Intelligent Agents

Epistemic Logic

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Todays lecture (and the following five) based on

- The AAMAS 2019 Tutorial **„EPISTEMIC REASONING IN MULTI-AGENT SYSTEMS“**
  http://people.irisa.fr/Francois.Schwarzentruber/2019AAMAStutorial/

MOTIVATION

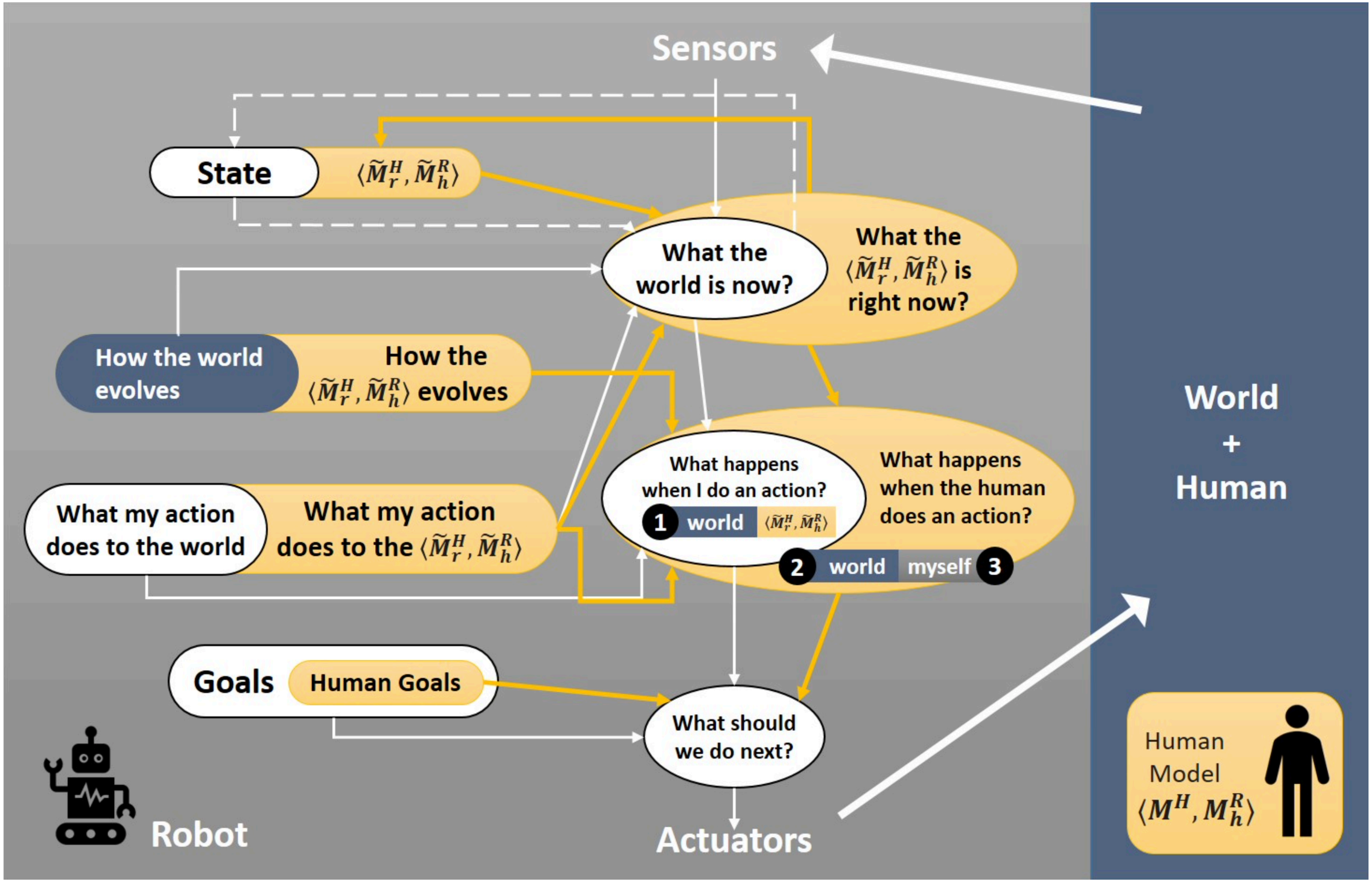
The need for knowledge

- Many multi-agent systems require to model knowledge of others‘ due to imperfect information
  - Agents have local view of environment
  - Agents communicate
  - Agents act -> Decisions taken w.r.t. knowledge
- Similar: Reasoning about other agents „knowledge“ in game theory (second half of this lecture thread Agents, Mechanism, and Collaboration (lecture))
  - Speculation about other one‘s strategies/values of things (and about their speculations on our values...)?
  - Collaborative agents (negotiation, communication)
  - Imperfect information

Interaction relies on knowledge

- **if** I know it is safe **then** I go
- **if** I know you are at the market place **then** I join you
- **if** (I know it is safe) and (I know you do not know it is safe) **then** I tell you it is safe
- **if** I know you know it is safe **then**
  I do not tell you it is safe
- **if** I know you know I know it is safe or not **then**
  I do not wait for a message from you

Reminder: Human-compatible AI



Towards XAI?

- XAI = explainable AI: Need to built understandable (human comprehensible) AI systems
- XAI for multi-agent systems?
  - Example: Robots interacting with humans
  - Legal issues in case of failures

Example (Explanations of a robot exploring a world)

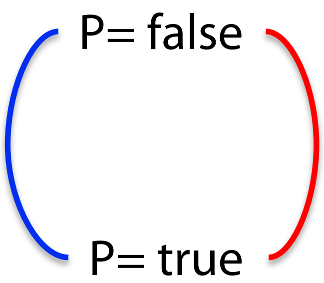
- I turned left because $x = 0$ and $y > 5$
  ⇒ not human understandable
- I turned left because my neuron 53 was activated.
  ⇒ not human understandable
- I turned left because I *knew* this area was not explored.
  ⇒ human understandable

The need for reasoning

- Given
  - what agents sense
  - The actions and communications that occurred
- What does each agent know?

Once upon a time ... In 2011 -212

- Schwarzentruber (2. presenter of AAMAS 2019) says: „I explained epistemic logic to other researchers in logic/AI/verification ...

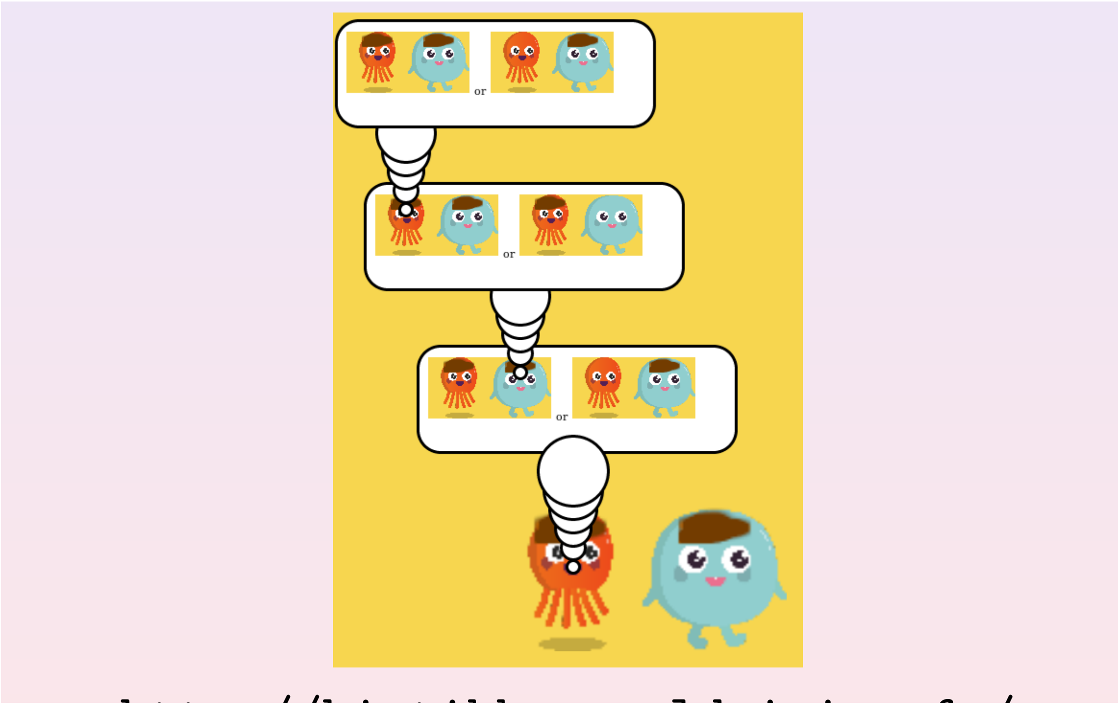


- ... but nobody understood me ..."

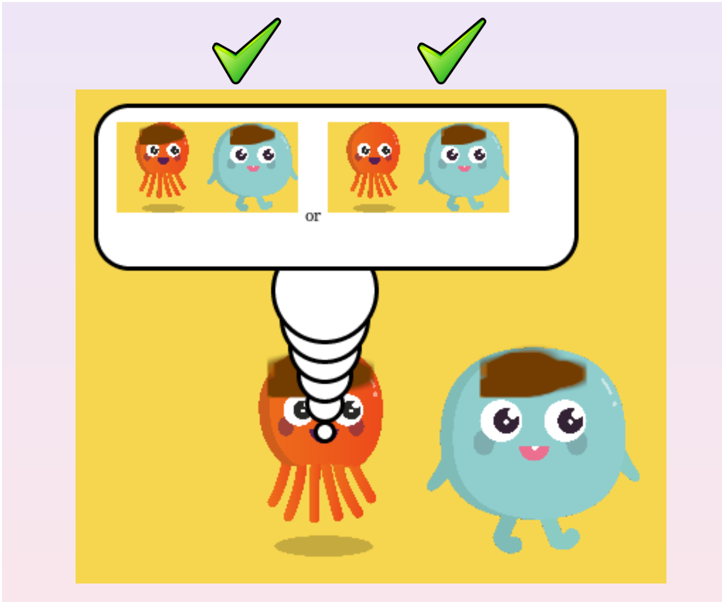
Possible worlds

- „... But, since 2017, everybody understood me with comics ...“

- Have a look at http://hintikkasworld.irisa.fr/



Semantics of knowing something



- Agent a knows that agent b is dirty
- Instance of the famous „muddy children puzzle“

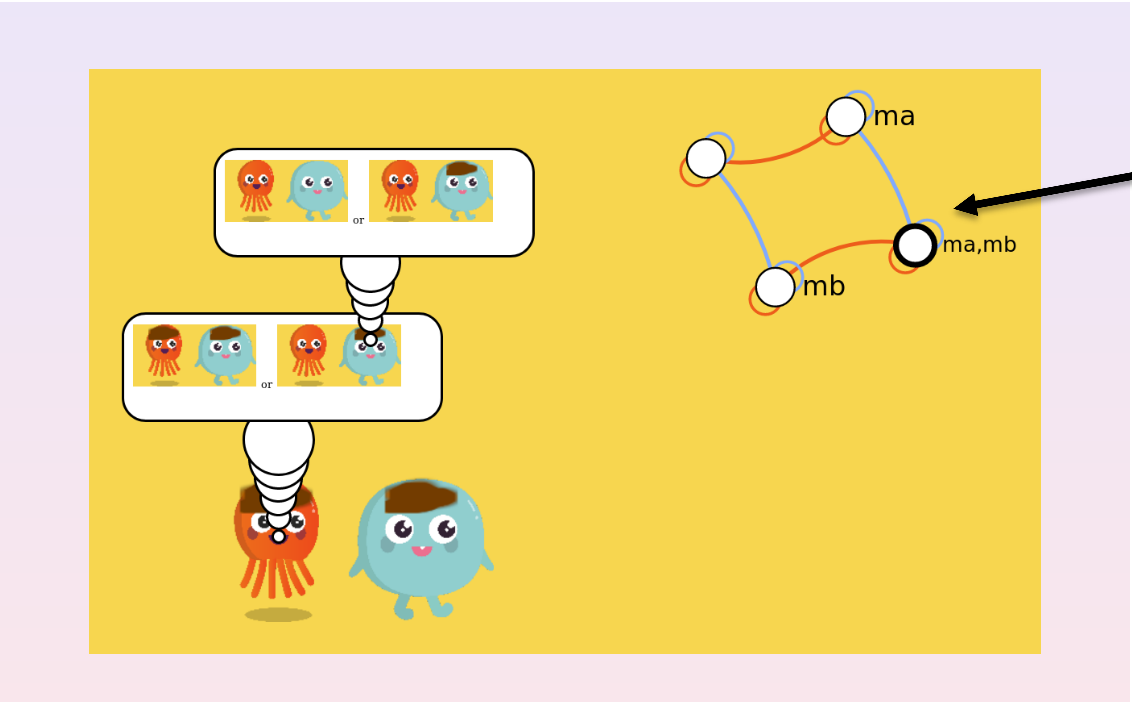
Muddy children Puzzle

„Three children (a,b,c) are playing in the mud. Father calls the children to the house, arranging them in a semicircle so that each child can clearly see every other child. "At least one of you has mud on your forehead", says Father. The children look around, each examining every other child's forehead. Of course, no child can examine his or her own. Father continues, "If you know whether your forehead is dirty, then step forward now". No child steps forward. Father repeats himself a second time, "If you know whether your forehead is dirty, then step forward now". Some (a,b) but not all of the children step forward. Father repeats himself a third time, "If you know whether your forehead is dirty, then step forward now". All of the remaining children step forward. Explain why a,b stepped forward after two requests. (In general show: if m children are muddy then after m requests of the father those will step forward"

We will reconsider this puzzle (in the context of dynamic epistemic logic (epistemic logic with operators changing epistemic models))

Epistemic state = pointed Kripke structure

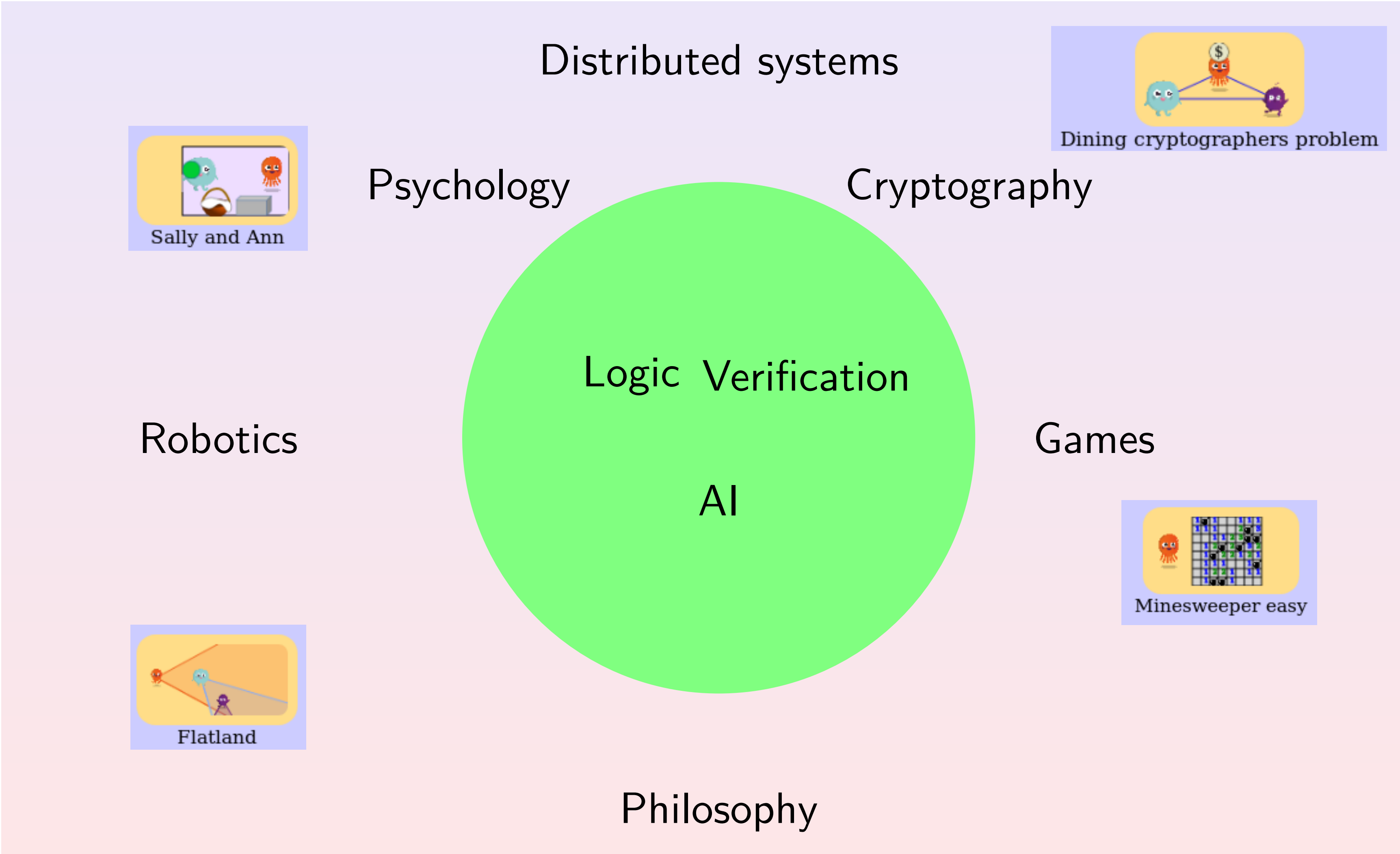
- Comics correspond to unravelling of a pointed Kripke structure



Actual word

ma = agent a has muddy forehead
mb = agent b has muddy forehead

Explaining these in many communities



Open-source project Hintikka‘s world

- http://hintikkasworld.irisa.fr/
- https://gitlab.inria.fr/ fschwarz/hintikkasworld

- Web app
- Modular source code in Typescript
- Easy to add examples
- Several contributors

EPISTEMIC LOGICS (SYNTAX AND SEMANTICS)

Epistemic states

- $AP = \{p, p_1, p_2, \dots\}$ countable set of atomic propositions
- $AGT = \{a, b, c, \dots\}$ finite set of agents

Definition

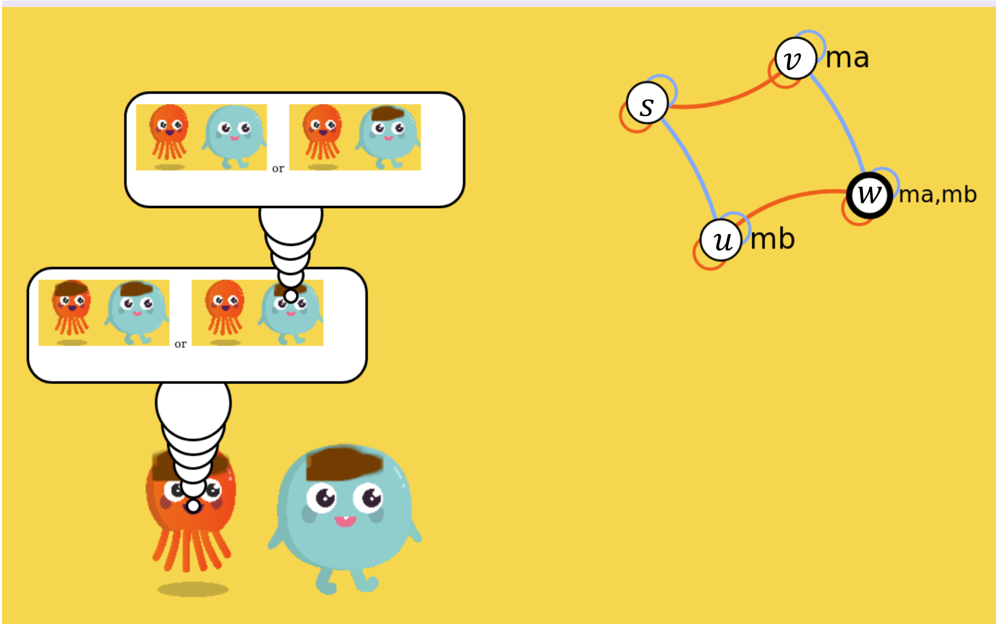
An epistemic model $M = (W, (R_a)_{a \in AGT}, V)$ is a tuple where

- $W = \{w, u, \dots\}$ is a non-empty set of possible worlds
- $R_a \subseteq W \times W$ is an accessibility relation for agent a
- $V: W \rightarrow 2^{AP}$ is a valuation function

A pair (M, w) is called an epistemic state, where w represents the actual world. A frame is an epistemic model withouth the valuation.

Example of an epistemic state

- Muddy children in Hintikka's world



- $W = \{ w, u, v, s \}$
- $R_a = \{ (w,w), (w,u), (u,w), (u,u), (v,v), (v,s), (s,v), (s,s) \}$
- $R_b = \{ (w,w), (w,v), (v,w), (v,v), (u,u), (u,s), (s,u), (s,s) \}$
- $V(w) = \{m_a, m_b\}; V(u) = \{m_b\}; V(v) = \{m_a\}; V(s) = \emptyset$

Syntax of $\mathcal{L}_{EL}$

Definition

- The syntax of $\mathcal{L}_{EL}$ (concretely, its set of well-formed formulae) is given by the following grammar:
  $\phi ::= p \mid \neg\phi \mid (\phi \vee \phi) \mid K_a\phi$
  where p ranges over AP and a ranges over AGT

- Other operators are defined as follows:
  - $\hat{K}_a$ abbreviates $\neg K_a \neg\phi$
  - ...

- $K_a\phi$ read as „agent a knows/believes that ϕ is true“
- $\hat{K}_a\phi$ read as „agent a considers ϕ as possible“

Syntax of $\mathcal{L}_{EL}$

Definition

- Other operators are defined as follows:
  - $(\phi \wedge \psi)$ abbreviates $\neg(\neg\phi \vee \neg\psi)$
  - $(\phi \rightarrow \psi)$ abbreviates $(\neg\phi \vee \psi)$
  - $\bot$ abbreviates $p \wedge \neg p$
  - $\top$ abbreviates $\neg\bot$

Length and Depth

Definition

The size/length and the modal depth of formulae are defined as follows:

- $|p| = 1$ $\qquad d(p) = 0$
- $|\neg\phi| = |\phi| + 1$ $\qquad d(\neg\phi) = d(\phi)$
- $|\phi \wedge \psi| = |\phi| + |\psi| + 1$ $\qquad d(\phi \wedge \psi) = \max\{d(\phi), d(\psi)\}$
- $|K_a\phi| = \phi + 1$ $\qquad d(K_a\phi) = 1 + d(\phi)$

Semantics of $\mathcal{L}_{EL}$

Definition

The semantics of $\mathcal{L}_{EL}$ (the modelling/satisfaction relation $\vDash$) is defined recursively by:

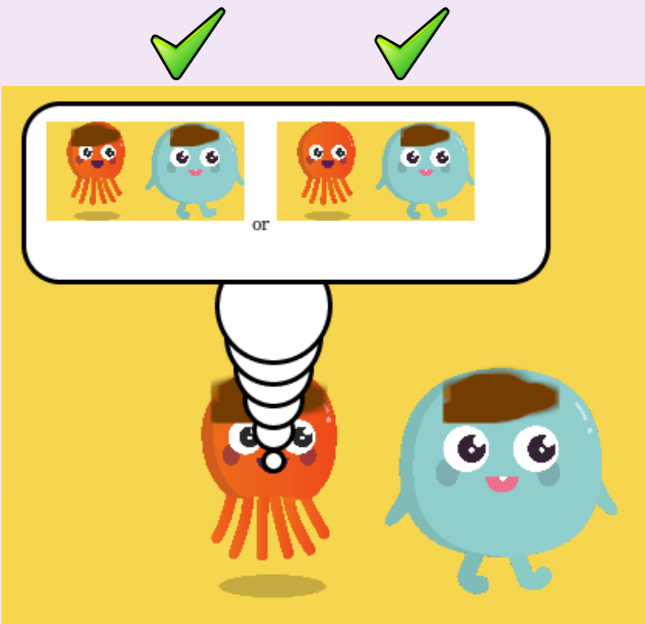
- $\mathcal{M}, w \vDash p$ if $p \in V(w)$
- $\mathcal{M}, w \vDash \neg\phi$ if not $\mathcal{M}, w \vDash \phi$
- $\mathcal{M}, w \vDash \phi \vee \psi$ if $\mathcal{M}, w \vDash \phi$ or $\mathcal{M}, w \vDash \psi$
- $\mathcal{M}, w \vDash K_a\phi$ if for all u s.t. wR_au: $\mathcal{M}, u \vDash \phi$

Wording: $\mathcal{M}, w$ models ϕ; $\mathcal{M}, w$ satisfies ϕ ; ϕ is true in $\mathcal{M}, w$; ϕ holds in world w in $\mathcal{M}$

If $\mathcal{M}, w \vDash \phi$ for all worlds, then we write $\mathcal{M} \vDash \phi$ and say ϕ is true/valid in $\mathcal{M}$

Semantics of dual operators $\widehat{K_a}$

- $\mathcal{M}, w \models K_a\phi$ if for all u s.t. wR_au: $\mathcal{M}, u \models \phi$
- $\mathcal{M}, w \models \widehat{K_a}\phi$ if there is u s.t. wR_au: $\mathcal{M}, u \models \phi$



$\mathcal{M}, w \models K_a m_b$

$\mathcal{M}, w \models \hat{K}_a m_a$

Common knowledge

Definition

The syntax of $\mathcal{L}_{ELCK}$ is given by the following grammar:

$$\phi ::= p \mid \neg\phi \mid (\phi \vee \phi) \mid K_a\phi \mid C_G\phi$$

where $p \in AP, a \in AGT, G \in 2^{AGT}$

Definition

The semantics of $\mathcal{L}_{ELCK}$ is that of $\mathcal{L}_{EL}$ extended by:

$\mathcal{M}, w \vDash C_G\phi$ iff for all $u \in W$: wR_Gu entails $\mathcal{M}, u \vDash \phi$

Here R_G denotes the transitive closure of $\bigcup_{a \in G} R_a$

MODEL CHECKING

Model checking problem

- Input:
  - An epistemic state $\mathcal{M}, w$
  - A formula ϕ
- Output: yes if $\mathcal{M}, w \vDash \phi$, no otherwise

Theorem

Model checking (both: with and without common knowledge operators) is P-complete

(Vanilla) Model checking algorithm

- Input: a Kripke model $\mathcal{M}$, a formula ϕ
- Output: set of worlds of $\mathcal{M}$ in which ϕ holds

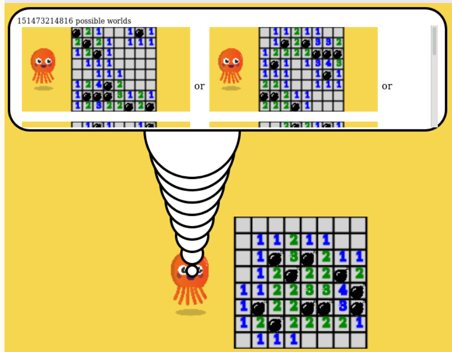
- **function** $mc(\mathcal{M}, \phi)$
  match ϕ **do**
  - **case** p: **return** $\{w \mid \mathcal{M}, w \vDash p\}$
  - **case** $\neg\psi$: **return** W \ $mc(\mathcal{M}, \psi)$
  - **case** $(\psi_1 \vee \psi_2)$: **return** $mc(\mathcal{M}, \psi_1) \cup mc(\mathcal{M}, \psi_2)$
  - **case** $K_a\psi$: **return** $\{w \mid R_a(w) \subseteq mc(\mathcal{M}, \psi)\}$

State explosion problem

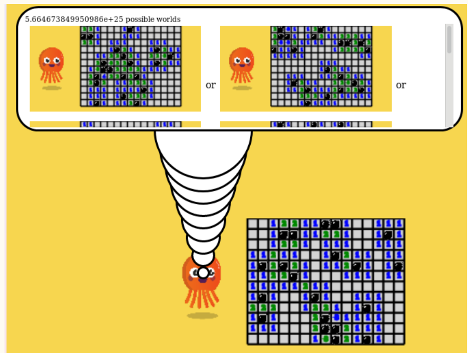
Example

Minesweeper

- 8×8 with 10 bombs:
  $> 10^{12}$ possible worlds



- 10×12 with 20 bombs:
  $> 10^{25}$ possible worlds



State explosion problem

- See (Benthem et al. 2015), (Benthem et al. 2018)

- Also see: (Charrier/S. 2017), (Charrier/S. 2018)
  - Succinct representations of epistemic states; **and** actions ($\Longrightarrow$ Dynamic Epistemic Logic);
  - Easy to specify by means of accessibility programs;
  - Succinct model checking Pspace-complete (and so stays in Pspace as for non-succinct case).

CALCULI

Satisfiability and validity

Definition

- A formula ϕ is satisfiable iff there is an epistemic state $\mathcal{M}, w$ s.t. $\mathcal{M}, w \vDash \phi$
- A formula ϕ is valid iff for all epistemic states $\mathcal{M}, w$: $\mathcal{M}, w \vDash \phi$

Clearly ϕ is valid iff $\neg\phi$ is not satisfiable

Example

- $K_a p$ is satisfiable but not valid
- $\big(K_a p \wedge K_a(p \rightarrow q)\big) \rightarrow K_a q$ is valid

Axiomatization

- Checking validity directly not trivial
- Solution: Calculus (with axioms and rules)
  - Axiom (should be valid); rule = „small“ correct inference
  - Derivation/inference: Finite sequence of formulae where
    - each formula is an axiom (instance) or
    - results from applying rule to formulae appearing before.

Definition (calculus K)

The basic calculus K is given by the following:

- All classical tautologies (and their uniform substitutions)
- Axiom K: $K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$
- Rule modus ponens: From ϕ and $\phi \to \psi$ infer ψ
- Rule of necessitation: From ϕ infer $K_a\phi$

Axiomatization

Theorem

A formula is valid (in the class of all epistemic states) iff it is provable in calculus K

Other wording: K is correct and complete for the class of all epistemic states

Example

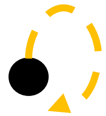
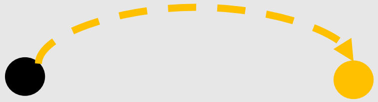
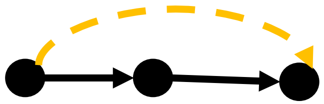
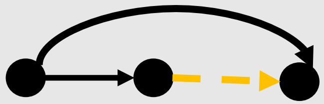
To show: $K_a(\phi \land \psi) \rightarrow K_a\phi$ is valid (by derivation in K)

1. $(\phi \land \psi) \rightarrow \phi$ (classical tautology)
2. $K_a\big((\phi \land \psi) \rightarrow \phi\big)$ (necessitation to 1.)
3. $K_a\big((\phi \land \psi) \rightarrow \phi\big) \rightarrow (K_a(\phi \land \psi) \rightarrow K_a\phi)$ (Axiom K)
4. $K_a(\phi \land \psi) \rightarrow K_a\phi$ (modus ponens to 2,3)

Why axiomatization

- the computation of knowledge is modeled;
- enables to explain why an agent knows something; (link with justification logic)
- axiomatization helps to understand the principle of the logics
- we do not have to design a specific epistemic state, as in model checking („open world“)

Classes of epistemic states

	Properties		Related axioms
K	any accessibility relation		
T	Reflexive		$K_a\phi \to \phi$
D	Serial		$\widehat{K}_a\top$
4	Transitive		$K_a\phi \to K_aK_a\phi$
5	Euclidean		$\neg K_a\phi \to K_a\neg K_a\phi$

Each row in the table is a completeness and correctness statement of calculi w.r.t. the given class of epistemic states

Definition

A formula ϕ is KD45-valid iff it is true in all epistemic states $\mathcal{M}, w$ in which accessibility relations are serial, transitive, and Euclidean

Theorem

A formula ϕ is KD45-valid iff it is provable in the axiomatization K extended with the axioms D, 4, 5.

(No it‘s not coffee time but) Time to wake up

- Show that if we confine Kripke structures to those with reflexive relations, then $K_a\phi \rightarrow \phi$ is valid w.r.t. that class

- Show that there might be pointed models which are not reflexive but make $K_a\phi \rightarrow \phi$ true

  (But at least you cannot find a frame F (i.e. a model without the evaluation V), such that
  - F is not reflexive and
  - for all models (F,V,w) based on it $K_a\phi \rightarrow \phi$ is made true)

Complexity of checking validity

- Without common knowledge:

	Singe agent	Several agents
K	PSPACE-complete	PSPACE-complete
KD45, S5	NP-complete	PSPACE-complete

- With common knowledge (and several agents): EXPTIME-complete

- In general and here: Model checking is more practical than theorem proving

LANGUAGE PROPERTIES

Expressivity

Definition

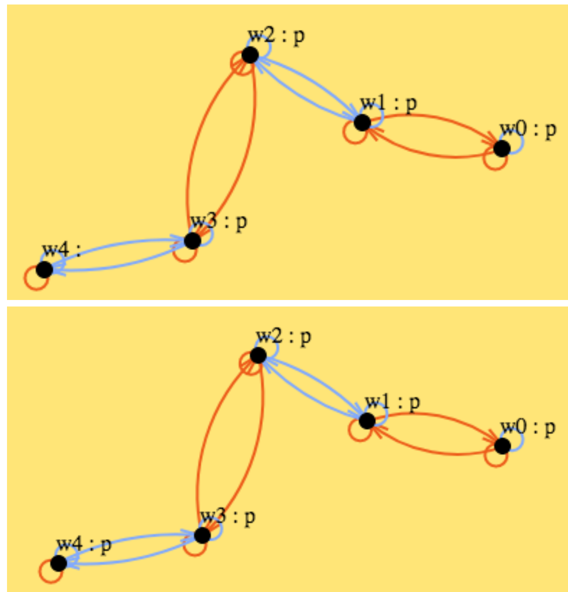
Two formuals ϕ, ψ are equivalent iff for all pointed models $\mathcal{M}, w$: $\quad \mathcal{M}, w \vDash \phi$ iff $\mathcal{M}, w \vDash \psi$

Theorem

$\mathcal{L}_{ELCK}$ is strictly more expressive than $\mathcal{L}_{EL}$: no formula in $\mathcal{L}_{EL}$ is equivalent to $C_{\{a,b\}}p$

Proof sketch:

- By contradiction, suppose $\phi \in \mathcal{L}_{EL}$ equivalent to $C_{\{a,b\}}p$.
- Let d be the modal depth of ϕ, e.g., $d = 3$
- Consider two models (from Hinntikka's world)
- ϕ has same value in both models but $C_{\{a,b\}}p$ can distinguish them



Expressivity

- Some operators are mere syntactic sugar such as operator $E_G\phi$, read as ``every agent in G knows ϕ''
- Define
  - $\mathcal{M},w \vDash E_G\phi$ iff for all agents in $a \in$ G: $\mathcal{M},w \vDash K_a\phi$

Theorem

$\mathcal{L}_{EL}$ augmented with E_G is equally expressive as $\mathcal{L}_{EL}$

Proof: $E_G\phi \equiv \bigwedge_{a\in G} K_a\phi$

- E_G gives intuitive reading for common knowledge: $C_G\phi$ means $E_G^n\phi$ for all $n \in \mathbb{N}$

Bisimulation

- Modal logics and epistemic logics cannot distinguish between structures with same „transition“ behaviour
- Captured by notion of bisimulation

Definition

For two models $M = (W, (R_a)_{a \in AGT}, V)$ and $M' = (W', (R'_a)_{a \in AGT}, V')$ a set $\mathcal{R} \subseteq W \times W'$ is a bisimulation iff for all $w \in W, w' \in W'$ with $(w, w') \in \mathcal{R}$

- $V(w) = V'(w')$
- For all $a \in AGT$, for all $v \in W$: If $\mathrm{R}_a(w, v)$ then there is $v' \in W'$ with $R'_a(w', v')$ and $(v, v') \in \mathcal{R}$
- For all $a \in AGT$, for all $v' \in W$: If $R'_a(w', v')$ then there is $v \in W$ with $R_a(w, v)$ and $(v, v') \in \mathcal{R}$
- $(M, w) \leftrightarrows (M', w')$ iff there is a bisimulation linking w and w

Bisimilarity preserves formulae

Theorem

Suppose that $(M, w) \leftrightarrows (M', w')$. Then, for all formulas $\phi \in \mathcal{L}_{ELCK}$ it holds that:$(M, w) \vDash \phi$ iff $(M', w') \vDash \phi$

So: Though the common knowledge operator can see arbitrarily far (transitive closure of accesibility relations !; see example on two models before), it can only do in a accessibility-guarded way

One big meta result regarding bisimulation in the so-called area of correspondence theory (but not directly relevant here)

Theorem

Modal logics are exactly those fragments of FOL whose formulae are invariant under bisimilarity

(see, e.g., Blackburn et al, 02)

Definition

Given a class X of models, L_1 is exponentially more succinct than L_2 on X iff the following conditions hold:

- for every formula $\beta \in L_2$ there is a formula $\alpha \in L_1$ such that $\alpha \equiv_X \beta$ and $|\alpha| \leq |\beta|$.
- there exist $k_1, k_2 > 0$, a sequence of formulas $\alpha_1, \alpha_2, \ldots \in L_1$, and a sequence of formulas $\beta_1, \beta_2, \ldots \in L_2$ such that, for all n, we have:
  - $|\alpha_n| \leq k_1 n$
  - $|\beta_n| \geq 2^{k_2 n}$
  - β_n is the shortest formula in L_2 that is equivalent to α_n on X

Succinctness

Theorem

$\mathcal{L}_{EL}$ augmented with E_G's is exponentially more succinct than $\mathcal{L}_{EL}$

- $E_{\{a,b\}}E_{\{a,b\}}E_{\{a,b\}}\phi \equiv K_aK_aK_a\phi \wedge K_aK_aK_b\phi \wedge K_aK_bK_a\phi$
  $K_aK_bK_b\phi \wedge K_bK_aK_a\phi \wedge K_bK_bK_a\phi \wedge K_bK_bK_b\phi$
- $E_{\{a,b\}} \ldots E_{\{a,b\}} \equiv \ \ldots$

- Proof is involved
  (French, van der Hoek, Illiev, Kooi 2013)

Take-Home Message

With epistemic logics there is a firm formal foundation for dealing with knowledge in multi-agents

Uhhh, a lecture with a hopefully useful

APPENDIX

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Color Convention in this course

- Formulae, when occurring inline
- Newly introduced terminology and definitions
- Important results (observations, theorems) as well as emphasizing some aspects
- Examples are given with standard orange with possibly light orange frame
- Comments and notes
- Algorithms