Web-Mining Agents
Game Theory and Social Choice

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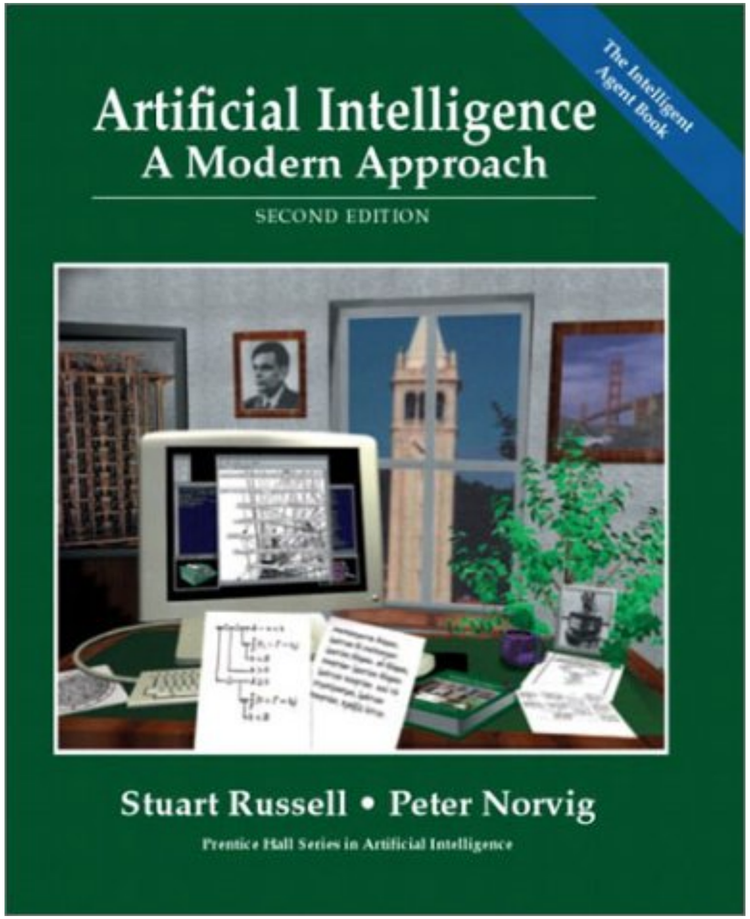
Literature



Chapter 17

Presentations from CS 886
Advanced Topics in
AI Electronic Market Design
Kate Larson
Waterloo Univ.

Multiagent Systems: Criteria

- Social welfare: $\max_{outcome} \sum_i u_i(outcome)$
- Surplus: social welfare of outcome – social welfare of status quo
  - Constant sum games have 0 surplus.
  - Markets are not constant sum
- Pareto efficiency: An outcome o is Pareto efficient if there exists no other outcome o′ s.t. some agent has higher utility in o′ than in o and no agent has lower
  - Implied by social welfare maximization
- Individual rationality: Participating in the negotiation (or individual deal) is no worse than not participating
- Stability: No agents can increase their utility by changing their strategies (aka policies)
- Symmetry: No agent should be inherently preferred, e.g. dictator

Game Theory: The Basics

- **A game:** Formal representation of a situation of strategic interdependence
  - Set of agents, I ($|I|=n$)
    - Aka players
  - Each agent, j, has a set of actions, A_j
    - Aka moves
  - Actions define outcomes
    - For each possible action there is an outcome.
  - Outcomes define payoffs
    - Agents' derive utility from different outcomes

Normal form game* (matching pennies)

Agent 2

Agent 1	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

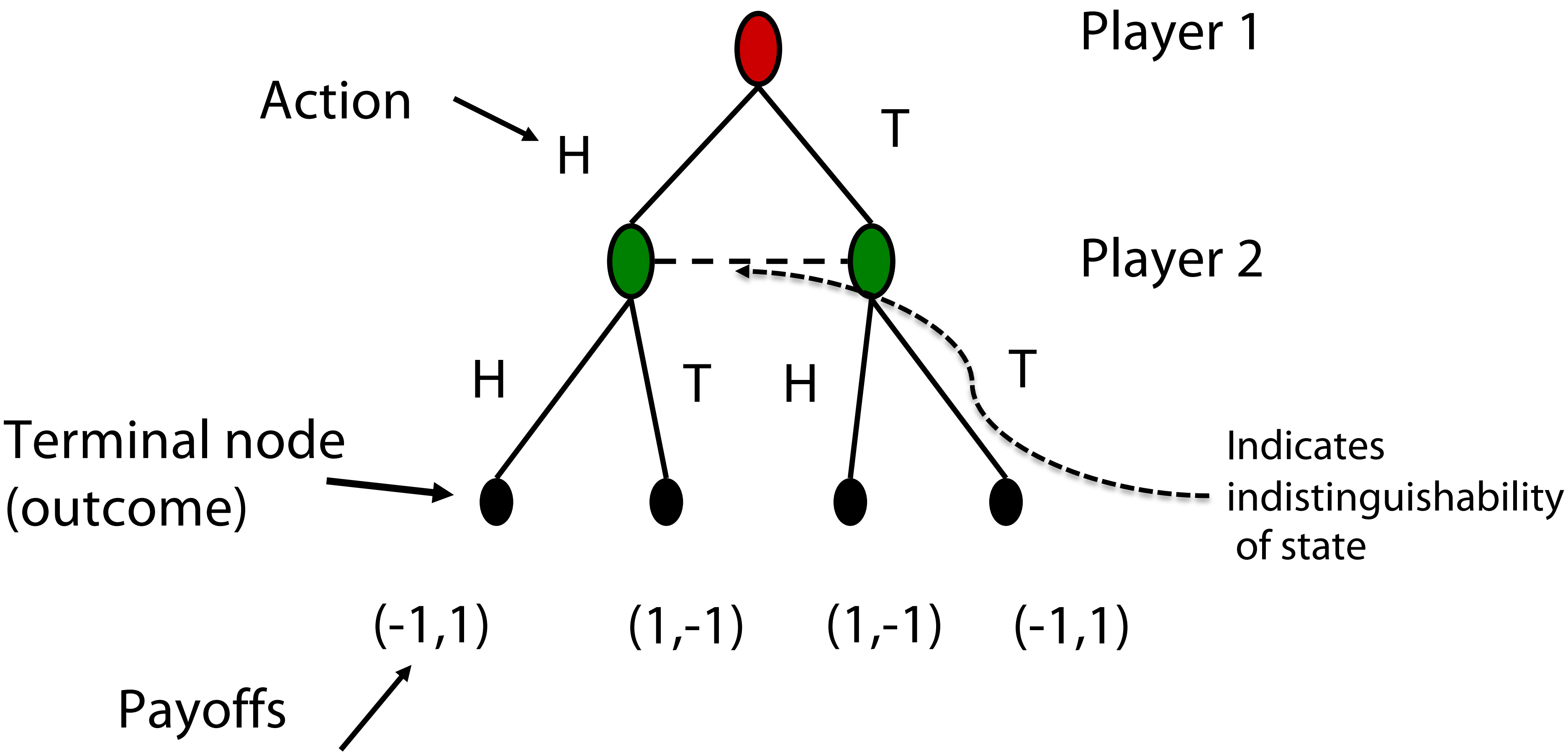
Action → H

Outcome → (cell)

Payoffs → 1, -1

*aka strategic form, matrix form

Extensive form game (matching pennies)



Strategies (aka Policies)

- Strategy:
  - A strategy, s_j, is a **complete contingency plan**; defines actions agent j should take for all possible states of the world
- Strategy profile: $s=(s_1,\ldots,s_n)$
  - $s_{-i} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$
- Utility function: $u_i(s)$
  - Note that the utility of an agent depends on the strategy profile, not just its own strategy
  - We assume agents are **expected utility maximizers**

Normal form game* (matching pennies)

Agent 2

Agent 1	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

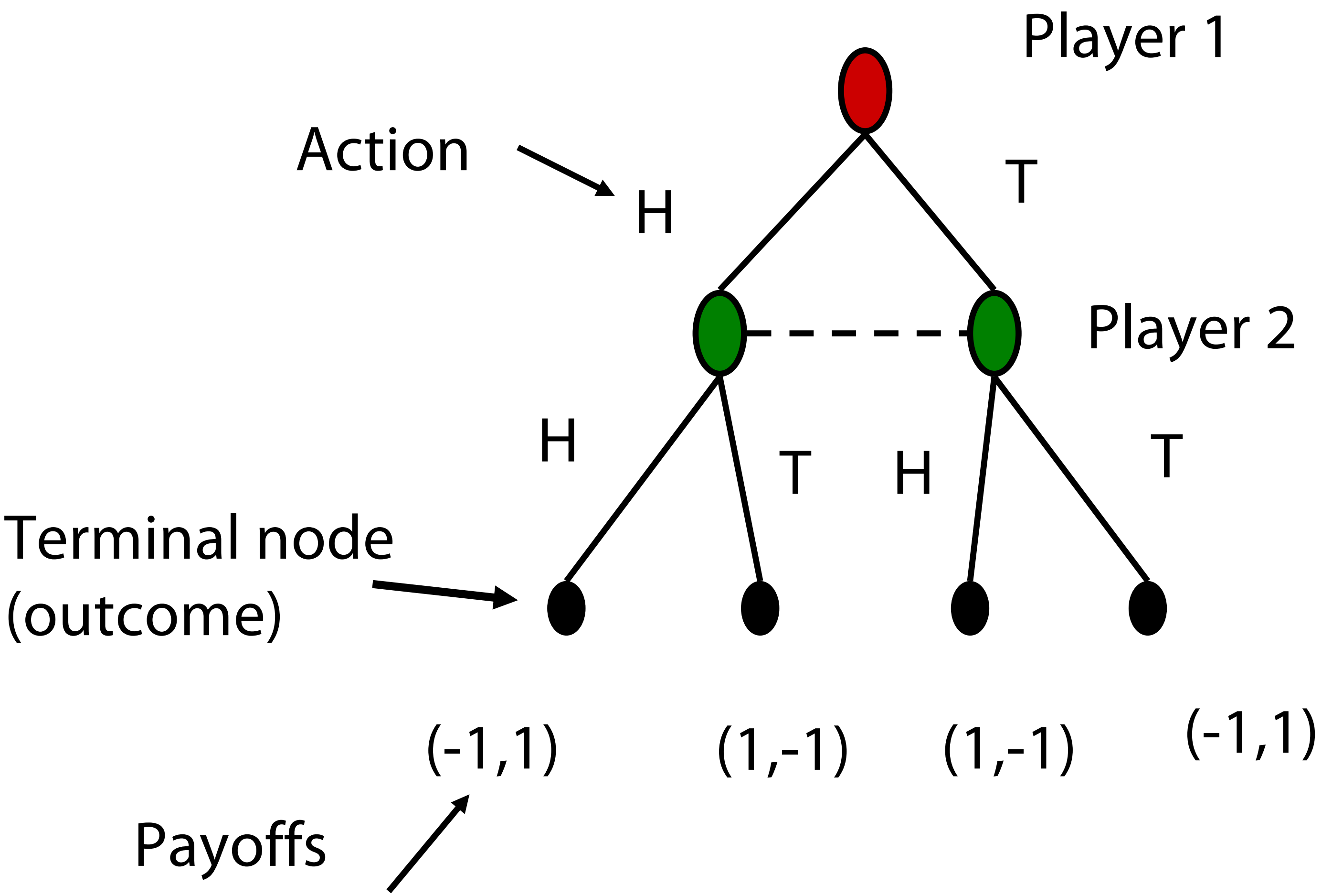
Strategy for agent 1: H

Strategy profile (H,T)

U1((H,T))=1
U2((H,T))=-1

*aka strategic form, matrix form

Extensive form game (matching pennies)



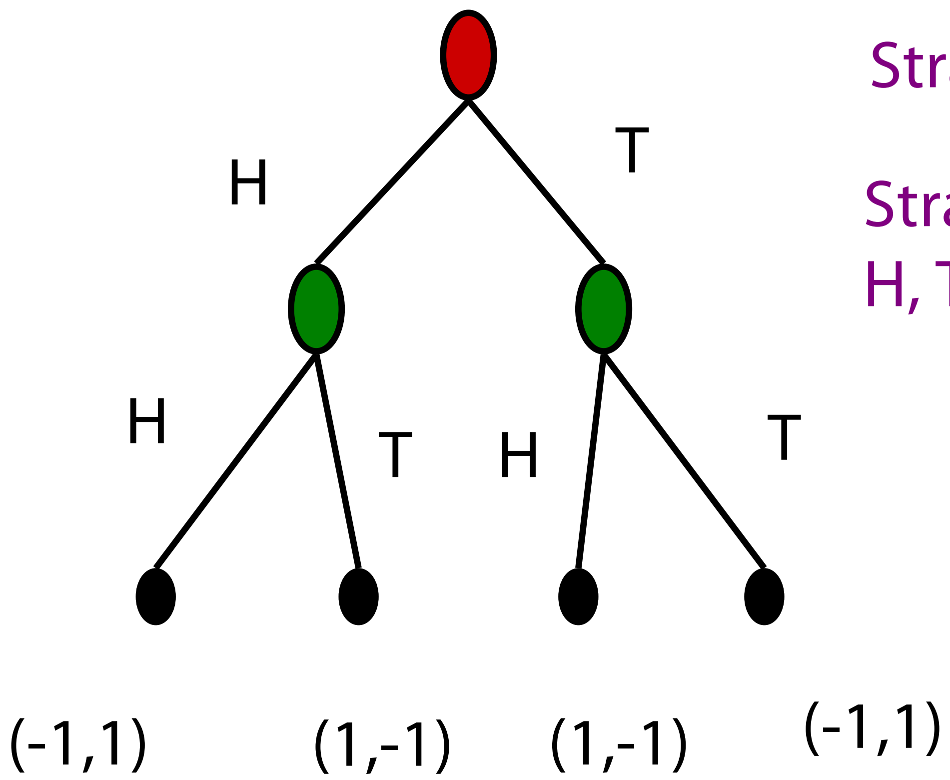
Strategy for agent 1: T

Strategy profile: (T,T)

U1((T,T))=-1

U2((T,T))=1

Extensive form game (matching pennies with sequential moves)



Recall: A strategy is a contingency plan for all states of the game

Strategy for agent 1: T

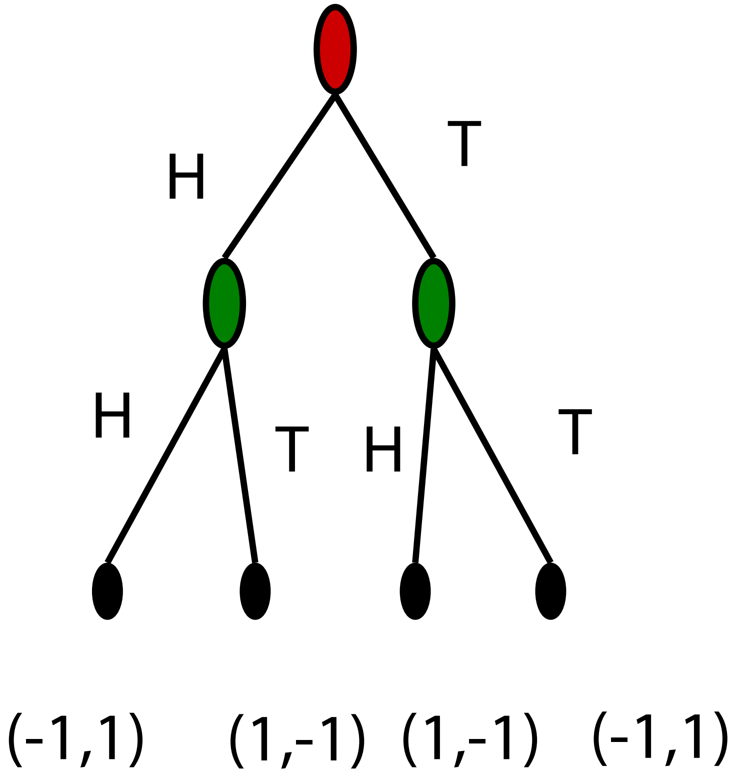
Strategy for agent 2: H if 1 plays H, T if 1 plays T (H,T)

Strategy profile: (T,(H,T))

U1((T,(H,T)))=-1

U2((T,(H,T)))=1

Game Representation



	H,H	H,T	T,H	T,T
H	-1,1	-1,1	1,-1	1,-1
T	1,-1	-1,1	1,-1	-1,1

Potential combinatorial explosion →

Example: Ascending Auction

- State of the world is defined by (x,p)
  - $x \in \{0,1\}$ indicates if the agent has the object
  - p is the current next price
- Strategy $s_i((x,p))$

$$s_i((x,p)) = \begin{cases} p, \text{ if } v_i \geqq p \text{ and } x=0 \\ \text{No bid otherwise} \end{cases}$$

(v_i is the value agent i ascribes to the object)

Dominant Strategies

- Recall that
  - Agents' utilities depend on what strategies other agents are playing
  - Agents are expected utility maximizers
- Agents will play best-response strategies

s_i^* is a best response if $u_i(s_i^*,s_{-i}) \geq u_i(s_i',s_{-i})$ for all s_i'

- A dominant strategy is a best-response for all s_{-i}
  - They do not always exist
  - Inferior strategies are called dominated

Dominant Strategy Equilibrium

- A dominant strategy equilibrium is a strategy profile where the strategy for each player is dominant
  - $s^*=(s_1^*,\ldots,s_n^*)$
  - $u_i(s_i^*,s_{-i})\geq u_i(s_i',s_{-i})$ for all i, for all s_i', for all s_{-i}

- **GOOD**: Agents do not need to counterspeculate!

Example: Prisoner's Dilemma

Two people are arrested for a crime.

- If neither suspect confesses, both are released (ÖÖ: but sentenced semi-heavy).
- If both confess then they get sent to jail.
- If one confesses and the other does not, then the confessor gets a light sentence and the other gets a heavy sentence.

	A: Confess	A: Don't Confess
B: Confess	B=-5, A=-5	B=-1, A=-10
B: Don't Confess	B=-10, A=-1	B=-2, A=-2

Dom. Str. Eq

Pareto Optimal Outcome

Dominant strategy exists but is not Pareto efficient

Example: Split or Steal

Does communication help?

Only if agents do not lie

	A: Steal	A: Split
B: Steal	B=0, A=0	B=100, A=-10
B: Split	B=-10, A=100	B=50, A=50

Dom. Str. Eq

Pareto Optimal Outcome

ÖÖ: Example from British Game Show „Golden Balls"
See http://blogs.cornell.edu/info2040/2012/09/21/split-or-steal-an-analysis-using-game-theory/
And may be...
https://www.youtube.com/watch?v=p3Uos2fzIJ0

Vickrey *) Auctions

- Vickrey auctions are:
  - *second-price*
  - *sealed-bid*
- Good is awarded to the agent that made the highest bid; at the price of the *second highest* bid
- *Bidding to your true valuation is dominant strategy in Vickrey auctions*
- Vickrey auctions susceptible to *antisocial* behavior

*) Russel/Norvig add in a FN:
Named after William Vickrey (1914–1996), who won the 1996 Nobel Prize in economics for this work and died of a heart attack three days later

Example: Vickrey Auction (2nd price sealed bid)

- Each agent i has value v_i
- Strategy $b_i(v_i) \in [0,\infty)$
- $b^* :=$ 2nd best bid.

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b^* & \text{if } b_i > b^* \\ 0 & \text{otherwise} \end{cases}$$

Given value v_i, $b_i(v_i)=v_i$ is dominant.

Let $b' = \max_{j \neq i} b_j$. If $b' < v_i$ then any bid $b_i(v_i) \geq b'$ is optimal. If $b' \geq v_i$, then any bid $b_i(v_i) \leq v_i$ is optimal. Bid $b_i(v_i)=v_i$ satisfies both constraints.

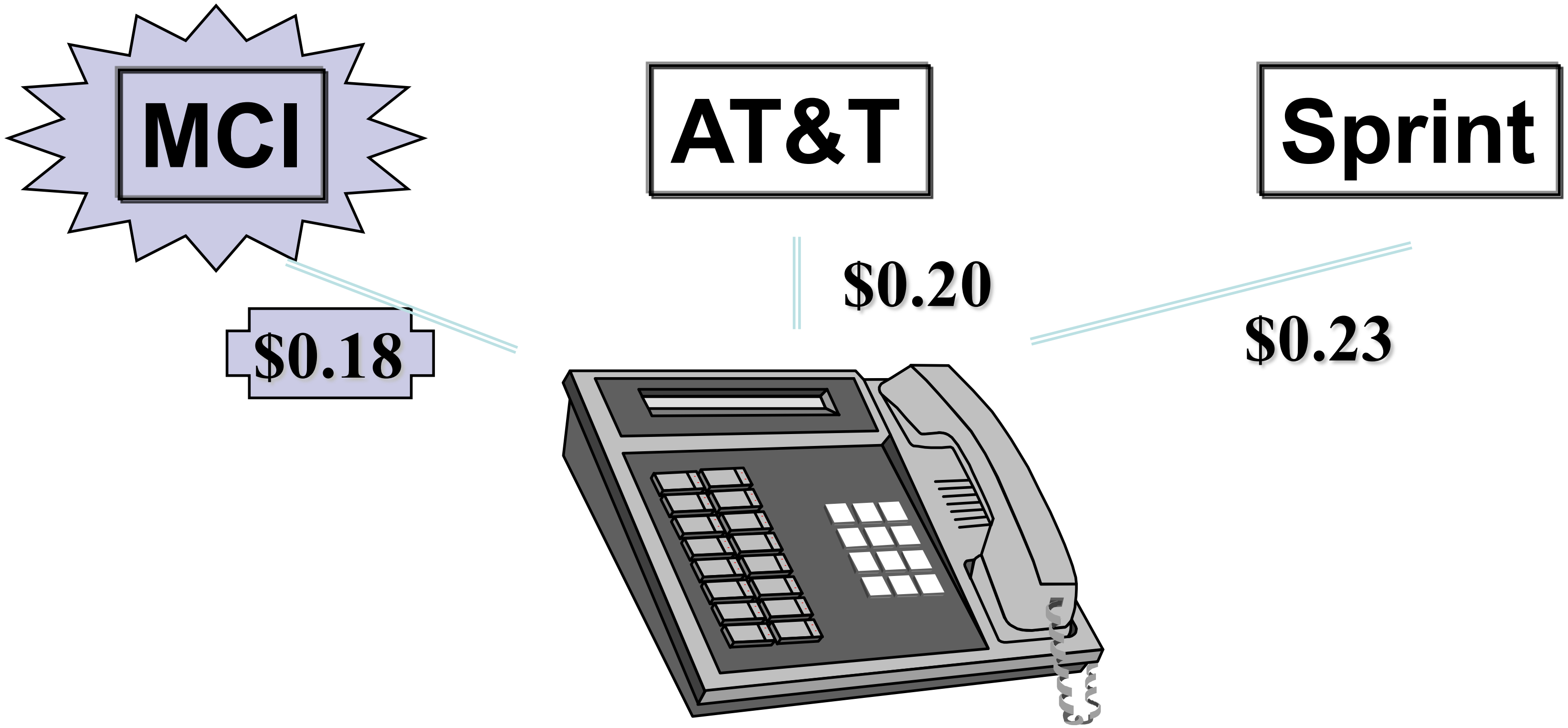
Dominant strategy is Pareto efficient

Phone Call Competition Example

- Customer wishes to place long-distance call
- Carriers simultaneously bid, sending proposed prices
- Phone automatically chooses the carrier (dynamically)

Best Bid Wins

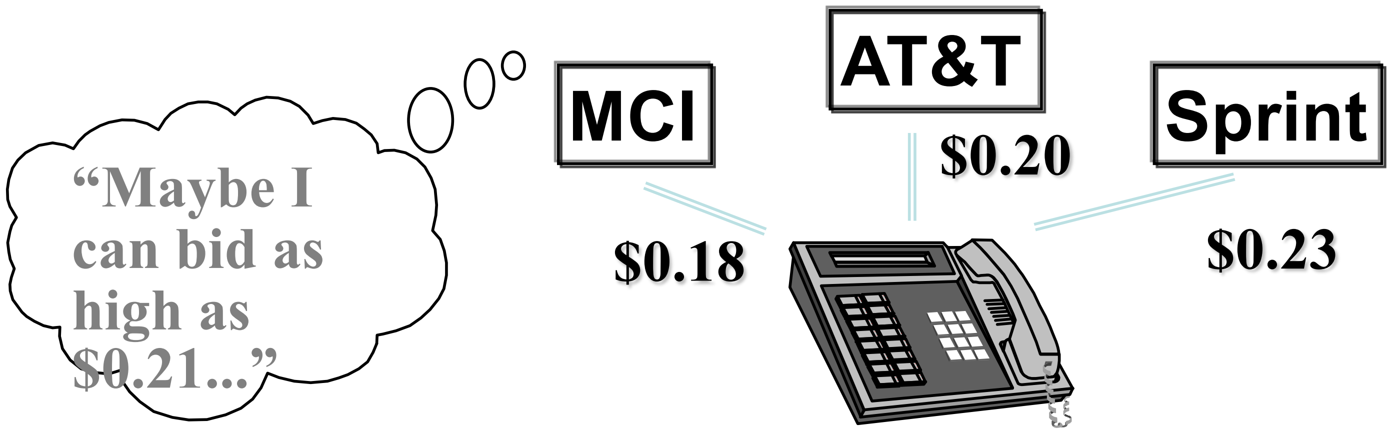
- Phone chooses carrier with lowest bid
- Carrier gets amount that it bid



Attributes of the Mechanism *)

- ✓ *Distributed*
- ✓ *Symmetric*
- ✗ *Stable*
- ✗ *Simple*
- ✗ *Efficient*

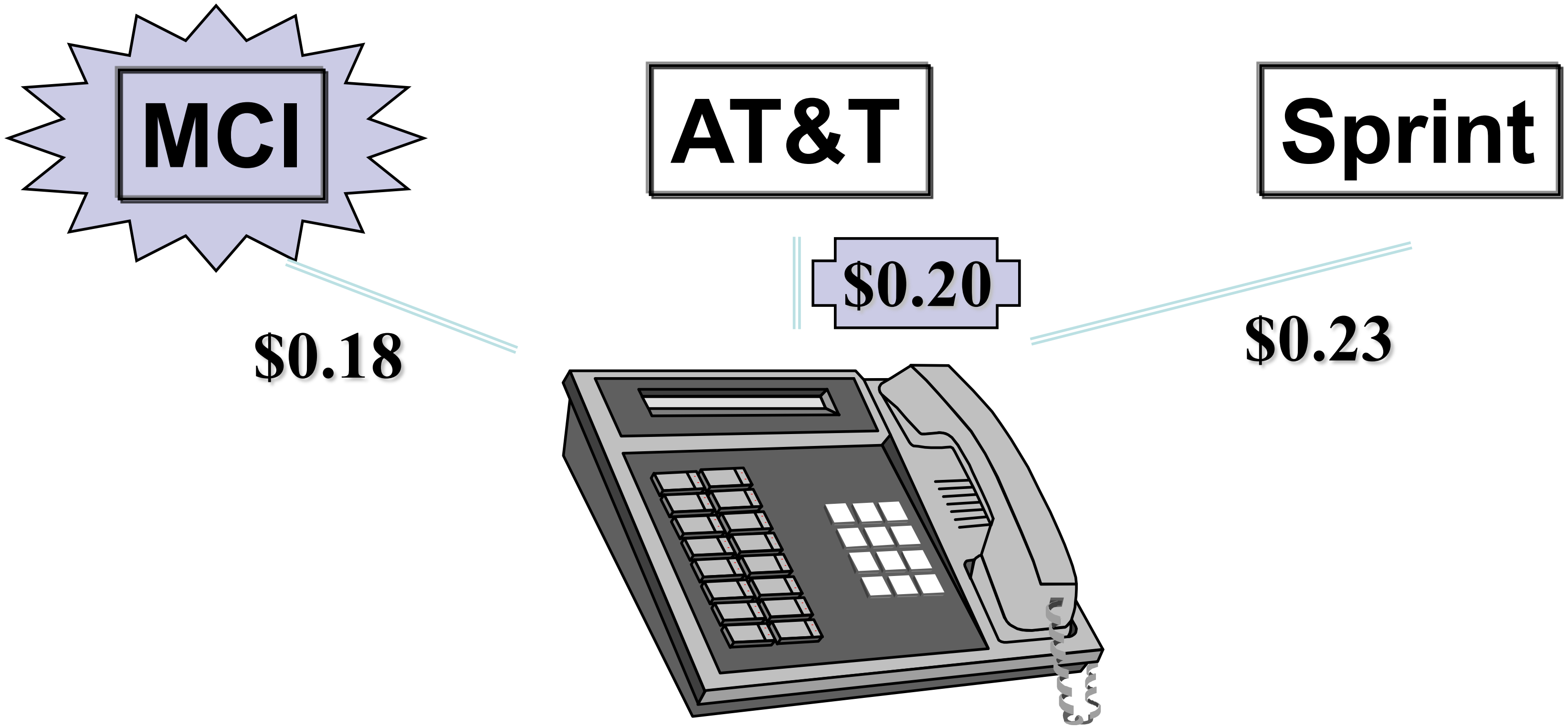
Carriers have an incentive to invest effort in strategic behavior



*) Mechanism design discussed later

Best Bid Wins, Gets Second Price (Vickrey Auction)

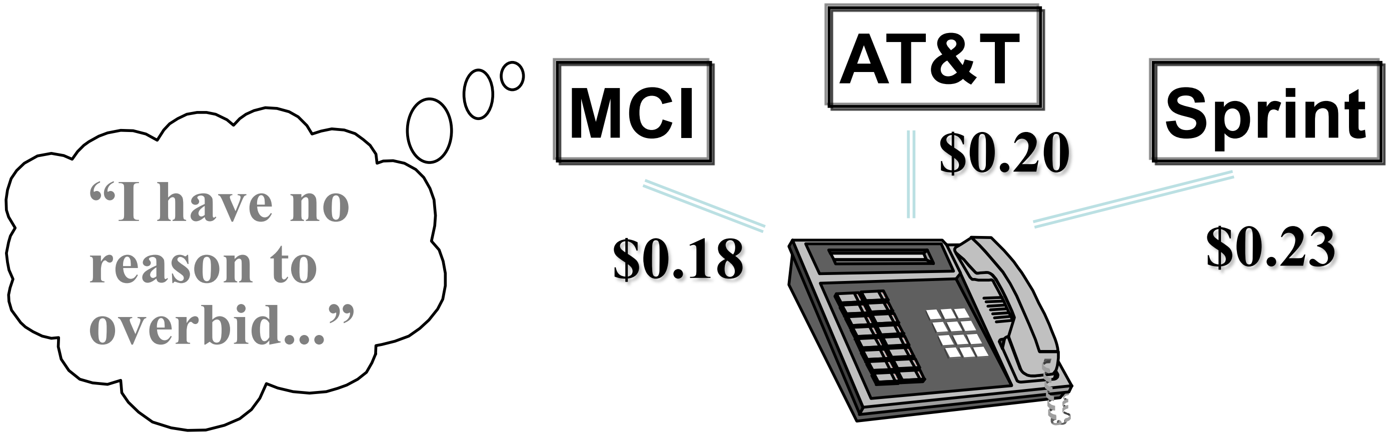
- Phone chooses carrier with lowest bid
- Carrier gets amount of second-best price



Attributes of the Vickrey Mechanism

- ✓ *Distributed*
- ✓ *Symmetric*
- ✓ *Stable*
- ✓ *Simple*
- ✓ *Efficient*

Carriers have *no* incentive to invest effort in strategic behavior



Example: Bach or Stravinsky

- A couple likes going to concerts together. One loves Bach but not Stravinsky. The other loves Stravinsky but not Bach. However, they prefer being together than being apart.

	B	S
B	2,1	0,0
S	0,0	1,2

No dom. str. equil.

Nash Equilibrium

- Sometimes an agent's best-response depends on the strategies other agents are playing
  - No dominant strategy equilibria
- A strategy profile is a **Nash equilibrium** if no player has incentive to deviate from his strategy given that others do not deviate:
  - for every agent i, $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*)$ for all s_i'

	B	S
B	2,1	0,0
S	0,0	1,2

Nash Equilibrium

- Interpretations:
  - Focal points, self-enforcing agreements, stable social convention, consequence of rational inference..
- Criticisms
  - They may not be unique (Bach or Stravinsky)
    - Ways of overcoming this
      - Refinements of equilibrium concept, Mediation, Learning
  - Do not exist in all games (in the form defined above)
  - They may be hard to find
  - People don't always behave based on what equilibria would predict (ultimatum games and notions of fairness,…)

Example: Matching Pennies

	H	T
H	−1, 1	1, −1
T	1, −1	−1, 1

So far we have talked only about **pure** (deterministic) strategy equilibria.

Not all games have pure strategy equilibria. Some equilibria are **mixed** (randomized) strategy equilibria.

Mixed strategy equilibria

- Let Σ_i be the set of probability distributions over S_i
- All possible pure strategy profiles: $S = S_1 \times \cdots \times S_n$
- σ_i in Σ_i
- Strategy profile: $\sigma=(\sigma_1, \ldots, \sigma_n)$
- Expected utility for pure strategy $s_i \in \sigma_i$ for agent i

$$u_i(s_i, \sigma_{-i}) = \sum_{s \in S_{-i}} \left(\prod_{1 \le j \le n, j \ne i} \sigma_j(s_j) \right) u_i(s_i, s)$$

- Expected utility for strategy profile σ:

$$u_i(\sigma) = \sum_{s \in S} \left(\prod_{1 \le j \le n} \sigma_j(s_j) \right) u_i(s)$$

Mixed strategy equilibria

- Nash Equilibrium:
  - σ^* is a (mixed) Nash equilibrium iff

    $u_i(\sigma^*_i, \sigma^*_{-i}) \geq u_i(\sigma_i, \sigma^*_{-i})$ for all $\sigma_i \in \Sigma_i$, for all i

Example: Matching Pennies

		q H	1-q T	: σ_2
σ_1:	p H	−1, 1	1, −1	
	1-p T	1, −1	−1, 1	

Want to play each strategy with a certain probability so that the competitor is indifferent between its own strategies.

$$u_2(H, \sigma_1) = u_2(T, \sigma_1)$$

1p+(-1)(1-p)=(-1)p+1(1-p) ⇒ p=1/2

q-(1-q)=-q+(1-q) ⇒ q=1/2

Mixed Nash Equilibrium

- **Theorem (Nash 50):**
  - Every game in which the strategy sets, $S_1,\ldots,S_n$ have a finite number of elements has a mixed strategy equilibrium.
- Complexity of finding Nash Equilibria
  - “Together with prime factoring, the complexity of finding a Nash Eq is, in my opinion, the most important concrete open question on the boundary of P today” (Papadimitriou)
  - (Daskalakis, Goldberg/Papadimitriou, 2005): Finding Nash equilibrium is very hard (though not NP complete): PPAD complete (Polynomial Parity Arguments on Directed graphs)

Imperfect Information about Strategies and Payoffs

- So far we have assumed that agents have complete information about each other (including payoffs)
  - Very strong assumption!
- Assume agent i has type $\theta_i \in \Theta_i$, which defines the payoff $u_i(s, \theta_i)$
- Agents have common prior over distribution of types $p(\theta)$
  - Conditional probability $p(\theta_{-i} | \theta_i)$ (obtained by Bayes Rule when possible)

Bayesian-Nash Equilibrium

- Strategy: $\sigma_i(\theta_i)$ is the (mixed) strategy agent i plays if its type is θ_i
- Strategy profile: $\sigma = (\sigma_1, \dots, \sigma_n)$
- Expected utility:

$$EU_i(\sigma_i(\theta_i), \sigma_{-i}(), \theta_i) = \sum_{\theta_{-i}} p(\theta_{-i}|\theta_i) u_i(\sigma_i(\theta_i), \sigma_{-i}(\theta_{-i}), \theta_i)$$

- Bayesian Nash Eq: Strategy profile σ^* is a Bayesian-Nash Eq iff for all i, for all θ_i,

$$EU_i(\sigma^*_i(\theta_i), \sigma^*_{-i}(), \theta_i) \geq EU_i(\sigma_i(\theta_i), \sigma^*_{-i}(), \theta_i)$$

(best responding w.r.t. its beliefs about the types of the other agents, assuming they are also playing a best response)

Harsanyi, John C., "Games with Incomplete Information Played by Bayesian Players, I-III." Management Science 14 (3): 159-183 (Part I), 14 (5): 320-334 (Part II), 14 (7): 486-502 (Part III) (1967/68)

John Harsanyi was a co-recipient along with John Nash and Reinhard Selten of the 1994 Nobel Memorial Prize in Economics

Social Choice Theory

Assume a group of agents make a decision

1. Agents have preferences over alternatives
   - Agents can rank order the outcomes
     - a>b>c=d is read as "a is preferred to b which is preferred to c which is equivalent to d"
2. Voters are sincere
   - They truthfully tell the center their preferences
3. Outcome is enforced on all agents

The problem

- Majority decision:
  - If more agents prefer a to b, then a should be chosen
- Two outcome setting is easy
  - Choose outcome with more votes!
- What happens if you have 3 or more possible outcomes?

Case 1: Agents specify their top preference

Election System

- Plurality Voting
  - One name is ticked on a ballot
  - One round of voting
  - One candidate is chosen

Is this a “good” system?

What do we mean by good?

Example: Plurality

- 3 candidates
  - Lib, NDP, C
- 21 voters with the preferences
  - 10 Lib>NDP>C
  - 6 NDP>C>Lib
  - 5 C>NDP>Lib
- Result: Lib 10, NDP 6, C 5
  - But a majority of voters (11) prefer all other parties more than the Libs!

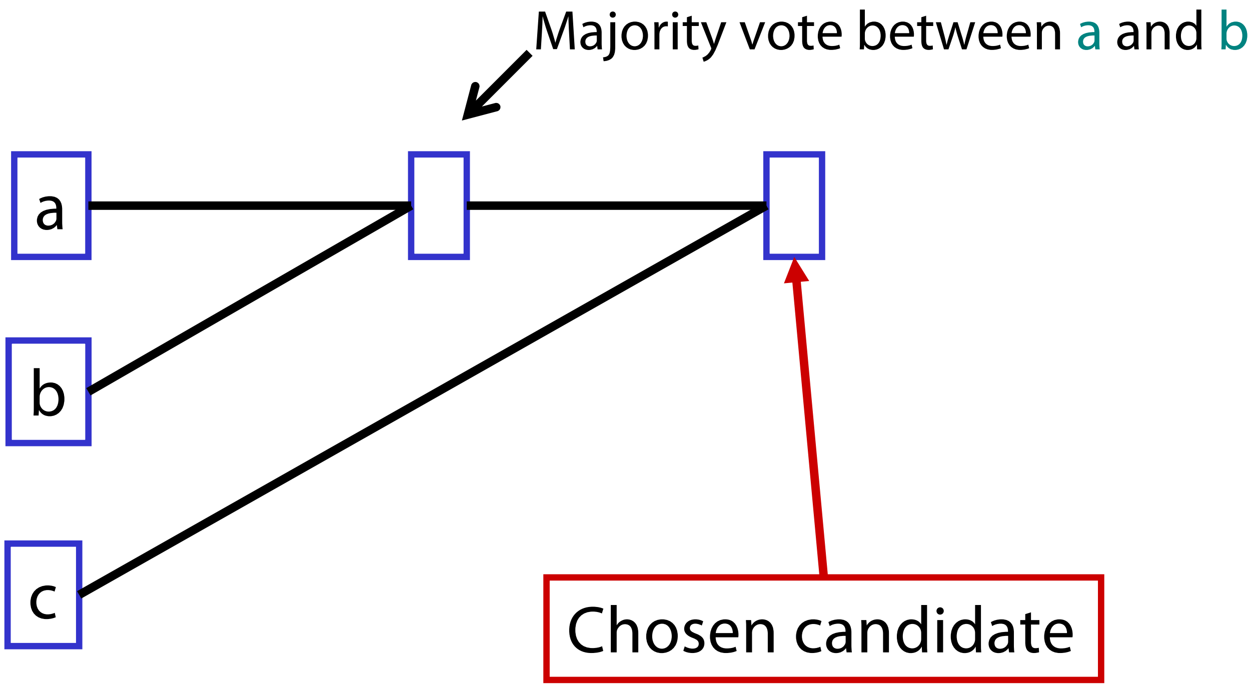
What can we do?

- Majority system
  - Works well when there are 2 alternatives
  - Not great when there are more than 2 choices

- Proposal:
  - Organize a series of votes between 2 alternatives at a time
  - How this is organized is called an agenda
    - Or a cup (often in sports)

Agendas

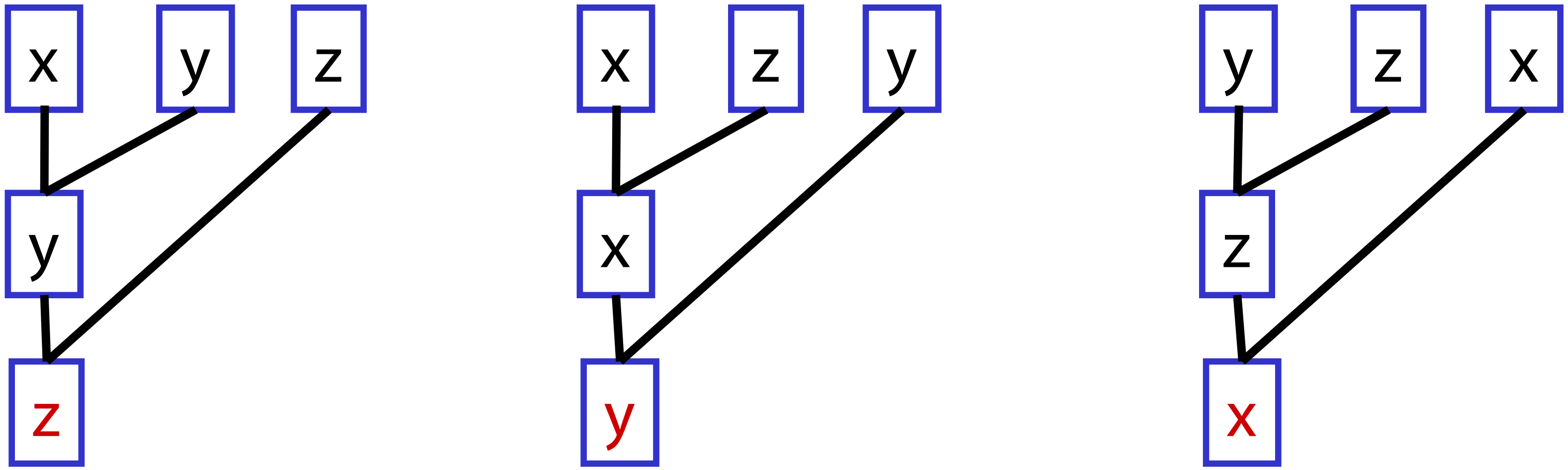
- 3 candidates {a,b,c}
- Agenda a,b,c



Agenda paradox

- *Binary protocol (majority rule) = cup*
- Three types of agents:

1. x > z > y (35%)
2. y > x > z (33%)
3. z > y > x (32%)

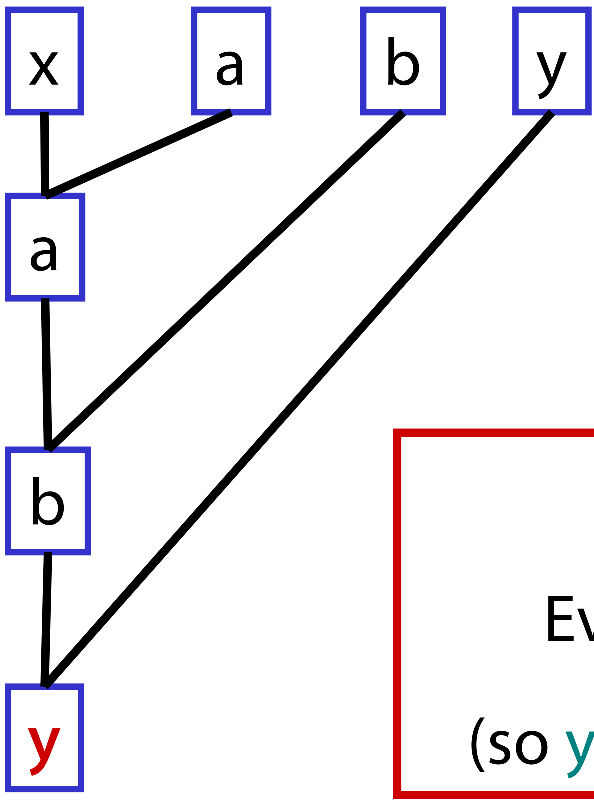


- Power of agenda setter (e.g. chairman)
- Vulnerable to irrelevant alternatives (z)
    - x vs. y only leads to winner y
    - But adding z may lead to x winning (last agenda)

Another problem: Pareto dominated winner paradox

Agents:

1. x > y > b > a
2. a > x > y > b
3. b > a > x > y



BUT

Everyone prefers x to y!

(so y pareto dominated by x)

Case 2: Agents specify their complete preferences

Maybe the problem was with the ballots!

Ballot

X>Y>Z

Now have more information

Condorcet

- Proposed the following
  - Compare each pair of alternatives
  - Declare “a” is socially preferred to “b” if more voters strictly prefer a to b

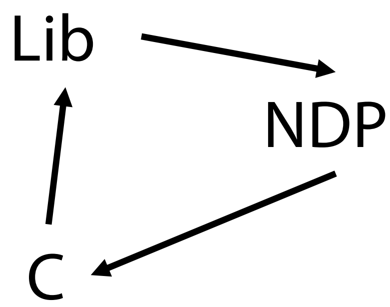
- Condorcet Principle: If one alternative is preferred to all other candidates then it should be selected

Example: Condorcet

- 3 candidates
  - Lib, NDP, C
- 21 voters with the preferences
  - 10 Lib>NDP>C
  - 6 NDP>C>Lib
  - 5 C>NDP>Lib
- Result:
  - NDP win! (11/21 prefer them to Lib, 16/21 prefer them to C)

A Problem

- 3 candidates
  - Lib, NDP, C
- 3 voters with the preferences
  - Lib>NDP>C
  - NDP>C>Lib
  - C>Lib>NDP
- Result:
  - No Condorcet Winner



Borda Count

- Each ballot is a list of ordered alternatives
- On each ballot compute the rank of each alternative
- Rank order alternatives based on decreasing sum of their ranks

Ballots		Result
A>B>C		A: 4
A>C>B	→	B: 8
C>A>B		C: 6

Borda Count

- Simple
- Always a Borda Winner
- BUT does not always choose Condorcet winner!
- 3 voters
  - 2: b>a>c>d
  - 1: a>c>d>b

Borda scores:

a:5, b:6, c:8, d:11

Therefore a wins

BUT b is the Condorcet winner

Inverted-order paradox

- Borda rule with 4 alternatives
  - Each agent gives 1 point to best option, 2 to second best...
- Agents:

  1. x > c > b > a
  2. a > x > c > b
  3. b > a > x > c
  4. x > c > b > a
  5. a > x > c > b
  6. b > a > x > c
  7. x > c > b > a
- x=13, a=18, b=19, c=20
- Remove x: c=13, b=14, a=15

Borda rule vulnerable to irrelevant alternatives

- Three types of agents:

  1. x > z > y (35%)
  2. y > x > z (33%)
  3. z > y > x (32%)

- Borda winner is x
- Remove z: Borda winner is y

Desirable properties for a voting protocol

- No dictators
- Universality (unrestricted domain)
  - It should work with any set of preferences
- Non-imposition (citizen sovereignty)
  - Every possible societal preference order should be achievable
- Independence of irrelevant alternatives (IIA)
  - The comparison of two alternatives should depend only on their standings among agents' preferences, not on the ranking of other alternatives
- Monotonicity
  - An individual should not be able to hurt an option by ranking it higher.
- Paretian
  - If all all agents prefer x to y then in the outcome x should be preferred to y

Arrow's Theorem (1951)

If there are 3 or more alternatives and a finite number of agents then there is **no** protocol which satisfies all desired properties

Take-home Message

- Despair?
  - No ideal voting method
  - That would be boring!

- A group is more complex than an individual
- Weigh the pro's and con's of each system and understand the setting they will be used in

- Do not believe anyone who says they have the best voting system out there!