Intelligent Agents

Dynamic Epistemic Logic – Part 1

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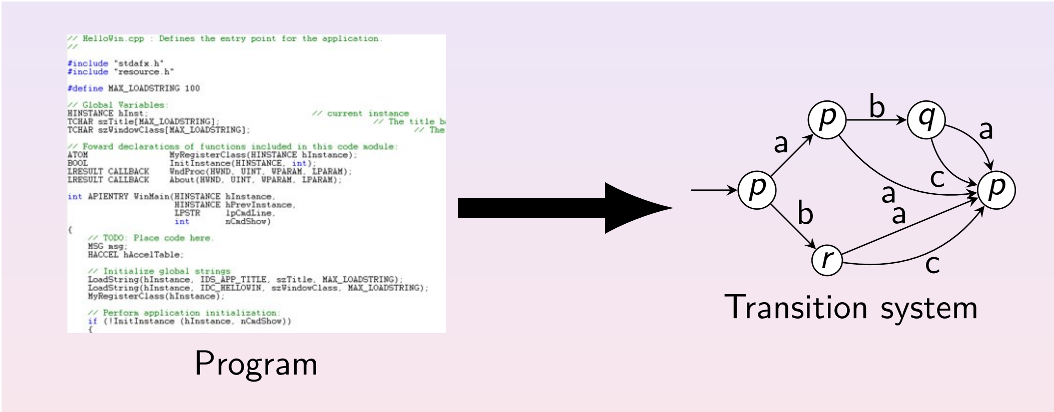
Todays lecture based on

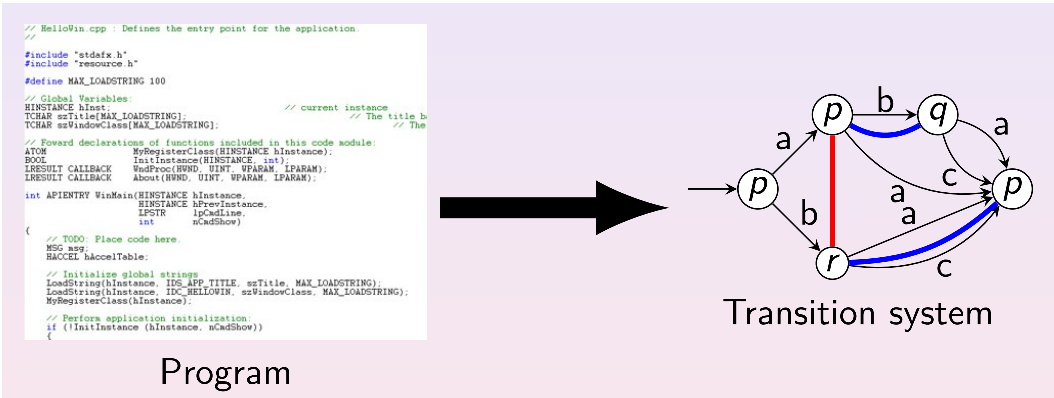
- The AAMAS 2019 Tutorial **„EPISTEMIC REASONING IN MULTI-AGENT SYSTEMS“, Part 4: Dynamic Epistemic Logic**
  http://people.irisa.fr/Francois.Schwarzentruber/2019AAMAStutorial/

MODELING ACTIONS

In the verification/model checking community



Action = an edge →



Action = an edge →

Epistemic =

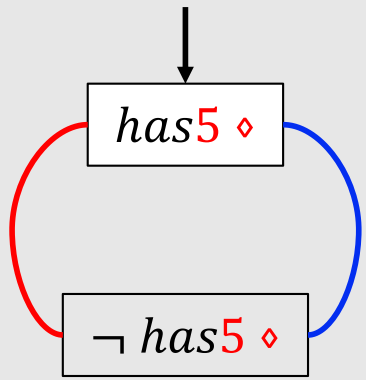
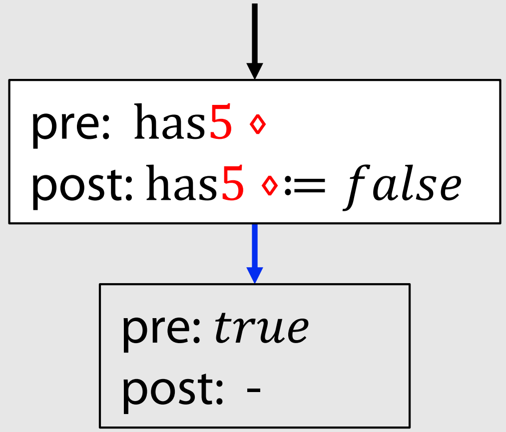
In philosophy and AI

Type of mechanism of actions is important

Type of mechanism	Example
Public/private announcement	She knows you hold 5 ♢
Public action	Play card 5 ♢
Private action	Secretely remove card 5 ♢
Belief revision	Revise believes (entailing $\neg p$) after being told p

- There is a dedicated logic for the first type of announcementes: PAL (Public announcement logic)
- What kind of formalism to use to handle all of them?

Dynamic Epistemic Logic (DEL)

	State	Action
Classical planning	→ has5 ♢	pre: has5 ♢ post: has5 ♢:= $false$
Logic DEL[1),2)] = Kripkean models of classical planning		

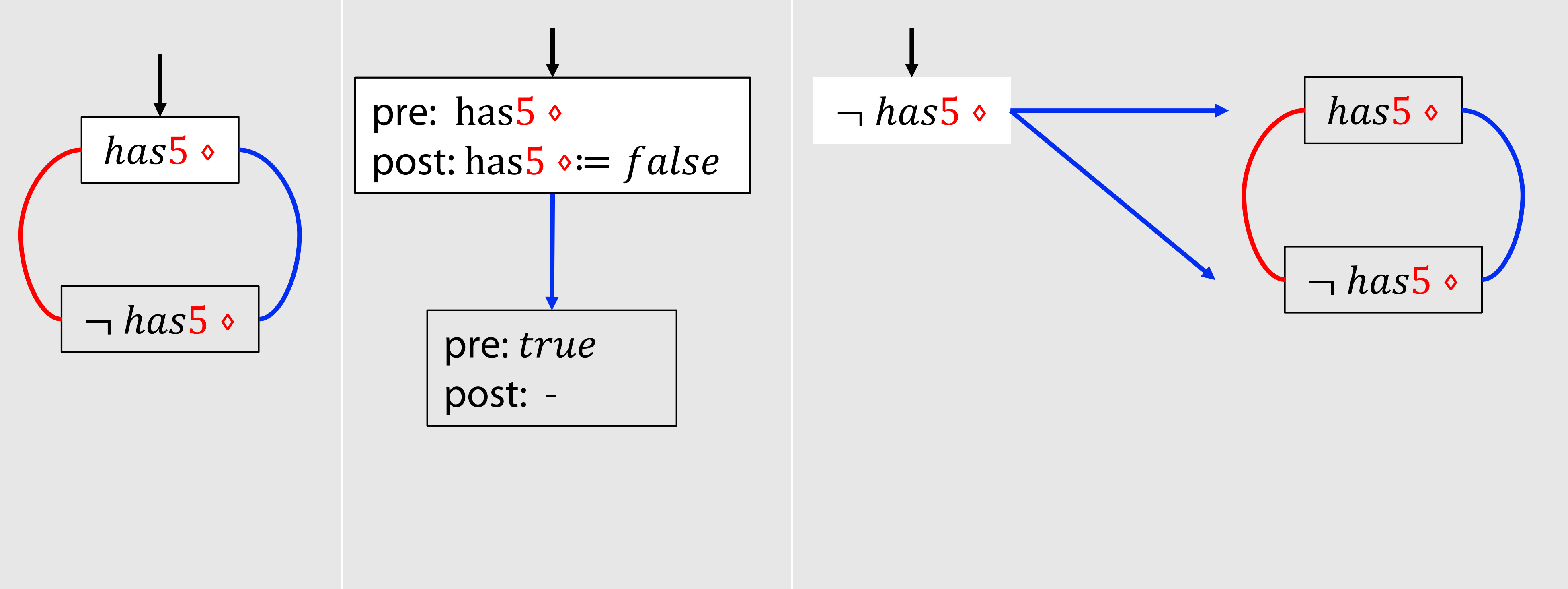
1) (Baltag et al., 1998)
2) (van Ditmarsch et al, 2007)

- Action: Privately announce to red agent removement of 5 ♢
- blue agent does not know

Note:
- Start states filled white
- Implicit self loops for red and blue agent

Computing the next state: product update

State	Action	Next State
(epistemic model)	(event model)	(updated epistemic model)



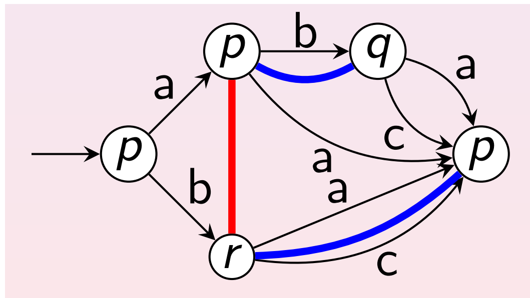
Some syntactic specifications/logics

Logic	Example sentence
Game description language (Love et al. 2008), (Thielscher, 2017)	Agent a sees the game position
Flatland (Babiani et al, 2021), (Gasquet et al. 2014), (Gasquet et al, 2016),	Agent a sees agent b
Visibility atoms (Charrier et al, 2016)	Agent a sees truth value of p
Paying attention to public announcements (Bolander et al, 2016)	$B_a payAtt(b) \rightarrow [p!] B_a B_b p$
Asynchronous announcements (Knight et al, 2019)	$[p!][read_a] K_a p$
Epistemic gossip (Ditmarsch et al 2017)	$[call_{ab}] K_a secret_b$

From DEL to Epistemic Logics

Syntactic Specification

Models of DEL

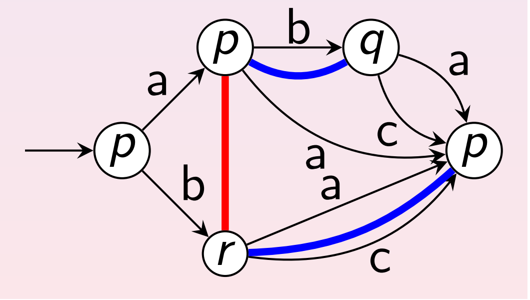


Epistemic temporal models

From DEL to Epistemic Logics

Syntactic Specification

+ Easy to specify
+ Succinct
- Ad-hoc languages
- Hand crafted semantics



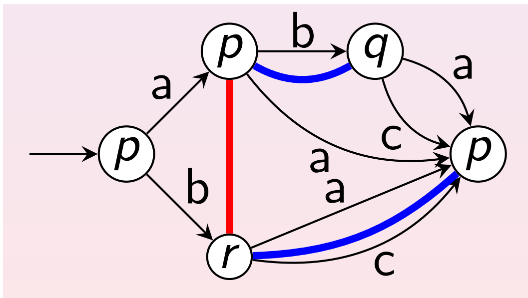
Models of DEL

Epistemic temporal models

From DEL to Epistemic Temporal Logics

Syntactic Specification

+ Elegant Kripkean extension of classical planning
+ Succinct
+ Classification in terms of action types
+ Has probabilistic extension
+ Has extensions that encompass belief revision
- Perfect-recall only
- Synchronous only



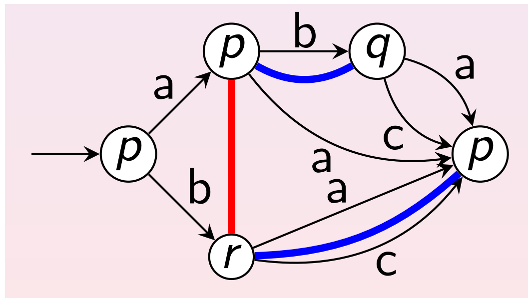
Models of DEL

Epistemic temporal models

From DEL to Epistemic Temporal Logics

Syntactic Specification

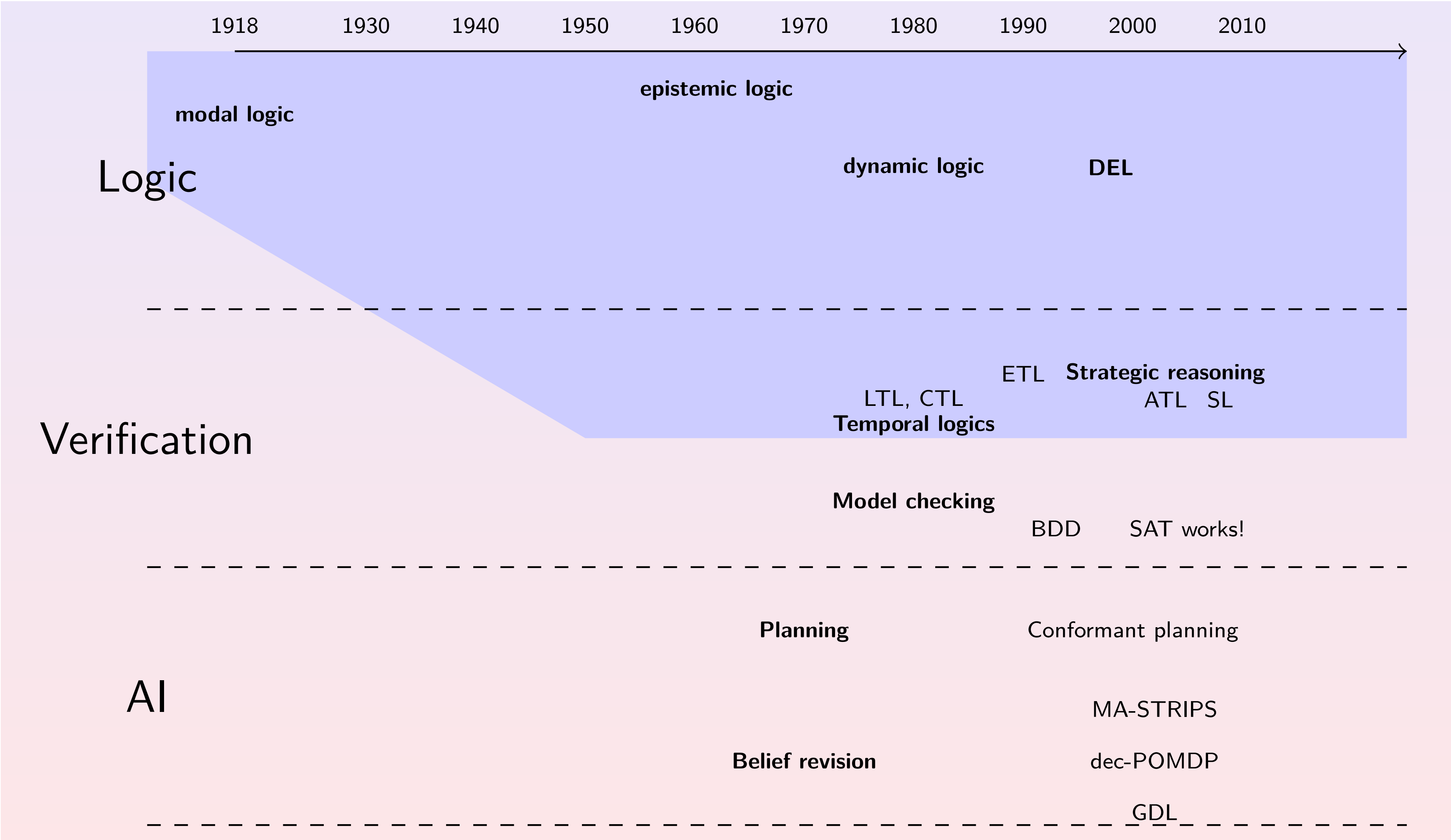
+ Elegant
+ Allows for async/no perfect recall semantics
-Type of actions lost
- Not succinct (usually infinite)



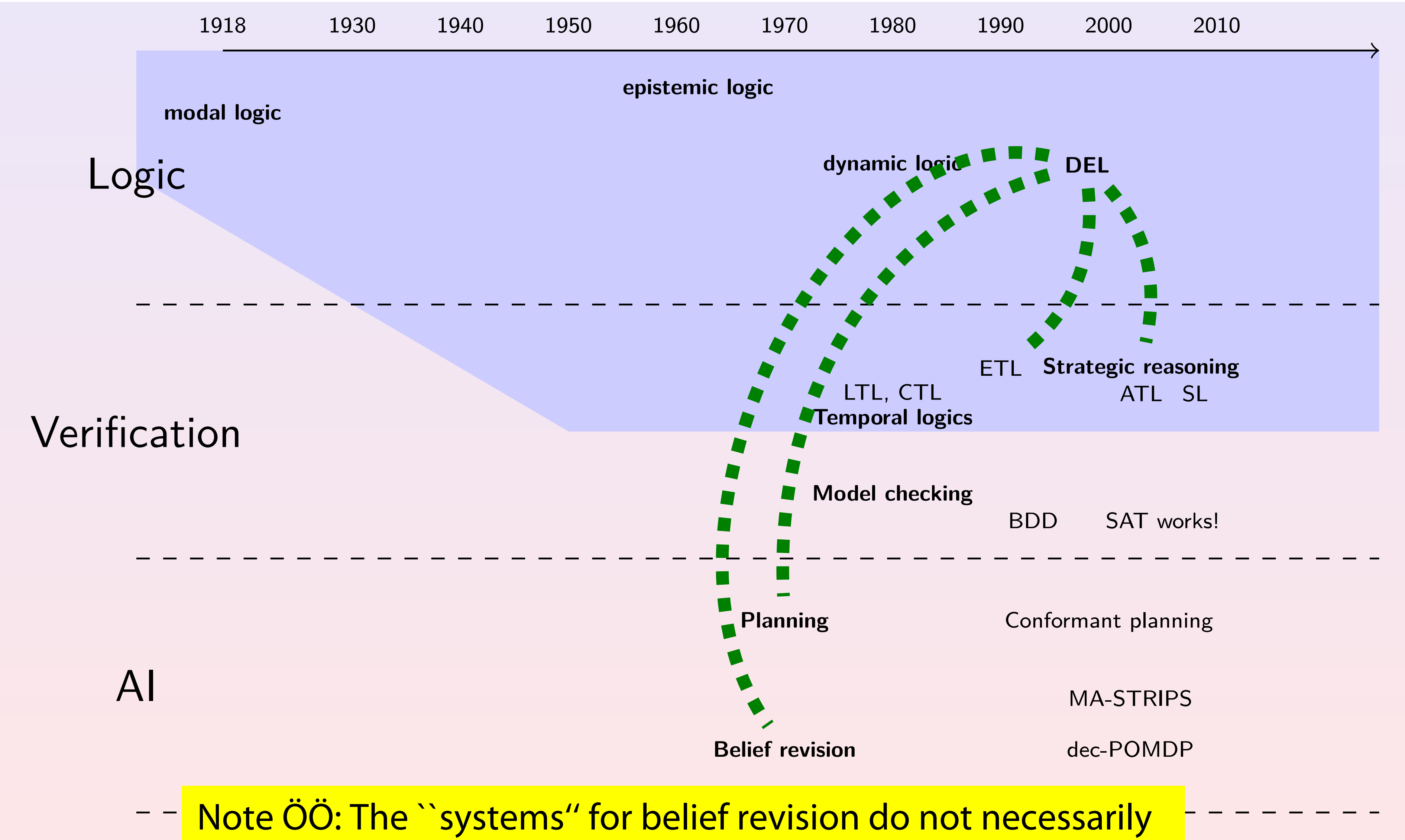
Models of DEL

Epistemic temporal models

Timeline


1918
1930
1940
1950
1960
1970
1980
1990
2000
2010
Logic
modal logic
epistemic logic
dynamic logic
DEL
Verification
ETL
Strategic reasoning
LTL, CTL
ATL
SL
Temporal logics
Model checking
BDD
SAT works!
AI
Planning
Conformant planning
MA-STRIPS
Belief revision
dec-POMDP
GDL

Timeline



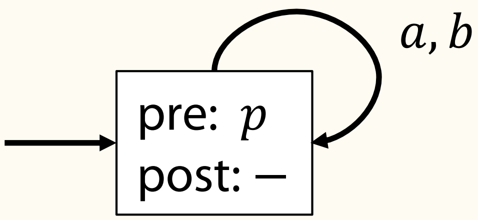
Note ÖÖ: The ``systems“ for belief revision do not necessarily reflect the classical approaches to belief revision

EVENT MODELS

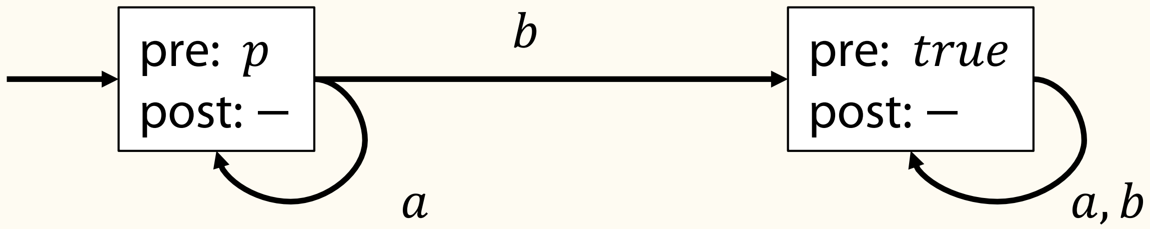
Examples of actions

(Baltag et al. 1998)

Example (public announcement of p)



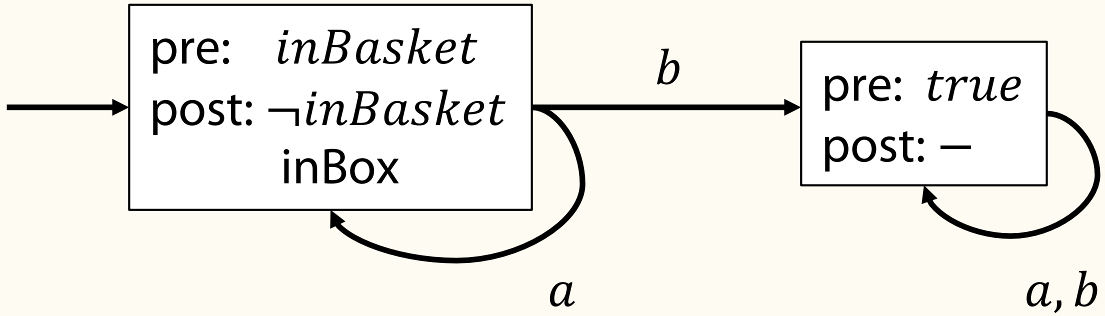
Example (Private announcement of p to a)



Examples of actions

Assume that agent a transfers a marble from a basket to a box - not seen by agent b

Example (Transfer marble from basket to box)

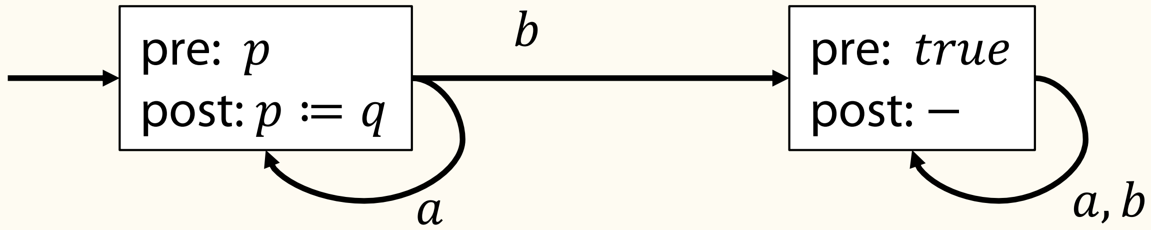


Formal Definition

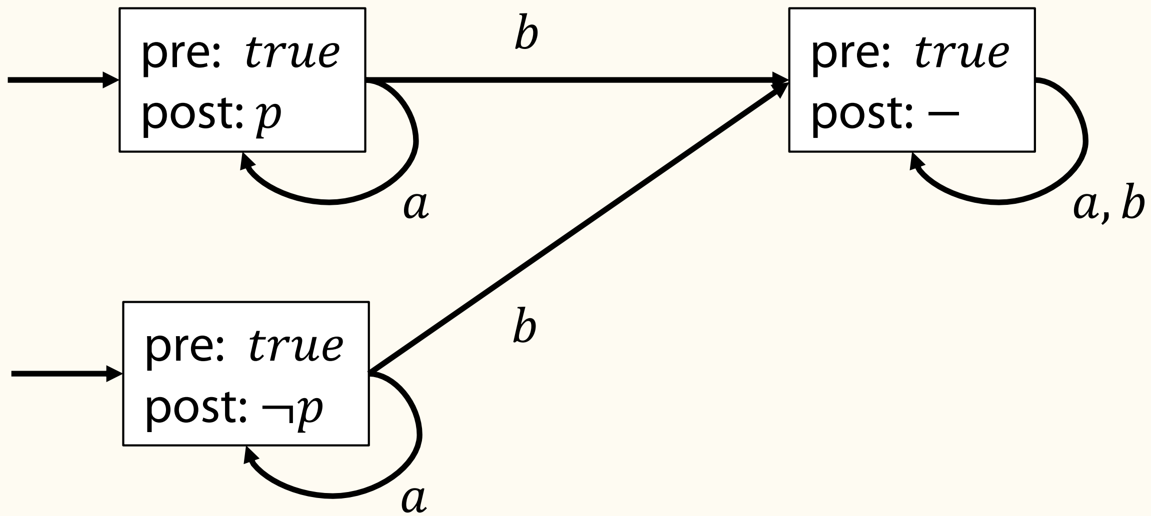
Definition

- An event model $\mathcal{E} = (E, (R_a^E)_{a \in AGT}, pre, post)$ is a tuple where
  - $E = \{e, e', \dots\}$ is a non-empty set of possible events
  - $R_a^{\mathcal{E}} \subseteq E \times E$ is an accessibility relation on E for agent a
  - $pre: E \to \mathcal{L}_{EL}$ is a precondition function
  - $post: E \times AP \to \mathcal{L}_{EL}$ is a postcondition function
- A pair $(\mathcal{E}, e)$ is called an action where e represents the actual event of $(\mathcal{E}, e)$
- A pair $(\mathcal{E}, E_0)$, for $E_0 \subseteq E$, is a non-deterministic action. The set E_0 is the set of triggerable events.

Example (Deterministic action = single-pointed event model)



Example (Non-deterministic action = multi-pointed event model)

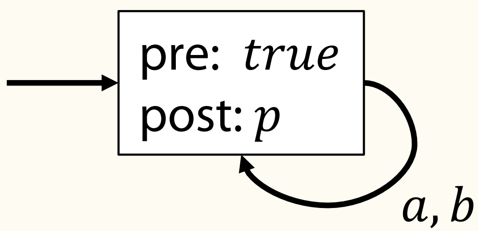


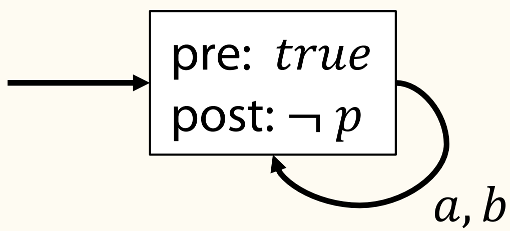
Public Actions

Definition

An action is said to be public if the accessibility relations in the underlying event model are self-loops

Example (public)



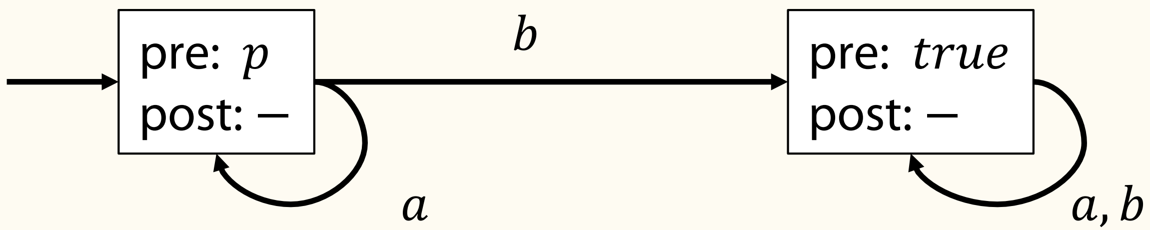


Non-ontic actions

Definition

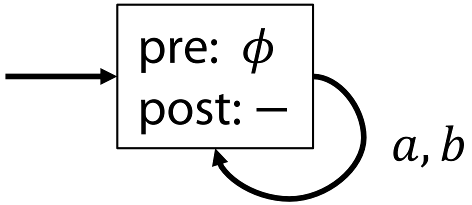
An action is said to be non − ontic if the postconditions are trivial: for all $e \in E$, for all propositions $p \in AP$: $post(e,p) = p$

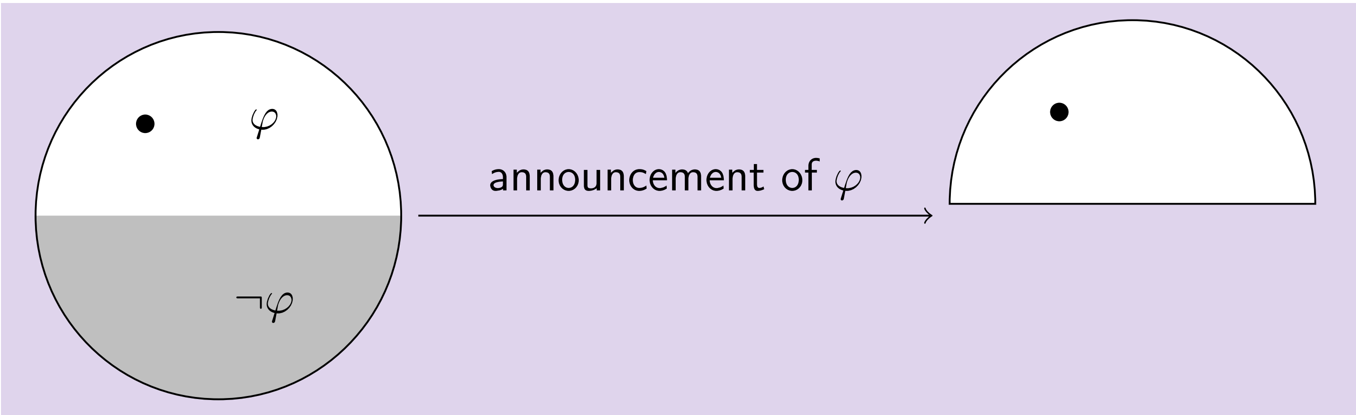
Example (non-ontic)



Effect of a public announcement

Publicly announcing ϕ leads to keeping only the ϕ worlds.





Can try this out on several examples in Hintikka's world.

Muddy children Puzzle

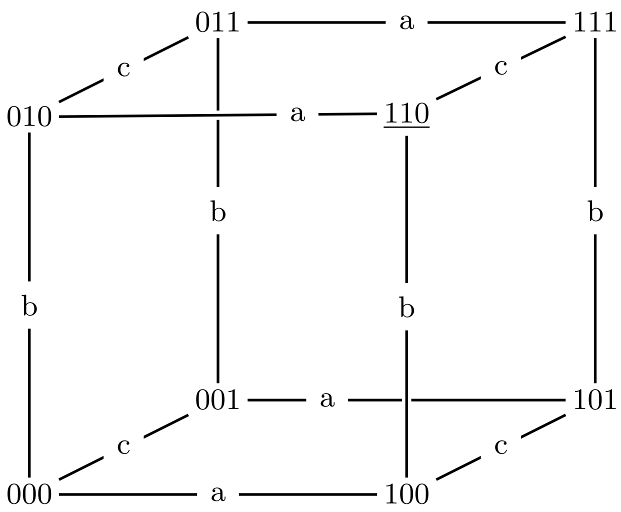
„Three children (a,b,c) are playing in the mud. Father calls the children to the house, arranging them in a semicircle so that each child can clearly see every other child. "At least one of you has mud on your forehead", says Father. The children look around, each examining every other child's forehead. Of course, no child can examine his or her own. Father continues, "If you know whether your forehead is dirty, then step forward now". No child steps forward. Father repeats himself a second time, "If you know whether your forehead is dirty, then step forward now". Some (a,b) but not all of the children step forward. Father repeats himself a third time, "If you know whether your forehead is dirty, then step forward now". All of the remaining children step forward. Explain why a,b stepped forward after two requests. (In general show: if m children are muddy then after m requests of the father those will step forward"

As promised we reconsider this puzzle in the context of dynamic epistemic logic

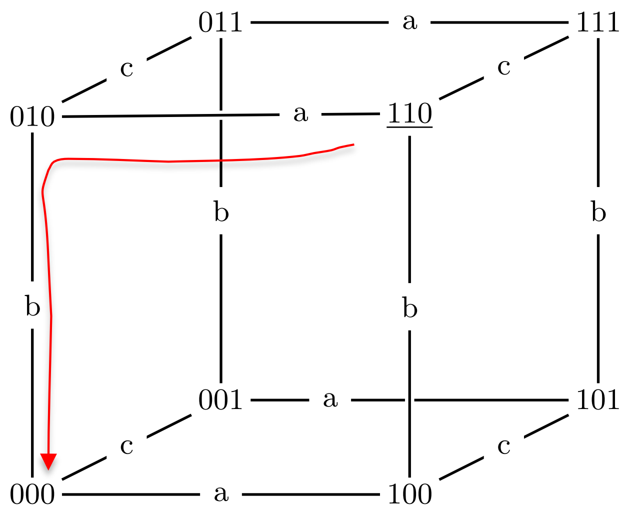
Muddy children: solution

- Children: Anne (a), Bill (b), Cath (c)
- Actual world: a, b muddy (m_a, m_b), c is not ($\neg m_c$)

- a argues:
  - $m_b, \neg m_c$
  - If $\neg m_a$ were the case, then b would see noone with mud on forehead and hence infer that m_b (due to the announcement of the father that someone has mud).
  - But b did not step forward so he does not know whether m_b
  - Therefore a steps forward next time
- b argues similarly



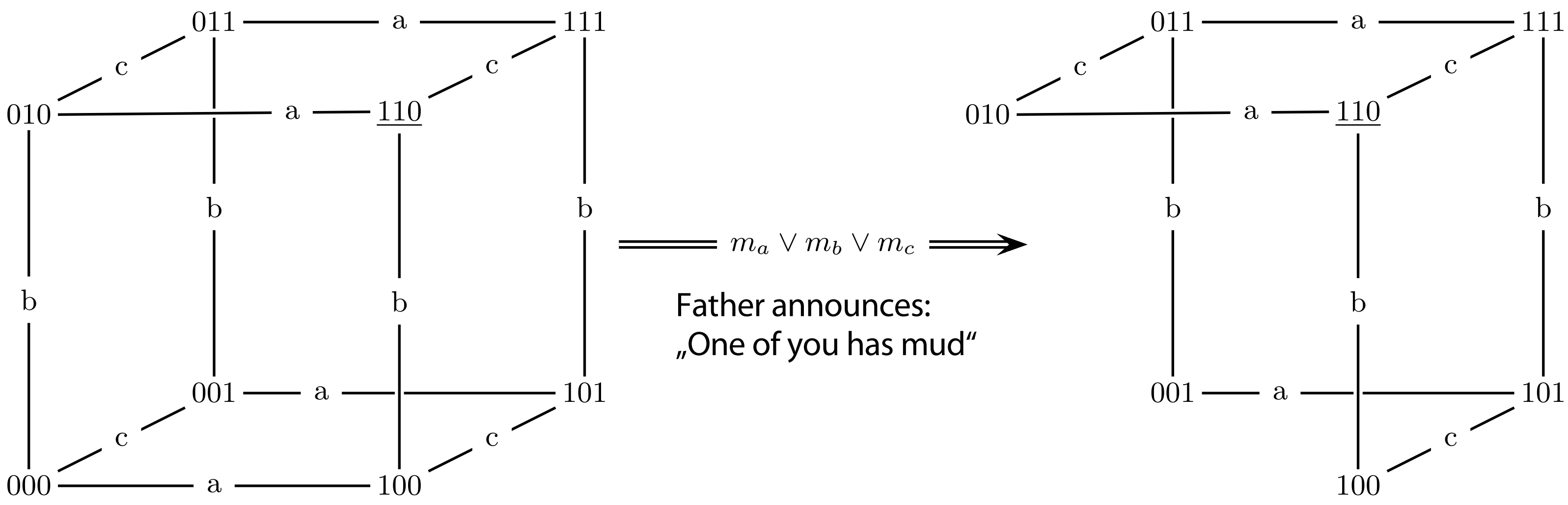
State 010 abbreviates
$\neg m_a \wedge m_b \wedge \neg m_c$, i.e.:
a has no mud;
b has mud;
c has no mud



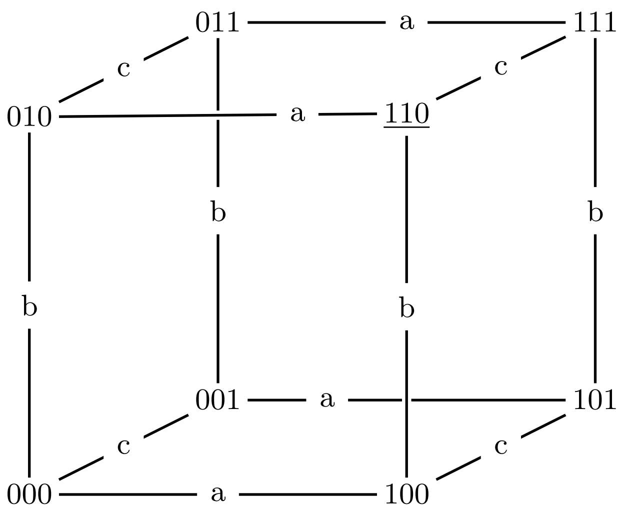
State 010 abbreviates
$\neg m_a \wedge m_b \wedge \neg m_c$, i.e.:
a has no mud;
b has mud;
c has no mud

Actual state: 110

- Here, everybody knows that there is someone with mud on his face
- But this is not common knowledge: a considers possible 010 where b considers 000 possible

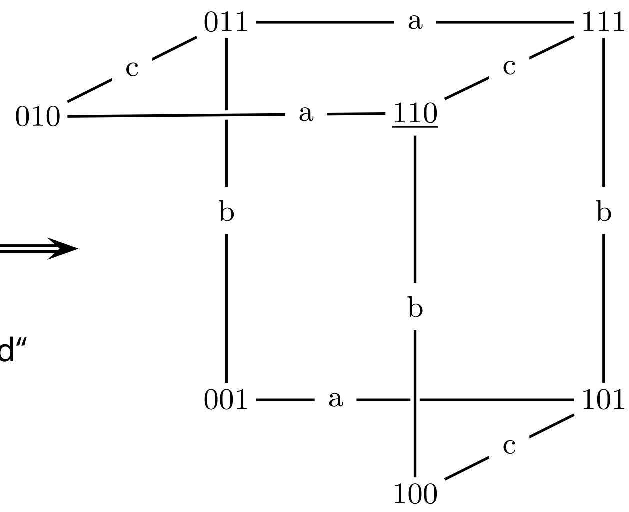


So world 000 gets eliminated



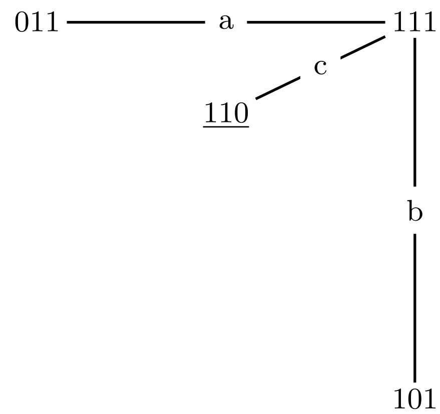
$$m_a \vee m_b \vee m_c$$

Father announces:
„One of you has mud“



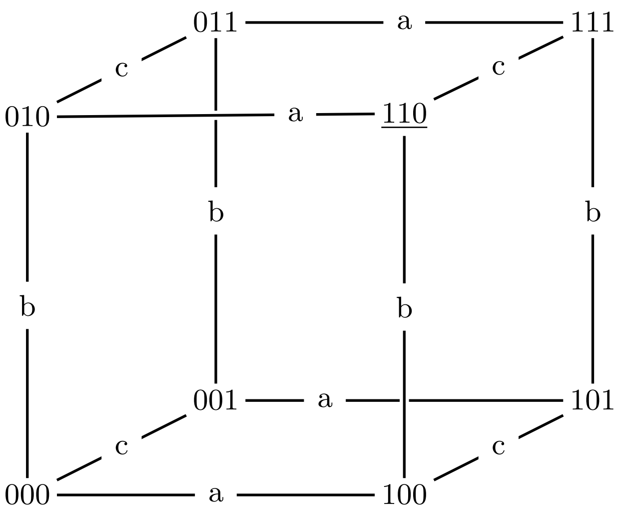
No child steps forward:
a, b, c: „I do not know whether I have mud on my face“

$$\neg(K_a m_a \vee K_a \neg m_a) \wedge \neg(K_b m_b \vee K_b \neg m_b) \wedge \neg(K_c m_c \vee K_c \neg m_c)$$



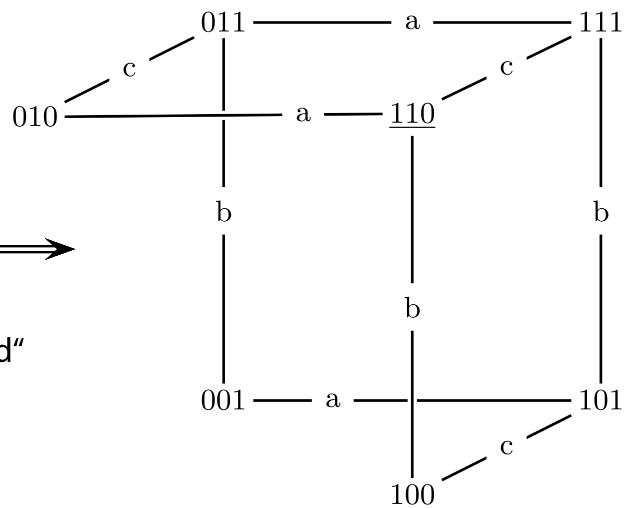
For example:

- $100 \vDash K_a m_a$
- Hence 100 gets eliminated



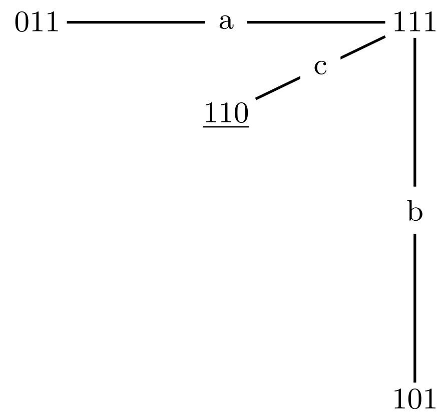
$$m_a \vee m_b \vee m_c$$

Father announces:
„One of you has mud“



$$\neg(K_a m_a \vee K_a \neg m_a) \wedge \neg(K_b m_b \vee K_b \neg m_b) \wedge \neg(K_c m_c \vee K_c \neg m_c)$$

No child steps forward:
a, b, c: „I do not know whether I have mud on my face“



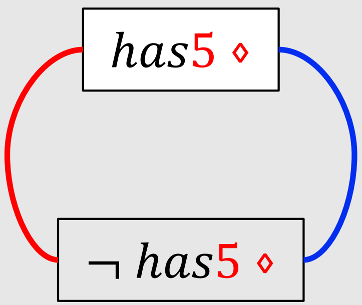
$$(K_a m_a \vee K_a \neg m_a) \wedge (K_b m_b \vee K_b \neg m_b)$$

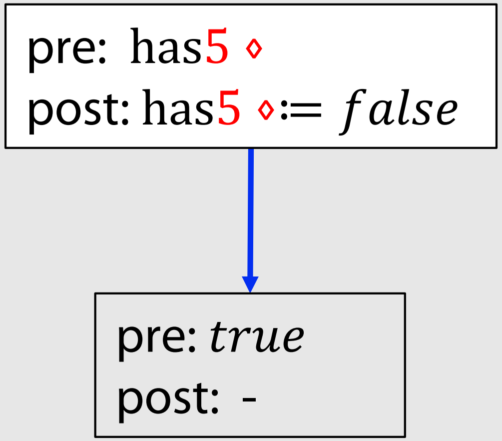
a and *b* step forward

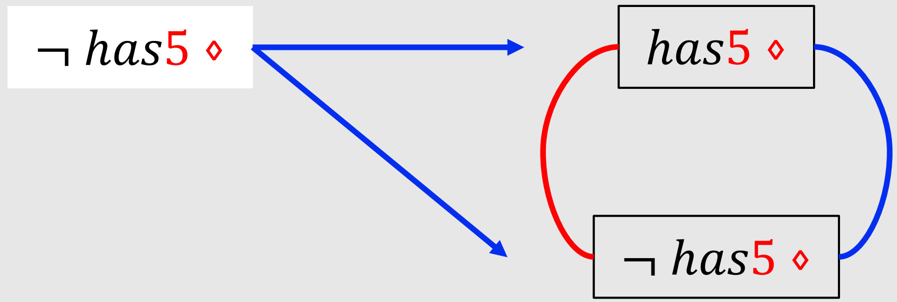
$\underline{110}$

Computing the next state: product update

State	Action	Next State
(epistemic model)	(event model)	(updated epistemic model)







Formal Definition of Update Products

Definition

- Given
  - $\mathcal{M} = (W, \{R_a\}_{a \in AGT}, V)$ (epistemic model)
  - $\mathcal{E} = (E, (R_a^E)_{a \in AGT}, pre, post)$ (event model)

- define the update product of M and $\mathcal{E}$ as the epistemic model $\mathcal{M} \otimes \mathcal{E} = (W^{\otimes}, \{R_a^{\otimes}\}_{a \in AGT}, V^{\otimes})$ where
  - $W^{\otimes} = \{(w, e) \in W \times E \mid \mathcal{M}, w \vDash pre(e)\}$
  - $R_a^{\otimes} = \{((w, e), (w', e')) \in W^{\otimes} \mid w\, R_a\, w' \quad and \quad e\, R_a^{\mathcal{E}} e'\}$
  - $V^{\otimes}((w, e)) = \{p \in AP \mid \mathcal{M}, w \vDash post(e, p)\}$

Pointed Update Products

Definition

The successor state of an epistemic state $(\mathcal{M}, w)$ by action $(\mathcal{E}, e)$ is

$$(\mathcal{M}, w) \otimes (\mathcal{E}, e) = (\mathcal{M} \otimes \mathcal{E}, (w, e))$$

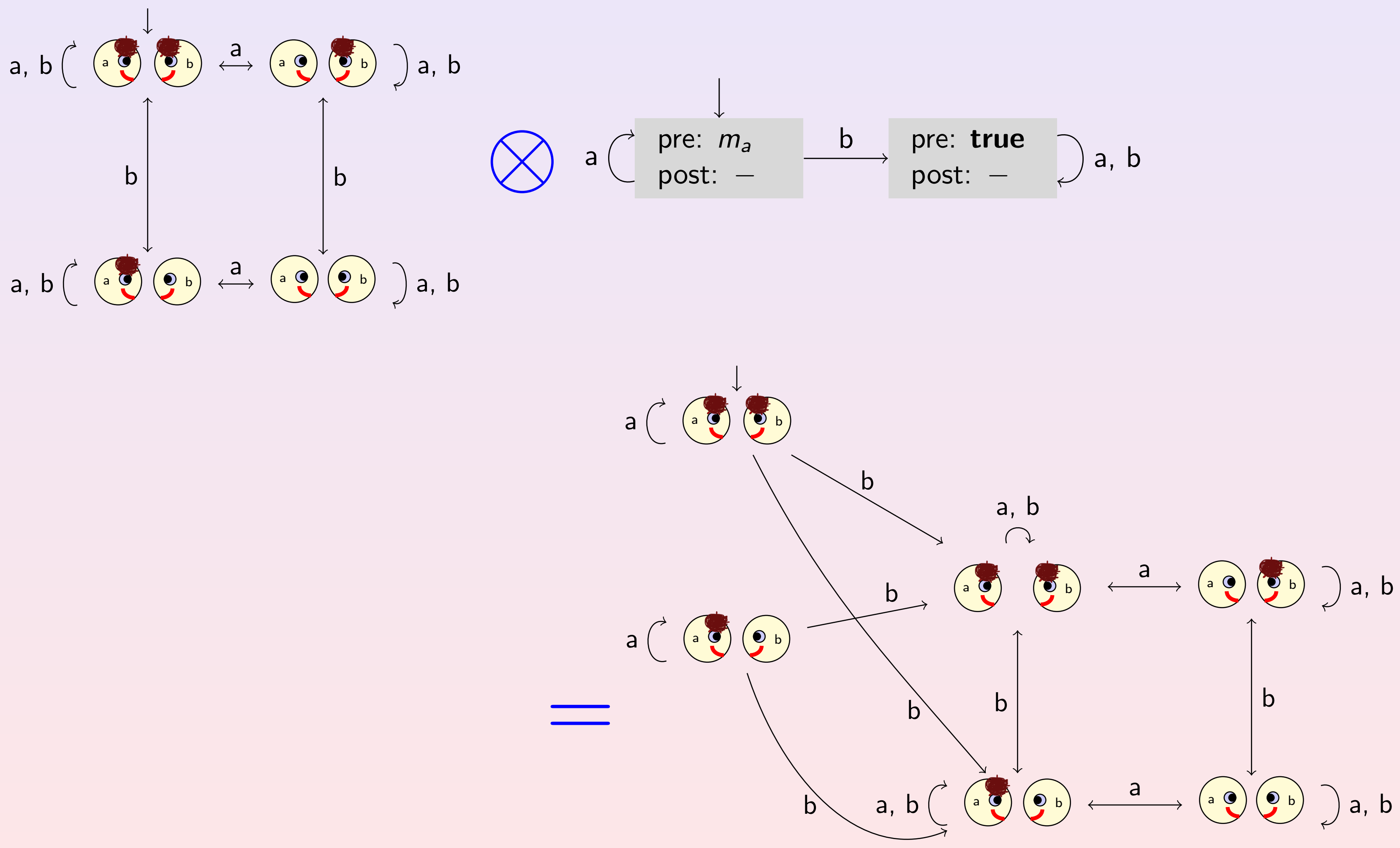
if $(\mathcal{M}, w) \vDash pre(e)$, otherwise it is undefined.

Notation

- Write e for $(\mathcal{E}, e)$
- Write \``we`‘ for (w, e)
- Write $\mathcal{M} \otimes \mathcal{E}^n$ for$\mathcal{M} \otimes \mathcal{E} \otimes \mathcal{E} \ldots \otimes \mathcal{E}$ (n-times)
- Write $we_1 \ldots e_n \vDash \phi$ for $\mathcal{M} \otimes \mathcal{E}^n, we_1 \ldots e_n \vDash \phi$,

Agent a gets private message about its mud

Example (Update Product)



Dynamic epistemic logic $\mathcal{L}_{DELCK}$

Definition

The language $\mathcal{L}_{DELCK}$ extends $\mathcal{L}_{ELCK}$ with dynamic (possibility) modalities $< \mathcal{E}, E_0 >$ according to the following BNF:

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \vee \phi) \mid K_a\phi \mid C_G\phi \mid < \mathcal{E}, E_0 > \phi$$

Definition

The modelling relation $\vDash$ for $\mathcal{L}_{ELCK}$ is extended with the following clause:

$\mathcal{M}, w \vDash < \mathcal{E}, E_0 > \phi$ iff there exists $e \in E_0$ such that $\mathcal{M}, w \vDash pre(e)$ and $\mathcal{M} \otimes \mathcal{E}, (w, e) \vDash \phi$

Dual operator

Definition (Dual operator)

$$[\mathcal{E}, E_0]\phi := \neg < \mathcal{E}, E_0 > \neg\phi$$

The induced semantics is

$\mathcal{M}, w \vDash [\mathcal{E}, E_0]\phi$ iff for all e $\in E_0$ we have:
If $\mathcal{M}, w \vDash pre(e)$ then $\mathcal{M} \otimes \mathcal{E}, (w, e) \vDash \phi$

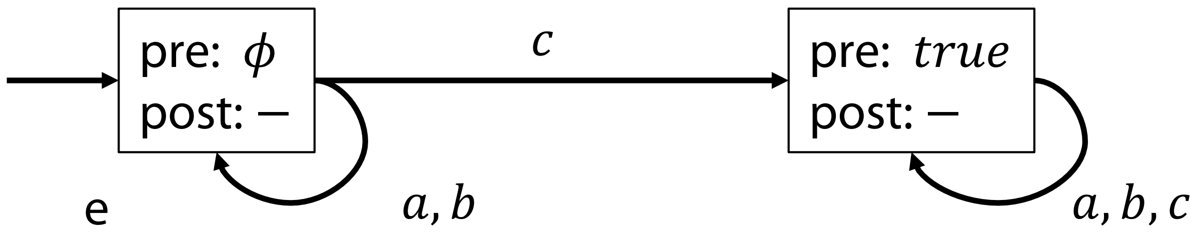
Wake-up: Group announcements

- Q: Consider the following secure group-announcements
  - Agents: $AGT = \{a, b, c\}$
  - ϕ announced publicly within group $\{a, b\}$;
    c does not even know about this announcement

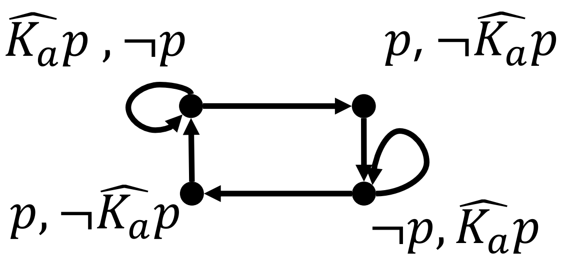
1. How to model this kind of announcement?
2. Once can show that the anouncement creates common knowledge for $\{a, b\}$ w.r.t. ϕ if ϕ is atomic. Give a counterexample for non-atomic ϕ
3. Model the fact that c does not know or even suspect that the announcement happened
4. How would you change the model of 1. to model that C is suspicious?

Answers

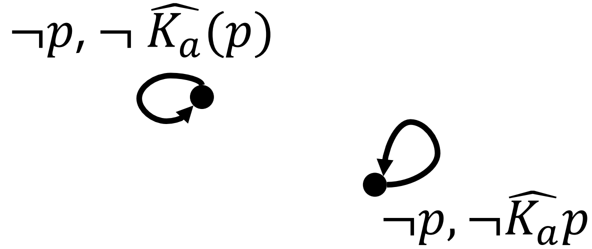
1. This is the model $(\mathcal{E}, \{e\})$ of the secure announcement



2. Creation of common knowledge means: $(\mathcal{M}, w) \vDash < \mathcal{E}, \{e\} > C_{\{a,b\}}\phi$. This does not hold if, e.g., $\phi = \widehat{K_a}p$. Take the simplest case of one agent a.



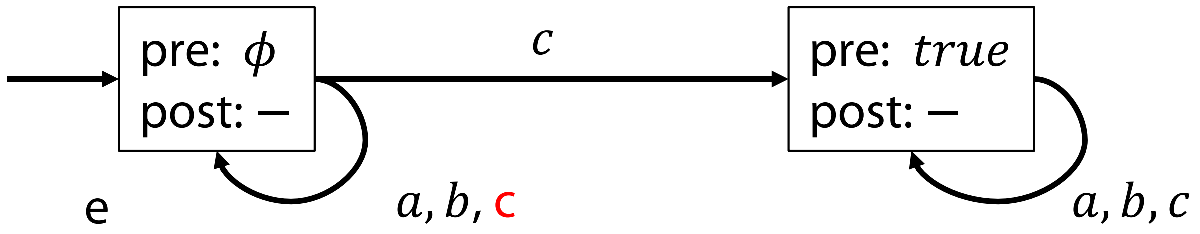
After public announcement: $\widehat{K_a}p$



Answers

3. c does not know or even suspect that the announcement happened: $\models K_c\phi \leftrightarrow < \mathcal{E}, \{e\} > K_c\phi$

4.



Expressivity and Succinctness

Theorem (Baltag 98)

DEL and EL have the same expressivity

Proof idea: Remove dynamic operators $[\mathcal{E}, E]$ as demonstrated here for public announcements:

- Rembember
  $[\phi!]\psi$: if ϕ holds then after having anounced ϕ publicly, ψ holds.
- $[\phi!]p$: says the same as $(\phi \rightarrow p)$
- $[\phi!](\psi \wedge \chi)$: says the same as $([\phi!]\psi \wedge [\phi!]\chi)$
- $[\phi!]\neg\psi$: says the same as $(\phi \rightarrow \neg[\phi!]\psi)$
- $[\phi!]K_a\psi$: says the same as $(\phi \rightarrow K_a[\phi!]\psi)$
- $[\phi!][\psi!]\chi$: says the same as $([\phi \wedge [\phi!]\psi!]\chi)$

Theorem (Lutz 2006)

DEL is more succinct

Uhhh, a lecture with a hoepfully useful

APPENDIX

References

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Color Convention in this course

- Formulae, when occurring inline
- Newly introduced terminology and definitions
- Important results (observations, theorems) as well as emphasizing some aspects
- Examples are given with standard orange with possibly light orange frame
- Comments and notes
- Algorithms