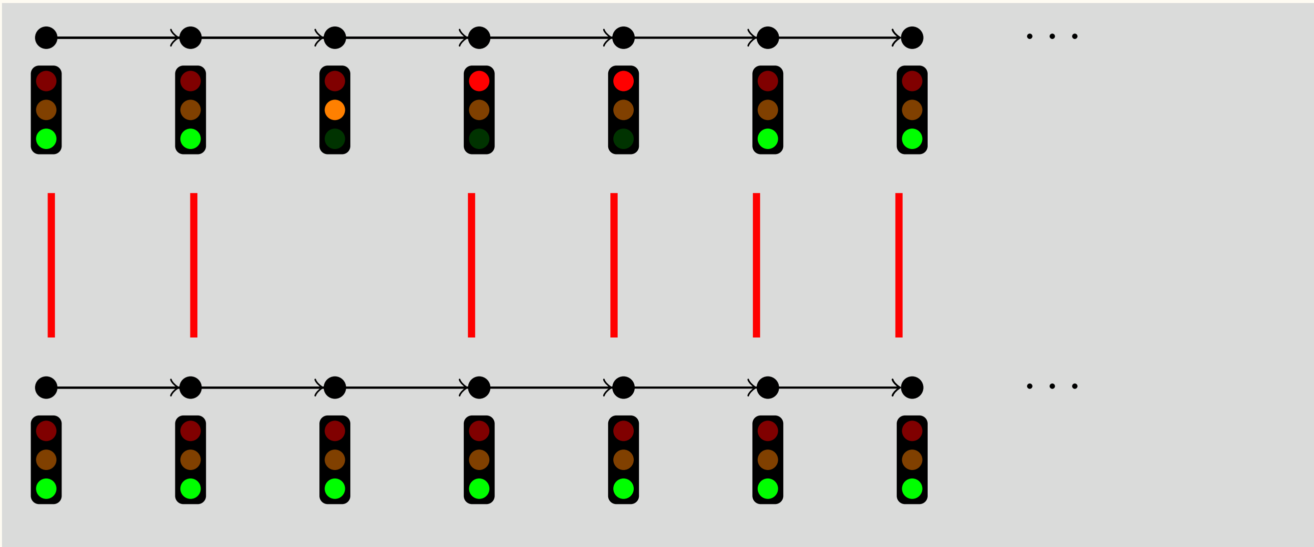
- TL is the set of paths of S starting in an initial state of S;
- For all agents a: $(\rho, t) \sim_a (\rho', t')$ if
  - $t = t'$ (synchrony)
  - $\rho[t] R_a \rho'[t]$ (memoryless)
- $V: TL \times \mathbb{N} \to 2^{AP}$ is defined by
  $V(\rho, t) =$ set of propositions true at $\rho[t]$

Example

Transition system S:



Memoryless



Theorem (Engelhardt et al. 2007)

The model checking problem for memoryless and synchronuos systems is PSPACE-complete

Theorem (van der Meyden and Shilov, 1999)

The model checking problem under perfect recall and synchrony is

- Undecidable if CK (common knowledge operator) and until (U)
- NON ELEM-c if until but not CK
- PSPACE-c if CK but not until

See also (Bozzelli et al 2019) for recent results.

Uhhh, a lecture with a hoepfully useful

APPENDIX

References

- P. Blackburn and M. De Rijke. Zooming in, zooming out. Journal of Logic, Language and Information, 6(1):5–31, 1997.
- W. Carnielli and M. E. Coniglio. Combining logics. The Stanford Encyclopedia of Philosophy (Win- ter 2008 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/win2008/entries/logic- combining/.
- J. Y. Halpern and M. Y. Vardi. The complexity of reasoning about knowledge and time i. lower bounds. Journal of Computer and System Sciences, 38(1):195–237, 1989.
- K. Engelhardt, P. Gammie, and R. Meyden. Model checking knowledge and linear time: Pspace cases. In Proceedings of the International Symposium on Logical Foundations of Computer Science, LFCS '07, pages 195—211, Berlin, Heidelberg, 2007. Springer-Verlag.
- L. Bozzelli, B. Maubert, and A. Murano. The complexity of model checking knowledge and time. In Proceedings of the 28th International Joint Conference on Artificial Intelligence, IJCAI'19, pages 1595– 1601. AAAI Press, 2019.

Color Convention in this course

- Formulae, when occurring inline
- Newly introduced terminology and definitions
- Important results (observations, theorems) as well as emphasizing some aspects
- Examples are given with standard orange with possibly light orange frame
- Comments and notes
- Algorithms