Intelligent Agents
Knowledge and Time

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Todays lecture based on

- The AAMAS 2019 Tutorial **„EPISTEMIC REASONING IN MULTI-AGENT SYSTEMS“, Part 3: Knowledge and Time**
  http://people.irisa.fr/Francois.Schwarzentruber/2019AAMAStutorial/
- Parts of „Formal Methods - Lecture III: Linear Temporal Logic“ 2010/11 by Allessandro Artale
  https://web.iitd.ac.in/~sumeet/slide3.pdf

„What then is time? If no one asks me, I know what it is. If I wish to explain it to him who asks, I do not know.“

(Augustine of Hippo- Confessiones)

TEMPORAL LOGIC

Temporal logic

- Temporal logic was originally developed in order to represent tense in natural language.
- Within CS, it has achieved a significant role in the formal specification and verification of concurrent reactive systems.
  - Reason: a number of useful concepts can be formally, and concisely, specified using temporal logics, e.g.
    - safety properties
    - liveness properties
    - fairness properties
  - When Vardi (Vardi 09) speaks of „industrial logics" he thinks mainly about temporal logics

Flow of Time

- Flow of time $(T, \leq_T)$ is a structure with a time domain T and a binary before relation $\leq_T$ over it.
  - Flow metaphor hints on directionality and dynamic aspect of time
  - Induced strictly before: $\mathrm{x} <_T y$ iff $x \leq_T y$ and not $y \leq_T x$
  - But still different forms of flow are possible
- Either consider concrete structures of flow of (time) (as done in LTL (or CTL))
- Or investigate them additionally axiomatically
  - An early model-theoretic and axiomatic treatise:
    Lit: J. van Benthem. The Logic of Time: A Model-Theoretic Investigation into the Varieties of Temporal Ontology and Temporal Discourse. Reidel, 2. edition, 1991.

Family of Flows of Time

- Domain T
  - points (atomic time instances)
  - pairs of points (application time, transaction time)
  - intervals etc.
- Properties of the before relation $\leq_T$
  - Non-branching (linear) vs. branching
    Linearity:
    - reflexive: $\forall t \in T: t \leq_T \ t$
    - antisymmetric: $\forall t1, t2 \ \in T: (t_1 \ \leq t_2 \ \wedge t_2 \ \leq_T \ t_1) \Rightarrow t_1 \ = t_2$
    - transitive: $\forall t_1, t_2, t_3 \ \in T \ : (t_1 \leq_T t_2 \wedge t_2 \leq t_3) \Rightarrow t_1 \leq t_3$.
    - total: $\forall t_1, t_2 \in T: t_1 \leq t_2 \vee t_2 \leq t_1 \vee t_1 = t_2$.

Family of Flows of Time (continued)

- Further possible properties of the before relation $\leq_T$
  - Existence of first or last element
  - discreteness (Example: $T = \mathbb{N}$)
  - density (Example: $\mathrm{T} = \mathbb{Q}$)
  - (Dedekind) continuity (Example: $T = \mathbb{R}$)
  - …

Family of Flows of Time (continued)

- One of the early expressivity results considers flows of time which are similar to $(\mathbb{R}, <_{\mathbb{R}})$

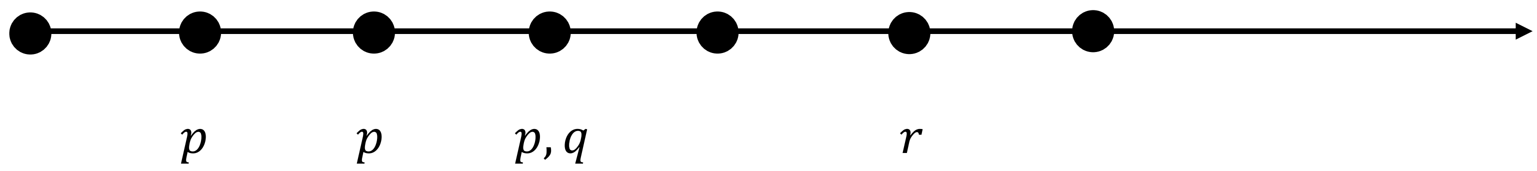
Theorem (Kamp 1968)

- The Logic L_{SU} based on binary modalities S(ince) and U(ntil) cannot be captured by modal logic based on F(uture) and G(lobally)
- Over Dedekind continuous strict total orders (such as $<_{\mathbb{R}}$) L_{SU} provides expressiveness of first order logic.

(see Chapter 7 in (Blackburn et al, 02)

LINEAR TEMPORAL LOGIC

Models



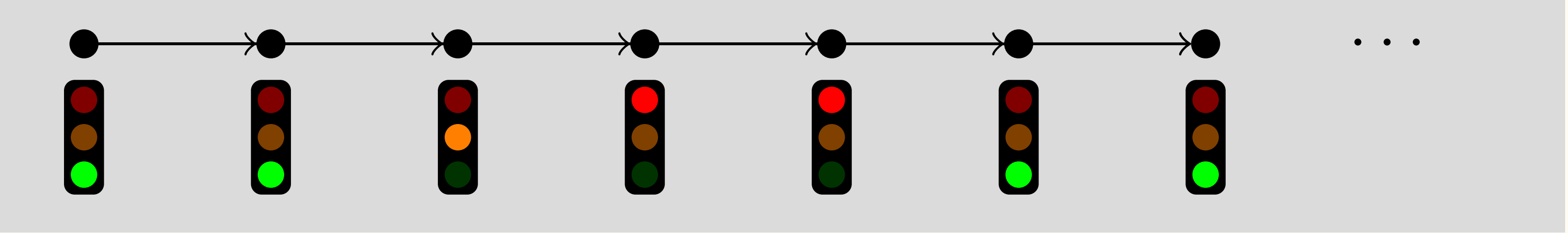
Definition

A linear temporal model is a structure $(\mathbb{N}, <, V)$ such that:

- $V: N \to 2^{AP}$
- $<$ is the natural order on $\mathbb{N}$

We sometimes do not mention the linear order $<$

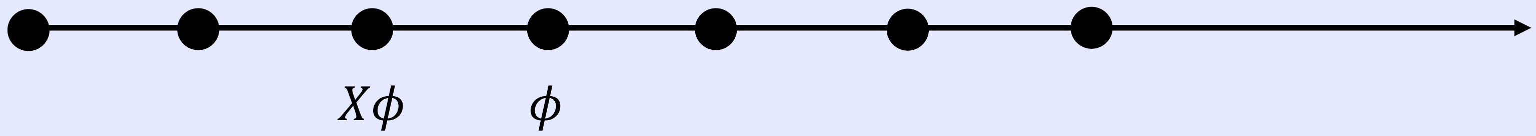
Example (traffic light)



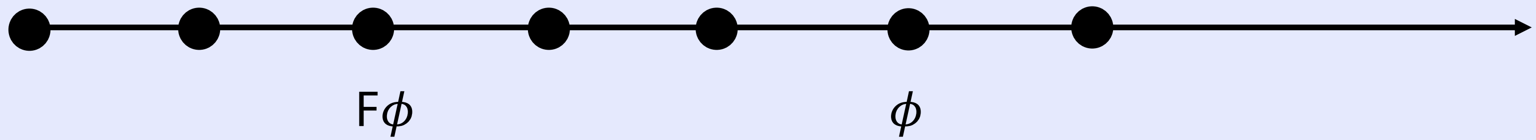
Syntax and semantics

Definition (Temporal Modalities of LTL)

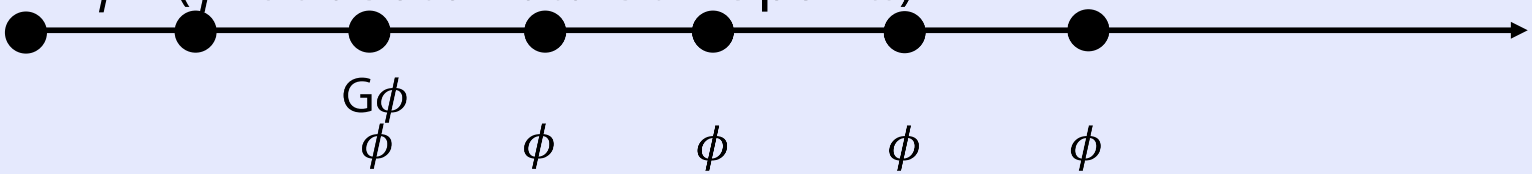
- $X\phi$ (ϕ is true at the next time)



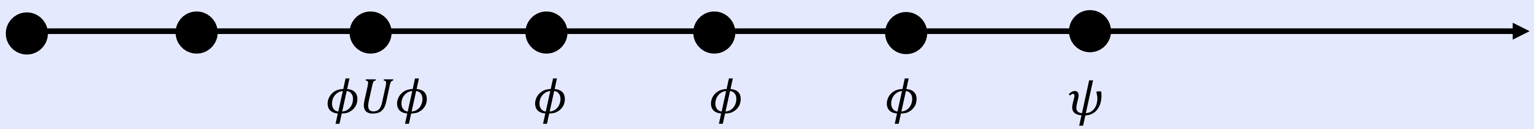
- Fϕ (ϕ is true at some point in the future)



- Gϕ (ϕ is true at all future time points)



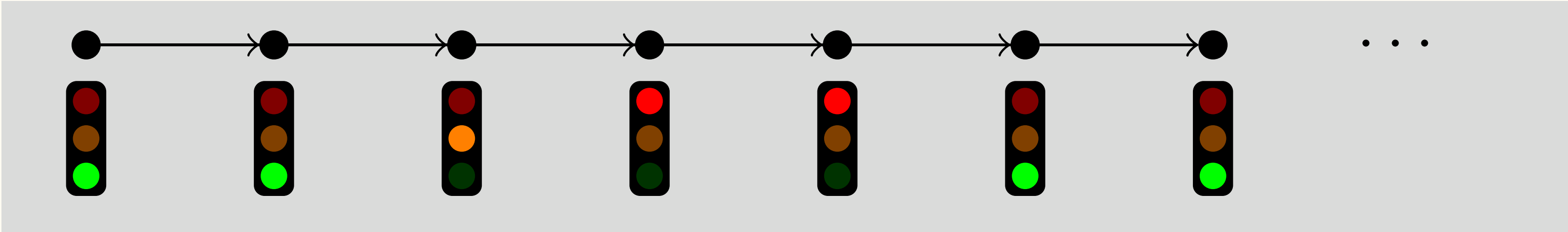
- $\phi U \psi$ (ψ is true at some future time point and ϕ holds until ψ)



Syntax and semantics

- $(\mathbb{N}, V), t \vDash p$ if $p \in V(t)$
- $(\mathbb{N}, V), t \vDash \neg\phi$ if not $(\mathbb{N}, V), t \vDash \phi$
- $(\mathbb{N}, V), t \vDash \phi \vee \psi$ if $(\mathbb{N}, V), t \vDash \phi$ or $(\mathbb{N}, V), t \vDash \psi$
- $(\mathbb{N}, V), t \vDash X\phi$ if $(\mathbb{N}, V), t + 1 \vDash \phi$
- $(\mathbb{N}, V), t \vDash F\phi$ if there is $t' \geq t$ such that $(\mathbb{N}, V), t' \vDash \phi$
- $(\mathbb{N}, V), t \vDash G\phi$ if for all $t' \geq t$: $(\mathbb{N}, V), t' \vDash \phi$
- $(\mathbb{N}, V), t \vDash \phi U \psi$ if there is $t' \geq t$ such that $(\mathbb{N}, V), t' \vDash \psi$ and $(\mathbb{N}, V), t'' \vDash \phi$ for all $t'' \in [t, t' - 1]$

Example (traffic light)



- Once red, the light cannot become green immediately
  $G\ (red \rightarrow \neg\, X\ green)$ (not fulfilled in model above)
- The light becomes green eventually
  $F\ green$ (fulfilled)
- Once red, the light becomes green eventually
  $G\ (red \rightarrow F\ green\ )$ (fulfilled in shown prefix)
- Once red, the light always becomes green eventually after being yellow for some time inbetween

$$G\ \left[red \rightarrow X\ \left(red\ U\ \left(yellow \wedge X\ (yellow\ U\ green\ )\right)\right)\right]$$

(not fulfilled)

Typical Properties for Verification

Example (safety)

„Something bad will not happen“

- $G \neg (reactor_{temp} > 1000)$
- $G \neg((x = 0) \wedge X\ X\ X\ (y = \frac{z}{x}))$

Example (liveness)

„Something good will happen“

- $F\ rich$
- $F\ (x > 5)$
- $G\ (start \rightarrow F\ terminate)$
- $G\ (trying \rightarrow F\ critical\ )$

Examples

Example (fairness)

„ if something is attempted/requested infinitely often, then it will be successful/allocated infinitely often“

- $G\ F\ ready \rightarrow G\ F\ run$

(Early) Wake-Up Exercise

- Q: Show that the following expansion properties hold
  - $\phi\ U\ \psi \equiv \psi \vee (\ \phi \wedge\ X(\phi\ U\ \psi)\ )$
  - $\mathrm{F}\phi\ \equiv \phi \vee X\ F\phi$
  - $G\phi \equiv \phi \wedge X\ G\phi$
- A: We show this for $\mathrm{F}\phi\ \equiv \phi \vee X\ F\phi$
  - $(\mathbb{N}, V), t \vDash F\,\phi$
  - iff there is t$' \geq t$ such that $(\mathbb{N}, V), t'\ \vDash \phi$
  - iff there is t‘ with $t‘\ =\ t$ or $t' \geq t+1$ s.t. $(\mathbb{N}, V), t'\ \vDash \phi$
  - iff $(\mathbb{N}, V), t\ \vDash \phi$ or there is t‘ with $t' \geq t+1$ s.t. $(\mathbb{N}, V), t'\ \vDash \phi$
  - iff $(\mathbb{N}, V), t\ \vDash \phi$ or there is $t‘$ s.t. $(\mathbb{N}, V), t' \vDash X\,F\,\phi$
  - iff $(\mathbb{N}, V), t\ \vDash \phi \vee X\,F\,\phi$

Satisfiability problem (reminder)

Definition

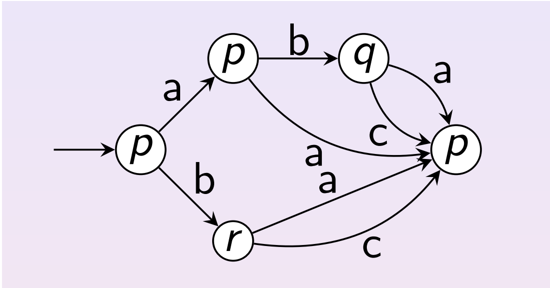
The satisfiability problem is:

- Input: a formula ϕ
- Output: yes if there is V such that $(\mathbb{N}, V), t \vDash \phi$

Theorem

The satisfiability problem is PSPACE-complete

Model checking (reminder)



Definition

The model checking problem is:

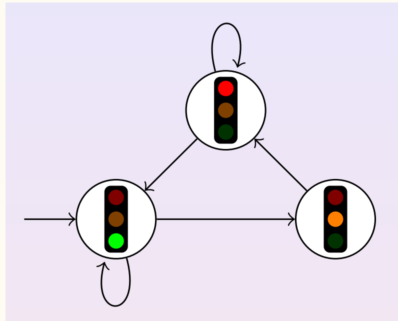
- Input: a transition system S; an LTL formula ϕ
- Output: yes if all paths of S starting from an initial state of S satisfy ϕ

Theorem

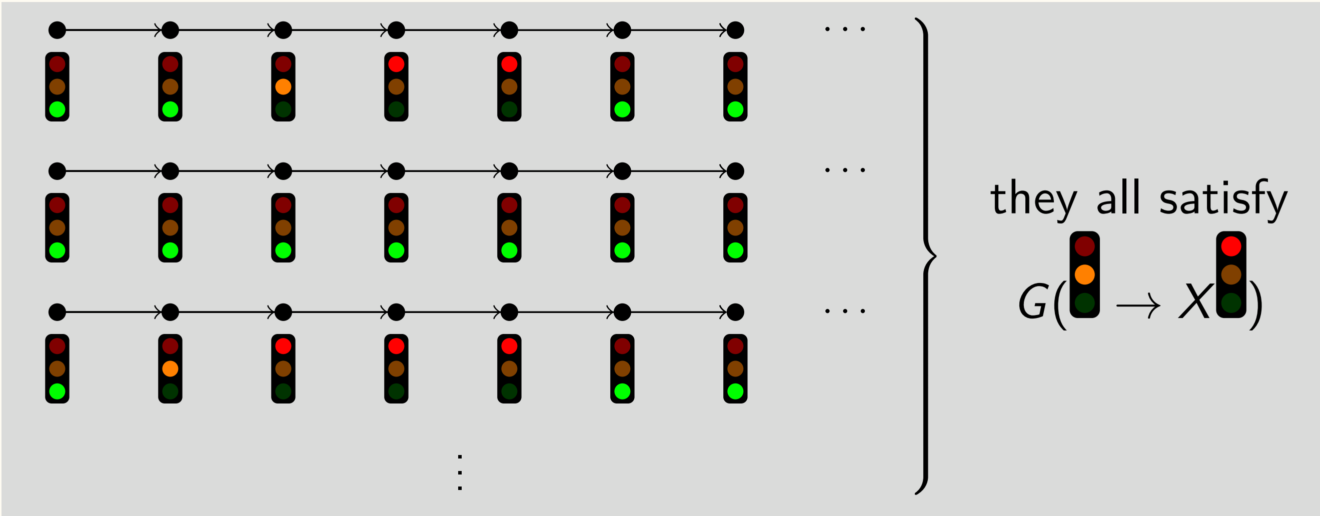
The model checking problem of LTL is PSPACE-complete

Example

Transition system S:



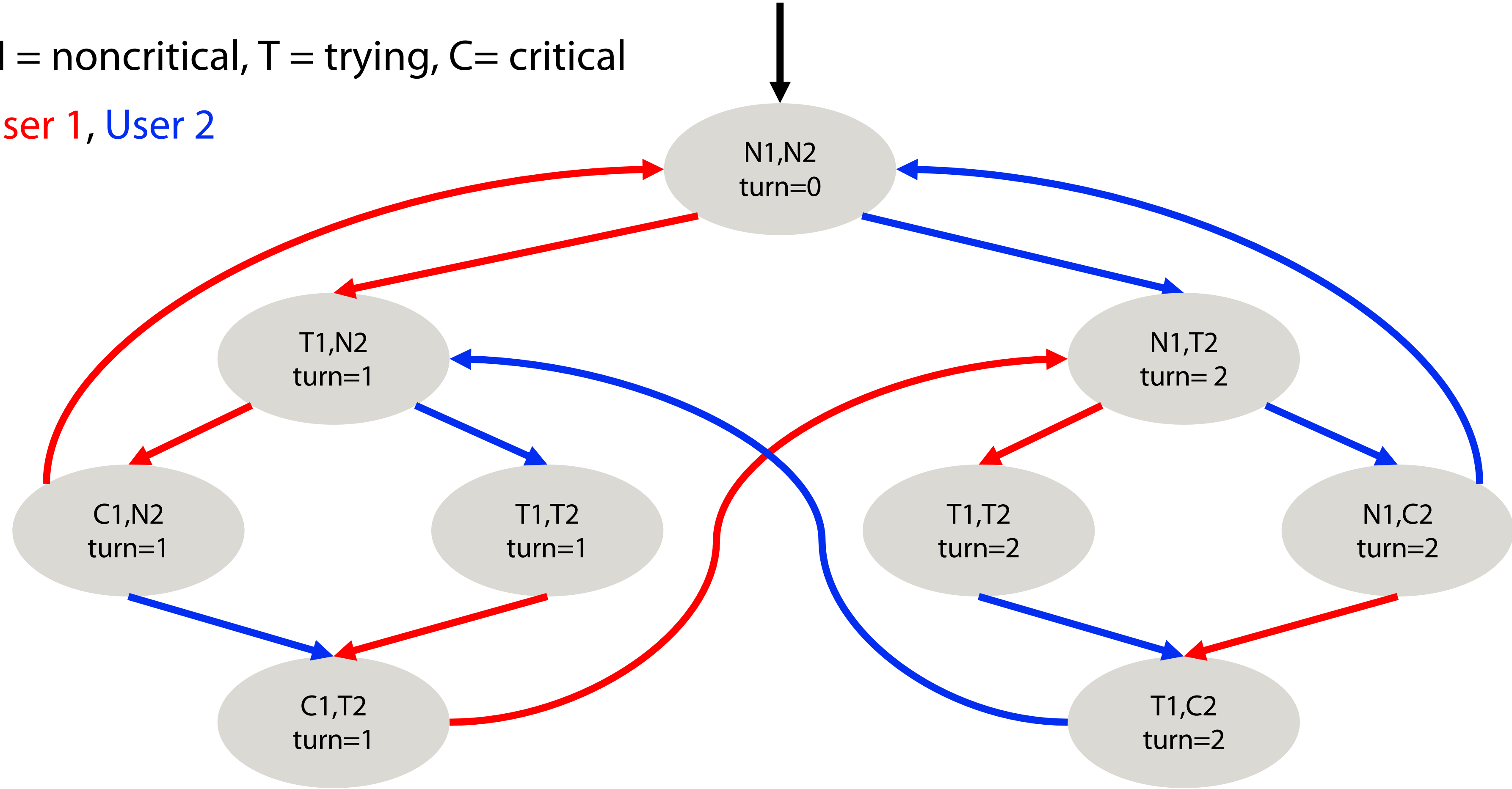
Paths of S starting from initial state



Example: Mutual Exclusion

N = noncritical, T = trying, C= critical

User 1, User 2



Example: Mutual Exclusion

N = noncritical, T = trying, C= critical

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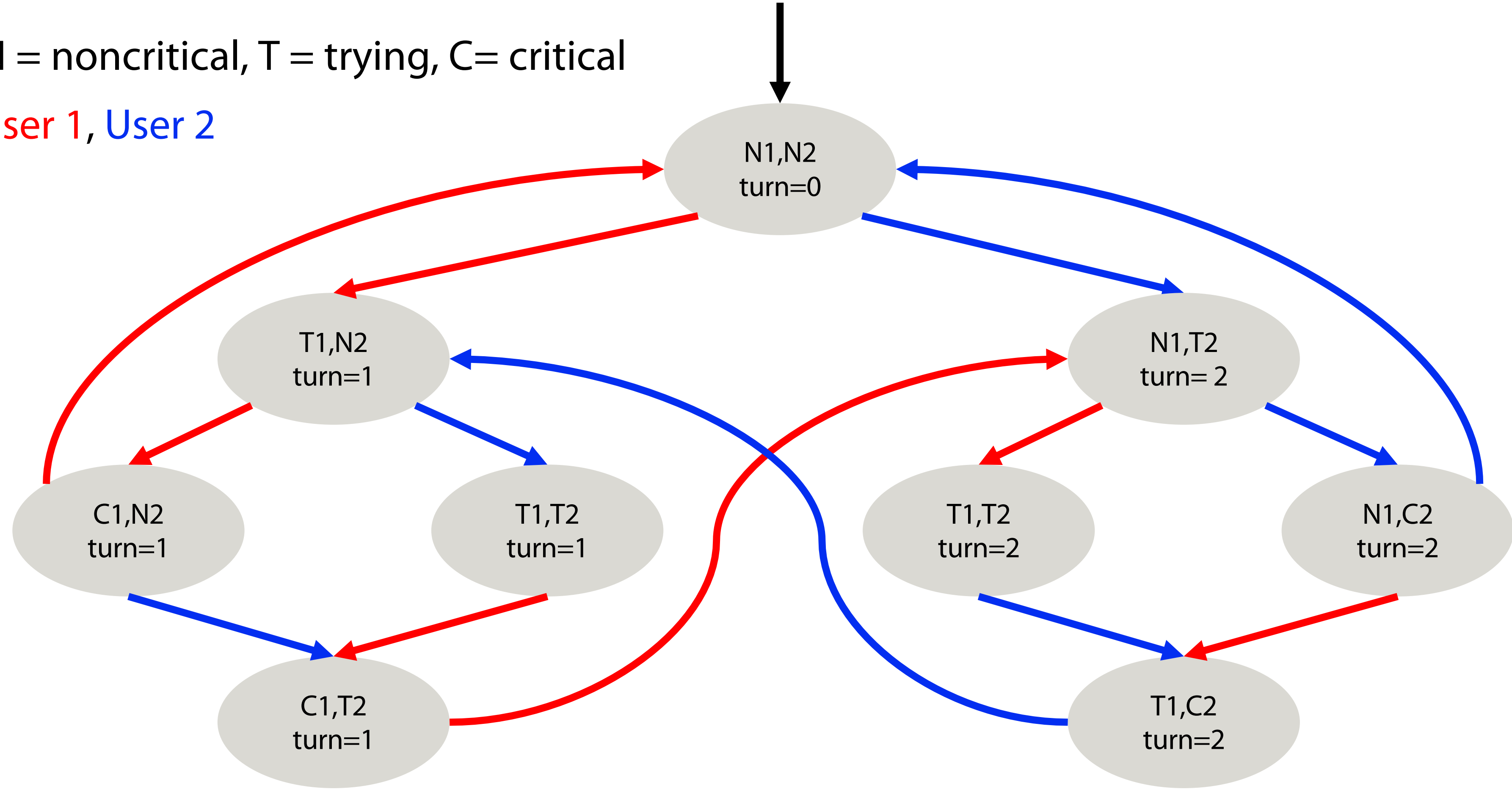
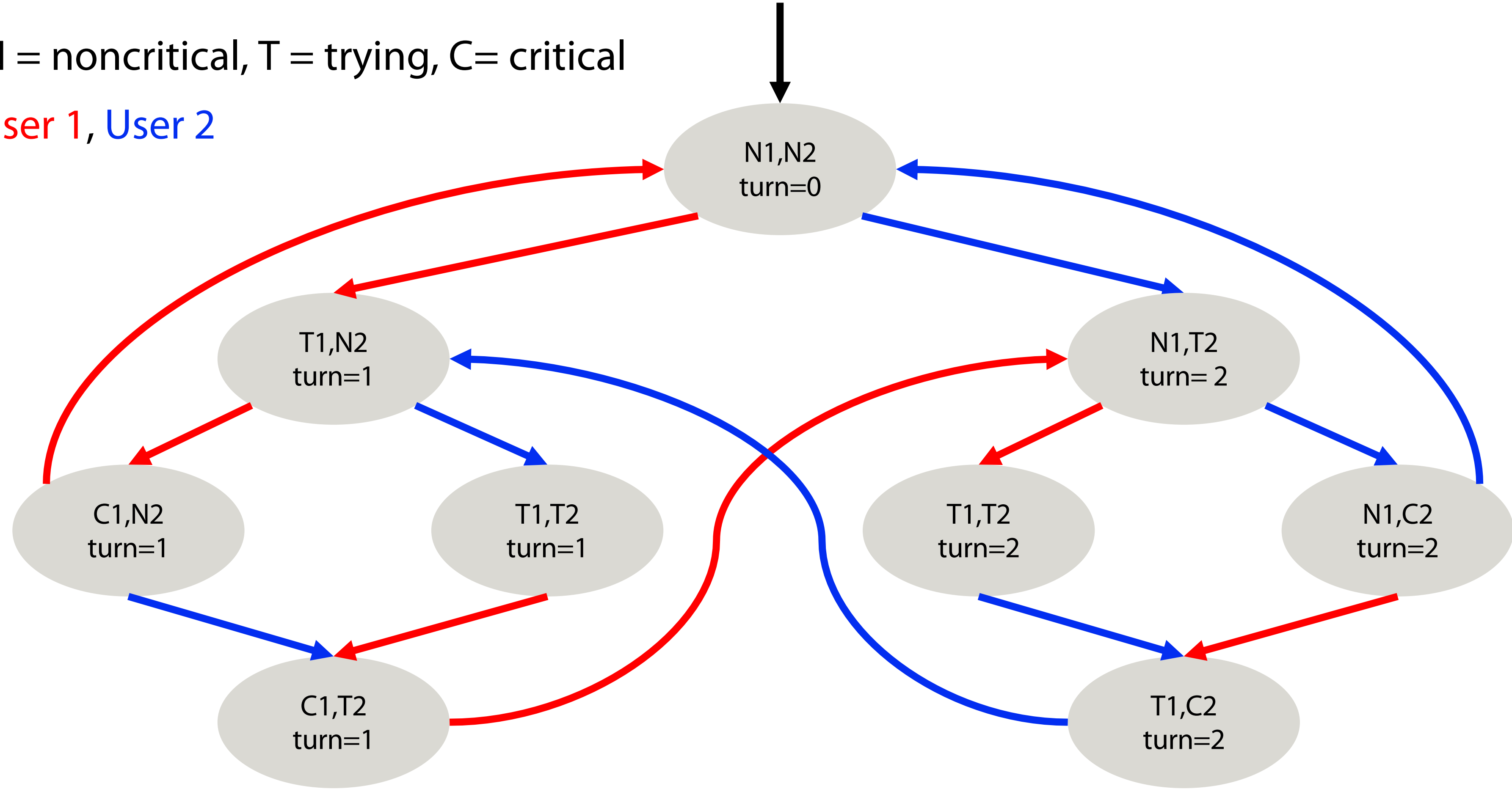


- Safety fulfilled? $S \vDash G\ \neg(C_1 \wedge C_2)$?
- Yes! There is no reachable state in which $\neg(C_1 \wedge C_2)$ holds

Example: Mutual Exclusion

N = noncritical, T = trying, C= critical

User 1, User 2

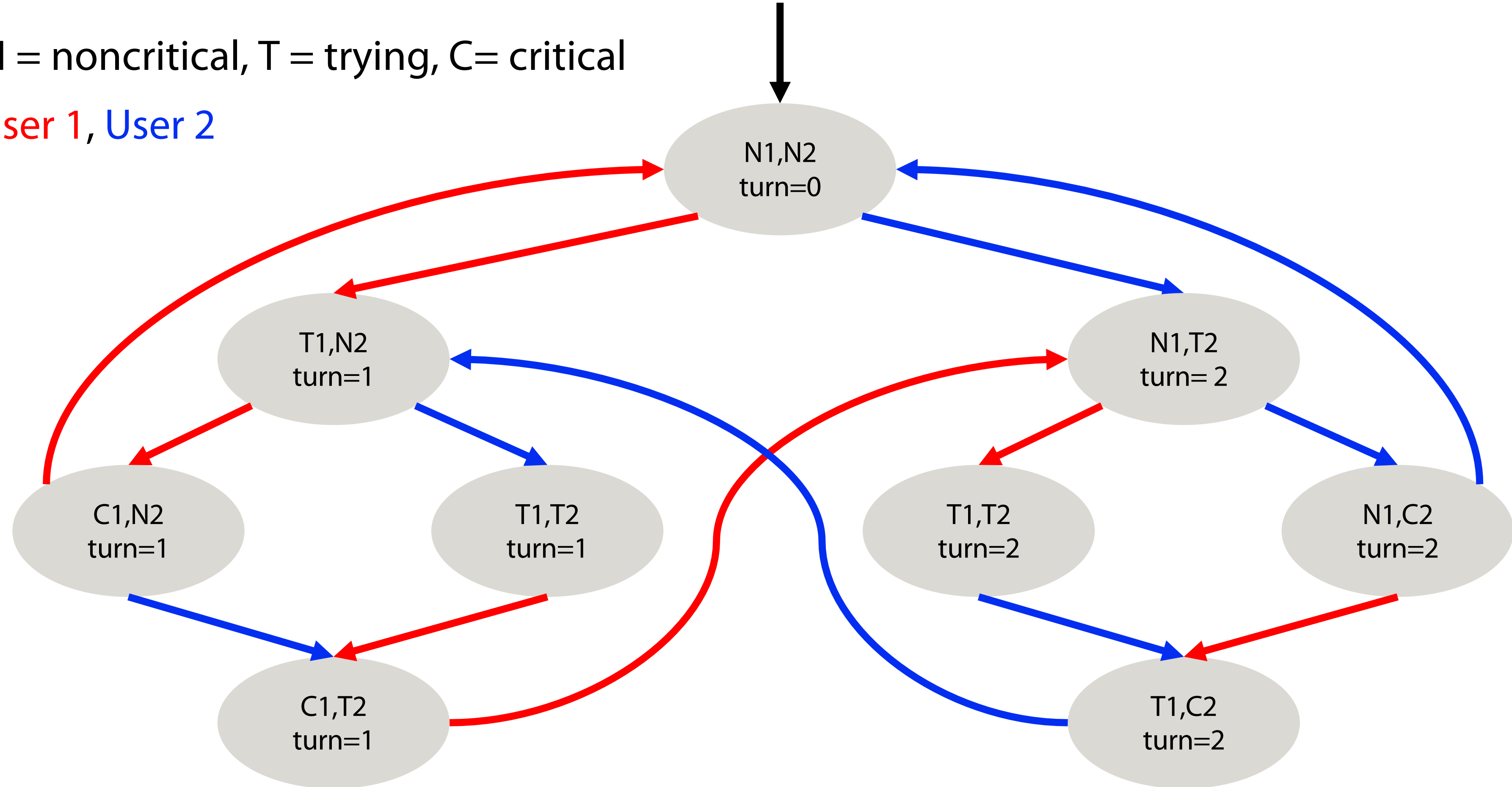


- (unconditioned) Liveness fulfilled? $S \models F\ C_1$?
- No! Blue cyclic path is counterexample

Example: Mutual Exclusion

N = noncritical, T = trying, C= critical

User 1, User 2

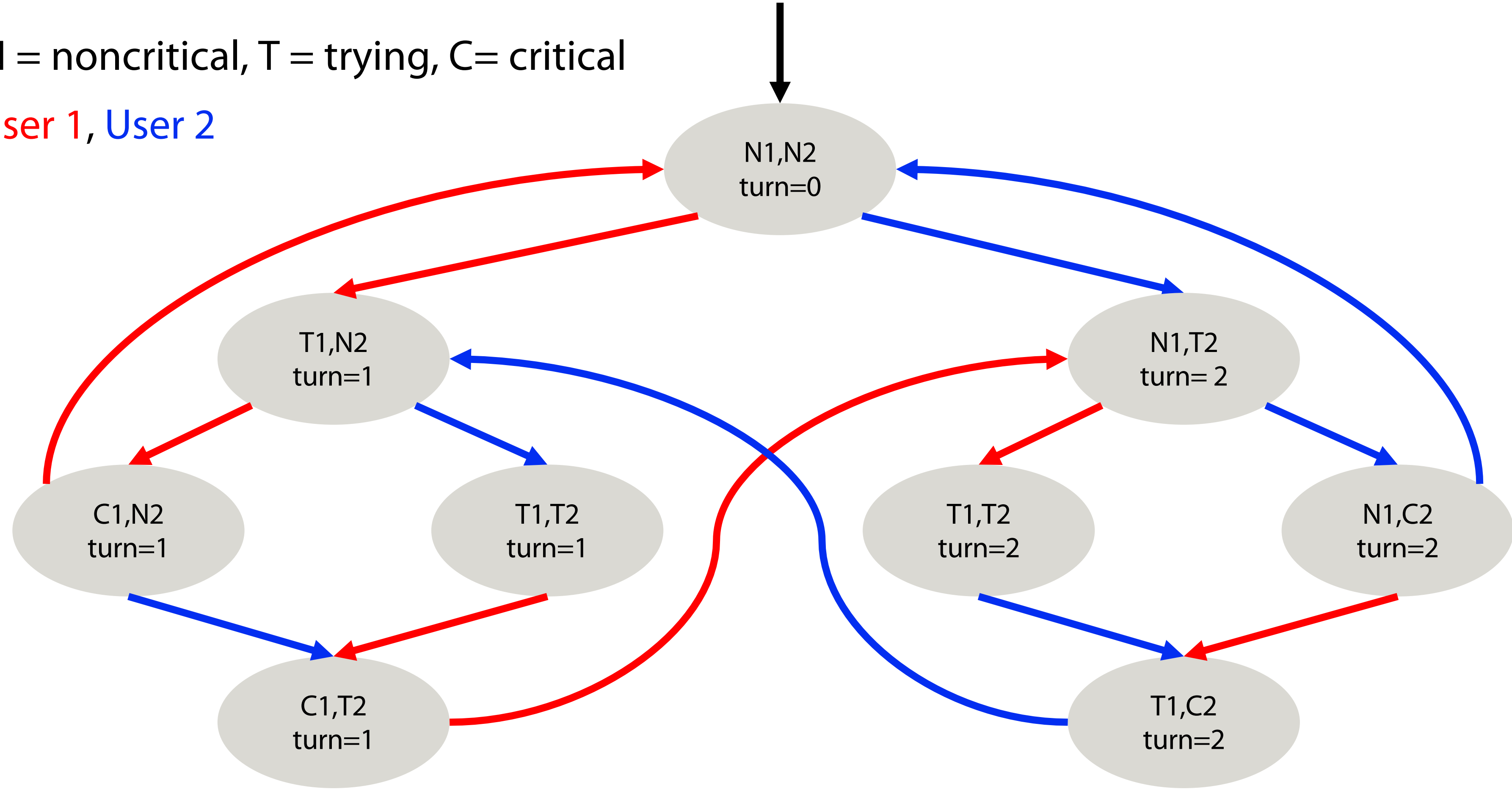


- Conditioned liveness fulfilled? $S \vDash G\ (T_1 \rightarrow F\ C_1)$?
- Yes! In every path: if T_1 holds, then eventually C_1 holds

Example: Mutual Exclusion

N = noncritical, T = trying, C= critical

User 1, User 2

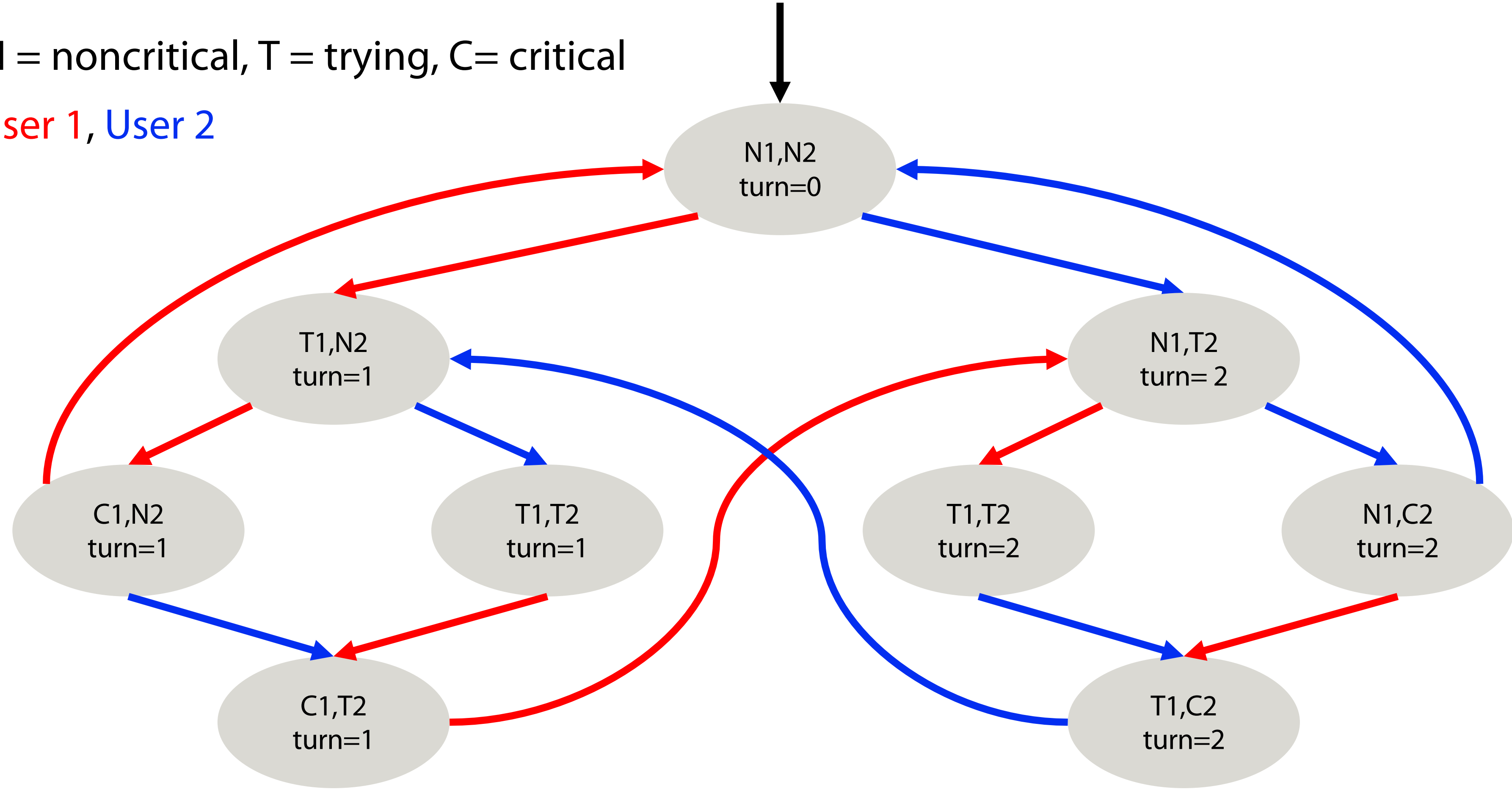


- Fairness fulfilled? $S \models G\ F\ C_1$
- No! Blue cyclic path is a counterexample.

Example: Mutual Exclusion

N = noncritical, T = trying, C= critical

User 1, User 2



- Strong fairness fulfilled? $S \vDash G\ F\ T_1 \rightarrow G\ F\ C_1$
- Yes! Every path which visits T_1 infinitely often also visits C_1 infinitely often

EPISTEMIC LINEAR TEMPORAL LOGIC

A combined logic

- Epistemic linear temporal logic (ELTL):
  - Epistemic logic (with epistemic operators K_a) combined with
  - Linear temporal logic (with temporal operators X, F, G, U)

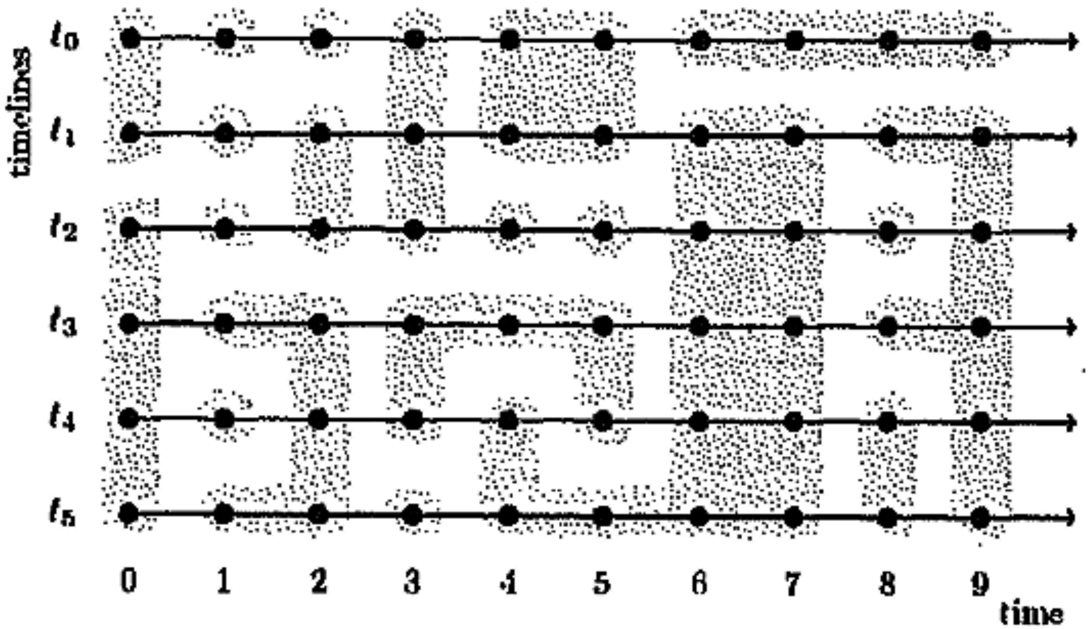
- Example of combining systems/logics
  - Conference series „Frontiers of combining systems“ (Frocos)
  - Interesting (ancient Dialogue-style) paper on combining systems : P. Blackburn and M. De Rijke., 1997
  - Overview in Stanford Encyclopedia of Philosophy: Carnielli and Coniglio: Combining Logics, 2020

Models

Definition

An ELTL model is a structure $\mathcal{M} = (TL \times \mathbb{N}, (\sim_a)_{a \in AGT}, V)$ such that

- TL is a non-empty set of timelines (runs)
- For all agents a, $\sim_a$ is an equivalence relation on $TL \times \mathbb{N}$
- $V: TL \times \mathbb{N} \rightarrow 2^{AP}$



Case of one agent a; regions denote equivalence classes of $\sim_a$

Think of run as a function from ticks of global clock to a global state, which is a variable assignment

INTERACTION BETWEEN KNOWLEDGE AND TIME

Axiomatisation in case of no interaction: Fusion

- All classical tautologies (and their uniform substitutions)

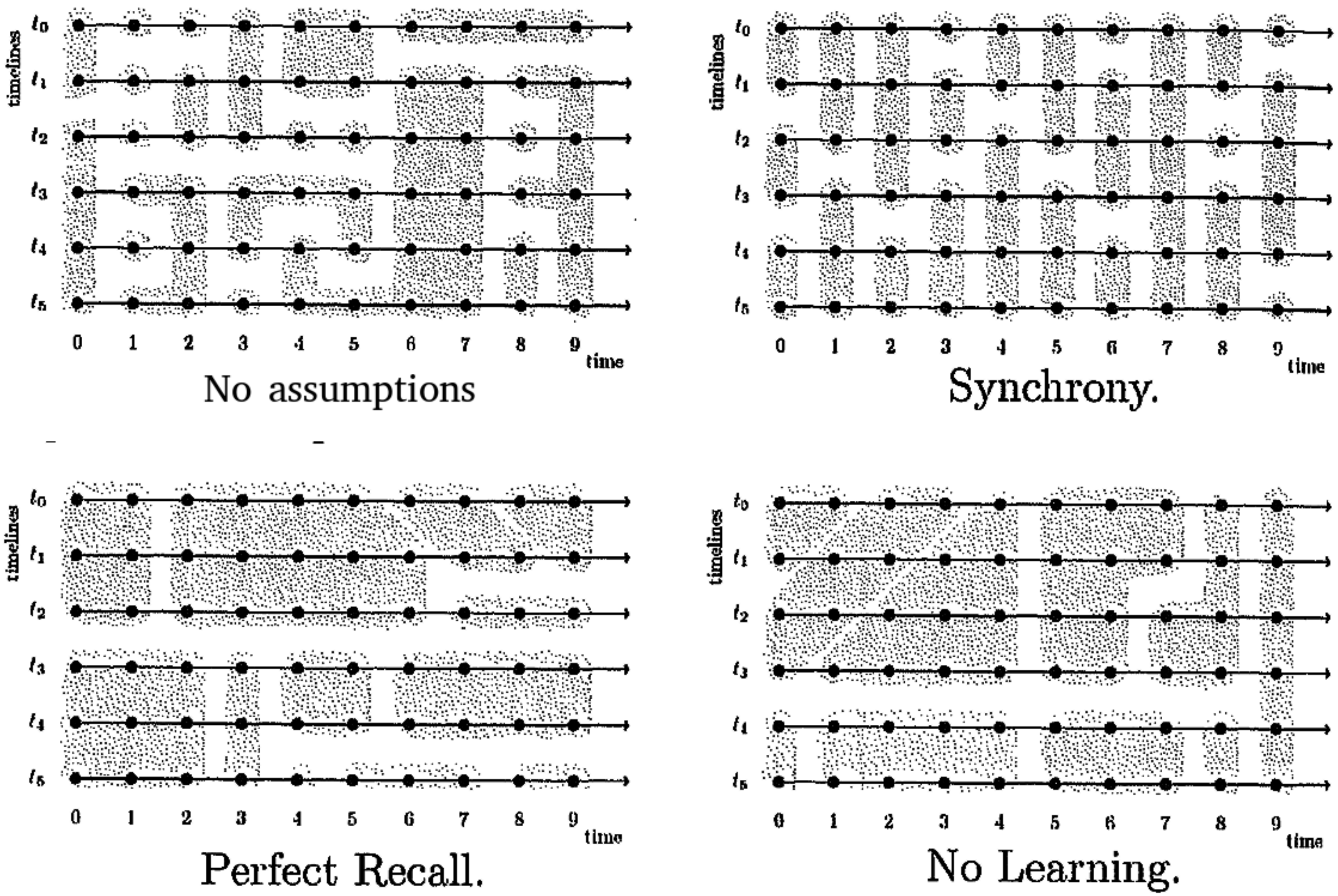
- $K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$
- $K_a\phi \to \phi$
- $\widehat{K}_a\top$
- $K_a\phi \to K_aK_a\phi$
- $\neg K_a\phi \to K_a\neg K_a\phi$

EL

- $G(\phi \to \psi) \to (G\phi \to G\psi)$
- $X(\phi \to \psi) \to (X\phi \to X\psi)$
- $X\neg\phi \leftrightarrow \neg X\phi$
- $G\phi \to (\phi \wedge XG\phi)$
- $G(\phi \to X\phi) \to (\phi \wedge G\phi)$
- $(\phi U\psi) \to F\psi$
- $(\phi U\psi) \leftrightarrow (\psi \vee X(\phi U\psi))$

LTL

Adding interaction



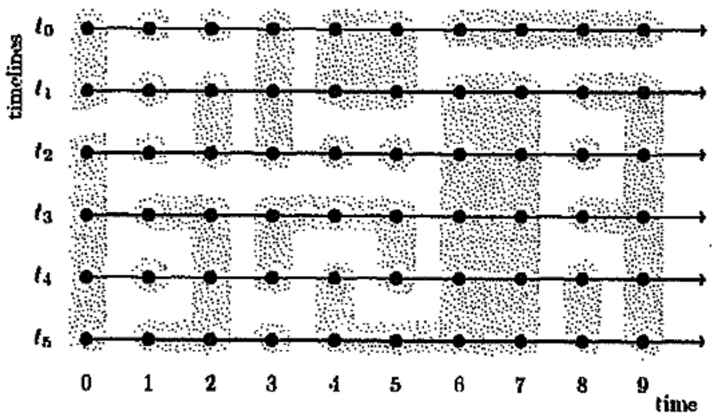
For additional criteria (resulting in 96 different epistemic temporal logics) see (Halpern/Vardi, 1989)

Properties

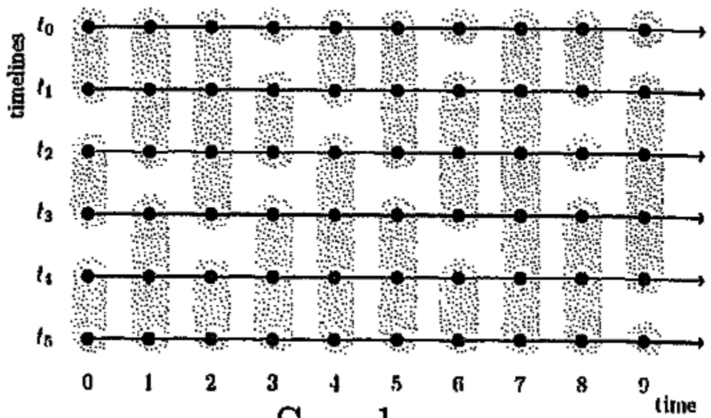
- Perfect recall/not forgetting: set of timelines agent a considers possible stays the same or decreases with time
  (Here we say agent a considers timeline t' possible at point (t, n) if for some n': $(t, n) \sim_a (t', n')$)

- Formally: for all timelines t, t' and times n, n', k:
  if $(t, n) \sim_a (t', n')$ and $k \leq n$, then there exists $k' \leq n'$ such that $(t, k) \sim_a (t', k')$.

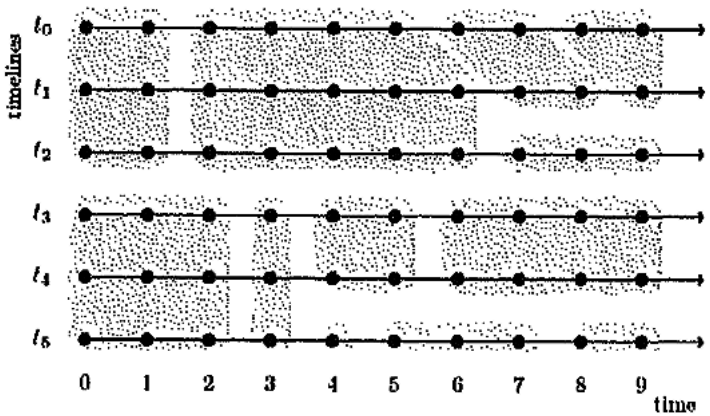
Counterexample Perfect recall



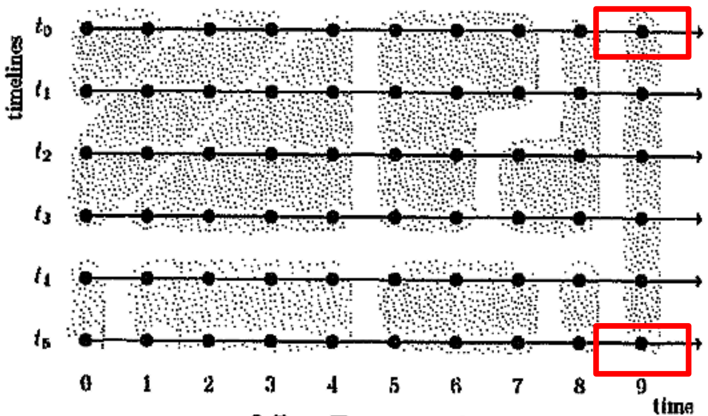
No assumptions



Synchrony.



Perfect Recall.



No Learning.

$(t_0, 9) \sim (t_5, 9)$.

But for no $k' < 9$: $(t_0,8) \sim (t_5, k')$

Properties

- No learning: set of timelines an agent a considers possible stays the same or increases over time.

- Formally: for all timelines t, t' and times n, n', k:
  if $(t, n) \sim_a (t', n')$ and $k \geq n$, then there exists $k' \geq n'$ such that $(t, k) \sim_a (t', k')$.

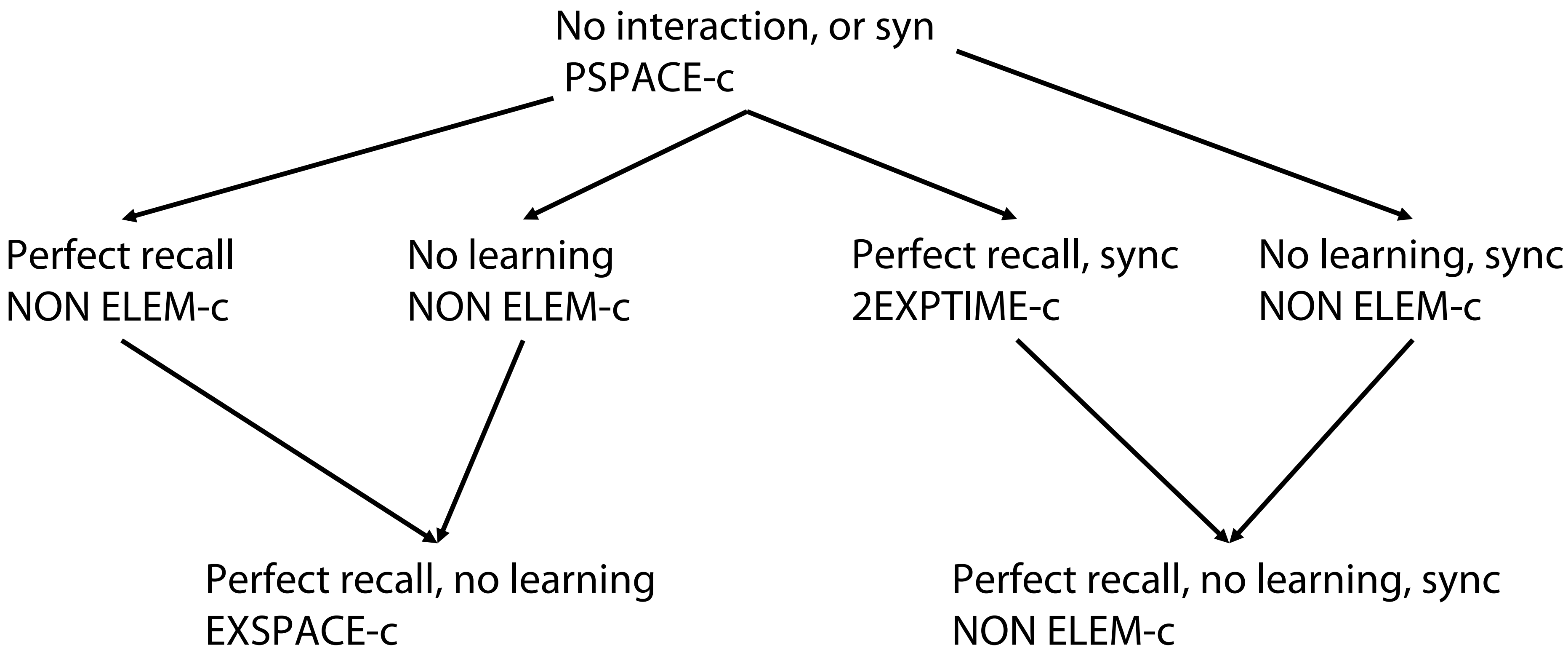
Corresponding properties/axioms

Synchronous	Agents know the time t (not an axiom)
Perfect recall, Synchronous	$K_a X\phi \rightarrow X\, K_a\phi$
Perfect recall	$K_a\phi \wedge X(K_a\psi \wedge \neg\, K_a\chi) \rightarrow \neg K_a \neg(K_a\phi\; U(K_a\psi U \neg\chi))$
No learning	$(K_a\phi\; U\; K_a\psi) \rightarrow K_a(K_a\phi\; U K_a\psi)$
No learning, Synchronous	$X K_a\phi \rightarrow K_a X\phi$

Combinations from a semantical point of view

- Input: classes of models M_1, M_2 of logics L_1, L_2
- Output: class of models M of combined logic
- Fusion: $\mathrm{M} = \{ (W, R_1, R_2, V) \mid (W, R_i, V) \in M_i \}$
- Product:
  $\mathrm{M} = \{ (W_1 \times W_2, S_1, S_2, V_1 \times V_2) \mid (W_i, R_i, V_i) \in M_i\}$
  where
  - $(u_1, u_2)\ S_1\ (w_1, w_2)$ iff $u_1 R_1\ w_1$ and $u_2 = w_2$;
  - $(u_1, u_2)\ S_2\ (w_1, w_2)$ iff $u_2 R_2\ w_2$ and $u_1 = w_1$;
  - $(V_1 \times V_2)(p) = V_1(p) \times V_2(p)$
- Fibring: More flexible combination based on on bitransfer-mappings h_i between worlds

Complexity of the satisfiability problem

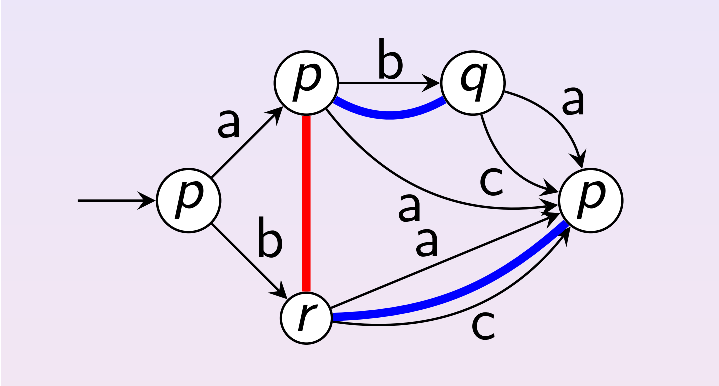


((Reminder:

Complexity Class ELEMENTARY= $\bigcup_{k\in\mathbb{N}} k-EXP = \; DTIME(2^n) \cup DTIME\left(2^{2^n}\right) \cup \cdots$))

MODEL CHECKING

Model checking



Definition

The model checking problem is:

- Input:
  - an epistemic transition system S, i.e. a transition system augmented with epistemic relations $(R_a)_{a \in AGT}$ with a set of initial states;
  - an LTL formula ϕ
- Output: yes if `` $\mathcal{M}_S, (\rho, 0) \models \phi$" for all paths ρ of S starting from an initial state of S

Possible Definition of $\mathcal{M}_S$

Definition

Given a transition system S, define
$\mathcal{M}_S = (\mathrm{TL} \times \mathbb{N}, (\sim_a)_{a \in AGT}, V)$ such tat

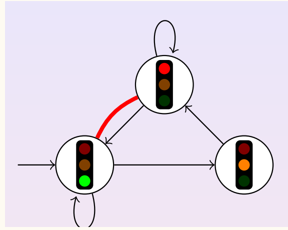
- TL is the set of paths of S starting in an initial state of S;
- For all agents a: $(\rho, t) \sim_a (\rho', t')$ if
  - $t = t'$ (synchrony)
  - $\rho[i] R_a \rho'[i]$ for all $i \in \{0, \ldots, t\}$ (perfect recall)
- $V: TL \times \mathbb{N} \to 2^{AP}$ is defined by
  $V(\rho, t) =$ set of propositions true at $\rho[t]$

Notes

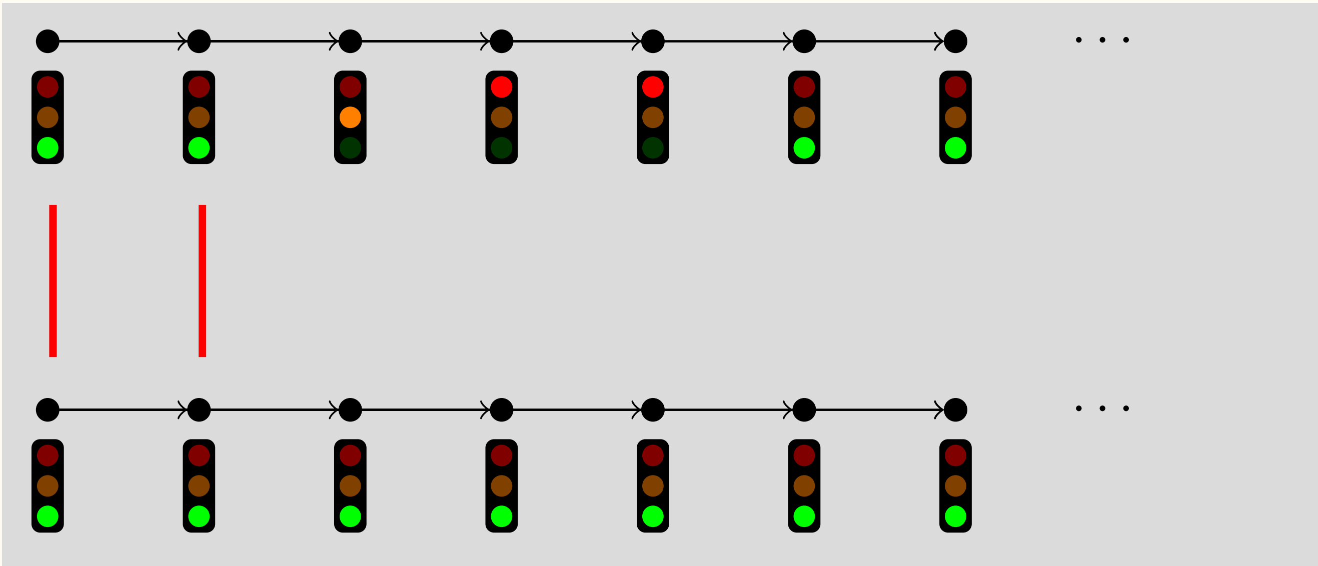
- Here instead of timelines we talk of runs (hence notation ρ)
- Note the difference: R_a defined on states; $\sim_a$ defined on pairs (ρ, i)

Example

Transition system S:



Perfect recall



Another Possible Definition of $\mathcal{M}_S$

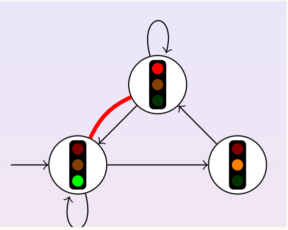
Definition

Given a transition system S, define
$\mathcal{M}_S = (\mathrm{TL} \times \mathbb{N}, (\sim_a)_{a \in AGT}, V)$ such tat

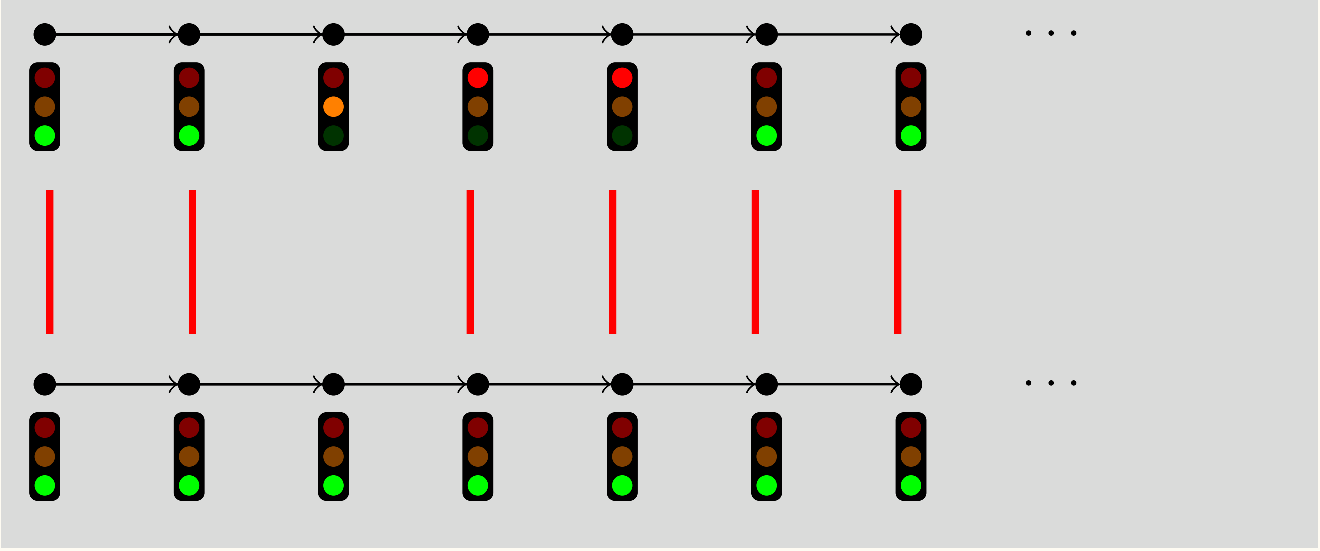
- TL is the set of paths of S starting in an initial state of S;
- For all agents a: $(\rho, t) \sim_a (\rho', t')$ if
  - $t = t'$ (synchrony)
  - $\rho[t] R_a \rho'[t]$ (memoryless)
- $V: TL \times \mathbb{N} \rightarrow 2^{AP}$ is defined by
  $V(\rho, t) =$ set of propositions true at $\rho[t]$

Example

Transition system S:



Memoryless



Theorem (Engelhardt et al. 2007)

The model checking problem for memoryless and synchronuos systems is PSPACE-complete

Theorem (van der Meyden and Shilov, 1999)

The model checking problem under perfect recall and synchrony is

- Undecidable if CK (common knowledge operator) and U (until)
- NON ELEM-c if U but not CK
- PSPACE-c if CK but not U

See also (Bozzelli et al 2019) for recent results.

Uhhh, a lecture with a hopefully useful

APPENDIX

References

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Color Convention in this course

- Formulae, when occurring inline
- Newly introduced terminology and definitions
- Important results (observations, theorems) as well as emphasizing some aspects
- Examples are given with standard orange with possibly light orange frame
- Comments and notes
- Algorithms